Achievable Rate for RIS-Assisted Sparse Channel with State Training

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***Abstract*—This paper studies the classical problem of communication across channels with state estimated at the receiver, in the context of wireless channels with reconfigurable intelligent surfaces (RIS). The RIS channels are characterized by numerous channel parameters but often have a sparse underlying structure. Under these conditions, the communication, channel training, and the characteristics of sparse recovery algorithms, interact in intricate ways. We calculate a training-based achievable rate for the RIS-induced sparse channel. We use an efficient sparse model for the RIS-aided channel that eliminates the need for recovering angles of arrival and departure at the RIS. We incorporate in the analysis the misalignment between the discrete parameter model of compressive sensing and the actual continuous-valued channel parameters, referred to as basis mismatch. Finally, we offer insights into designing RIS size and compressive sensing-based channel estimation parameters for RIS-aided communication systems.**

I. Introduction

In channels with estimated states at the receiver, allocation of channel resources to estimation and communication tasks has been a classical problem in wireless information theory [1]. This paper studies a new manifestation of this problem with many channel parameters (as in the case of reconfigurable intelligent surface (RIS) [2], [3]) while the underlying channels are sparse. A key point of interest in this paper is to uncover how the inherent features of sparse channel estimation, such as grid mismatch, influence the above-mentioned tradeoff in capacity optimization, and the resulting overall system behavior and performance.

For coherent reception, the channel state must be known at the receiver. In the context of RIS, some works consider capacity analysis without requiring channel estimation [4], [5] (i.e., with genie-aided channel states). In practice, channel estimates are obtained by pilot transmission that involves the expenditure of power and degrees of freedom. The large number of RIS parameters complicates pilot transmission and channel estimation [6]–[9]. The effect of channel estimation overhead on the capacity of RIS-aided channel is studied in [10], [11].

The use of RIS is most compelling in millimeter wave (mmWave) frequencies when the channels are sparse. Compressive sensing is often used for sparse recovery, including in mmWave channel estimation [12], and is computationally more efficient than competing algorithms [13]. However, compressive sensing is inherently finite-dimensional, while channel parameters such as angles of arrival and departure are continuous-valued. This results in a basis mismatch problem [14] arising from the differences between true and quantized angles. The compressive RIS channel estimation has been studied in [15]–[19]. However, the effect of basis mismatch on the performance of the channel estimation and connections to spectral efficiency is not examined in the context of communication using RIS link.

This paper derives a training-based lower bound on the capacity of RIS-aided communication, subject to channel state recovery by compressive sensing. We use an efficient representation of the end-to-end sparse channel that avoids the recovery of the angles of arrival and departure at the RIS. Only the angles at the transmitter and receiver are discretized for compressive sensing and consequently estimated. This results in smaller dimensions, lower complexity, and smaller estimation errors, compared with earlier representations [18]. We analyze errors due to grid mismatch. Our results also highlight the effect of parameters such as the number of dominant paths, quantization resolution, and power allocation on the spectral efficiency of the mmWave RIS-aided communication. Simulation results provide valuable insight into the optimal training overhead, RIS size, and the design criteria for compressive sensing channel estimation.

II. RIS-aided mmWave System Model

Assume an N_t-antenna transmitter and N_r-antenna receiver communicate through a wireless link assisted by RIS with N passive elements. Let $\mathbf{G} \in \mathbb{C}^{N \times N_t}$ denote the channel between the transmitter and the RIS and $\mathbf{H} \in \mathbb{C}^{N_r \times N}$ denote the channel between the RIS and the receiver. The RIS reflection coefficient matrix is denoted by $\mathbf{\Psi} = \mathrm{diag}(\boldsymbol{\psi})$, with the passive elements $\boldsymbol{\psi} = [\psi_1, \ldots, \psi_N]^T$, of the form $\psi_i = \nu_i e^{j\tau_i}$, where $\nu_i \in [0, 1]$, and $\tau_i \in [0, 2\pi)$. Let $\mathbf{x}$ be the input signal and ρ be the transmit power, the received signal is given by

$$\mathbf{y} = \sqrt{\rho}\mathbf{W}^H\mathbf{H}\mathbf{\Psi}\mathbf{G}\mathbf{F}\mathbf{x} + \mathbf{W}^H\mathbf{n}, \tag{1}$$

where $\mathbf{F} \in \mathbb{C}^{N_t \times M_t}$ is the precoding matrix employed at the transmitter to beamform the signal and $\mathbf{W} \in \mathbb{C}^{N_r \times M_r}$ is the combining matrix preprocesses the signal at the receiver and $\mathbf{W}^H\mathbf{n}$ is the additive white Gaussian noise with variance σ_n^2.

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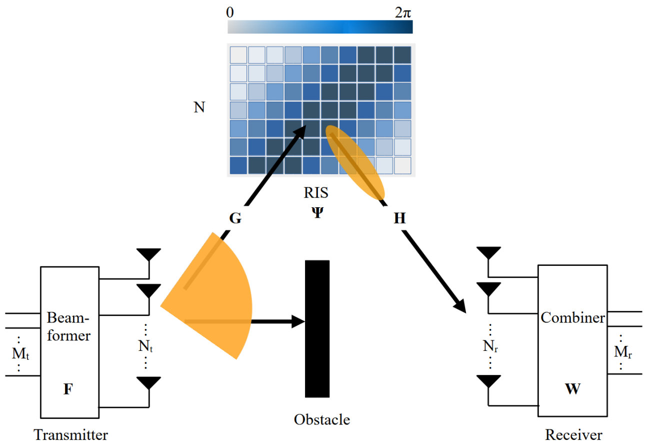

Fig. 1. RIS-assisted system model

The direct channel between the transmitter and receiver is assumed to be blocked. The corresponding system model is illustrated in Fig. 1.

To estimate the channel, the transmitter sends M_t pilot symbols at M_t successive time slots (represented by $\mathbf{S}_p$), resulting in the received signal $\mathbf{Y}_p \in \mathbb{C}^{M_r \times M_t}$:

$$\mathbf{Y}_p = \sqrt{\rho_p}\mathbf{W}^H\mathbf{H}\mathbf{\Psi}\mathbf{G}\mathbf{F}\mathbf{S}_p + \mathbf{N}_p, \tag{2}$$

where the transmit power dedicated to pilot transmission is denoted by ρ_p. We assume the geometric channel model based on the propagation path gains of each link and their angles of arrival and departure [20]. The transmitter and receiver are considered to be equipped with a uniform linear array with antenna spacing d_t, and d_r respectively, and without loss of generality, we model the RIS as a uniform linear array with element spacing d_n. L is the number of paths between the transmitter and RIS and P is the number of paths between the RIS and receiver. The channel $\mathbf{G}$ can be written as follows

$$\mathbf{G} = \sum_{l=1}^{L} \alpha_l \mathbf{a}_2(\theta_l)\mathbf{a}_1^H(\phi_l), \tag{3}$$

where α_l is the complex gain of the l-th path, ϕ_l and θ_l are the l-th azimuth angles of departure and arrival at the transmitter and the RIS, respectively, uniformly distributed over the range of $[-\pi, \pi]$. The vectors $\mathbf{a}_1(.)$ and $\mathbf{a}_2(.)$ are the steering and response vectors which are defined using $\mathbf{f}_M(x) = [1, e^{j\frac{2\pi}{\lambda}x}, \ldots, e^{j(M-1)\frac{2\pi}{\lambda}x}]$ where λ is the carrier wavelength, as $\mathbf{a}_1(\phi_l) = \mathbf{f}_{N_t}(d_t \sin(\phi_l))$, $\mathbf{a}_2(\theta_l) = \mathbf{f}_N(d_n \sin(\theta_l))$. Similarly, the channel $\mathbf{H}$ is written as

$$\mathbf{H} = \sum_{p=1}^{P} \beta_p \mathbf{b}_2(\vartheta_p)\mathbf{b}_1^H(\varphi_p), \tag{4}$$

where β_p is the complex gain of the p-th path, φ_p and ϑ_p are the azimuth angles of departure and arrival at the RIS and receiver, respectively, and steering and response vectors are $\mathbf{b}_1(\varphi_p) = \mathbf{f}_N(d_n \sin(\varphi_p))$ and $\mathbf{b}_2(\vartheta_p) = \mathbf{f}_{N_r}(d_r \sin(\vartheta_p))$.

One can rewrite the channel matrices $\mathbf{G}$ and $\mathbf{H}$ as follows

$$\mathbf{G} = \mathbf{A}_2\mathbf{\Lambda}_g\mathbf{A}_1^H, \tag{5}$$

$$\mathbf{H} = \mathbf{B}_2\mathbf{\Lambda}_h\mathbf{B}_1^H, \tag{6}$$

where $\mathbf{\Lambda}_g \triangleq \text{diag}([\alpha_1, \ldots, \alpha_L]^T)$ and $\mathbf{\Lambda}_h \triangleq \text{diag}([\beta_1, \ldots, \beta_P]^T)$ are matrices corresponding to the path gains. The steering matrix $\mathbf{A}_1$ is obtained from the corresponding steering vectors of L paths, as $\mathbf{A}_1 = [\mathbf{a}_1(\phi_1) \ \ \mathbf{a}_1(\phi_2) \ \ \cdots \ \ \mathbf{a}_1(\phi_L)]$ and matrices $\mathbf{A}_2, \mathbf{B}_1$ and $\mathbf{B}_2$ are similarly obtained.

III. Sparse Channel Estimation

The sparse channel estimation employing compressive sensing is as follows. Without loss of generality, we consider the pilot signal to be $\mathbf{S}_p = \mathbf{I}_{M_t}$. By vectorizing the received signal $\mathbf{y}_p = \text{vec}(\mathbf{Y}_p)$, and using the Kronecker product $\otimes$ and Khatri-Rao product $\odot$, we have

$$\begin{aligned}\mathbf{y}_p =& \sqrt{\rho_p}(\mathbf{F}^T \otimes \mathbf{W}^H)\text{vec}(\mathbf{H}\mathbf{\Psi}\mathbf{G}) + \mathbf{n}_p\\ =& \sqrt{\rho_p}(\mathbf{F}^T \otimes \mathbf{W}^H)(\mathbf{G}^T \odot \mathbf{H})\boldsymbol{\psi} + \mathbf{n}_p\\ =& \sqrt{\rho_p}(\mathbf{F}^T \otimes \mathbf{W}^H)(\mathbf{A}_1^*\mathbf{\Lambda}_g^T\mathbf{A}_2^T \odot \mathbf{B}_2\mathbf{\Lambda}_h\mathbf{B}_1^H)\boldsymbol{\psi} + \mathbf{n}_p,\end{aligned} \tag{7}$$

where $\mathbf{n}_p = \text{vec}(\mathbf{N}_p)$. One can write $\mathbf{y}_p$ as

$$\sqrt{\rho_p}\underbrace{(\mathbf{F}^T \otimes \mathbf{W}^H)}_{\mathbf{\Phi}}\underbrace{(\mathbf{A}_1^* \otimes \mathbf{B}_2)}_{\mathbf{\Gamma}_a}\underbrace{(\mathbf{\Lambda}_g^T \otimes \mathbf{\Lambda}_h)(\mathbf{A}_2^T \odot \mathbf{B}_1^H)\boldsymbol{\psi}}_{\mathbf{c}_a} + \mathbf{n}_p, \tag{8}$$

where $\mathbf{c}_a \in \mathbb{C}^{LP \times 1}$ is the signal to be recovered which is represented with respect to the basis $\mathbf{\Gamma}_a \in \mathbb{C}^{N_tN_r \times LP}$ and $\mathbf{\Phi} \in \mathbb{C}^{M_tM_r \times N_tN_r}$ is the measurement matrix.

To model the system for the compressed sensing procedure, a representation of the channel in a quantized angular domain is required. In this paper, we employ a channel representation where the sparse signal to be recovered includes both the cascaded channel and the RIS configuration matrix. The basis for this signal is obtained by discretizing only the angular domain at the transmitter and receiver. This approach simplifies the problem by avoiding the need to discretize the angles at the RIS, resulting in lower complexity and reduced errors due to the basis mismatch.

Therefore, the problem of estimating the channel is formed as the following compressive sensing problem

$$\mathbf{y}_p = \sqrt{\rho_p}\mathbf{\Phi}\mathbf{\Gamma}_v\mathbf{c}_v + \mathbf{n}_p, \tag{9}$$

where $\mathbf{\Gamma}_v = \widetilde{\mathbf{A}}_1^* \otimes \widetilde{\mathbf{B}}_2 \in \mathbb{C}^{N_tN_r \times GH}$ is the quantized basis defined by the product of transmitter and receiver dictionary matrices $\widetilde{\mathbf{A}}_1$ and $\widetilde{\mathbf{B}}_2$ which are unitary matrices obtained from the uniform sampling of the angular domain with resolution G and H. In this paper, these matrices are $N_t \times G$ and $N_r \times H$ DFT matrices, respectively (orthonormal DFT matrix for $G = N_t$, $H = N_r$ and overcomplete DFT matrix for $G > N_t$ and $H > N_r$). The vector $\mathbf{c}_v$ is a sparse vector in the basis $\mathbf{\Gamma}_v$, with non-zero values on the elements corresponding to the grid points where the true angle of arrival and departure lie. Note that the number of grid points GH is far greater than the level of sparsity.

To recover the sparse vector $\mathbf{c}_v$, assume that $\mathbf{c}_v = \mathbf{R}\boldsymbol{\psi}$ where $\mathbf{R}$ is a row sparse matrix with LP non-zero rows containing the virtual representation of the steering vectors of RIS-side as well as the product of the channel gains. To estimate all the elements in the matrix $\mathbf{R}$, N observations are needed. In this representation, the pilot signal is received in M_t time slots, repeated N times using N different RIS configuration vector $\boldsymbol{\psi}$, i.e. $\mathbf{R}\boldsymbol{\Psi}' = \mathbf{C}_v$. $\boldsymbol{\Psi}'$ is $N \times N$ RIS coefficient matrix with N orthogonal columns (e.g. columns of the DFT matrix) and $\mathbf{C}_v \in \mathbb{C}^{GH\times N}$ is a row sparse matrix obtained from concatenating different $\mathbf{c}_v$ column vectors corresponding to N different RIS coefficients. Therefore, the compressive sensing problem results in estimating the channel coefficients in $\mathbf{C}_v$ from the observation $\mathbf{Y}'_p \in \mathbb{C}^{M_tM_r\times N}$

$$\mathbf{Y}'_p = \sqrt{\rho_p}\boldsymbol{\Phi}\boldsymbol{\Gamma}_v\mathbf{C}_v + \mathbf{N}'_p, \tag{10}$$

where $\mathbf{N}'_p \in \mathbb{C}^{M_tM_r\times N}$ is the additive noise. Employing compressive sensing algorithms such as orthogonal matching pursuit to obtain $\mathbf{C}_v$, one can recover the matrix $\mathbf{R}$ from the set of equations $\mathbf{R}'\boldsymbol{\Psi}' = \mathbf{C}'_v$ where $\mathbf{R}' \in \mathbb{C}^{LP\times N}$ and $\mathbf{C}'_v \in \mathbb{C}^{LP\times N}$ are attained from omitting the zero rows in the matrices $\mathbf{R}$ and $\mathbf{C}_v$, respectively. In this way, one can estimate the cascaded channel $\mathbf{C} \triangleq \mathbf{G}^T \odot \mathbf{H} = (\widetilde{\mathbf{A}}_1^* \otimes \widetilde{\mathbf{B}}_2)\mathbf{R}$ whose estimation is necessary and sufficient for coherent detection at the receiver and beamforming at the transmitter and RIS.

The advantage of employing this method is that one can recover the cascaded channel using the fact that the path gains in the quantized basis is sparse, annihilating many elements in the RIS side steering matrices $\mathbf{A}_2$ and $\mathbf{B}_1$. This prevents quantizing the RIS-end angular domain and reduces the complexity.

The discretization of the angular domain may result in an estimation error if the true angles do not lie on the discretized angle and the power leakage to the other neighboring quantized angle deteriorates the recovery of all the parameters. This issue is referred to as basis mismatch [14]. The problem of basis mismatch arises when the quantized representation of the basis does not match the true angle domain basis, i.e., $\mathbf{c}_v = \boldsymbol{\Gamma}_v^\dagger\boldsymbol{\Gamma}_a\mathbf{c}_a \triangleq \boldsymbol{\Gamma}\mathbf{c}_a$ where $\boldsymbol{\Gamma}$ is the mismatch between the true basis $\boldsymbol{\Gamma}_a$ and the quantized basis $\boldsymbol{\Gamma}_v$ and † is the Moore-Penrose inverse.

The element $(H(m-1)+k, LP(l-1)+p)$ of the matrix $\boldsymbol{\Gamma}$ is calculated as

$$\frac{1}{GH}e^{j\frac{1}{2}(\eta(N_t-1)+\zeta(N_r-1))}\frac{\sin(\eta N_t/2)\sin(\zeta N_r/2)}{\sin(\eta/2)\sin(\zeta/2)}, \tag{11}$$

where $\eta \triangleq 2\pi(\frac{(m-1)}{G} - \phi_l)$ and $\zeta \triangleq 2\pi(\vartheta_p - \frac{(k-1)}{H})$. Every element of the basis mismatch matrix is a Dirichlet kernel function, which is two-dimensional due to the discretization of the angular domain at both transmitter and receiver. This means that any mismatch between true angles ϕ_l and ϑ_p and their corresponding DFT element $\frac{(m-1)}{G}$ and $\frac{(k-1)}{H}$ lead to leakage to the adjacent cells, which is shown by Dirichlet kernel function. This results in an error in the channel estimation stage defined as $\mathbf{E} \triangleq \boldsymbol{\Gamma} - \overline{\boldsymbol{\Gamma}}$ in which $\overline{\boldsymbol{\Gamma}}$ is the case where the quantized basis and true basis are perfectly matched. We consider this error in the capacity analysis in the following section.

IV. Achievable Rate

During channel estimation, T_p time slots are assigned to pilots, making the pilot transmission time equal to $T_p = NM_t$. The remaining time slots in the coherence interval T are dedicated to data transmission, denoted by $T_d \triangleq T - T_p$. Consider the received data given as $\mathbf{Y}_d = \sqrt{\rho_d}\mathbf{W}^H\mathbf{H}\boldsymbol{\Psi}\mathbf{G}\mathbf{F}\mathbf{S}_d + \mathbf{N}_d$, where ρ_d is the data transmission power and $\mathbf{S}_d$ is composed of vectors of data symbols of size $M_t \times 1$ sent over T_d time slots. By vectorizing the received signal $\mathbf{Y}_d$, we have $\mathbf{y}_d = \sqrt{\rho_d}(\mathbf{S}_d^T \otimes \mathbf{I}_{M_r})\boldsymbol{\Phi}\mathbf{c} + \mathbf{n}_d$ where $\mathbf{c} = \mathbf{C}\boldsymbol{\psi}$ and $\mathbf{n}_d = \mathrm{vec}(\mathbf{N}_d)$ is the additive noise.

Proposition 1. *The capacity lower bound for the RIS-assisted sparse channel with channel state obtained via compressive sensing is*

$$C \geq (\frac{T - NM_t}{T})\mathbb{E}\{\log_2(1 + \frac{\rho_d\widetilde{\boldsymbol{\psi}}_{\hat{\mathbf{C}}}^H\hat{\mathbf{C}}^H\boldsymbol{\Phi}^H\boldsymbol{\Phi}\hat{\mathbf{C}}\widetilde{\boldsymbol{\psi}}_{\hat{\mathbf{C}}}}{\sigma_{\widetilde{n}}^2})\}, \tag{12}$$

where $\hat{\mathbf{C}}$ is the estimate of the cascaded channel using the orthogonal matching pursuit algorithm, $\widetilde{\boldsymbol{\psi}}_{\hat{\mathbf{C}}}$ is the optimal RIS phase shift and

$$\sigma_{\widetilde{n}}^2 = (1 + \frac{\rho_d M_t}{\rho_p(T - NM_t)})\sigma_n^2 + \frac{2LPGH\rho_d}{N_tN_rM_r(T - NM_t)}\sigma_e^2\sigma_\alpha^2, \tag{13}$$

where $\sigma_e^2 \triangleq \mathbb{E}\{\mathbf{E}^H\mathbf{E}\}$ and σ_α^2 is the variance of the cascaded channel path gains.

Proof. To obtain the capacity of the above system, one needs to calculate:

$$C = \max_{p(\mathbf{S}_d,\boldsymbol{\psi})} I(\mathbf{S}_d; \mathbf{Y}_d, \hat{\mathbf{C}}), \tag{14}$$

where $p(\mathbf{S}_d, \boldsymbol{\psi})$ is the joint distribution of the input signal $\mathbf{S}_d$ and the RIS coefficient vector $\boldsymbol{\psi}$. We have:

$$\begin{aligned} C &= \max_{p(\mathbf{S}_d,\boldsymbol{\psi})} I(\mathbf{S}_d; \mathbf{Y}_d|\hat{\mathbf{C}}) + I(\mathbf{S}_d; \hat{\mathbf{C}}) \\ &\geq \max_{p(\mathbf{S}_d),\boldsymbol{\psi}_{\hat{\mathbf{C}}}} I(\mathbf{S}_d; \mathbf{Y}_d|\hat{\mathbf{C}}), \end{aligned} \tag{15}$$

where (15) holds since the maximization over $p(\mathbf{S}_d, \boldsymbol{\psi})$, is replaced with the maximization over the distribution of the data, and the RIS phase shifts vector $\boldsymbol{\psi}_{\hat{\mathbf{C}}}$ which is set employing the channel estimate. Also, $I(\mathbf{S}_d; \hat{\mathbf{C}}) = 0$ holds, since the transmit data is independent of the estimated channel. To calculate the capacity lower bound, the received signal $\mathbf{y}_d = \sqrt{\rho_d}(\mathbf{S}_d^T \otimes \mathbf{I}_{M_r})\boldsymbol{\Phi}\mathbf{C}\boldsymbol{\psi} + \mathbf{n}_d$ can be rewritten as

$$\sqrt{\rho_d}(\mathbf{S}_d^T \otimes \mathbf{I}_{M_r})\boldsymbol{\Phi}\hat{\mathbf{C}}\boldsymbol{\psi} + \sqrt{\rho_d}(\mathbf{S}_d^T \otimes \mathbf{I}_{M_r})\boldsymbol{\Phi}\widetilde{\mathbf{c}} + \mathbf{n}_d, \tag{16}$$

where $\widetilde{\mathbf{c}} = \mathbf{c} - \hat{\mathbf{C}}\boldsymbol{\psi}$ is the cascaded channel estimation error which is due to the error caused by employing orthogonal matching pursuit algorithm and the mismatch error $\mathbf{E}$. Let the sparse estimation error in (16) denoted by $\widetilde{\mathbf{n}} \triangleq \sqrt{\rho_d}(\mathbf{S}_d^T \otimes$

$\mathbf{I}_{M_r})\mathbf{\Phi}\widetilde{\mathbf{c}}$. The corresponding combined noise for the data transmission phase is defined as $\overline{\mathbf{n}} \triangleq \widetilde{\mathbf{n}} + \mathbf{n}_d$. To evaluate the effect of the combined noise $\overline{\mathbf{n}}$ on data transmission, the correlation between the combined noise and the transmitted signal is calculated as

$$\begin{aligned}\mathbb{E}\{\mathbf{S}_d\overline{\mathbf{N}}^H|\hat{\mathbf{C}}\} &= \mathbb{E}\{\mathbf{S}_d(\sqrt{\rho_d}\mathbf{W}^H\widetilde{\mathbf{C}}\mathbf{F}\mathbf{S}_d + \mathbf{N}_d)^H|\hat{\mathbf{C}}\} \quad (17)\\ &= \sqrt{\rho_d}\mathbb{E}\{\mathbf{S}_d\mathbf{S}_d^H\mathbf{F}^H\widetilde{\mathbf{C}}^H\mathbf{W}|\hat{\mathbf{C}}\} \quad (18)\\ &= \sqrt{\rho_d}\mathbb{E}\{\mathbf{S}_d\mathbf{S}_d^H|\hat{\mathbf{C}}\}\mathbb{E}\{\mathbf{F}^H\widetilde{\mathbf{C}}^H\mathbf{W}|\hat{\mathbf{C}}\}\\ &= 0, \quad (19)\end{aligned}$$

where in (17) we have $\overline{\mathbf{N}} \in \mathbb{C}^{M_r\times T_d}$ and $\widetilde{\mathbf{C}} \in \mathbb{C}^{N_r\times N_t}$ which are equivalent noise matrix obtained from $\text{vec}(\overline{\mathbf{N}}) = \overline{\mathbf{n}}$ and the cascaded channel estimation error derived from $\text{vec}(\widetilde{\mathbf{C}}) = \widetilde{\mathbf{c}}$, respectively. (18) holds since the transmitted signal and the receiver noise are independent, and the noise is Gaussian with zero mean. Due to randomly generating the precoding and combining matrices from a discrete uniform distribution with values $\{\pm 1, \pm j\}$ which has zero mean [21], the equality in (19) is obtained resulting in the signal and noise to be uncorrelated. Therefore, one can utilize the worst-case uncorrelated noise theorem, to calculate a training-based lower bound for the capacity [1]

$$C \geq \max_{p(\mathbf{S}_d),\boldsymbol{\psi}_{\hat{\mathbf{C}}}} (\frac{T-T_p}{T})\mathbb{E}\{\log_2(1+\frac{\rho_d\boldsymbol{\psi}_{\hat{\mathbf{C}}}^H\hat{\mathbf{C}}^H\mathbf{\Phi}^H\mathbf{Q}\mathbf{\Phi}\hat{\mathbf{C}}\boldsymbol{\psi}_{\hat{\mathbf{C}}}}{\sigma_{\overline{n}}^2})\}, \quad (20)$$

where $\mathbf{Q} \triangleq \mathbb{E}\{(\mathbf{S}_d^T \otimes \mathbf{I}_{M_r})^H(\mathbf{S}_d^T \otimes \mathbf{I}_{M_r})\} \in \mathbb{C}^{M_tM_r\times M_tM_r}$ is the equivalent transmit signal covariance matrix and $\sigma_{\overline{n}}^2$ is the variance of the equivalent additive noise. The distribution of the transmit signal $\mathbf{S}_d$ which maximizes the capacity is Gaussian. We consider the data transmission is done through transmit antennas independently and with equal power, the covariance matrix of $\mathbf{S}_d$ is $\mathbb{E}\{\mathbf{S}_d^*\mathbf{S}_d^T\} = \mathbf{I}_{M_t}$, resulting in $\mathbf{Q} = \mathbf{I}_{M_tM_r}$.

Therefore, the maximization problem in (20) is simplified to maximizing the capacity with respect to $\boldsymbol{\psi}_{\hat{\mathbf{C}}}$. The maximization can be performed on $\boldsymbol{\psi}_{\hat{\mathbf{C}}}^H\hat{\mathbf{C}}^H\mathbf{\Phi}^H\mathbf{\Phi}\hat{\mathbf{C}}\boldsymbol{\psi}_{\hat{\mathbf{C}}}$ with respect to $\boldsymbol{\psi}_{\hat{\mathbf{C}}}$ assuming each RIS coefficient $\psi_i = \nu_i e^{j\tau_i}$ has amplitude $\nu_i = 1$ and phase $\tau_i \in [0, 2\pi)$ for $i = 1, \dots, N$. Since this is, in general, an NP-hard problem, by defining $\mathbf{D} \triangleq \hat{\mathbf{C}}^H\mathbf{\Phi}^H\mathbf{\Phi}\hat{\mathbf{C}}$ and using $\boldsymbol{\psi}_{\hat{\mathbf{C}}}^H\mathbf{D}\boldsymbol{\psi}_{\hat{\mathbf{C}}} = \text{tr}(\mathbf{D}\widetilde{\mathbf{\Psi}})$ with $\widetilde{\mathbf{\Psi}} = \boldsymbol{\psi}_{\hat{\mathbf{C}}}\boldsymbol{\psi}_{\hat{\mathbf{C}}}^H$, one can solve the equivalent semi-definite problem

$$\begin{aligned}\max_{\widetilde{\mathbf{\Psi}}} \quad & \text{tr}(\mathbf{D}\widetilde{\mathbf{\Psi}})\\ \text{s.t.} \quad & [\widetilde{\mathbf{\Psi}}]_{i,i} = 1, \quad \widetilde{\mathbf{\Psi}} \succeq 0, \quad \text{rank}(\widetilde{\mathbf{\Psi}}) = 1,\end{aligned} \quad (21)$$

where $[\widetilde{\mathbf{\Psi}}]_{i,i}$ is the element (i, i) in $\widetilde{\mathbf{\Psi}}$ [22]. One can relax the rank-one constraint on $\widetilde{\mathbf{\Psi}}$ to obtain the solution. Therefore, the optimization problem becomes a convex semi-definite programming where the solution can be obtained using existing optimization solvers. The answer to the relaxed problem may have higher ranks. One can employ the method in [23] to find a rank-one solution, denoted by $\widetilde{\boldsymbol{\psi}}_{\hat{\mathbf{C}}}$ from the attained optimal higher-rank.

The equivalent noise variance in (20) is calculated as, $\sigma_{\overline{\mathbf{n}}}^2$

$$\begin{aligned}\sigma_{\overline{\mathbf{n}}}^2 &= \frac{1}{M_rT_d}\mathbb{E}\{||\mathbf{n}_d + \widetilde{\mathbf{n}}||_2^2\}\\ &= \frac{1}{M_rT_d}\big(\text{tr}\{\mathbb{E}\{\mathbf{n}_d\mathbf{n}_d^H\}\} + \text{tr}\{\mathbb{E}\{\widetilde{\mathbf{n}}\widetilde{\mathbf{n}}^H\}\}\big)\\ &= \frac{1}{M_rT_d}\text{tr}\{\sigma_n^2\mathbf{I}_{M_rT_d}\}\\ &+ \frac{\rho_d}{M_rT_d}\text{tr}\big\{\mathbb{E}\{(\mathbf{S}_d^T \otimes \mathbf{I}_{M_r})\mathbf{\Phi}\widetilde{\mathbf{c}}\widetilde{\mathbf{c}}^H\mathbf{\Phi}^H(\mathbf{S}_d^T \otimes \mathbf{I}_{M_r})^H\}\big\}\\ &= \sigma_n^2 + \frac{\rho_d}{M_rT_d}\text{tr}\big\{\mathbb{E}\{\mathbf{S}_d^*\mathbf{S}_d^T \otimes \mathbf{I}_{M_r}\}\mathbb{E}\{\mathbf{\Phi}\widetilde{\mathbf{c}}\widetilde{\mathbf{c}}^H\mathbf{\Phi}^H\}\big\}\\ &= \sigma_n^2 + \frac{\rho_d}{M_rT_d}\mathbb{E}\big\{\text{tr}\{\widetilde{\mathbf{c}}\widetilde{\mathbf{c}}^H\}\big\}, \quad (22)\end{aligned}$$

where $\mathbb{E}\{\mathbf{\Phi}^H\mathbf{\Phi}\} = \mathbf{I}_{N_tN_r}$ which is due to the characteristics of combining and precoding matrices, and $\mathbb{E}\big\{\text{tr}\{\widetilde{\mathbf{c}}\widetilde{\mathbf{c}}^H\}\big\}$ is the channel estimation error variance. This error can be upper bounded by $\frac{M_rM_t}{\rho_p}\sigma_n^2 + \frac{2LPGH}{N_tN_r}\sigma_e^2\sigma_\alpha^2$ [24]. The first term in this upper bound is the estimation error of the employed orthogonal matching pursuit algorithm, and the second term is due to the basis mismatch error $\mathbf{E}$. This proves the proposition. □

For the setting where the sparsity of the channel is of the order LP and the size of the sparse signal is GH, since there are M_r measurements for each pilot which is sent over M_t time slots, with N times transmission using N different RIS coefficients, M_t can be calculated according to $T_p = NM_t = \mathcal{O}(\frac{LP}{M_r}\log(GH))$ for the orthogonal matching pursuit algorithm [25].

V. Simulation Results

Throughout the simulations, a 32×32 communication system is assumed and the data and training powers are set to be equal. Fig. 2 investigates the effect of choice of the training length T_p on the spectral efficiency for RIS with 32 elements and the coherence interval $T = 300$. As a reference, we provide the spectral efficiency of the case where the channel state information at the receiver (CSIR) is perfectly known. The figure demonstrates that as the length of the training increases, the gap between the training-based capacity bound and the capacity with perfect knowledge of CSIR becomes more pronounced.

Fig. 3 demonstrates the effect of basis mismatch on the training-based bound on the capacity as a function of block length T. It can be noticed that the capacity bound is improved as the mismatch error variance is decreased, which is achieved through selecting an appropriate dictionary size (setting G and H to integer multiples of the number of transmit and receive antennas). Therefore, the overall capacity expression is increased due to the reduced mismatch error and the improved cascaded channel estimate.

In Fig. 4, the training-based bound is shown as a function of the number of RIS elements for block lengths $T = 300$ under different SNR levels. It can be seen that the spectral efficiency is increased up to a certain value. Beyond the corresponding N, increasing the number of RIS elements does not improve the capacity bound. This limitation arises because,

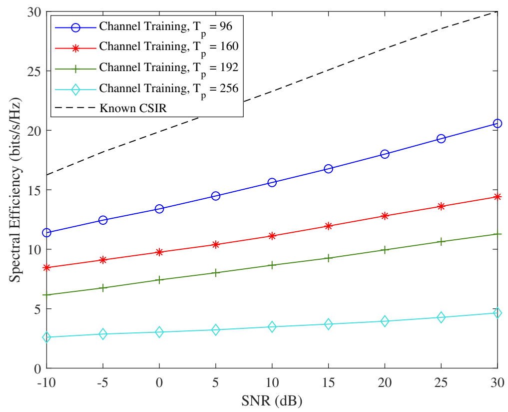

Fig. 2. Training-based capacity lower bound as a function of SNR for different training length

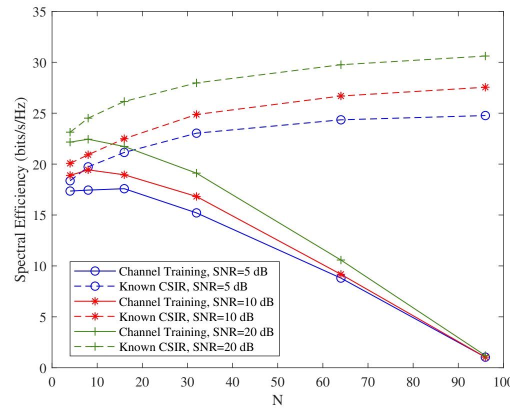

Fig. 4. Training-based capacity lower bound as a function of number of RIS elements

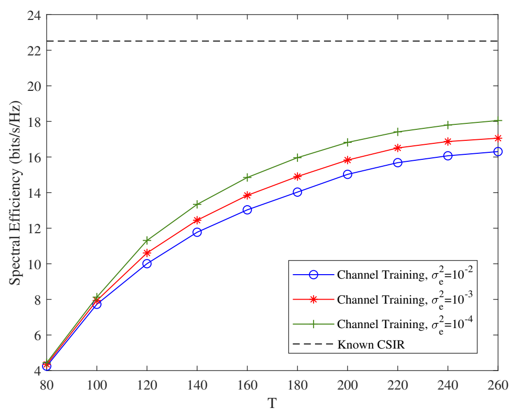

Fig. 3. Training-based capacity lower bound as a function of block length

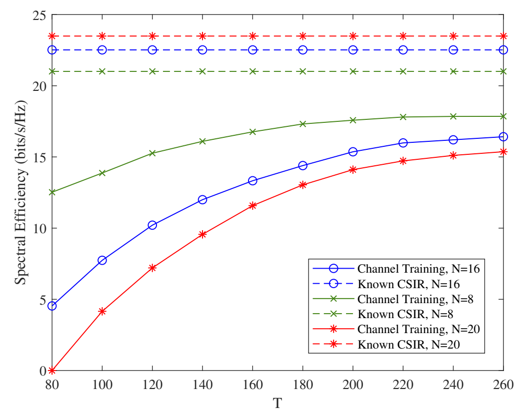

Fig. 5. Training-based capacity lower bound as a function of block length

beyond the optimal RIS sizes shown in the figure, increasing N results in significantly higher training overhead, reducing the available time for effective RIS beamforming. Consequently, there is a decline in spectral efficiency as the resources are predominantly consumed by the training phase, restricting the potential for optimal RIS beamforming exploitation.

To further illustrate these observations, Fig. 5 demonstrates the effect of increasing the number of RIS elements beyond the optimal value, on the spectral efficiency as a function of the block length. The existing gap between the training-based capacity bound and the capacity with perfect knowledge of CSIR for RIS of size 8 is remarkably lower than the case where the RIS size is 20. Therefore, increasing the number of RIS elements higher than a certain amount is ineffective in training-based systems and results in performance degradation.

VI. Conclusion

This study investigated the achievable rate for sparse RIS-assisted communication channels when channel states are known at the receiver. Employing the compressive sensing algorithm to obtain the channel estimate, we derived a training-based capacity lower bound based on the training phase parameters. Our results illustrated the impact of the misalignment between the assumed discrete parameter model for compressive sensing and the actual continuous channel parameters, on the spectral efficiency of RIS-aided channels.

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