Optimal Bayesian Persuasion for Containing SIS Epidemics

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***Abstract*— We consider a susceptible-infected-susceptible (SIS) epidemic model in which a large group of individuals decide whether to adopt partially effective protection without being aware of their individual infection status. Each individual receives a signal which conveys noisy information about its infection state, and then decides its action to maximize its expected utility computed using its posterior probability of being infected conditioned on the received signal. We first derive the static signal which minimizes the infection level at the stationary Nash equilibrium under suitable assumptions. We then formulate an optimal control problem to determine the optimal dynamic signal that minimizes the aggregate infection level along the solution trajectory. We compare the performance of the dynamic signaling scheme with the optimal static signaling scheme, and illustrate the advantage of the former through numerical simulations.**

***Index Terms*— Game theory, Agents-based systems, Decentralized control**

I. Introduction

Effective containment of spreading processes or epidemics is a challenging problem. Prior works have developed optimal control techniques for containing epidemics via non-pharmaceutical interventions (such as wearing masks or social distancing) [1], [2], allocation of limited medical resources including vaccines and testing kits [3], [4], and limiting mobility among individuals [5]. However, implementing the optimal control actions in the above settings is not straightforward since the suggested actions often require a large number of individuals to adhere to them, many of whom may not even be aware of their true infection status.

Past works have investigated the impacts of decentralized decision-making by a large group of selfish and strategic individuals on epidemic containment in the framework of game theory [6]–[9]. In most of these prior works, the agents were assumed to be aware of their true infection status. However, there are epidemics with asymptomatic infections and in some cases, similar symptoms are observed for multiple diseases; both the above characteristics were true for the recent COVID-19 pandemic. In addition, limited availability of testing kits made it difficult for individuals to learn about their true infection status in frequent intervals.

Consequently, individuals often relied on certain smartphone applications [10] to learn about their infection status. These applications collected information about several health indicators (such as pulse rate, body temperature, and sleep patterns) from the user, and whether the user was in proximity to other infected users to determine its risk of infection. However, the information conveyed by such applications is not free from error, and is potentially biased. Nevertheless, these applications provide a scalable manner in which individuals can be influenced to adopt protection measures and limit the spread of the epidemic. In this letter, we investigate how to design suitable signals to influence the behavior of strategic individuals towards the optimal containment of SIS epidemics in the framework of Bayesian persuasion [11], [12].

Although the Bayesian persuasion and information design frameworks have been applied in diverse settings, such as routing games [13], [14], resource allocation games [15], and modeling deception and privacy [16], there are only a few works that explore their application in the context of epidemics. Early attempts in this direction include [17] and [18], neither of which consider any specific compartmental epidemic model. In a recent work [19] by a subset of authors of this letter, the authors consider a game-theoretic setting where a large population of individuals adopt protection against the SIS epidemic with asymptomatic infections by relying on a noisy signal they receive from the sender. The authors characterize the stationary Nash equilibrium (SNE) of the game when the signaling scheme remains unchanged throughout the duration of the epidemic.

In this work, we build upon [19] and investigate the *optimal* signaling scheme to minimize the proportion of infected individuals. First we derive the optimal static signal that minimizes the proportion of infected individuals at the SNE under suitable assumptions. We then formulate an optimal control problem which aims to minimize the integral of the time-varying infected proportion subject to constraints that include the SIS epidemic dynamics as well as an evolutionary learning dynamics that capture the evolution of the actions by the individuals. We numerically illustrate the trajectories of infected proportion obtained under the optimal static and dynamic signaling schemes. The results show that the tra-

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jectory of infected proportion is smaller under the dynamic (time-varying) signaling scheme compared to the optimal static signaling scheme, though the difference reduces with time as the trajectories converge to the corresponding SNE.

II. SIS Epidemic under Bayesian Persuasion

In this section, we briefly review the setting and main result from [19]. We consider a large group of agents, each of whom is either *susceptible* ($\mathtt{S}$) or *infected* ($\mathtt{I}$) at a given point of time, but is unaware of its exact infection state. The proportion of infected individuals at time t is denoted by $y(t)$. Each agent receives a noisy signal x from the sender from the set $\{\bar{\mathtt{S}}, \bar{\mathtt{I}}\}$. The signaling scheme is governed by two parameters $\mu_{\mathtt{S}}, \mu_{\mathtt{I}} \in [0,1]$ with

$$\mu_{\mathtt{S}} := \mathbb{P}[x = \bar{\mathtt{S}} \mid \mathtt{S}], \quad \mu_{\mathtt{I}} := \mathbb{P}[x = \bar{\mathtt{I}} \mid \mathtt{I}].$$

The agents are aware of the signaling scheme, i.e., the values of $\mu_{\mathtt{S}}$ and $\mu_{\mathtt{I}}$. After receiving the signal, each agent computes its posterior probability of being infected as

$$\pi^+[\mathtt{I}|x] = \frac{\mathbb{P}[x|\mathtt{I}]y(t)}{\mathbb{P}[x|\mathtt{I}]y(t) + \mathbb{P}[x|\mathtt{S}](1-y(t))},$$

with $\pi^+[\mathtt{S}|x] = 1 - \pi^+[\mathtt{I}|x]$ and $y(t)$ acts as the prior.

Each individual chooses among two available actions: adopting protection and remaining unprotected, denoted by $a \in \{\mathtt{P}, \mathtt{U}\}$. Adopting protection comes with a cost $C_{\mathtt{P}} > 0$ while an infected individual that remains unprotected incurs cost $C_{\mathtt{U}} > 0$. An infected agent that adopts protection (or remains unprotected) spreads infection with probability $\beta_{\mathtt{P}}$ (respectively, $\beta_{\mathtt{U}}$), with $0 < \beta_{\mathtt{P}} < \beta_{\mathtt{U}} < 1$. A susceptible agent, upon adopting protection, reduces its likelihood of becoming infected by a factor $\alpha \in (0,1)$. Further, $\gamma \in (0,1)$ represents the recovery rate from the disease. Let $z_{\bar{\mathtt{S}}}(t)$ (respectively, $z_{\bar{\mathtt{I}}}(t)$) denote the proportion of agents who remain unprotected among the agents that receive signal $\bar{\mathtt{S}}$ (respectively, $\bar{\mathtt{I}}$) at time t. Consequently, the proportion of infected agents evolves as

$$\begin{aligned}\dot{y} &= ((1-y)[\beta_{\mathtt{P}} + (\beta_{\mathtt{U}} - \beta_{\mathtt{P}})(z_{\bar{\mathtt{I}}}\mu_{\mathtt{I}} + z_{\bar{\mathtt{S}}}(1-\mu_{\mathtt{I}}))] \\ &\quad \times [\alpha + (1-\alpha)(z_{\bar{\mathtt{S}}}\mu_{\mathtt{S}} + z_{\bar{\mathtt{I}}}(1-\mu_{\mathtt{S}}))] - \gamma)y \qquad (1) \\ &=: ((1-y)\beta_{\mathtt{eff}}(z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}}) - \gamma)y,\end{aligned}$$

where the dependence on t is omitted for better readability.

Analogous to the classical SIS epidemic model [20], the above dynamics has two equilibrium points: a *disease-free equilibrium*, $y_{\mathtt{DFE}} = 0$ which always exist; and an *endemic equilibrium*, $y_{\mathtt{EE}}(z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}}) := 1 - \frac{\gamma}{\beta_{\mathtt{eff}}(z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}})}$ which exists only when $\beta_{\mathtt{eff}}(z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}}) > \gamma$, and is strictly positive. We now impose the following assumptions.

Assumption 1: The signal revealed to infected agents is truthful, i.e., $\mu_{\mathtt{I}} = 1$; $C_{\mathtt{P}} < C_{\mathtt{U}}$, which incentivizes infected agents to adopt protection; and the recovery rate satisfies $\gamma < \alpha\beta_{\mathtt{P}}$.

The assumption $\mu_{\mathtt{I}} = 1$ is motivated by the fact that during an actual pandemic, the signaling scheme will err on the side of being safe at the expense of a larger false alarm rate. Since $0 < \beta_{\mathtt{P}} < \beta_{\mathtt{U}} < 1, \alpha \in (0,1)$, and $(\mu_{\mathtt{S}}, \mu_{\mathtt{I}}, z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}}) \in [0,1]^4$, it follows from (1) that $\alpha\beta_{\mathtt{P}} \leq \beta_{\mathtt{eff}}(z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}})$. Thus, the assumption $\gamma < \alpha\beta_{\mathtt{P}}$ guarantees the existence of the endemic equilibrium $y_{\mathtt{EE}}$ with a nonzero infected proportion.[1]

The expected utility (with respect to posterior π^+) received by an agent upon receiving signal $x \in \{\bar{\mathtt{S}}, \bar{\mathtt{I}}\}$ and choosing action $a \in \{\mathtt{P}, \mathtt{U}\}$ is denoted by $U[x,a]$, and the dependence of $U[x,a]$ on the tuple $(y, z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}})$ is suppressed for better readability. Following [19, Section III], the difference in the expected utilities for each x is given by

$$\begin{aligned}\Delta U[\bar{\mathtt{S}}] &= (1-\alpha)L[\beta_{\mathtt{P}} + (\beta_{\mathtt{U}} - \beta_{\mathtt{P}})z_{\bar{\mathtt{I}}}]y - C_{\mathtt{P}}, \\ \Delta U[\bar{\mathtt{I}}] &= \pi^+[\mathtt{S}|\bar{\mathtt{I}}]((1-\alpha)L[\beta_{\mathtt{P}} + (\beta_{\mathtt{U}} - \beta_{\mathtt{P}})z_{\bar{\mathtt{I}}}]y - C_{\mathtt{U}}) \\ &\qquad + C_{\mathtt{U}} - C_{\mathtt{P}}, \qquad (2)\end{aligned}$$

where $L > 0$ is the loss incurred by an agent upon infection.[2]

Since the utility of an agent depends on the strategies chosen by all other agents $(z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}})$, we study their interaction within a game-theoretic framework. A tuple $(y^\star, z_{\bar{\mathtt{S}}}^\star, z_{\bar{\mathtt{I}}}^\star)$ is said to be a stationary Nash equilibrium (SNE) when (i) $y^\star$ is the unique nonzero endemic equilibrium of (1) with $z_{\bar{\mathtt{I}}} = z_{\bar{\mathtt{I}}}^\star$ and $z_{\bar{\mathtt{S}}} = z_{\bar{\mathtt{S}}}^\star$, and (ii) the following mixed complementarity conditions are satisfied:

$$\Delta U[\bar{\mathtt{S}}] \perp (0 \leq z_{\bar{\mathtt{S}}}^\star \leq 1), \quad \Delta U[\bar{\mathtt{I}}] \perp (0 \leq z_{\bar{\mathtt{I}}}^\star \leq 1).$$

A more elaborate discussion on the SNE is provided in [19]. Before stating the main result on the characterization of SNE, we state the following notation from [19]:

$$\begin{aligned}y_{\mathtt{EE}}(z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}}) &:= 1 - \frac{\gamma}{\beta_{\mathtt{eff}}(z_{\bar{\mathtt{S}}}, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}})}, \qquad (3) \\ h(z_{\bar{\mathtt{I}}}, \mu_{\mathtt{S}}) &:= (1-\alpha)L(\beta_{\mathtt{P}} + (\beta_{\mathtt{U}} - \beta_{\mathtt{P}})z_{\bar{\mathtt{I}}})y_{\mathtt{EE}}(1, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}}) \\ &\quad - C_{\mathtt{P}}, \\ g(z_{\bar{\mathtt{I}}}, \mu_{\mathtt{S}}) &:= y_{\mathtt{EE}}(1, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}})(C_{\mathtt{U}} - C_{\mathtt{P}}) \\ &\quad - (1-\mu_{\mathtt{S}})(1 - y_{\mathtt{EE}}(1, z_{\bar{\mathtt{I}}}; \mu_{\mathtt{S}}))(-h(z_{\bar{\mathtt{I}}}, \mu_{\mathtt{S}})). \qquad (4)\end{aligned}$$

The quantity $\mu_{\mathtt{S}}^{\max}$ is defined in [19, Equation (11)] as:

$$\mu_{\mathtt{S}}^{\max} := \begin{cases} 0, & \text{if} \quad g(0,0) \geq 0, \\ \mu_{\mathtt{S}}^\star, & \text{where } g(0, \mu_{\mathtt{S}}^\star) = 0, \mu_{\mathtt{S}}^\star \in (0,1). \end{cases} \qquad (5)$$

Finally, we define

$$\begin{aligned}y_{\mathtt{P}}^* &:= 1 - \frac{\gamma}{\alpha\beta_{\mathtt{P}}}, \quad y_{\mathtt{INT}}^* := \frac{C_{\mathtt{P}}}{L(1-\alpha)\beta_{\mathtt{P}}}, \\ z_{\bar{\mathtt{S}}}^\dagger &:= \frac{\gamma - \alpha\beta_{\mathtt{P}}(1 - y_{\mathtt{INT}}^*)}{\beta_{\mathtt{P}}(1-\alpha)(1 - y_{\mathtt{INT}}^*)\mu_{\mathtt{S}}}, \\ \mu_{\mathtt{S}}^{\min} &:= 1 - \frac{(\beta_{\mathtt{U}} - \gamma)(C_{\mathtt{U}} - C_{\mathtt{P}})}{\gamma(C_{\mathtt{P}} - (1-\alpha)L(\beta_{\mathtt{U}} - \gamma))}.\end{aligned}$$

With the above notations in hand, the characterization of the SNE $(y^\star, z_{\bar{\mathtt{S}}}^\star, z_{\bar{\mathtt{I}}}^\star)$ as established in [19] is stated below.

Theorem 1: ([19, Theorem 1]) Under Assumption 1, we have the following characterization of the SNE:

1) $(y_{\mathtt{P}}^*, 0, 0)$ is the SNE if and only if $y_{\mathtt{P}}^* > y_{\mathtt{INT}}^*$.

[1] Since the infected proportion is zero at the disease-free equilibrium, our main objective is to minimize the nonzero infected proportion at the endemic equilibrium.

[2] A summary of important notations introduced above is provided in Table I of the extended version of this work [21].

2) $(y^*_{\mathtt{INT}}, z^\dagger_{\mathtt{S}}, 0)$ is the SNE if and only if $z^\dagger_{\mathtt{S}} \in (0,1)$ or equivalently,

$$1 - \frac{\gamma}{\alpha\beta_{\mathrm{P}}} < y^*_{\mathtt{INT}} < 1 - \frac{\gamma}{\beta_{\mathrm{P}}(\alpha + (1-\alpha)\mu_{\mathrm{S}})}.$$

3) $(y_{\mathrm{EE}}(1,0;\mu_{\mathrm{S}}),1,0)$ is the SNE if and only if

$$\mu_{\mathrm{S}} \in [\mu_{\mathrm{S}}^{\max}, 1], \quad 1 - \frac{\gamma}{\beta_{\mathrm{P}}(\alpha + (1-\alpha)\mu_{\mathrm{S}})} \le y^*_{\mathtt{INT}}.$$

4) $(y_{\mathrm{EE}}(1, z^\dagger_{\mathrm{I}}; \mu_{\mathrm{S}}), 1, z^\dagger_{\mathrm{I}})$ with $z^\dagger_{\mathrm{I}} \in (0,1)$ being the unique value satisfying $g(z^\dagger_{\mathrm{I}}, \mu_{\mathrm{S}}) = 0$ is the SNE if and only if $\mu_{\mathrm{S}} < \mu_{\mathrm{S}}^{\max}$, and either of the following two conditions are satisfied:

$$L(1-\alpha)(\beta_{\mathrm{U}} - \gamma) < C_{\mathrm{P}} \quad \text{with} \quad \mu_{\mathrm{S}} > \mu_{\mathrm{S}}^{\min}, \quad \text{or}$$
$$L(1-\alpha)(\beta_{\mathrm{U}} - \gamma) > C_{\mathrm{P}}.$$

5) $(1 - \frac{\gamma}{\beta_{\mathrm{U}}}, 1, 1)$ is the SNE if and only if $L(1-\alpha)(\beta_{\mathrm{U}} - \gamma) < C_{\mathrm{P}}$, and $\mu_{\mathrm{S}} < \mu_{\mathrm{S}}^{\min}$.

The above result characterizes the SNE for a given set of game parameters and a given value of μ_{S}.

III. Optimal Static Signal

In this section, we consider the case where the principal or the sender chooses the signaling scheme in order to minimize the infected proportion at the SNE. The optimal static signal μ_{S} that achieves the smallest proportion of infected individuals at the SNE is denoted by $\mu^\star_{\mathtt{S(stat)}}$. Our analysis holds under the following assumption.

Assumption 2: The protection cost C_{P} satisfies $C_{\mathrm{P}} > (1-\alpha)L(\beta_{\mathrm{U}} - \gamma)$.

Remark 1: Note that Assumption 2 implies

$$C_{\mathrm{P}} > (1-\alpha)L(\beta_{\mathrm{U}} - \gamma) \implies \frac{C_{\mathrm{P}}}{(1-\alpha)L\beta_{\mathrm{U}}} > 1 - \frac{\gamma}{\beta_{\mathrm{U}}}$$
$$\implies y^*_{\mathtt{INT}} := \frac{C_{\mathrm{P}}}{(1-\alpha)L\beta_{\mathrm{P}}} > \frac{\beta_{\mathrm{U}}}{\beta_{\mathrm{P}}} - \frac{\gamma}{\beta_{\mathrm{P}}} > 1 - \frac{\gamma}{\alpha\beta_{\mathrm{P}}} =: y^*_{\mathrm{P}},$$

where the last inequality follows since $\beta_{\mathrm{U}} > \beta_{\mathrm{P}}$ and $\alpha \in (0,1)$. Consequently, the SNE $(y^*_{\mathrm{P}}, 0, 0)$ (Case 1 of Theorem 1) does not exist under Assumption 2. Now, observe that a necessary condition for existence of the SNE $(y^*_{\mathtt{INT}}, z^\dagger_{\mathtt{S}}, 0)$ is

$$\frac{C_{\mathrm{P}}}{(1-\alpha)L\beta_{\mathrm{P}}} < 1 - \frac{\gamma}{\beta_{\mathrm{P}}(\alpha + (1-\alpha)\mu_{\mathrm{S}})} < 1 - \frac{\gamma}{\beta_{\mathrm{U}}},$$

which is not possible under Assumption 2. Nevertheless, at these two SNEs, the proportions of infected agents are y^*_{P} and $y^*_{\mathtt{INT}}$, both of which do not depend upon μ_{S}, and hence there is no notion of an optimal signal. More importantly, the existence of the remaining three SNEs, whose infected proportion depends on μ_{S} is not affected by Assumption 2.

We start with the following lemma.

Lemma 1: Suppose Assumptions 1 and 2 hold. Let $\mu_{\mathrm{S}}^{\max} > 0$ and $\mu_{\mathrm{S}}^{\min} < \mu_{\mathrm{S}}^{\max}$. Then, the endemic infection level $y_{\mathrm{EE}}(1, z^\dagger_{\mathrm{I}}; \mu_{\mathrm{S}})$ is strictly decreasing in μ_{S} over the range $\mu_{\mathrm{S}} \in (\mu_{\mathrm{S}}^{\min}, \mu_{\mathrm{S}}^{\max})$.

Proof: It follows from Theorem 1 that when $\mu_{\mathrm{S}}^{\max} > 0$ and $\mu_{\mathrm{S}} \in (\mu_{\mathrm{S}}^{\min}, \mu_{\mathrm{S}}^{\max})$, the SNE is given by $(y_{\mathrm{EE}}(1, z^\dagger_{\mathrm{I}}; \mu_{\mathrm{S}}), 1, z^\dagger_{\mathrm{I}})$. In addition, $C_{\mathrm{P}} > (1-\alpha)L(\beta_{\mathrm{U}} - \gamma)$ implies $\mu_{\mathrm{S}}^{\min} < 1$. Further, it can be shown that $\mu_{\mathrm{S}}^{\max} \in (0,1)$. The infected proportion at the SNE is given by

$$y_{\mathrm{EE}}(1, z^\dagger_{\mathrm{I}}; \mu_{\mathrm{S}}) = 1 - \frac{\gamma}{(\beta_{\mathrm{P}} + (\beta_{\mathrm{U}} - \beta_{\mathrm{P}})z^\dagger_{\mathrm{I}})(\alpha + (1-\alpha)(z^\dagger_{\mathrm{I}} + (1 - z^\dagger_{\mathrm{I}})\mu_{\mathrm{S}}))}, \tag{6}$$

where $z^\dagger_{\mathrm{I}} \in (0,1)$ is the unique value satisfying $g(z^\dagger_{\mathrm{I}}, \mu_{\mathrm{S}}) = 0$. As μ_{S} varies in the range $(\mu_{\mathrm{S}}^{\min}, \mu_{\mathrm{S}}^{\max})$, $z^\dagger_{\mathrm{I}} \in (0,1)$ varies while preserving $g(z^\dagger_{\mathrm{I}}, \mu_{\mathrm{S}}) = 0$. In the rest of the proof, we denote $z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}})$ to indicate that $z^\dagger_{\mathrm{I}}$ is a function of μ_{S}.

With a slight abuse of notation, we use $y_{\mathrm{EE}}(\mu_{\mathrm{S}})$ to denote the endemic infection level $y_{\mathrm{EE}}(1, z^\dagger_{\mathrm{I}}; \mu_{\mathrm{S}})$ at the SNE. It should be noted that $y_{\mathrm{EE}}(\mu_{\mathrm{S}})$ is a function of only μ_{S}, since $z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}})$ itself is a function of μ_{S}. We now introduce the following functions:

$$\begin{aligned}
\beta_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}}) &:= \beta_{\mathrm{P}} + (\beta_{\mathrm{U}} - \beta_{\mathrm{P}})z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}}), \\
\alpha_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}}) &:= \alpha + (1-\alpha)(z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}}) + (1 - z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}}))\mu_{\mathrm{S}}), \\
L_{\mathrm{eq}}(\mu_{\mathrm{S}}) &:= (1-\alpha)L(\beta_{\mathrm{U}} - \beta_{\mathrm{P}})(1 - \mu_{\mathrm{S}})(1 - y_{\mathrm{EE}}(\mu_{\mathrm{S}})), \\
w(\mu_{\mathrm{S}}) &:= \beta_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}}) \cdot \alpha_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}}).
\end{aligned}$$

Note that when $\alpha \in (0,1)$, $\mu_{\mathrm{S}} \in (0,1)$ and $0 < \beta_{\mathrm{P}} < \beta_{\mathrm{U}} < 1$, $w(\mu_{\mathrm{S}})$ is a non-zero, continuous and differentiable function of μ_{S}. Therefore, $y_{\mathrm{EE}}(\mu_{\mathrm{S}}) = 1 - \frac{\gamma}{w(\mu_{\mathrm{S}})}$ is also continuous and differentiable in μ_{S}. We now introduce the operator $\nabla_a(b)$ to denote the derivative of b w.r.t a, i.e., $\nabla_a(b) := \frac{\mathrm{d}}{\mathrm{d}a}(b)$. By applying the operator $\nabla_{\mu_{\mathrm{S}}}(\cdot)$ on both sides of the above identity, we obtain $\nabla_{\mu_{\mathrm{S}}}(y_{\mathrm{EE}}) = \frac{\gamma}{(w(\mu_{\mathrm{S}}))^2}\nabla_{\mu_{\mathrm{S}}}(w)$. Thus, when $w(\mu_{\mathrm{S}})$ is strictly decreasing in μ_{S}, $y_{\mathrm{EE}}(\mu_{\mathrm{S}})$ is also strictly decreasing in μ_{S}.

By substituting $h(z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}}), \mu_{\mathrm{S}})$ in $g(z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}}), \mu_{\mathrm{S}})$, and equating $g(z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}}), \mu_{\mathrm{S}}) = 0$, we obtain

$$z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}}) = \frac{C_{\mathrm{P}} w(\mu_{\mathrm{S}})}{L(w(\mu_{\mathrm{S}}) - \gamma)(1-\alpha)(\beta_{\mathrm{U}} - \beta_{\mathrm{P}})} - \frac{C_{\mathrm{U}} - C_{\mathrm{P}}}{L_{\mathrm{eq}}(\mu_{\mathrm{S}})} - \frac{\beta_{\mathrm{P}}}{\beta_{\mathrm{U}} - \beta_{\mathrm{P}}}. \tag{7}$$

The R.H.S. of the above equation is well-defined, and continuous in μ_{S} over the specified range. Differentiating the above equation, we obtain $\nabla_{\mu_{\mathrm{S}}}(z^\dagger_{\mathrm{I}})$ as

$$-\gamma\nabla_{\mu_{\mathrm{S}}}(w(\mu_{\mathrm{S}}))\Big(\frac{C_{\mathrm{P}}}{L(w(\mu_{\mathrm{S}}) - \gamma)^2(1-\alpha)(\beta_{\mathrm{U}} - \beta_{\mathrm{P}})} + \frac{C_{\mathrm{U}} - C_{\mathrm{P}}}{\gamma w(\mu_{\mathrm{S}}) L_{\mathrm{eq}}(\mu_{\mathrm{S}})}\Big) - \frac{C_{\mathrm{U}} - C_{\mathrm{P}}}{(1 - \mu_{\mathrm{S}}) L_{\mathrm{eq}}(\mu_{\mathrm{S}})}. \tag{8}$$

Detailed derivation of both the above equations is provided in the extended version [21].

Now, differentiating $w(\mu_{\mathrm{S}}) = \beta_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}}) \cdot \alpha_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}})$, we obtain

$$\nabla_{\mu_{\mathrm{S}}}(w) = \nabla_{\mu_{\mathrm{S}}}(z^\dagger_{\mathrm{I}})[\beta_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}})(1-\alpha)(1-\mu_{\mathrm{S}}) + \alpha_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}})(\beta_{\mathrm{U}} - \beta_{\mathrm{P}})] + \beta_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}})(1-\alpha)(1 - z^\dagger_{\mathrm{I}}(\mu_{\mathrm{S}})).$$

Substituting $\nabla_{\mu_{\mathrm{S}}}(z^\dagger_{\mathrm{I}})$ from (8) into the above equation yields

$$\nabla_{\mu_{\mathrm{S}}}(w) = \Big[\beta_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}})(1-\alpha)(1-\mu_{\mathrm{S}}) + \alpha_{z^\dagger_{\mathrm{I}}}(\mu_{\mathrm{S}})(\beta_{\mathrm{U}} - \beta_{\mathrm{P}})\Big]$$

$$\times\Big[-\gamma\nabla_{\mu_S}(w)\Big(\frac{C_P}{L(w(\mu_S)-\gamma)^2(1-\alpha)(\beta_U-\beta_P)} + \frac{C_U-C_P}{\gamma w(\mu_S)L_{eq}(\mu_S)}\Big) - \frac{C_U-C_P}{(1-\mu_S)L_{eq}(\mu_S)}\Big] + \beta_{z_{\bar{I}}^\dagger}(\mu_S)(1-\alpha)(1-z_{\bar{I}}^\dagger(\mu_S)).$$

By rearranging the terms, we can write $\nabla_{\mu_S}(w) = \frac{\mathcal{N}}{\mathcal{D}}$ where

$$\begin{aligned}\mathcal{N} =& -\Big[\beta_{z_{\bar{I}}^\dagger}(\mu_S)(1-\alpha)(1-\mu_S) \\ &+ \alpha_{z_{\bar{I}}^\dagger}(\mu_S)(\beta_U-\beta_P)\Big]\frac{C_U-C_P}{(1-\mu_S)L_{eq}(\mu_S)} \\ &+ \beta_{z_{\bar{I}}^\dagger}(\mu_S)(1-\alpha)(1-z_{\bar{I}}^\dagger(\mu_S)), \\ \mathcal{D} =& 1 + \Big[\beta_{z_{\bar{I}}^\dagger}(\mu_S)(1-\alpha)(1-\mu_S) + \alpha_{z_{\bar{I}}^\dagger}(\mu_S)(\beta_U-\beta_P)\Big] \\ &\times\gamma\Big(\frac{C_P}{L(w(\mu_S)-\gamma)^2(1-\alpha)(\beta_U-\beta_P)} \\ &+ \frac{C_U-C_P}{\gamma w(\mu_S)L_{eq}(\mu_S)}\Big). \end{aligned} \tag{9}$$

Note that the denominator (9) is a positive quantity. Therefore, for $w(\mu_S)$ to be a strictly decreasing function of μ_S, we must have $\mathcal{N} < 0$, or equivalently

$$\begin{aligned}&\beta_{z_{\bar{I}}^\dagger}(\mu_S)(1-\alpha)\Big(1 - z_{\bar{I}}^\dagger(\mu_S) - \frac{C_U-C_P}{L_{eq}(\mu_S)}\Big) \\ &< \alpha_{z_{\bar{I}}^\dagger}(\mu_S)(\beta_U-\beta_P)\frac{C_U-C_P}{(1-\mu_S)L_{eq}(\mu_S)}. \end{aligned}\tag{10}$$

We now show that (10) is satisfied under Assumption 2. Since, $C_P > (1-\alpha)L(\beta_U-\gamma)$ holds and $y_{EE}(\mu_S) < 1-\frac{\gamma}{\beta_U}$, the following holds true:

$$\begin{aligned}&\frac{C_P}{(1-\alpha)Ly_{EE}(\mu_S)} > \beta_U \\ \implies& \frac{C_P w(\mu_S)}{(1-\alpha)L(w(\mu_S)-\gamma)} > \beta_U \\ \implies& \frac{\beta_U}{\beta_U-\beta_P} - \frac{C_P w(\mu_S)}{(1-\alpha)L(w(\mu_S)-\gamma)(\beta_U-\beta_P)} < 0 \\ \implies& 1 + \frac{\beta_P}{\beta_U-\beta_P} - \frac{C_P w(\mu_S)}{(1-\alpha)L(w(\mu_S)-\gamma)(\beta_U-\beta_P)} \\ &+ \frac{C_U-C_P}{L_{eq}(\mu_S)} < \frac{C_U-C_P}{L_{eq}(\mu_S)}. \end{aligned}\tag{11}$$

On careful examination of (7) and (11), we find that the L.H.S. of (11) is equal to $1 - z_{\bar{I}}^\dagger(\mu_S)$, which implies $1 - z_{\bar{I}}^\dagger(\mu_S) < \frac{C_U-C_P}{L_{eq}(\mu_S)}$. Substituting this inequality into (10), we obtain the L.H.S. of (10) is a negative quantity, whereas, the R.H.S. is a positive quantity.

As a result, $\nabla_{\mu_S}(w) < 0$, and consequently, $\nabla_{\mu_S}(y_{EE}) < 0$. Thus, the endemic infection level $y_{EE}(\mu_S)$ is strictly decreasing in μ_S within the interval $\mu_S \in (\mu_S^{\min}, \mu_S^{\max})$. ■

Now we state our main result which reveals the optimal static persuasion signal.

Theorem 2: Suppose Assumptions 1 and 2 hold. Then, the optimal static signal which leads to the smallest proportion of infected individuals at the SNE is $\mu^\star_{\mathtt{S(stat)}} = \mu_S^{\max}$.

Proof: It follows from Remark 1 that the SNEs described in the first two cases of Theorem 1 do not exist under Assumption 2. We now focus on the remaining equilibria.

First, we observe that $\mu_S < \mu_S^{\min}$ cannot be the optimal signal, since it results in the SNE $(1-\frac{\gamma}{\beta_U}, 1, 1)$, which has the highest infection level compared to other SNEs; note that the entire population remains unprotected in this SNE.

It follows from Lemma 1 that when μ_S increases from $\mu_S^{\min}$ till $\mu_S^{\max}$, the infected fraction $y_{EE}(1, z_{\bar{I}}^\dagger; \mu_S)$ is monotonically decreasing in μ_S. Thus, the range $\mu_S \in (\mu_S^{\min}, \mu_S^{\max})$ does not contain the optimal signal.

Now, at the SNE $(y_{EE}(1,0;\mu_S), 1, 0)$, the infection level

$$y_{EE}(1,0;\mu_S) = 1 - \frac{\gamma}{\beta_P(\alpha + (1-\alpha)\mu_S)}$$

is strictly increasing in $\mu_S \in [\mu_S^{\max}, 1]$. In other words, the infected proportion at the SNE is monotonically decreasing in μ_S when $\mu_S \in (\mu_S^{\min}, \mu_S^{\max})$ and monotonically increasing when $\mu_S \in [\mu_S^{\max}, 1]$. Finally, leveraging the monotonicity and continuity of $g(z_{\bar{I}}, \mu_S)$ in both of its arguments, it can be shown[3] that as μ_S approaches $\mu_S^{\max}$ from below, $z_{\bar{I}}^\dagger(\mu_S)$ approaches 0. Consequently, the infected proportion at the SNE is continuous at $\mu_S^{\max}$.

The above analysis for $\mu_S \in [\mu_S^{\max}, 1]$ also subsumes the corner case where $\mu_S^{\max} = 0$, under which the SNE is $(y_{EE}(1,0;\mu_S), 1, 0)$ for the entire range of $\mu_S \in [0,1]$. In this case, $(y_{EE}(1,0;\mu_S), 1, 0)$ is monotonically increasing in μ_S for $\mu_S \in [0,1]$. Thus, $\mu_S^{\max}$ is the optimal static signal. ■

The above result can be interpreted as the Stackelberg equilibrium where the principal or the leader aims to minimize the infected proportion, and the agents act as followers who respond to the signaling scheme of the leader by playing the SNE strategy profile. In Section V, we numerically show that when Assumption 2 is violated, then $\mu_S^{\max}$ is not necessarily the optimal signal.

IV. Optimal Dynamic Signal

We now introduce the *dynamic signaling scheme* where the sender is allowed to vary or modulate the persuasive signal over time depending on the states. To this end, we formulate a finite-horizon optimal control problem where the goal is to minimize the integral of the infected proportion $y(t)$ subject to the epidemic dynamics (1) and evolutionary learning dynamics adopted by the individuals to update their strategies $z_{\bar{S}}, z_{\bar{I}}$. Specifically, we adopt the Smith dynamics [22] to capture the evolution of the strategies since the stationary points of the Smith dynamics corresponds to the Nash equilibrium of the game, and vice versa [23].

However, the Smith dynamics is not smooth due to the presence of a max operator. Consequently, we relax this operator with a soft-max function with parameter $\sigma \in [0,\infty)$; when σ increases, the relaxed dynamics closely approximates the Smith dynamics. We now formally state the finite-time horizon optimal control problem as

$$\begin{aligned}\min_{\mu_S} \int_0^T y(t)\,dt& \\ \text{s.t. } \dot{y} = ((1-y)&[\beta_P + (\beta_U-\beta_P)z_{\bar{I}}] \\ &\times[\alpha + (1-\alpha)(z_{\bar{S}}\mu_S + z_{\bar{I}}(1-\mu_S))] - \gamma)y,\end{aligned}$$

[3]Please refer to the extended version [21] for a more detailed proof.

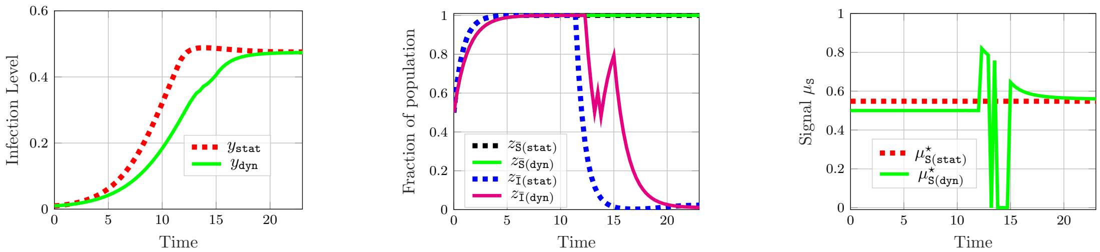

Fig. 1. Evolution of proportion of infected individuals (left), the proportions of individuals who remain unprotected (middle), and the signal $\boldsymbol{\mu}_{\mathrm{S}}$ (right) under both the static (dashed) and dynamic (solid) signaling schemes.

$$
\begin{aligned}
\dot{z}_{\bar{\mathrm{S}}} &= \frac{1 - z_{\bar{\mathrm{S}}}}{1 + e^{(\sigma \Delta U[\bar{\mathrm{S}}])}} - \frac{z_{\bar{\mathrm{S}}}}{1 + e^{(-\sigma \Delta U[\bar{\mathrm{S}}])}}, \\
\dot{z}_{\bar{\mathrm{I}}} &= \frac{1 - z_{\bar{\mathrm{I}}}}{1 + e^{(\sigma \Delta U[\bar{\mathrm{I}}])}} - \frac{z_{\bar{\mathrm{I}}}}{1 + e^{(-\sigma \Delta U[\bar{\mathrm{I}}])}}, \\
0 &\le \mu_{\mathrm{S}} \le 1, \qquad (12)
\end{aligned}
$$

where $e^{(\cdot)}$ is the exponential function and $\Delta U[\bar{\mathrm{S}}]$ and $\Delta U[\bar{\mathrm{I}}]$ are defined in (2). We denote the optimal dynamic signal by $\mu^{\star}_{\mathtt{S(dyn)}}(t)$, and omit t when required for better readability.

V. Numerical Results

We now compare the state trajectory under the static and dynamic signaling schemes via numerical simulations. The parameters whose values remain unaltered throughout this section are given in the following table.

α	γ	L	$y(0)$	$z_{\bar{\mathrm{S}}}(0)$	$z_{\bar{\mathrm{I}}}(0)$	σ
0.45	0.2	80	0.01	0.5	0.5	20

We choose the value of L to be large enough to capture the adverse health and financial impacts of infection. For $\sigma = 20$, dynamics of $z_{\bar{\mathrm{S}}}$ and $z_{\bar{\mathrm{I}}}$ closely approximate the Smith dynamics. In [24], the authors showed that an ideal surgical mask reduces COVID-19 infection risk by approximately 70%. In order to model partial effectiveness, we choose $\alpha = 0.45$ which corresponds to 55% reduction.

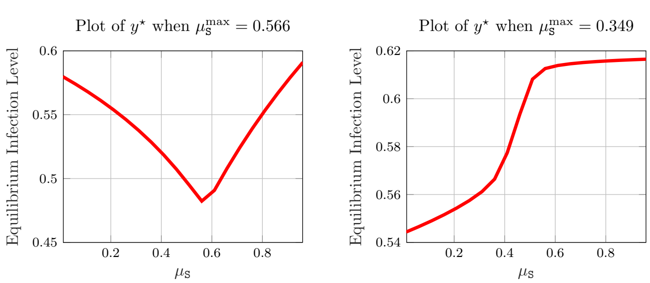

Fig. 2. Infected proportion at the SNE with respect to $\boldsymbol{\mu}_{\mathrm{S}}$ when Assumption 2 is satisfied (left); and not satisfied (right).

1) Static Signaling: Here, we vary μ_{S} from $\mu_{\mathrm{S}} = 0.01$ to $\mu_{\mathrm{S}} = 0.96$, and at each value of μ_{S}, we obtain the infected proportion at the SNE. First, we choose the following parameter values: $\beta_{\mathrm{U}} = 0.65$, $\beta_{\mathrm{P}} = 0.5$, $C_{\mathrm{P}} = 25$, $C_{\mathrm{U}} = 32$, which satisfy Assumption 2. For these parameters, we have $\mu_{\mathrm{S}}^{\min} = -2.028$ and $\mu_{\mathrm{S}}^{\max} = 0.566$. The left panel of Figure 2 shows the infection level at the SNE with respect to μ_{S} under the above parameters. When $\mu_{\mathrm{S}} \in [0.01, 0.566)$, the SNE is given by $(y_{\mathrm{EE}}(1, z^{\dagger}_{\bar{\mathrm{I}}}; \mu_{\mathrm{S}}), 1, z^{\dagger}_{\bar{\mathrm{I}}})$ following Theorem 1, and the equilibrium infection level decreases as μ_{S} increases from 0.01 to 0.566. As μ_{S} grows beyond $\mu_{\mathrm{S}} = \mu_{\mathrm{S}}^{\max} = 0.566$, the SNE has the form $(y_{\mathrm{EE}}(1, 0; \mu_{\mathrm{S}}), 1, 0)$, with $y^\star$ strictly increasing in μ_{S}. Thus, under Assumption 2, the infected proportion at the SNE is smallest when $\mu_{\mathrm{S}} = \mu_{\mathrm{S}}^{\max}$.

We now consider parameter values which violate Assumption 2. We choose $\beta_{\mathrm{U}} = 0.9$, $\beta_{\mathrm{P}} = 0.7$, $C_{\mathrm{P}} = 19$, $C_{\mathrm{U}} = 20$ under which $\mu_{\mathrm{S}}^{\max} = 0.349$. The right panel of Figure 2 shows that the infected proportion at the SNE is not minimized at $\mu_{\mathrm{S}} = \mu_{\mathrm{S}}^{\max}$; rather increasing with μ_{S}.

2) Dynamic Signaling: We now illustrate the effectiveness of the optimal dynamic signaling scheme compared to the optimal static signaling scheme. We set the time horizon $T = 23$ and parameters $\beta_{\mathrm{U}} = 0.65$, $\beta_{\mathrm{P}} = 0.5$, $C_{\mathrm{P}} = 20$, and $C_{\mathrm{U}} = 25$ in accordance with Assumption 2. For the above parameters, the reproduction number is 2.5 under protection and 3.25 when protection is not adopted. These values fall within the empirically estimated range of the reproduction number for COVID-19 [25, Figure 1].

For the above set of parameters, we obtain $\mu^{\star}_{\mathtt{S(stat)}} = \mu_{\mathrm{S}}^{\max} = 0.548$, and determine the corresponding SNE following Theorem 1. We then solve the optimal control problem (12) via the numerical solver Quasi-Interpolation based Trajectory Optimization (QuITO) [26] which uses a direct multiple shooting (DMS) technique. We plot $y(t), z_{\bar{\mathrm{S}}}(t), z_{\bar{\mathrm{I}}}(t)$, and the optimal control input under both static and dynamic signaling in Figure 1. The dashed (respectively, solid) lines represent the quantities corresponding to the static (respectively, dynamic) signaling scheme. The rightmost plot in Figure 1 shows the optimal signal chosen by the sender under both schemes; the optimal dynamic signal converges to the optimal static signal $\mu_{\mathrm{S}}^{\max}$ towards the end of the simulation. The middle panel of Figure 1 represents the evolution of strategies $z_{\bar{\mathrm{S}}}$ and $z_{\bar{\mathrm{I}}}$, whereas the left panel plot shows the trajectory of the infected proportion under both schemes. As expected, the infected proportion under the dynamic signaling scheme is smaller than the infected proportion under the static signaling scheme throughout the simulation; indeed the former aims to minimize the infection level along the entire trajectory while the latter is only concerned with the SNE.

Remark 2: While we discuss numerical solutions of (12) in this section, the nonlinearity in the dynamics of $y, z_{\bar{\mathrm{S}}}$, and $z_{\bar{\mathrm{I}}}$ (particularly in the denominators in the dynamics of $z_{\bar{\mathrm{S}}}$ and $z_{\bar{\mathrm{I}}}$), renders the analysis of the Lagrangian function extremely challenging. Thus, deriving closed-form analytical solutions of the optimal dynamic signal is beyond the scope of this letter, and remains a challenging open problem.

3) Relaxing the Assumption $\mu_{\mathtt{I}} = 1$: We now examine the consequence of relaxing the assumption $\mu_{\mathtt{I}} = 1$. We discretize both $\mu_{\mathtt{S}}$ and $\mu_{\mathtt{I}}$ in steps of 0.005 over the interval $[0.01, 1]$. For each $\mu_{\mathtt{I}}$, we compute the infected proportions at the SNE over all possible values of $\mu_{\mathtt{S}}$, denoted by $y_{\mathtt{EE}}(\mu_{\mathtt{S}}, \mu_{\mathtt{I}})$ with a slight abuse of notation. On the left panel of Figure 3, we plot $\mu_{\mathtt{S}}^{\mathtt{opt}}(\mu_{\mathtt{I}}) := \arg\min_{\mu_{\mathtt{S}}} y_{\mathtt{EE}}(\mu_{\mathtt{S}}, \mu_{\mathtt{I}})$, i.e., the value of $\mu_{\mathtt{S}}$ that results in the smallest infected proportion at the SNE with respect to $\mu_{\mathtt{I}}$. The corresponding infected proportion $(\min_{\mu_{\mathtt{S}}} y_{\mathtt{EE}}(\mu_{\mathtt{S}}, \mu_{\mathtt{I}}))$ is shown on the right panel for three combinations of $\beta_{\mathtt{P}}$ and $C_{\mathtt{P}}$ satisfying Assumptions 1 and 2. All other parameter values are kept identical to those in Section V.1 that satisfy Assumption 2.

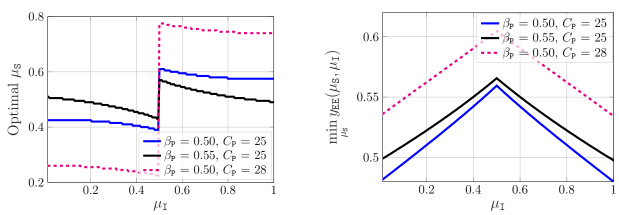

Fig. 3. Variation of $\boldsymbol{\mu}_{\mathtt{S}}^{\mathtt{opt}}(\boldsymbol{\mu}_{\mathtt{I}})$ (left), and the infected proportion at the SNE under $(\boldsymbol{\mu}_{\mathtt{S}}^{\mathtt{opt}}(\boldsymbol{\mu}_{\mathtt{I}}), \boldsymbol{\mu}_{\mathtt{I}})$ (right) with respect to $\boldsymbol{\mu}_{\mathtt{I}}$.

As expected, the optimal $\mu_{\mathtt{S}}$ at $\mu_{\mathtt{I}} = 1$ coincides with the corresponding $\mu_{\mathtt{S}}^{\max}$. Now observe that as $\mu_{\mathtt{I}}$ decreases below 1, $\mu_{\mathtt{S}}^{\mathtt{opt}}(\mu_{\mathtt{I}})$ does not see significant change compared to $\mu_{\mathtt{S}}^{\max}$. Thus, the optimal signal $\mu_{\mathtt{S}}$ is somewhat robust with respect to $\mu_{\mathtt{I}}$. However, the magnitude of the minimum infection level $(\min_{\mu_{\mathtt{S}}} y_{\mathtt{EE}}(\mu_{\mathtt{S}}, \mu_{\mathtt{I}}))$ increases as $\mu_{\mathtt{I}}$ decreases from 1 to 0.5. Furthermore, the plots exhibit symmetry around $\mu_{\mathtt{I}} = 0.5$ because the posterior probabilities satisfy

$$\pi^{+}[\mathtt{s} \mid \bar{\mathtt{S}}](\mu_{\mathtt{S}}, \mu_{\mathtt{I}}) = \pi^{+}[\mathtt{s} \mid \bar{\mathtt{I}}](1 - \mu_{\mathtt{S}}, 1 - \mu_{\mathtt{I}}),$$

for $\mathtt{s} \in \{\mathtt{S}, \mathtt{I}\}$. Finally, we note that the smallest value of $y_{\mathtt{EE}}(\mu_{\mathtt{S}}^{\mathtt{opt}}(\mu_{\mathtt{I}}), \mu_{\mathtt{I}})$ is obtained at $\mu_{\mathtt{I}} = 1$. Establishing the optimality of $\mu_{\mathtt{I}} = 1$ remains an open problem.

VI. Conclusions

In this work, we investigated SIS epidemic containment via Bayesian persuasion of a large population of agents who strategically adopt a partially effective protection measure. We first derived the optimal static signal which minimizes the infected proportion at the stationary Nash equilibrium. We then formulated a finite-horizon optimal control problem to determine the optimal dynamic signaling scheme to minimize the infected proportion along the solution trajectory. Simulation results show that the dynamic signaling scheme is more effective in containing the infection prevalence over the entire trajectory. This work can be extended along several directions, including to networked epidemic models, and to settings where the parameters of the epidemic dynamics and cost functions of the players are not known to the sender.

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