Improving Approximation Guarantees for Maximin Share

HANNANEH AKRAMI, Max Planck Institute for Informatics, Graduate school of Saarland University, Germany
JUGAL GARG, University of Illinois at Urbana-Champaign, USA
EKLAVYA SHARMA, University of Illinois at Urbana-Champaign, USA
SETAREH TAKI, Grubhub, USA

We consider fair division of a set of indivisible goods among n agents with additive valuations using the fairness notion of maximin share (MMS). MMS is the most popular share-based notion, in which an agent finds an allocation fair to her if she receives goods worth at least her (1-out-of-n) MMS value. An allocation is called MMS if all agents receive their MMS values. However, since MMS allocations do not always exist [Kurokawa et al., JACM'18], the focus shifted to investigating its ordinal and multiplicative approximations.

In the ordinal approximation, the goal is to show the existence of 1-out-of-d MMS allocations (for the smallest possible $d > n$). A series of works led to the state-of-the-art factor of $d = \lfloor 3n/2 \rfloor$ [Hosseini et al., JAIR'21]. We show that 1-out-of-$4\lceil n/3 \rceil$ MMS allocations always exist, thereby improving the state-of-the-art of ordinal approximation. In the multiplicative approximation, the goal is to show the existence of α-MMS allocations (for the largest possible $\alpha < 1$), which guarantees each agent at least α times her MMS value. A series of works in the last decade led to the state-of-the-art factor of $\alpha = \frac{3}{4} + \frac{3}{3836}$ [Akrami and Garg, SODA'24].

We introduce a general framework of *approximate MMS with agent priority ranking*. We order the agents, and agents earlier in the order are considered more *important*. An allocation is said to be T-MMS, for a non-increasing sequence $T := (\tau_1, \ldots, \tau_n)$ of numbers, if the agent at rank i in the order gets a bundle of value at least τ_i times her MMS value. This framework captures both ordinal approximation and multiplicative approximation as special cases. We show the existence of T-MMS allocations where $\tau_i \geq \max(\frac{3}{4} + \frac{1}{12n}, \frac{2n}{2n+i-1})$ for all i. Furthermore, by ordering the agents randomly, we can get allocations that are $(\frac{3}{4} + \frac{1}{12n})$-MMS ex-post and $(0.8253 + \frac{1}{36n})$-MMS ex-ante. We also investigate the limitations of our algorithm and show that it does not give better than $(0.8631 + \frac{1}{2n})$-MMS ex-ante.

The full version of this paper is available at https://arxiv.org/abs/2307.12916v2.

CCS Concepts: • **Theory of computation → Algorithmic game theory**.

Additional Key Words and Phrases: Fair division, Maximin share, Ordinal maximin share

ACM Reference Format:
Hannaneh Akrami, Jugal Garg, Eklavya Sharma, and Setareh Taki. 2024. Improving Approximation Guarantees for Maximin Share. In *The 25th ACM Conference on Economics and Computation (EC '24), July 8–11, 2024, New Haven, CT, USA.* ACM, New York, NY, USA, 1 page. https://doi.org/10.1145/3670865.3673544

Acknowledgments

Jugal Garg and Eklavya Sharma were supported by NSF Grants CCF-1942321 and CCF-2334461.

Authors' Contact Information: Hannaneh Akrami, hakrami@mpi-inf.mpg.de, Max Planck Institute for Informatics, Graduate school of Saarland University, Saarbrücken, Germany; Jugal Garg, jugal@illinois.edu, University of Illinois at Urbana-Champaign, Urbana, USA; Eklavya Sharma, eklavya2@illinois.edu, University of Illinois at Urbana-Champaign, Urbana, USA; Setareh Taki, Staki@grubhub.com, Grubhub, Chicago, USA.

EC '24, July 8–11, 2024, New Haven, CT, USA

ACM ISBN 979-8-4007-0704-9/24/07
https://doi.org/10.1145/3670865.3673544