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# Transfer Learning from Math to Engineering and Using Scaffolds through Hands-on Learning to Build New Engineering Skills in Sensors and Systems Course

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# Transfer Learning from Math to Engineering and Using Scaffolds through Hands-on Learning to Build New Engineering Skills in Sensors and Systems Course


## Abstract

Transfer of learning from mathematics to engineering entails relating and applying theoretical concepts learned in mathematics courses to engineering concepts and courses. The project team investigated engineering students' skills in transferring learning from mathematics to engineering in an engineering Sensors and Systems course, and, based on the results, developed scaffolding exercises to lead students to their team- and project-based final project that incorporated the targeted skills through hands-on engaged student learning. This work targeted the following research question: Can transfer of learning be successfully achieved in remote hands-on engaged student learning (ESL) scenarios?

First, the mathematics faculty studied the sensors and systems course material, and identified relevant mathematical background that the students should remember and build on in the engineering course. Three assignments were prepared for the Sensors and Systems course to assess the students' readiness to transfer the learned math skills to the sensors and systems engineering concepts: **1)** Linearization, **2)** Units (and unit conversions), and **3)** Calibration (by calculating the transfer function from data).

Students were assisted by the engineering course instructor to build on what they had learned in math to develop the targeted engineering skills in a problem-based learning assignment encapsulated in the course's hands-on sensor-related team project. This team-based final project entailed hands-on engagement with sensors and required interfacing sensors to microcontrollers, or designing circuitry to drive an actuator based on sensor data.

This paper will present the details of the relevant math concepts for sensors and systems that were targeted for transfer of learning, and scaffolds faculty built to guide the students towards developing a team-based final project through hands-on engaged student learning in the students' chosen location and time, giving students flexibility to succeed to answer the posed research question.


## Introduction

Engineering curricula rely heavily on mathematics, and students' mathematical skills and knowledge. A sound foundation of mathematics concepts is necessary for the students to be successful in engineering. Transfer of learning from mathematics to engineering, therefore, plays a significant part in students' academic development and achievement in engineering programs. Transfer of learning from math to engineering entails relating and applying theoretical concepts learned in mathematics courses to engineering concepts and courses. Learners can sometimes excel at learning factual information but lack the depth to apply them in future contexts [1]. Transfer of knowledge, this ability of students to apply existing knowledge to future problems [2], can be especially difficult for engineering students as they progress from science and mathematics courses, which tend to involve more theoretical information, into engineering courses where they need to apply previously learned concepts from these courses. The Engineering faculty can use *scaffolding* to assist students in addressing these challenges.

Scaffolding of learning or building scaffolds refers to using and building on existing foundational knowledge and skills to help a learner develop new ones. Scaffolding involves providing cognitive and motivational support to a learner towards creating a structure and understanding problems [1]. Scaffolding is an important approach for cultivating the development of expertise by a novice learner in a particular area of learning. Scaffolding can be conceptual, strategic, or metacognitive [2]. As students get better at applying their knowledge, the instructor can gradually take a step back from providing support or the "scaffold" [3]. This approach helps students learn at a comfortable level while increasing their confidence as they advance in their learning.

In this paper the authors explore transfer of learning from mathematics to a senior-level Sensors and Systems course taken as a technical elective typically by mechanical and electrical engineering students. The goal of the work includes identifying bottlenecks in learning new concepts, and to assist the students by building scaffolds to fill in identified gaps, strengthen the students' mathematical knowledge and skills, and then expand on their conceptual learning in engineering through engaged student learning that incorporate sensors and embedded devices as part of Internet of Things (IoT) systems.

In the rest of this paper, the authors describe mathematics assignments and exercises developed for the Sensors and Systems class to gauge students transfer of learning. This discussion highlights scaffolds and project-based learning assignments (PBL) with hands-on components employed in the classroom for engaged student learning in a collaborative environment.

**Background**

Engineering and science educators have recognized the significance of transfer of learning from mathematics to engineering and sciences, because without the understanding and application of the underlying math concepts in these disciplines, students cannot effectively develop new knowledge and skills in their major [4]-[18]. For example, Alpár *et al.* performed a qualitative analysis of a cohort of computer science students' responses to assess these students' perceptions of mathematics and to investigate if mathematics can be a bottleneck to learning in computer science [5]. Students generally perceived mathematics background as significant and relevant (and transferrable) to software engineering, algorithm analysis, logical thinking and continuous learning in computer science.

Ayyagari discusses the significance of math in the control systems education in selected institutions of higher education in India, and the importance of demonstrating theory through practice in laboratory experiments, since students have a general reluctance to algebra [6]. De Andrade *et al.* describes a mathematical assessment task hierarchy (MATH) taxonomy for engineering math assessment to transfer academic, and in particular math, knowledge to real-life problems through realistic engineering scenarios [7]. The taxonomy emphasizes active engagement, enquiry and creativity in the assessment of engineering mathematics in first-year students. Activities ranged from recalling definitions to applying mathematical concepts and making deductions from obtained results. The authors report that 89% of students in the study found contextual assessment adds value to their education, and 81% of students agreed that they learned new skills with the assessment exercises. The authors activities relate to translating theoretical knowledge to applied engineering concepts.

More specific pedagogies involve transfer of learning that allow connecting concepts during problem solving. In one of the earlier papers, Dixon and Brown discuss Project Lead the Way (PLTW) which investigates students' ability to relate concepts learned through PLTW with those required to solve problems in mathematics, science, and design [8]. According to National Research Council report, transfer of knowledge is dependent on the organization of learning and how this learning relates to what the students already know [1]. Standardized tests were used to assess the percentage of concepts connected to test items. The preliminary results showed that the highest percentage of connections occurred with design problems at 96% followed by science questions (17%) and math questions (16%).

Britton *et al.* investigated first-year science students' ability to transfer math skills and knowledge to science contexts at the University of Sidney [9], using Barnett and Ceci's taxonomy for far transfer [10]. Far transfer is defined as the ability to transfer skills and knowledge to different or dissimilar contexts such as in different disciplines that is not close to the contexts or disciplines where those skills and knowledge were first acquired [10]. Britton and her colleagues investigated the correlation between university as well as test variables and test and high school variables, and found high correlations between all university (average test scores in university math and university science) and test variables perhaps suggesting high transfer of learning in these environments.

Rebello *et al.* investigate the transfer of learning in problem solving in the context of math and physics [11], [12]. The authors use Redish's theoretical framework [19] where transfer learning is considered to be "the dynamic creation of associations by the learner in a new problem situation" [11]. The authors discuss two types of transfer, namely, horizontal and vertical. In horizontal transfer, learning is applied to similar problems already seen before – "mapping of new information onto existing knowledge structure". In vertical transfer, on the other hand, transfer of learning occurs where the student builds on their prior knowledge to create new knowledge. The authors found that most students understand the mathematics concepts but experience challenge in appropriately applying these mathematics concepts to physics. The authors, therefore, recommend that the mathematics and physics course are taught in an integrated fashion or at least concurrently, to enable students to transfer internal knowledge structures learned in math to solving problems in physics.

Nakakoji and Wilson explore transfer of learning from first semester mathematics learning [13], [14] to second semester problem solving, and conclude that it is important to address transfer of learning at institutional levels to equip students with this 21st century skill to ensure success of students in STEM fields [14].

Concepts of scaffolds and building scaffolds are also numerous in the literature, which go hand-in-hand with knowledge transfer when supporting student learning [18]-[27] as well as hands-on learning and related exercises [28]-[30].

## Methods

Based on the need for transfer of learning from math to engineering for student success, the authors developed mathematics assignments to gage **1)** student's retention of mathematical conceptual knowledge and skills from previous math courses, **2)** identify gaps and weakness in students' math knowledge and skills required to be successful in the Sensors and Systems course, and **3)** help students relate apply the relevant theoretical math concepts to the engineering contexts.

***Identifying Relevant Concepts from Mathematics That Apply to Sensors and Systems Engineering Course***

To identify relevant concepts from math that were necessary for transfer of learning into the targeted engineering course, first the project's mathematics faculty reviewed the textbook for the sensors and systems course [31]. Then, together with the engineering faculty, identified the mathematics foundations necessary to work with the sensor models that capture the behavior of the sensors. Two major topics were selected during the first year of the project as necessary for transfer of learning from math to engineering [28], namely, *linearization* (being able to determine a linear model for a sensor behavior based on sensor input / output data pairs), and *units* (being able to handle mathematical operations with units associated with variables and their numeric values). The third major math assignments targeted transfer of learning from math on error analysis necessary in sensor and system calibration. These three assignments allowed the assessment of the students' retention of mathematical concepts and helped them relate these math concepts to the engineering principles applicable to sensors and systems, thus fostering transfer of learning from math, as further explained below.

***Developed Assignments for Transfer of Learning from Mathematics***

The characteristics of the sensors or actuators start with their transfer functions (in LTI systems, impulse responses). The transfer function characterizes the relationship between the input and output of the device. The function can be linear or nonlinear and represents the relationship between one or multiple inputs and outputs. The transfer function describes the response of a sensor or actuators to a given input [31]:

$$y = f(x), \ \ x \in D, \tag{1}$$

where $x$ is the input and $y$ is the output, and $D$ is the domain for $x$.

For example, we may have the following linear equation describing the behavior of a thermocouple, from which the transfer function can be determined from its linear equation,

$$y = a_1 x + a_0, \tag{2}$$

where $y = V$ is the output voltage of a thermocouple, and $x = T$ is the input temperature defined in the range $T_{min} \leq T \leq T_{max}$. In this simple example, it is important to assist students in understanding that the theoretical dependent and independent variables, $x$ and $y$ in Equation (2), can be replaced with meaningful physical quantities, time, $T$, and voltage, $V$, as they apply to physics and engineering principles.

In a general case, we may have a nonlinear transfer function that can be interpolated by a polynomial fit [31]-[33]

$$y = \sum_{i=0}^{m} a_i x^i. \tag{3}$$

This would correspond to a linear (sensor) system in engineering. To establish the transfer function for the sensor, we should specify the coefficients $a_i$ for $i = \overline{0, m}$ based on some given dataset $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ (or sensor input/output pair) with $n \geq m$. To find a polynomial relation between the output and inputs, we form a matrix and right-hand side vector based on the given dataset [33],

$$A = \begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^m \\ 1 & x_1 & x_1^2 & \dots & x_1^m \\ \dots & \dots & \dots & \dots & \dots \\ 1 & x_n & x_n^2 & \dots & x_n^m \end{bmatrix}, \; b = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix}, \tag{4}$$

and define the following to solve the system of equations for $x = [a_0, a_1, \dots, a_n]^T$

$$Ax = b, \tag{5}$$

with matrix $n$ by $(m+1)$ matrix A. Then, to minimize the mean square error, $e$, of a system of linear equations ($e = ||Ax - b||^2 \rightarrow min$), we solve the following system,

$$\tilde{A}\,\tilde{x} = \tilde{b} \tag{6}$$

with square $n$ by $n$ matrix with $\tilde{A} = A^T A$, $\tilde{b} = A^T b$ and $\tilde{x} = [\tilde{a}_0, \tilde{a}_1, \dots, \tilde{a}_n]^T$.

Finally, we can evaluate the approximation error using absolute or relative norm:

$$e = \sqrt{\frac{\sum_{i=0}^{n}(y_i - \tilde{y}_i)^2}{n}} \text{ and } e = \sqrt{\frac{\sum_{i=0}^{n}(y_i - \tilde{y}_i)^2}{\sum_{i=0}^{n} y_i^2}}\, 100\%, \tag{7}$$

where $\tilde{y}_i = \sum_{k=0}^{m} \tilde{a}_k x_i^k$ for each $i = \overline{0, n}$.

This concept is discussed in a Linear Algebra course [33], and more details about interpolation and polynomial fit are discussed in a Numerical Analysis courses [32].

***Scaffolding***

Once the students remember and connect the mathematical foundations to engineering applications in Sensors and Systems course, they are ready to build on this knowledge, beyond near transfer (transfer to a similar context [10]) and far transfer (transfer to a dissimilar context [10]) learning, to develop new knowledge and skills. This is achieved through scaffolding exercises that incrementally build on students' learning through hands-on mini projects that lead to a final project that incorporate collaborative learning in teams using IoT devices, including sensors and embedded systems, such as microcontrollers.

Two mini-projects that were incorporated into this course entailed the use of sensor kits to individually analyze the behavior of sensors based on physics equations that represent the individual sensors. Through hands-on exercises, the students collected sensor data using independent and dependent variables, relating output to changing input. The students then created calibration models, and calculated transfer functions as well as errors in experimental data sets. The students were able to use mathematical equations and concepts representing sensor output based on sensor input. They were also able to compare theoretical equations (math/science) to their experimental data and their *own* developed calibration models and transfer functions based on their data, thus connecting the math concepts to engineering principles and experimental outcomes in hands-on exercises. These exercises qualify for vertical transfer learning [11] through scaffolding.

After two mini-projects covering individual sensor behavior through calibration models that the students determine through experimental data collection, the students then choose a final project topic that incorporate multiple sensors in a sensor system that performs sensing and actuation functions though microcontroller interfacing that serves a purpose (e.g. monitoring (sensing) and response (actuation)). These IoT devices are lent to the students so that they can work collaboratively with their teammates in remote environments. This would qualify for vertical as wall as far transfer learning. (We note that although IoT devices are used, most student teams did not explore internet connectivity or cloud access in these final projects.)

***Problem- and Project-Based Learning (PBL): Final Class Team Project***

The final project is the final test to address the research question posed earlier, whether transfer of learning can be successfully achieved in remote hands-on engaged student learning (ESL) scenarios, in this case through final projects that entail the development of a sensor system that addresses an engineering/science problem in collaborative teams and remotely, at the students own space and time, outside the classroom. Some of the projects the students selected in the past three years include “Smart Home”, “Weather Station”, “First Responder UAV”, “Car Engine Sensor System”, “Health Monitoring System”, “Integrated Push Boat Safety System”, “Home Security System”, and “Water Tank Control”. In each of these project-based learning assignments, the students had to demonstrate the calibration of the sensors they used in their system, obtain the transfer function based on sensor input/output relationships, and error estimation. The students effectively demonstration utilizing the targeted mathematical concepts such as unit conversions and linearization, demonstrating effective near and far transfer of learning. The final projects also achieve ABET student learning outcomes, in particular student outcomes 1, 2, 5, 6 and 7 (See [34] for details of ABET/EAC student outcomes). These methods and concepts can be adopted in other institutions that cover sensors, systems, or related topics.

***Assessment***

Assessment was conducted over two years (2022, 2023) for the Linearization and Units transfer learning exercises intended to assess students retention of these math concepts and mathematical problem solving related to these concepts from previous mathematics courses. Error Analysis/Calibration assignment has so far been conducted once in 2023. Used assessment methods included **1)** student success on the assignments, and **2)** student perception surveys. The results are presented in Results section.

## Results

Two categories of results are reported here. First category involves the three mathematics assignments that were developed to assess transfer of learning, learning gaps, and scaffolding activities. The second category of results reported here involves assessment based on student perceptions of and student performance in the developed math homework assignments. These assignments were *Linearization*, *Units*, and *Calibration/Error Analysis.* These assignments are presented below. Assessment of the final project, which contributes to the posed research questions, is presented at the end of this section.

***Developed Assignments for Transfer of Learning from Mathematics***

**HW #1: Linearization** (see [28], developed in 2021, implemented in 2022 and 2023)

The textbook says it "linearizes" the transfer functions.

In your math classes you have met two types of linearization:

1) In Calculus: linearization by using the tangent line to a graph, and
2) In Linear Algebra: fitting a finite number of points with a least squares line.

Assume our function is exponential decay, let's say $f(x) = 2\,e^{-10x}$

Use calculus methods to find the equation of the tangent line to the graph at the point $(1, \frac{2}{e^{10}})$.

How can you tell without a calculator whether the tangent line is above or below the graph of the function (except at the point x=1)?

Use the tangent line to get an estimates for f(0.98) and for f(1.01) and f(10).

Which of those estimates is the least trustworthy? What is the reason for that?

Then use 11 equally spaces points in the interval [0,2], call them $x_0, \dots x_{10}$, calculate the function values0
$f(x_0), \dots f(x_{10})$ and use the 11 pairs of points to find a least squares line for them. To do that you want to solve the overdetermined system of the form $\boldsymbol{A}x = \boldsymbol{b}$

$$\begin{bmatrix} 1 & x_0 \\ \vdots & \vdots \\ 1 & x_{10} \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} f(x_0) \\ \vdots \\ f(x_{10}) \end{bmatrix}$$

This involves the normal equations $A^T A x = A^T b$ . Write the equations for this example and solve them.

Which slope $m$ and $y$-intercept $b$ do you get? Show your work.

Now that you have two different linearization of the same function, which one is better? Justify your decision.

Calculate errors, and graph your results.

Reflection:

1. Did you remember what linearization was? (Please elaborate)
2. Does this exercise refresh your understanding of linearization from Calculus I or Linear Algebra/Engineering Analysis for ME (least square fit)? (Please elaborate).

**HW #2: Units** (see [28]. Developed in 2022, implemented in 2022 and 2023)
**Transfer Learning Units** [units based on derivatives vs. integrals]

Assume f(x) is a function, where x is measured in units of v (as in variable) and the function values are measured in units of o (as in outputs).

Write down the definition of the derivative of f(x) as a limit of the difference quotient.

What are the units of the numerator and denominator of this difference quotient? What does that make the units of f'(x)?

The second derivative, f''(x), is the derivative of f'(x). What does that make its units?

The integral is defined as a limit of Riemann sums. White down the limit form and then decide on the units of $\int_a^b f(x)dx$.

Fancier version: assume g(s,t) is a function of two variables, where s is measured in v units and t is measured in w units and g is measured in o units (for output) .

Write down the limit and difference quotient that is used to find ∂g/∂s.

What does that make the units of ∂g/∂s ?

What would be the units for the double integral $\int_a^b \int_c^d g(s,t)ds\, dt$ ?

Reflection:

1. Did you remember how to obtain units on derivatives and integrals? (Please elaborate)
2. Does this exercise refresh your understanding of calculating units from Calculus I or Linear Algebra/Engineering Analysis for ME? (Please elaborate).

**HW #3. Calibration and Error Analysis** (Developed in 2023. Implemented in 2023)

**Solution of system of linear equations and least square approximation (with application to sensors calibration)**

***Input Data:***

We consider a temperature sensor in the range 0°C - 500°C. The output (voltage) of a temperature sensor for a given temperature (input) is given by the following table:

| I | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_i$[°C] | 0 | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 |
| $y_i$[mV] | 0 | 0.002 | 0.033 | 0.092 | 0.178 | 0.291 | 0.431 | 0.596 | 0.786 | 1.002 | 1.241 |

***Method 1.*** (Linear fit)

Assumes that the equation of the transfer function is known, in which case the constants in the equation must be determined experimentally. Suppose that the sensor above has a linear transfer function between the range points given as

$$\tilde{y} = a_0 + a_1 \mathrm{x}$$

where $x$ is the temperature and $y$ is the voltage, with $a_0$ and $a_1$ constants.

To establish the transfer function for the sensor we must specify the constants $a_0$ and $a_1$.

**1.1** Find $a_0$ and $a_1$ using two points $(x_1, y_1)$ and $(x_{11}, y_{11})$ from the Table and calculate the error using formula given in **A1** and graph your results.
**1.2** Find $a_0$ and $a_1$ by solution of the least square problem (see **A2**) and calculate the error of approximation.

Which of these methods is better? What is the reason for this?

***Method 2.*** (Polynomial fit)

Assume that the transfer function is described by a more complex function.

$$\tilde{y} = a_0 + a_1 \mathrm{x} + a_2 x^2 + a_3 x^3$$

Where the constants $a_0$, $a_1$, $a_2$ and $a_3$ need to be determined.

To establish the transfer function for the sensor we must specify the constants $a_0$, $a_1$, $a_2$ and $a_3$ .

**2.1** Find $a_0$, $a_1$, $a_2$ and $a_3$ using four points $(x_1, y_1)$, $(x_4, y_4)$, $(x_7, y_7)$ and $(x_{11}, y_{11})$ from the Table and calculate the error of approximation and graph your results.
**2.2** Find $a_0$, $a_1$, $a_2$ and $a_3$ by solution of the least square problem (see **A3**) and calculate the error of approximation.

Which of these methods is better? What is the reason for this?

**A1.** An absolute and relative errors: $\mathrm{e} = \sqrt{\frac{\sum_{i=1}^{11}(y_i - \tilde{y}_i)^2}{11}}$ and $\mathrm{e} = \sqrt{\frac{\sum_{i=1}^{11}(y_i - \tilde{y}_i)^2}{\sum_{i=1}^{11} y_i^2}}\ 100\%$

**A2.** Least squares problem or find a "best fit" line or linear regression (Linear Algebra, [33]).

Dataset: points $(\mathrm{x}_i, \mathrm{y}_i)$, $\mathrm{i} = 1, \ldots, \mathrm{N}$.

We fit our observed data using the linear model

$$\mathrm{y} = \mathrm{a}_0 + \mathrm{a}_1 \mathrm{x}$$

where $a_0$ and $a_1$ are intercept and slope of the linear equation.

1. Based on the given dataset, we first form matrix and right-hand side vector

$$\begin{bmatrix} 1 & x_0 \\ 1 & x_1 \\ \dots & \dots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \dots \\ y_n \end{bmatrix} \rightarrow \mathrm{Ax} = \mathrm{b},\ \mathrm{A} = \begin{bmatrix} 1 & x_0 \\ 1 & x_1 \\ \dots & \dots \\ 1 & x_n \end{bmatrix},\ \mathrm{b} = \begin{bmatrix} y_0 \\ y_1 \\ \dots \\ y_n \end{bmatrix},\ \mathrm{x} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}.$$

2. To minimize the mean square error of a system of linear equations we can get our parameter estimates in the form of matrix multiplications

$$A^T A\mathrm{x} = A^T \mathrm{b}$$

**A3.** Polynomial fit (polynomial regression)

You can regard polynomial regression as a generalized case of linear regression. You assume the polynomial dependence between the output and inputs and, consequently, the polynomial estimated regression function.

$$\mathrm{y} = \mathrm{a}_0 + \mathrm{a}_1\mathrm{x} + \mathrm{a}_2 x^2 + \cdots + +\mathrm{a}_m x^m$$

where $a_0$, $a_1$ … $a_m$ are coefficients with $m \leq n$.

1. Based on the given dataset, we first form matrix and right-hand side vector

$$\mathrm{Ax} = \mathrm{b},\ \mathrm{A} = \begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^m \\ 1 & x_1 & x_1^2 & \dots & x_1^m \\ \dots & \dots & \dots & \dots & \dots \\ 1 & x_n & x_n^2 & \dots & x_n^m \end{bmatrix},\ \mathrm{b} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix},\ \mathrm{x} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \dots \\ a_m \end{bmatrix}$$

2. To minimize the mean square error of a system of linear equations we can get our parameter estimates in the form of matrix multiplications

$$A^T A\mathrm{x} = A^T \mathrm{b}$$

Reflection:

1. Did you remember linear and polynomial curve fitting? (Please elaborate)
2. Did this exercise help you build on mathematical concepts you previously learned to learn and apply new concepts in engineering contexts?
3. Did this exercise create structure for you to build on to determine mean square error?

*Assessment*

Figures 1, 3 and 5 show the assessment results of math assignments intended for transfer of learning assessment based on student performance. Figures 2, 4 and 6 demonstrate student perceptions based on transfer learning and related criteria presented to the students regarding the three homework assignments. Figures 2, 4 and 6 represent student perception results based on the prompt “Rate the following statements from1 to 5 (poor to excellent) based on their contribution to your learning.” The students were asked to rate each of the four components of each assignment (assignment overall, remembering concepts from previous math courses, transfer of learning from math, and adapting/applying math to engineering problems) based on their personal agreement with each component’s contribution to their learning.

Figure 7 shows the evaluation of the PBL final class project based on student performance on final report, oral presentation, and real-time demonstration. Figure 7 is an indicator of the effectiveness of transfer learning and scaffolding to assist student learning using PBL and collaborative learning in teams, with hands-on engaged student learning exercises that utilize IoT sensors and embedded systems.

*Linearization* assignment was the first assignment given to the class on the first day of classes two years in a row. Figure 1 shows that more than half of the students struggled with this assignment. This is attributed to the students needing to brush up on linearization methods from math at the beginning of the semester.

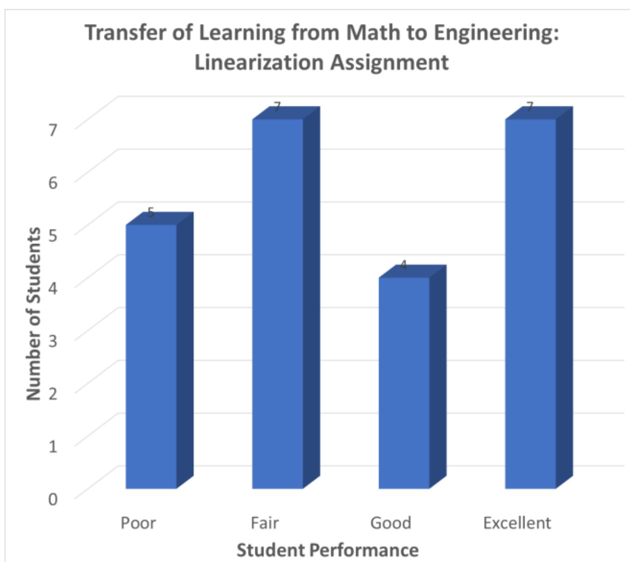


**Figure 1.** Student Performance in Linearization mathematics assignment used for gaging student’s retention of math concepts and whether they can apply it to mathematical problems (horizontal, near transfer learning). This homework was assigned at the beginning of the semester.
Student Performance:
1 – Poor (<70%); 2 – Fair (70-79%); 3 – Good (80-89%); 4 – Excellent (90-100%)
Sample size: $n$ = 23. $n = n_1 + n_2$, where $n_1 = 12$ (Summer 2022), and $n_2 = 11$ (Summer 2023)

Based on Figure 2, more than half of the students (15 out of 19) were either neutral (9/19) or thought more positively (6/19) about the linearization assignment helping them in remembering how to perform linearization from prior math courses. Three out of 19 students did not think the linearization assignment contributed to their remembering the linearization concepts. One student did not respond to survey question.

Most students agreed that the *Linearization* assignment contributed to the transfer of learning from mathematics to sensors and systems class, with 18 out of 19 students being neutral (7/19) or positive (10/19) about this assignment contributing to transfer of learning from math, with two students assessing the assignments contribution to be at a level of fair.

Similar responses were observed for student perceptions of the contribution of the Linearization assignment to their ability to adapt mathematical methods to sensors concepts. However, overall, the linearization assignment was not a popular assignment among students possibly due to the fact that it was the first assignment after the students returned to school from a semester break and had to spend time in reviewing the mathematical concepts and methods to complete the assignment.

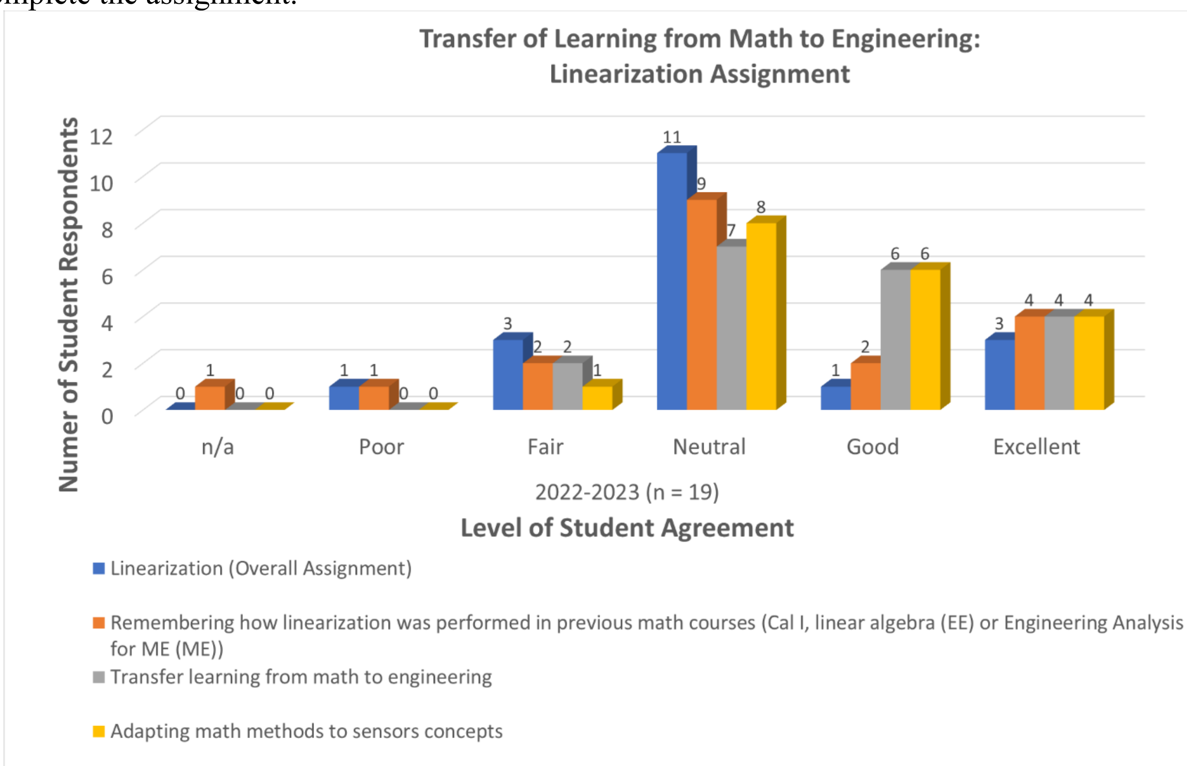


**Figure 2.** Student Perceptions on the *Linearization* mathematics assignment and its contribution to learning from mathematics to engineering (Sensors and Systems course), at different learning taxonomies (remembering, adapting/applying, transfer of learning, overall) (horizontal, near transfer learning).

Sample size, $n = 19$. $n = n_1 + n_2$, where $n_1 = 10$ (Summer 2022), and $n_2 = 9$ (Summer 2023)

Level of student agreement:

n/a - not applicable (did not respond); 1 – poor; 2 – fair; 3 – neutral; 4 – good; 5 – excellent

*Units* assignment was the second of the math assignments, given also towards the beginning of the semester. Student performance much improved in this second assignment (see Fig. 3). By this time student had identified the course content they had to revise.

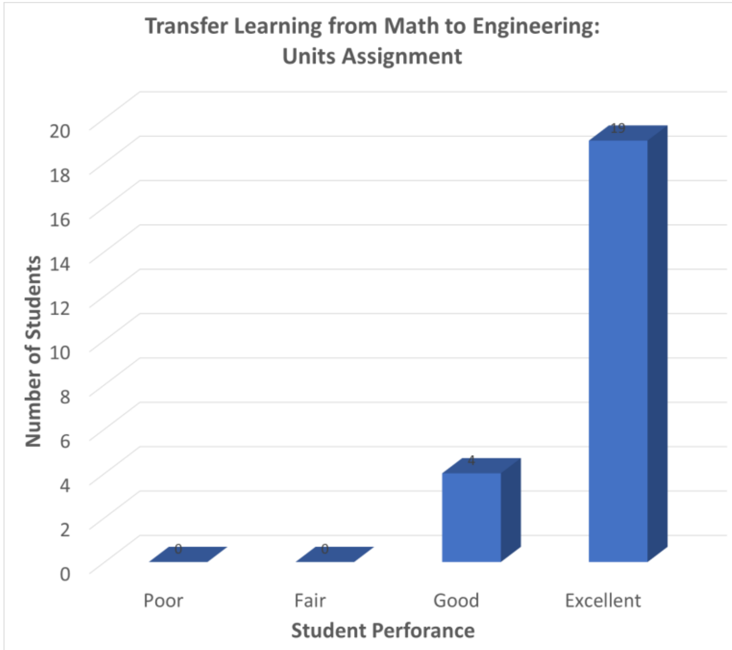


**Figure 3.** Student Performance in Units mathematics assignment used for gaging student's retention of math concepts and whether they could apply it to mathematical problems using arbitrary symbology (horizontal, near transfer learning). This homework was also assigned at the beginning of the semester.
Sample size: $n$ = 23. $n = n_1 + n_2$, where $n_1$ = 12 (Summer 2022), and $n_2$ = 11 (Summer 2023)
Student Performance:
1 – Poor (<70%); 2 – Fair (70-79%); 3 – Good (80-89%); 4 – Excellent (90-100%)

Figure 4 shows that students were mostly neutral (8/19) or positive at the level of good (6/19) or excellent (4/19) about the contribution of *Units* assignment to remembering units from previous math courses. Only one student rated this assignment to have contributed to learning at a level of fair.

This assignment's contribution to students' ability to determine units using derivatives and integrals, as well as to transfer learning from determining units in math to determining units in engineering was the same with more than half the students (11/19) considering the assignment effective at a level of good or excellent with four students remaining neutral (4/19), and four students indicating low level of agreement at a level of poor (1/19) or fair (3/19).

Seventeen students expressed opinions that were neutral (8/19) or good (4) or excellent (5) in this assignment helping to build on what the students learned in math to determine units for sensors and systems. Overall, the *Units* assignment was received more positively with only two students indicating agreement at a level of fair regarding the contribution of this assignment to building on math units to get sensor units.

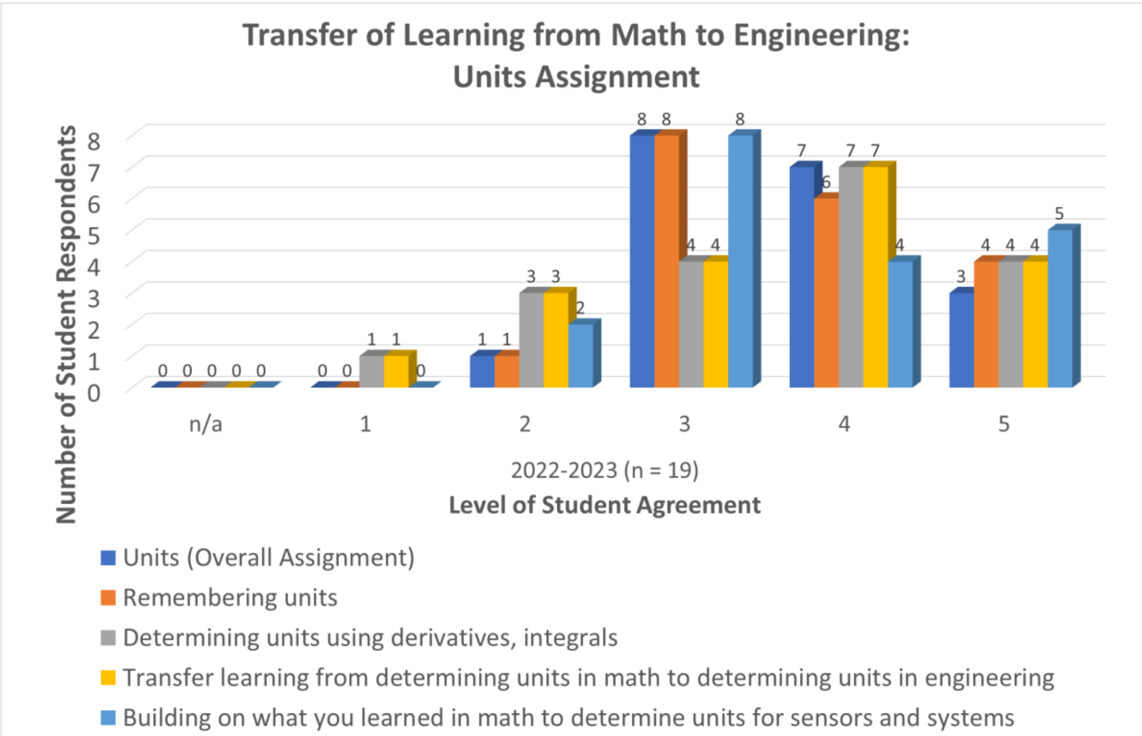


**Figure 4.** Student Perceptions on the *Units* mathematics assignment, and its contribution to transfer of learning from mathematics to engineering (Sensors and Systems course), at different learning taxonomies (remembering, adapting/applying, transfer of learning, overall) (horizontal, near transfer learning). Level of student agreement:
n/a - not applicable (did not respond); 1 – poor; 2 – fair; 3 – neutral; 4 – good; 5 – excellent
Sample size, $n = 19$. $n = n_1 + n_2$, where $n_1 = 10$ (Summer 2022), and $n_2 = 9$ (Summer 2023)

*Calibration* assignment was assigned towards mid-semester when students had already covered the basics of sensors. Figure 5 summarizes the student performance results for the *Calibration / Error Calculation* assignment which was conducted only once and with limited number of participants (n = 6) in this optional assignment. Five out of 6 participating students performed at a level of good or excellent while one student performed at a level of poor in this assignment.

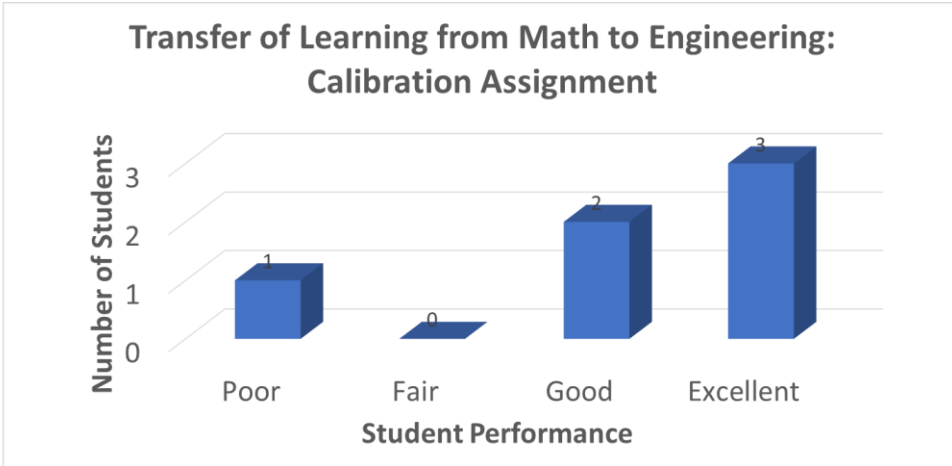


**Figure 5.** Student Performance in Calibration mathematics assignment used for gaging student's retention of math concepts and whether they can apply it to mathematical problems (vertical, far transfer learning). This homework was assigned at the beginning of the semester.
Sample size, $n$ = 6 (Summer 2023)
Student Performance:
1 – Poor (<70%); 2 – Fair (70-79%); 3 – Good (80-89%); 4 – Excellent (90-100%)

Similarly, student perception survey results show that, though statistically not significant, the majority of the participants evaluated the overall contribution of this math assignment to learning to be at a level of good (2/6) or excellent (3/6), while only one student rated the assignment at a level of fair (1/6).

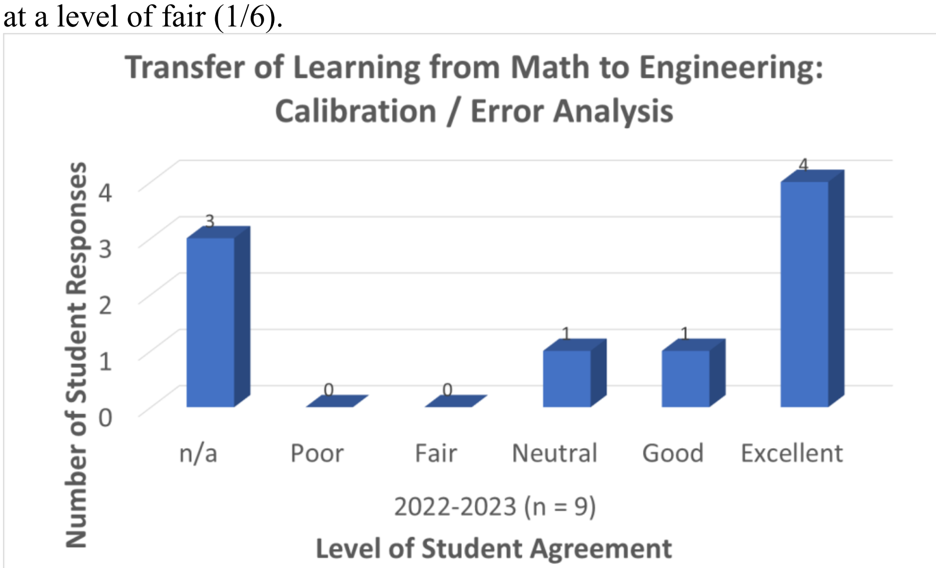


**Figure 6.** Student Perceptions on the *Calibration* mathematics assignment, and its contribution to transfer of learning from mathematics to engineering (Sensors and Systems course), at different learning taxonomies (remembering, adapting/applying, transfer of learning, overall) (vertical, far transfer learning). Sample size, $n$ = 9 (Summer 2023)
Note: Only 6 students completed assignment and survey from $n$ = 9.
Level of student agreement:
n/a - not applicable (did not respond); 1 – poor; 2 – fair; 3 – neutral; 4 – good; 5 – excellent

Student performance was evaluated in the final class project, which was a collaborative and collaborative PBL team assignment fostering engaged student learning through hands-on IoT devices, including sensors and microcontrollers, to assess the students' performance as an indicator of the response to the posed research question. Final projects were evaluated for final reports, oral presentations, and real-time demonstrations.

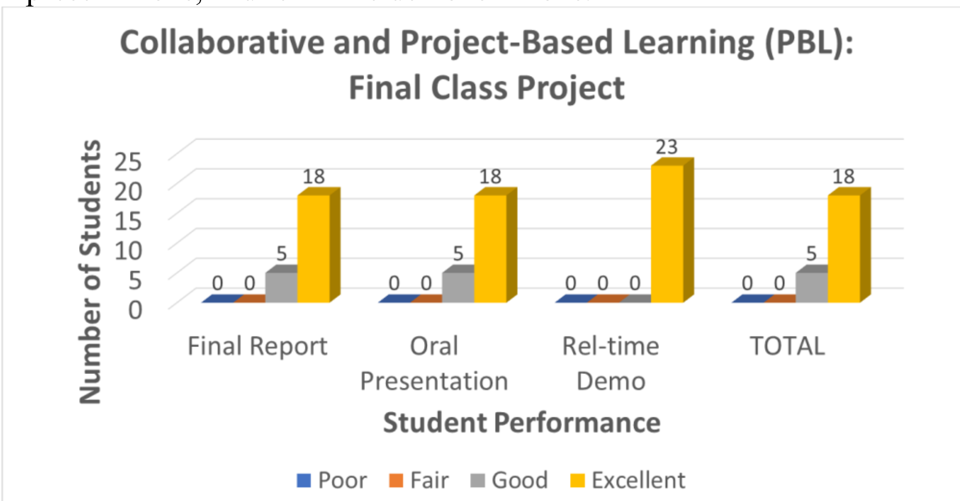


**Figure 7.** Student Performance on PBL Final Class Project
Sample size: $n = 23$. $n = n_1 + n_2$, where $n_1 = 11$ (Summer 2022), and $n_2 = 12$ (Summer 2023)
Student Performance:
1 – Poor (<70%); 2 – Fair (70-79%); 3 – Good (80-89%); 4 – Excellent (90-100%)

As can be seen from Figure 7, all students performed at a high level (good or excellent) in all categories of the final project, supporting a positive answer to the research question.

**Findings and Conclusion**

The results suggest that most students had forgotten and had to review and remember some of the math concepts if they took the math course over a year before the senior-level Sensors and Systems course. Some of students experienced challenges relating the mathematical concepts they learned in mathematics courses to cases and examples presented in the engineering Sensors and Systems course that utilized those same mathematical concepts to solve, describe or analyze an engineering process or application due to different symbology used in the math and engineering courses. Scaffolding exercises assisted the students in making connections to and building on their previously learned knowledge and skills to ultimately succeed in delivering a high-quality final project prototype through collaborative and problem-based learning that allowed hands-on experiences through sensors, actuators and embedded systems to integrate these devices for monitoring (sensing) and control (activation) applications. With the addition of mathematics exercises added to the Sensors and Systems course, the students were able to remember and transfer the learning of mathematics to engineering contexts.

By the time the final project was completed, students demonstrated that transfer of learning CAN be successfully achieved in remote hands-on engaged student learning (ESL) scenarios that involved, in this case, student access to IoT devices (sensors, actuators, microcontrollers) in their own environment and time that allowed flexibility and learning, including collaborative and PBL, in this course.

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