Competitive Equilibrium for Chores: from Dual Eisenberg-Gale to a Fast, Greedy, LP-based Algorithm

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We study the computation of competitive equilibrium for Fisher markets with n agents and m divisible chores. Prior work showed that competitive equilibria correspond to the nonzero KKT points of the Nash welfare *minimization* program, which is a non-convex analogue of the Eisenberg-Gale convex program. We introduce an analogue of the Eisenberg-Gale *dual* for chores: we show that all KKT points of this dual correspond to competitive equilibria, and while it is not a dual of the non-convex primal program in a formal sense, the objectives touch at all KKT points. Similar to the primal program, the dual has problems from an optimization perspective: there are many feasible directions where the objective tends to positive infinity, and these attract iterative optimization methods. We then derive a new constraint for the dual, which restricts optimization to a hyperplane that avoids all these directions. We show that restriction to this hyperplane retains all KKT points, and surprisingly, does not introduce any new ones. This allows, for the first time ever, application of iterative optimization methods over a convex region for computing competitive equilibria for chores.

We next introduce a greedy Frank-Wolfe algorithm for optimization over our program and show a state-of-the-art convergence rate to competitive equilibrium. In the case of equal incomes, we show a $\tilde{O}(n/\epsilon^2)$ rate of convergence, which improves over the two prior state-of-the-art rates of $\tilde{O}(n^3/\epsilon^2)$ for an exterior-point method and $\tilde{O}(nm/\epsilon^2)$ for a combinatorial method. Moreover, our method is significantly simpler: each iteration of our method only requires solving a simple linear program. We show through numerical experiments on simulated data and a paper review bidding dataset that our method is extremely practical: it easily solves every instance we tried, usually in 5-20 iterations, is simple to implement, and has no numerical issues. This is the first highly practical method for solving competitive equilibrium for Fisher markets with chores.

The full version can be found at https://arxiv.org/abs/2402.10439.