

Exploring Noise-Resilient Spiking Neural Encoding using $\Sigma\Delta\Sigma$ Neurons

R. Sreekumar[†], Faiyaz E. Mullick[†], Md Golam Morshed, Avik W. Ghosh, Mircea R. Stan[†]
(equal contributors to this work)

Department of Electrical and Computer Engineering, University of Virginia
Email: {rs2xd, fm4fv, mm8by, ag7rq, mrs8n}@virginia.edu

Abstract—Researchers are increasingly focusing on spiking neurons to enhance the energy efficiency of edge machine learning (ML) models. Spiking neural encoding has evolved from traditional methods like Integrate and Fire and Time to First Spike to techniques such as Delta and Sigma-delta modulation (SD), enabling sparser and energy-efficient feature representation. In this work, we introduce the Sigma-Delta-Sigma ($\Sigma\Delta\Sigma$ /SDS) neuron, a noise-invariant spiking neural encoding technique. Our aim with this neural encoding is to emulate the robustness of a specific class of biological neurons to input noise by effectively filtering out noise-like features from the input stimuli. While noise injection in training prevents overfitting, improper noise profiles can be detrimental to the inference accuracy of a model. Our objective with the SDS encoding is to develop noise-resilient spiking neural network models capable of being trained with ideal features, while still being able to extract features from low-quality data during the inference phase. To assess the robustness of our technique, we implement an information encoding ensemble model that demonstrates a 6.2x improvement in robustness when the Signal-to-Noise Ratio (SNR) of the incoming signal is 1 dB. Furthermore, we evaluate the performance of the SDS neurons in more complex models such as a Liquid State Machine (LSM), where the model exhibits a 3.87% improvement in the predictive accuracy against the baseline model when tested against input features with SNR degradation from 55 to 1 dB.

Index Terms—Sigma-Delta-Sigma, overfitting, spiking neural network, Liquid State Machine

I. INTRODUCTION

Spiking neural models have garnered considerable attention recently due to their efforts to mimic the energy efficiency observed in biological systems [1]. These models encode information in sparse dimensions, offering enhanced energy efficiency and facilitating efficient feature representation by minimizing redundancy. In the nascent period of spiking-based neural architectures, the primary effort was focused on the biomimicry of neurons present within vertebrates. This led to the extensive implementation of networks based on encoding techniques such as Integrate-and-Fire (IF), Leaky-Integrate-and-Fire (LIF), time-to-first spike (TTFS), etc. Recently, we have seen a paradigm shift in encoding techniques, wherein the focus has shifted from biomimicry to emulating certain characteristics observed within biological neurons. For instance, the research community has extensively explored various neural encoding techniques to replicate properties observed in biological systems, such as those found in audio cochlear [2] and dynamic vision-based systems [3]. This has

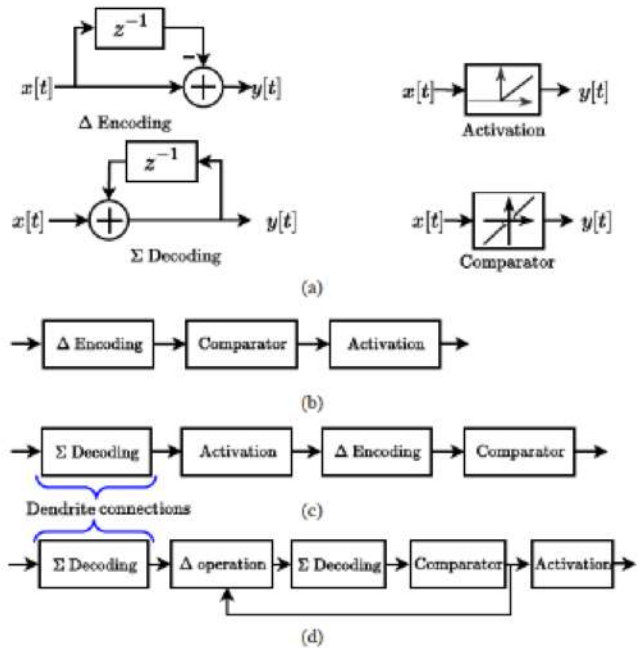


Fig. 1. (a) Symbolic representation of fundamental building blocks for Δ encoded neurons and signal flow representation for (b) Δ (c) $\Sigma\Delta$ and (d) proposed $\Sigma\Delta\Sigma$ neural encoding.

led to sparser schemes such as Delta modulation (Δ) and Sigma-delta ($\Sigma\Delta$) encoding. These schemes enable feature communication between neurons only when they exceed a certain threshold, leading to even sparser feature representations between layers within a network.

In this work, we extend the capabilities of delta-modulated neurons by introducing a novel evolution in this encoding approach termed Sigma-Delta-Sigma ($\Sigma\Delta\Sigma$ /SDS) neural encoding. Our proposed encoding method stands out for its adeptness in filtering out noise-like components from critical features, achieved through a unique feedback mechanism and intrinsic delay within the encoding technique. Our inspiration for developing this schema stems from observations of noise-invariant neurons found in specific bird species [4] and resonant frequency selective traits observed in primate auditory systems [5]. Our research aims to emulate and enhance these functionalities using our proposed scheme, aiming to construct noise-robust spiking models capable of feature extraction even in the presence of noisy input stimuli.

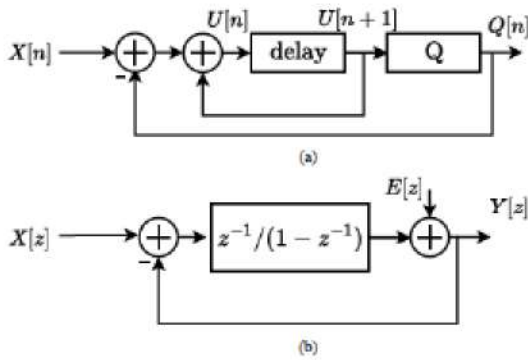


Fig. 2. Schematic representation of (a) Time domain signal, and (b) z-transform diagram for the encoding scheme.

From an applications perspective, it is imperative to understand the motivation for developing a noise-invariant neural encoding schema. In Machine Learning (ML) models, noise is often introduced into training features to prevent overfitting and to ensure robust inferences when faced with non-deterministic variations within test features. However, injecting an inaccurate noise profile during training can result in underfitting and distorted feature representations, ultimately undermining model performance [6]. Through our proposed neural encoding schema, we aim to train models on ideal feature sets without the need for noise injection, while still be able to perform feature extraction with distorted, low-quality data. During testing, the neural connections prioritize the transmission of significant features while attenuating noise components. By implementing this neural encoding approach, our objective is to develop noise-resilient spiking neural network models capable of performing classification tasks even with lower-quality incoming features.

The remainder of this paper is organized as follows: Section II provides a detailed discussion on the SDS neural encoding and its functional adaptations from existing delta-modulated neurons. In Section III, we delve into the noise-robustness property of the schema by implementing an ensemble model to assess the model's ability to accurately encode and decode information even in the presence of noise. Additionally, we evaluate the robustness of SDS neurons within the framework of a Liquid State Machine (LSM) and quantify the network's performance in predicting time-series features when exposed to inferior features from a Signal-to-Noise Ratio (SNR) perspective, as outlined in Section IV.

II. EVOLUTION OF Δ MODULATED NEURONS

Recent years have seen an emergence of advanced spiking neural encoding techniques tailored to specific applications. The evolving landscape of temporal encoding methods has witnessed a divergence, particularly marked by approaches that rely on the differential amplitude between consecutive feature samples in a time series format. This paradigm shift is exemplified in notable instances such as the adoption of spiking neural encoding-based Dynamic Vision Sensor (DVS) cameras [7] and audio cochlear feature extractors [8], both

integrating spiking networks with Asynchronous Delta Modulation (Fig. 1b). Within this class of neurons, researchers have introduced another encoding technique, the asynchronous Sigma Delta (SD) neuron (Fig. 1c), distinguished by its partitioning of neural encoding into two distinct phases: Sigma decoding and Delta encoding. The versatility of the SD encoding technique is underscored by its successful application in video processing applications [9], demonstrating substantial enhancements in energy efficiency.

A notable commonality between these two neurons lies in the foundational encoding blocks employed within the schema, as illustrated in Fig. 1a. This progression marks a substantial advancement in the temporal encoding landscape, with promising implications across diverse applications. The adoption of these techniques in current state-of-the-art neuromorphic hardware, such as Loihi 2.0, further validates their impact [9]. From an encoding perspective, a distinguishing characteristic of these neural encoding techniques compared to existing methods is the increasing sparsity of activations as information traverses successive layers of neurons. This phenomenon is driven by the mechanisms of the encoding techniques, where a post-neural spike occurs when the incoming feature/activation surpasses a predefined Δ threshold or deviates from its previous activation value by a certain threshold.

A. $\Sigma\Delta\Sigma$ neuron: Noise-robust Encoding schema

In this work, we draw upon the aforementioned encoding techniques and propose our novel SDS neural encoding (Fig. 1d). At this juncture, it is important to understand how the SDS neuron is an evolution in the class of delta-modulated neurons. To begin with, the SD neuron is essentially a Δ modulated neuron with a Σ decoding block that integrates dendrite connections from multiple pre-synaptic neurons. Our proposed SDS neuron is essentially a $\Delta\Sigma$ modulated neuron with a similar pre-synaptic Σ decoding block. Another critical difference within our encoding technique is the internal Δ operation block. Within the previously mentioned Delta and SD neuron classes, the delta operation is essentially performed on the current input and a delayed version of the input feature. However, within the SDS neuron, this delta operation performs a negated addition between the input feature and the output spike. From an encoding perspective, the SDS neural encoding stands out from its predecessors due to two distinct signal processing features. Firstly, a portion of the soma output from the comparator element is subtracted from the input stimuli at the dendrite terminal. This introduces a negative feedback mechanism, represented by the Δ operation block within the encoding scheme. Secondly, an additional integration phase is embedded within the encoding process which accumulates the aforementioned Δ operation over multiple time steps. These elements effectively impart two different filtering characteristics to the incoming stimuli and noise-like features.

To examine the filtering properties of the neural encoding scheme, we construct the time-domain representation and the equivalent discrete Z-transform signal flow diagram, as

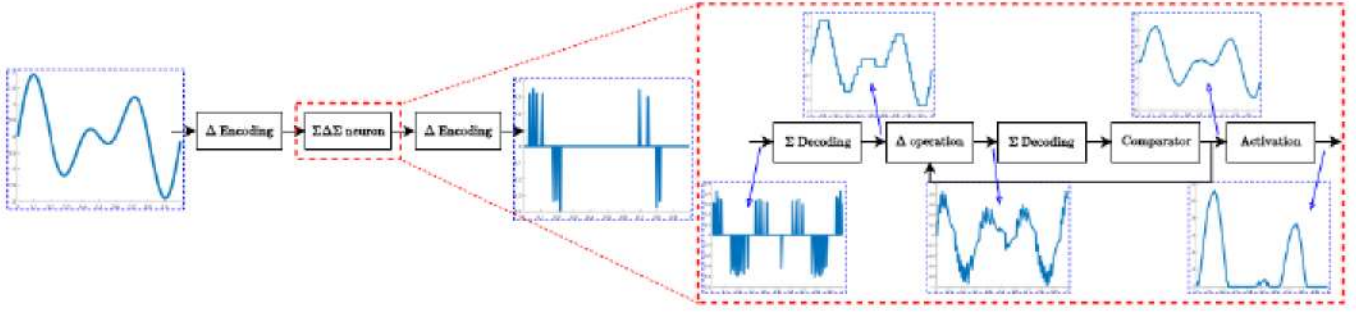


Fig. 3. Internal feature transformation and transient operative characteristics of the proposed SDS encoding schema.

depicted in Fig. 2. By analyzing the signal flow, we can derive the output $Q[n]$ as follows:

$$\begin{aligned} Q[n] &= X[n-1] + Q[n] - U[n] - Q[n-1] + U[n-1] \\ &= X[n-1] + e[n] - e[n-1] \end{aligned}$$

, where $X[n]$, $U[n]$, and $e[n]$ represent the discrete input stimuli, integrator input, and deduced error vector, respectively. The equivalent z-transform for the designated output $Y(z)$ is:

$$\begin{aligned} Y(z) &= X(z)z^{-1} + E(z)(1 - z^{-1}) \\ \Rightarrow \frac{Y(z)}{X(z)} &= z^{-1} \quad (1) \\ \Rightarrow \frac{E(z)}{X(z)} &= 1 - z^{-1} \quad (2) \end{aligned}$$

, where Eq. 1 and Eq. 2 represent the deduced signal and noise transfer function, respectively. As evident from these equations, the input signal features undergo a low-pass filtering operation while the noise features are shifted towards higher frequency regions and attenuated at the lower end of the spectrum. This mechanism allows us to diminish noisy components from critical features, provided these elements lie beyond the corner frequency of the low-pass filter and are modulated by the feedback strength and integration period of the neuron. It is noteworthy that, to maintain a similar sparse feature density profile to its predecessors, each SDS neuron is linked with a delta encoding block depicted in Fig. 3, showcasing the internal transformation occurring to an incoming feature at each encoding step within the neuron.

III. NOISE-ROBUST ENSEMBLE MODEL WITH SDS ENCODING

To evaluate the noise-filtering capabilities of our proposed encoding, we utilize an ensemble-based signal regenerative model. This ensemble model serves the dual purpose of assessing feature preservation and noise filtering simultaneously while extracting features from the incoming signal. By employing the ensemble model, we aim to replicate real-world scenarios where the nature of noise during inference is uncertain necessitating training with idealized features and subsequent fine-tuning for non-deterministic noise conditions. However, using our proposed neural encoding, models trained

with idealistic features still demonstrate robust noise performance during testing, obviating the need for fine-tuning. In Fig. 4a, we illustrate the pair of ensemble models employed to validate our hypothesis. Ref. Ensemble 1 processes the incoming time-series feature through a fully connected layer of neurons, generating a spike train. They are then fed into the second ensemble with a similar structure, which, under ideal conditions, reconstructs the original signal from the incoming spike activations. By implementing the ensemble model, we ensure that the SDS neuron can faithfully encode information in the spiking domain and decode the spike train to retrieve the original signal.

During the training phase, the first reference ensemble model weights are updated using idealistic features with an SNR value $\geq 100\text{dB}$. The weights for the second ensemble model are obtained from the first ensemble through a transpose operation and a scaling factor g_{tj} . In the subsequent testing phase, Reference Ensemble I encounters degraded signal features through the injection of artificial noise profiles and is characterized by their respective SNR values. The signal output regenerated by Ensemble II is then assessed for its fidelity against the ideal signal, measured by the normalized mean square error (NMSE). The ensemble model is trained using the Nengo PyTorchSpiking package, which facilitates the emulation of spiking neuron models based on the $\Sigma\Delta$ neural encoding scheme. To evaluate the performance of the neural encoding, we compare our results with a baseline ensemble model featuring $\Sigma\Delta$ neuron banks. Additionally, we vary the ensemble size to assess the network's performance as more features are extracted from a larger neuron bank. Fig. 4b demonstrates the network's performance across different test cases, with the reconstruction error (NMSE) calculated for input features spanning a range of SNR values. The critical findings within this simulation can be summarized as follows:

- In a constant ensemble size setting, the SDS neuron-based ensemble model consistently surpasses the performance of the SD baseline model by an average factor of 6.2x when the SNR of the input feature reaches 1 dB. This underscores the robustness of the neural encoding scheme in effectively filtering out noise-like features, even as the noise spectral power approaches that of the signal.
- For a fixed ensemble size ($N=50$), as we vary the Signal-to-Noise Ratio (SNR) of the incoming features from 20

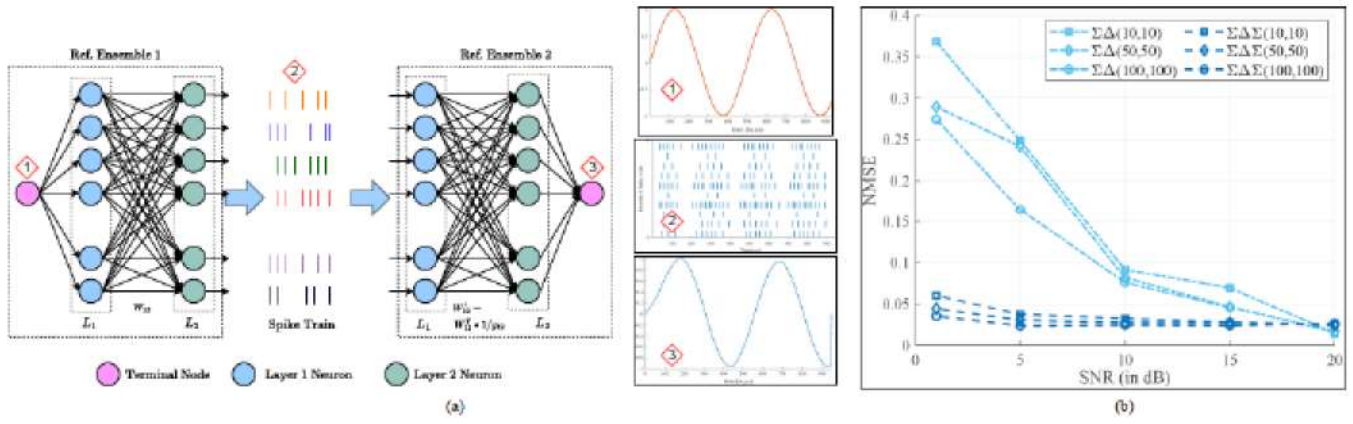


Fig. 4. (a) Ensemble model with transient feature transformation and (b) Robustness (NMSE variation) of ensemble model against SNR degradation.

to 1 dB, the degradation in model performance with SDS encoding is approximately 1.482x compared to the 57.56x degradation for the baseline. This confirms the neural encoding scheme's capability to maintain consistent regenerative performance across different noise levels. It demonstrates the effectiveness of the SDS-based ensemble in operating efficiently even with low-quality data, despite being trained under ideal conditions.

- It is essential to assess whether the ensemble's feature extraction capabilities improve with an increase in the neuron bank size. As we raise N from 10 to 100 neurons per layer, the average reduction in Normalized Mean Square Error (NMSE) is 1.418 times. In comparison to the baseline model, this figure is approximately 1.425 times, indicating that our encoding scheme maintains the ensemble's feature extraction capabilities. This observation confirms the neuron's ability to extract meaningful features while effectively filtering out noisy components.

IV. LIQUID STATE MACHINE BASED PREDICTOR MODEL FOR TIME-SERIES DATA

To further evaluate the noise-invariant properties of the SDS neural encoding within a practical machine learning (ML) framework, we construct a Liquid State Machine (LSM) based on the original Echo State Network (ESN) model outlined in [10]. The LSM model comprises three layers (see Fig. 5): an input layer responsible for mapping incoming time-series features to a collection of recurrently connected neurons constituting the *reservoir* layer and an output layer. Randomized weights are assigned to both the input and reservoir layers based on an initial seed matrix. Connections from the reservoir layer extend to the output layer, where weights are trained using the Moore-Penrose pseudo-inverse method and optimized to minimize the least squares fitting (LSQ) error. We employ a training methodology similar to that detailed in Section III for training the LSM model. In this approach, the output layer weights are updated using an input feature dataset characterized by a signal-to-noise ratio (SNR) value of ≥ 55 dB. Subsequently, during the testing phase, the network is subjected to various time-series features with SNR values

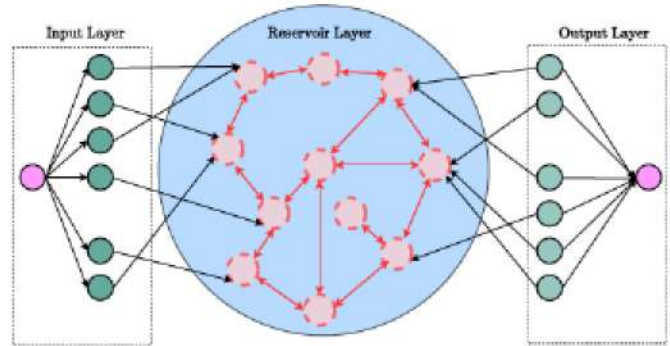


Fig. 5. Implemented LSM with predictive modeling for time series data based on $\Sigma\Delta$ (baseline) and $\Sigma\Delta\Sigma$ encoding.

ranging from 55 (ideal) to 1 dB. Through the evaluation of the LSM model incorporating the SDS neuron, we aim to scrutinize two fundamental aspects of the encoding process. Firstly, within the context of a randomly connected network, it is crucial to assess the effectiveness of the noise-filtering characteristics inherent in the proposed technique. Secondly, compared to the ensemble model, the LSM network embodies a more intricate feature abstraction model, necessitating a thorough examination of the performance enhancement facilitated by our technique in a prediction-based paradigm. In our simulations, we assess the model across different reservoir sizes: specifically 10, 20, and 50, and analyze the normalized error between the predicted output series and the ground truth (Figure 6a-c). The model is trained using the open-source Lava framework developed by Intel Labs. We individually train the network based on our proposed SDS neural encoding and benchmark it against the existing SD-based encoding schema. From our simulation of the LSM model, the following observations emerge across various reservoir sizes:

- **Noise robustness is more pronounced in smaller reservoirs:** When the reservoir size is small ($N=10$), the degradation in predictive performance for the baseline SD neuron is notably higher ($\geq 3.8\%$) compared to the SDS implementation, as the input feature SNR is reduced from 55 to 1 dB. This indicates that when the model cannot extract more features due to a limited network size, the

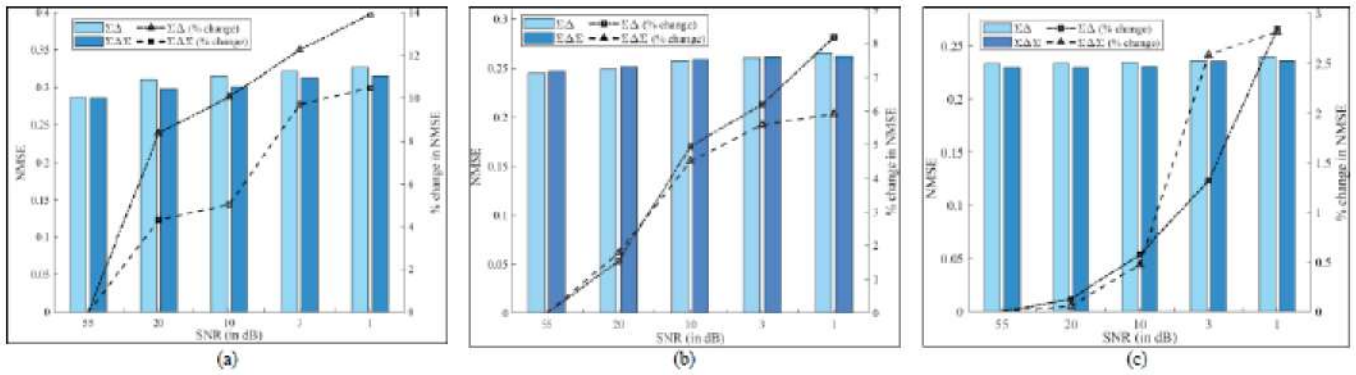


Fig. 6. Performance of the LSM model against SNR degradation for reservoir sizes of (a) 10, (b) 20, and (c) 50 neurons.

noise-filtering characteristic of the SDS neuron ensures improved reliability in terms of accuracy. Another critical aspect to highlight is the increased performance degradation when the incoming feature is slightly contaminated ($25 \leq \text{SNR} \leq 10$ dB). Figure 6a shows an average 1.86x improvement over the SD implementation when the incoming SNR is 20 dB. However, these benefits diminish as the noise power approaches the signal power, leading to significant corruption of the predicted output label.

- **Resilience to noise degradation has diminishing returns:** LSM models are renowned for their inherent noise robustness, attributed to the feature extraction capabilities of the reservoir layer. Hence, as we augment the reservoir size ($N=50$), the enhancement in reliability diminishes to a point where both models exhibit similar performance degradation as the SNR approaches 1 dB. This trend is evident in Fig. 6c, where the percentage change in NMSE follows a comparable trajectory for both encoding schemes. Moreover, the overall improvement in NMSE as reservoir sizes increase from 10 to 50 highlights a 1.18x enhancement in the network's predictive capabilities. This underscores the model's robustness, regardless of the encoding scheme employed.

V. CONCLUSION & FUTURE WORK

In this study, we introduced a novel spiking neural encoding scheme, $\Sigma\Delta\Sigma$ aimed at developing noise-resilient spiking neurons. Our current approach has shown a significant enhancement in the neuron's capability to encode and decode features, even amidst heavy noise corruption, particularly within the framework of an ensemble model. Our future research endeavors include assessing the performance of the $\Sigma\Delta\Sigma$ neuron in an edge machine learning (ML) application. Specifically, we plan to implement a continuous audio-based digit recognition model based on an LSM network. Previous reservoir models have demonstrated a Word Error Rate (WER) of less than or equal to 3% with fewer than 30,000 parameters [11], [12]. However, these implementations suffer from notable performance degradation (greater than or equal to 10%) when the incoming signal quality decreases by 15 dB. Our objective is to implement this model utilizing our SDS encoding technique, aiming to mitigate performance

degradation in the presence of external noise. To this end, our work also showcased an LSM-based predictive model that exhibited increased noise robustness for lower network sizes, even under heavily corrupted signal conditions. This ensured a higher level of reliability in terms of performance, a critical consideration for real-world applications.

VI. ACKNOWLEDGEMENT

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