Laplacian-ORBGRAND: Decoding for Impulsive Noise

Jiewei Feng
Dept. of Elec. & Comput. Engineering
Northeastern University
Boston, USA
feng.ji@northeastern.edu

Ken R. Duffy
Dept. of ECE & Dept. of Mathematics
Northeastern University
Boston, USA
k.duffy@northeastern.edu

Muriel Médard
Research Laboratory for Electronics
Massachusetts Institute of Technology
Cambridge, USA
medard@mit.edu

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***Abstract*—While many communication systems experience extraneous noise that is well-modelled as Gaussian, experimental studies have shown that large values are more common when noise is impulsive and the Laplace distribution has been proposed as a more appropriate statistical model in that setting. Guessing Random Additive Noise Decoding is a class of forward error correction decoders that can avail of channel knowledge to improve decoding. Here we introduce a GRAND decoder that is specifically tailored to impulsive noise, which we call Laplace Ordered Reliability Bits GRAND (LORBGRAND). By adapting GRAND to the characteristics of Laplace noise we find an improvement of the order of ∼1dB in block error rate, highlighting the benefits of noise-specific decoding strategies. Additionally, we extend the algorithm to provide soft output to indicate the probability estimation of correct decoding, which can be used to identify unreliable decoded signals.**

***Index Terms*—GRAND, soft decoding, Laplacian noise, universal decoding**

I. Introduction

In many communication scenarios, additive noise is typically modeled using a Gaussian distribution. However, some channels experience impulsive noise which is better to be described by a distribution with a heavier tail in its probability density function (pdf) resulting in more frequent outliers [1]–[3], which may confound forward error correction decoding. The Laplace distribution is commonly employed to model impulsive noise in these contexts, including ultra-wideband and multi-user interference environments [4]–[12]. Fig. 1 illustrates the difference of the two noise distributions. With considerable interest in modeling impulsive noise using the Laplace distribution, there has been a significant amount of related research analyzing channel performance [13]–[15] and proposing detectors [4]–[8], [16]–[18].

Soft input decoding relies on reliability information about the received signal and its performance can be degraded by greater number of outliers caused by impulsive noise. Guessing Random Additive Noise Decoding (GRAND) and its variants [19]–[26] are particularly well-suited to incorporate knowledge of the channel. Here we investigate how GRAND decoding can be tailored to Laplacian noise. Soft detection

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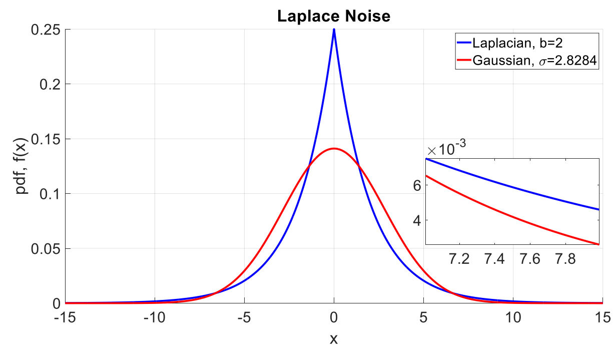

Fig. 1. Gaussian distribution and Laplace distribution with the same variance and zero mean. Laplacian pdf has higher value near the mean and a heavier tail compared to Gaussian as shown in the large plot and the zoom-in.

variants of GRAND provide near Maximum Likelihood decoder and are computationally efficient for any moderate redundancy code of any structure [27], [28]. GRAND algorithms can be highly parallelized, making them efficient when implemented in circuits [29]–[34]. The main idea behind GRAND algorithms is to sequentially generate putative binary noise effect sequences in decreasing order of likelihood based on statistical models or soft input, invert them from the received demodulated bits and check if what remains is in the codebook. The first instance that is found is the decoding.

Ordered Reliability Bits Guessing Random Additive Noise Decoding (ORBGRAND) [21] is a variant of GRAND that uses soft input, in the form of reliabilities of received bits, to determine its query order, resulting in higher accuracy of decoding. Its algorithm, by design, is suitable for implementation in hardware. It has been employed to assess the applicability of both conventional and non-traditional code structures for ultra reliable low latency communication [27], [35].

With the consideration of impulsive noise, we introduce a variant of ORBGRAND called Laplacian-ORBGRAND (LORBGRAND) as a soft input decoder specifically suited to Laplacian Noise. This variant demonstrates the suitability of GRAND algorithms to tackle the challenges posed by specific types of noise in communication systems. Additionally, it has recently been established that soft input GRAND algorithms can readily provide soft output in the form of an accurate estimate of the posterior probability of correct

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decoding [36]. This desirable feature can be used to inform decoding failure or inform iterative decoding of long, low-rate codes [37]. This approach is versatile and can be applied with any soft input GRAND algorithm. Computation of the soft output only requires knowledge of the code's dimensions and the accumulation of probabilities for each binary noise effect query during GRAND decoding. Consequently, this calculation does not add to algorithmic complexity or memory requirements. We validate the precision of the soft output with LORBGRAND, demonstrating its suitability for use in this new noise environment.

The remainder of this paper is organized as follows. Section II provides an overview of the model setup, detailing the assumptions and formulations for decoding in Laplacian noise channels. Section III presents the decoding algorithm, explaining the implementation of LORBGRAND. Section IV discusses the simulation results including performance comparison of LORBGRAND and ORBGRAND in various CRC codes, the comparison of LORBGRAND and CRC-Aided Successive Cancellation List (CA-SCL) decoding in CRC-Aided Polar (CA-Polar) code, and the justification of soft output algorithm paired with LORBGRAND. Section VI concludes the paper with a discussion.

II. Model Setup

Let $X^n = (X_1, ..., X_n) \in \{0,1\}^n$ be a binary codeword chosen uniformly at random from a codebook $\mathcal{C} \subset \{0,1\}^n$ of 2^k codewords. A codeword X^n is sent through a channel using binary modulation and the detector receives signal $R^n = (R_1, ..., R_n) = 1 - 2X^n + Z^n$ where $Z^n = (Z_1, ..., Z_n)$ and $\{Z_i\}$ are i.i.d real value noise effects.

Given an assumption on the distribution of noise, let $f_{R|X}$ be the pdf of R conditioned on X. The log-likelihood ratio (LLR) of a received signal R_i is defined to be

$$\text{LLR}(R_i) = \log \frac{f_{R|X}(R_i|1)}{f_{R|X}(R_i|0)}, \tag{1}$$

which is can be calculated using the pdf of the noise distribution. With (1), we use $\Gamma(R_i) = |\text{LLR}(R_i)|$ to denote the reliability of R_i. Let $Y_i = (\text{sign}(\text{LLR}(R_i)) + 1)/2$, we use $Y^n = (Y_1, Y_2, ..., Y_n)$ to denote signal demodulated from R^n.

In many systems, noise is well modeled as Gaussian, in which case (1) leads to the well-known formulae

$$\text{LLR}_N(R_i) = \log \frac{e^{-(R_i+1)^2/(2\sigma^2)}}{e^{-(R_i-1)^2/(2\sigma^2)}} = \frac{-2R_i}{\sigma^2}, \tag{2}$$

$$\Gamma_N(R_i) = \frac{2|R_i|}{\sigma^2}. \tag{3}$$

The Laplace distribution, however, has a heavier tail and hence is widely used to model impulsive noise. Since i.i.d. noise is a special case of white noise, we will call the channel an Additive White Laplacian Noise (AWLN) channel. LLRs in an AWLN differ from those in (2). Using the definition in (1) and the pdf for Laplace distribution we obtain

$$\text{LLR}_L(R_i) = \log \frac{\frac{1}{2b} e^{-\frac{|R_i+1|}{b}}}{\frac{1}{2b} e^{-\frac{|R_i-1|}{b}}} = \frac{|R_i - 1| - |R_i + 1|}{b}, \tag{4}$$

$$\Gamma_L(R_i) = \frac{2I_{(|R_i|>1)} + |2R_i|I_{(|R_i|\leq 1)}}{b}, \tag{5}$$

where $I_{(\cdot)}$ is the indicator function. By (5), $\Gamma_L(R_i)$ achieves its maximum $2/b$ when $|R_i| \geq 1$. The demodulated likelihood of the hard decision bit Y_i is in error can be represented as

$$B_i = P(Y_i \neq X_i | R_i) = \frac{e^{-\Gamma(R_i)}}{1 + e^{-\Gamma(R_i)}},$$

from which we can evaluate the likelihood of a binary noise effect sequence z^n

$$P(Z^n = z^n | R^n) = \prod_{i=1}^{n} (1 - B_i) \prod_{i:z_i=1} \frac{B_i}{1 - B_i} \tag{6}$$

$$\propto \prod_{i:z_i=1} \frac{B_i}{1 - B_i} = \exp\left(-\sum_{i=1}^{n} \Gamma(R_i) z_i\right). \tag{7}$$

Equation (7) says that, regardless of the distribution of noise, it suffices to find the rank order of the values $\sum_{i=1}^{n} \Gamma(R_i) z_i$ to rank order binary noise effect patterns by their likelihood. This idea is one of the essential foundations of ORBGRAND, which uses a statistical model and resulting solution of an integer partition problem to create an efficient algorithm to generate noise patterns from approximate highest to lowest probability. When considering AWLN, $\Gamma_L(R_i)$ replaces $\Gamma_N(R_i)$ in (7) and a new pattern generation algorithm needs to be developed.

Suppose $\Lambda_1, \Lambda_2, .., \Lambda_n$ are i.i.d random variables having the same distribution, $P(\Lambda_i \leq u) = F(u)$, with $F^{-1}(s) = \inf\{u : F(u) \geq s\}$ denoting the inverse of the cumulative distribution function (cdf). Let $\Lambda_{(1)}, \Lambda_{(2)}, ..., \Lambda_{(n)}$ be the corresponding order statistics such that $\Lambda_{(i)} \geq \Lambda_{(j)}$ for all $i > j$. The Law of Large Numbers can be used to show the following, for which a sketch prove will be provided later: for $\alpha \in [0,1]$, the α^{th} sample quantile $\Lambda_{(\lfloor \alpha n \rfloor)} \to F^{-1}(\alpha)$ in probability with converge rate n^2 [38]. If we fixed a number of quantiles $\alpha_1, \alpha_2, ..., \alpha_k$ and use the fact that F^{-1} is monotonic, the convergence result says that the plot of order statistics will be well-approximated by the function F^{-1}. Therefore, by substituting Λ_i with $\Gamma(R_i)$ using either (3) or (5), depending on the assumption of noise, the plot of rank ordered reliability versus the rank will be similar to the inverse of $P(\Gamma(R_i) \leq u)$ for a random signal R^n. A sketch proof is as follows: for any $\epsilon > 0$, let $I_{(\cdot)}$ be the indicator function,

$$\begin{aligned}
&P\left(\Lambda_{(\lfloor \alpha n \rfloor)} \geq F^{-1}(\alpha) + \epsilon\right) \\
&= P\left(\sum_{j=1}^{n} I_{(\Lambda_j \geq F^{-1}(\alpha)+\epsilon)} \geq n - \lfloor \alpha n \rfloor + 1\right) \\
&= 1 - P\left(\frac{1}{n}\sum_{j=1}^{n} I_{(\Lambda_j \geq F^{-1}(\alpha)+\epsilon)} < \frac{n+1}{n} - \frac{\lfloor \alpha n \rfloor}{n}\right).
\end{aligned}$$

By the law of large numbers,

$$\frac{1}{n}\sum_{j=1}^{n} I_{(\Lambda_j \geq F^{-1}(\alpha)+\epsilon)} \to 1 - P(\Lambda < F^{-1}(\alpha)+\epsilon)$$

in probability and $1 - P(\Lambda < F^{-1}(\alpha)+\epsilon) \leq 1-\alpha < (n+1)/(n) - (\lfloor \alpha n \rfloor)/(n)$. Therefore, $P(\Lambda_{(\lfloor \alpha n \rfloor)} \geq F^{-1}(\alpha)+\epsilon) \to 0$ as $n \to \infty$. Similarly, $P(\Lambda_{(\lfloor \alpha n \rfloor)} \leq F^{-1}(\alpha)-\epsilon) \to 0$. As a result, $\Lambda_{(\lfloor \alpha n \rfloor)} \to F^{-1}(\alpha)$ in probability for any given $\alpha \in [0,1]$. Such convergence has rate n^2 by the proof of strong law of large numbers assuming the indicator variable has finite fourth moment, which it has as the indicator variable is a Bernoulli variable. Therefore we would expect that when a random signal R^n is given, the plot of rank ordered reliability versus the rank will be similar to the inverse of cdf of Γ_L.

Simulations shown in Fig. 2 contrast rank-ordered likelihoods of reliabilities in AWGN and AWLN channels at different SNRs. While they are approximately linear in an AWGN, in an AWLN they are piecewise linear and go flat after a given index.

To understand this, as X_i is independent and has equal probability of being 0 or 1, then $P(\Gamma_L(R_i) < 2/b) = P(|R_i| < 1 | X_i = 0) = (1-e^{-2/b})/2$. This implies that when given n bits, we would expect around $(1-e^{-2/b})n/2$ bits to have reliability less than the maximum, and therefore we would anticipate that the plot of rank ordered reliability will go flat after approximately the first $(1-e^{-2/b})n/2$ bits. Fig. 2 also verifies the relation between maximum value of reliability, the number of bits that achieves maximum and the value of b. For example, for the case where $b = 0.315$, the maximum value of reliability is $2/b \approx 6.34$ and $(1-e^{2/b})/2 \approx 0.5$ which means half of the bits have maximum reliability. When rank ordering reliabilities, we call the lowest rank such that the corresponding reliability reaches maximum as the change point and denote it by $\theta = \sum_{i=1}^{n} I_{\{\Gamma_L(R_i)<2/b\}} + 1$. Fig. 3 also provides a demonstration that the expected value of change point is near $n/2$ within proper range of the SNR and its corresponding range of b.

III. Decoding Algorithm

It is proposed by [21] that if the rank ordered reliability plot has simple structure, it is possible to design an algorithm to generate possible noise sequences from the highest probability to the lowest with efficiency and low complexity.

For notational simplicity, we shall assume that the reliabilities, $\{\Gamma_L(R_i) : i \in \{1,2,...,n\}\}$ happen to be received in increasing order of bit position, so that $\Gamma_L(R_i) \leq \Gamma_L(R_j)$ for $i \leq j$. In practice, for each received block we sort the reliabilities and store the permutation, $\pi^n = (\pi_1, ..., \pi_n)$, such that π_i records the received order index of the i^{th} least reliable bit. The permutation π^n enables us to map all considerations back to the original order that the bits were received in.

As it has been shown in Fig. 2, the rank ordered reliability can be approximated for some θ, c, β by

$$\Gamma_L(R_i) \approx \begin{cases} \beta(i+c) & \text{if } i \leq \theta \\ \beta(\theta+c) & \text{if } i > \theta \end{cases}. \quad (8)$$

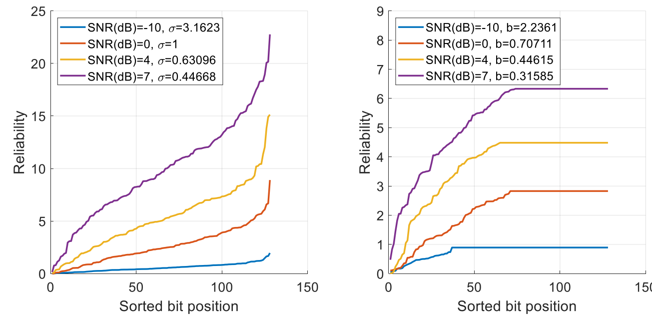

Fig. 2. Both plots are for $n = 128$ at different SNR. The left plots rank ordered reliability for signals from a AWGN channel, where the reliability is calculated by (3). The right plot uses (5) with signal from AWLN. In each plot, the parameter of the noise distribution corresponding to the value of SNR is also provided. In an AWGN channel, the plot can be approximated by a straight line which is one of the motivation of ORBGRAND [21]. In contrast, when considering AWLN, the plot must be approximated by a piece-wise linear function that goes flat beyond a given index.

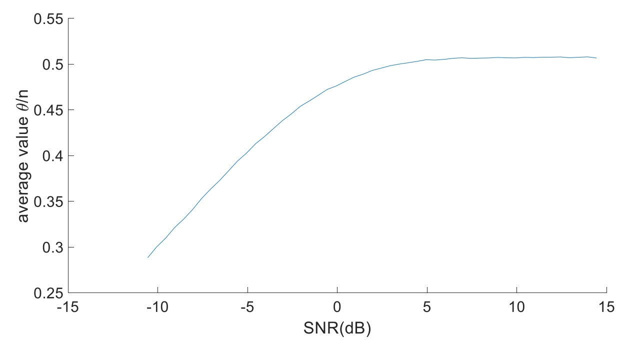

Fig. 3. The graph plots the average value of change point divided by n versus different values of SNR. This plot shows that for SNR in the range of $[4, +\infty)$, the average value of change point is around half of n, which coincides with the fact that $P(\Gamma_L(R_i) < 2/b) = (1-e^{-2/b})/2$.

This enables us to generate noise sequences z^n with probability from highest to lowest.

For a noise sequence z^n, we define its Total Weight to be

$$W^*(z^n) = \sum_{i:z_i=1} \Gamma_L(R_i)/\beta. \quad (9)$$

For any z^n, define its Logistic Weight to be $W_L(z^n) = \sum_{i=1}^{n} i z_i$ and its Hamming Weight to be $W_H(z^n) = \sum_{i=1}^{n} z_i$. By (7) and (9), generating the noise with highest probability to lowest is the same as generating the noise with lowest Total Weight to highest. Equation (8) leads to the following relationship: For any $z^n = (z_1, z_2, ..., z_n)$, the first part $z_l^{\theta-1} = (z_1, z_2, ..., z_{\theta-1})$ contributes to the Total Weight as the sum of Logistic Weight and Hamming Weight $W_L(z_l^{\theta-1}) + cW_H(z_l^{\theta-1})$ while the second part $z_r^{n-\theta+1} = (z_\theta, ..., z_n)$ contributes to the Total Weight as Hamming Weight $(\theta+c)W_H(z_r^{n-\theta+1})$ which says that $W^*(z^n)$ can be approximated by $W_L(z_l^{\theta-1}) + cW_H(z_l^{\theta-1}) + (\theta+c)W_H(z_r^{n-\theta+1})$. Note that the zero sequence 0^n is always the first error sequence to be tested since it has zero Total Weight.

We now provide our Laplacian noise soft decoder LORBGRAND in Algorithm 1. In the algorithm, noise sequences with increasing Total Weight w^* are generated until the first codeword is identified. For each value of w^*, line 3 to line 8 in the algorithm generates the set $\mathcal{S}(w^*) = \{z^n : W^*(z^n) = w^*\}$ where each element in the set will be tested to see if the recovered sequence is in the code-book.

Algorithm 1 LORBGRAND.

Inputs: $\Gamma_L(r^n)$ as soft information for received signal $r^n = (r_1, r_2, ..., r_n)$. Demodulated bits y^n based on r^n. Code-book membership function $C : \{0,1\}^n \to \{0,1\}$ with 1 if and only if the decoded signal is in the code book.

Output: decoded signal $c^{n,*}$

1: $\theta, c \leftarrow$ calculate parameters to approximate ordered reliability based on (8) and received reliabilities $\Gamma_L(r^n)$.
2: **for** $w^* = 1, 2, ...$ **do**
3: $\quad \mathcal{S}(w^*) \leftarrow \emptyset$.
4: $\quad$ **for** each $j = 0, 1, ..., \lfloor w^*/(\theta + c) \rfloor$ **do**
5: $\quad\quad \mathcal{S}_r(j) \leftarrow \{z_r^{\theta-n+1} : W_H(z_r^{\theta-n+1}) = j\}$.
6: $\quad\quad \mathcal{S}_l(w^*, j) \leftarrow \{z_l^{\theta-1} : W_L(z_l^{\theta-1}) + cW_H(z_l^{\theta-1}) = w^* - j(\theta + c)\}$
7: $\quad\quad \mathcal{S}(w^*) \leftarrow \mathcal{S}(w^*) \cup \{z_l \bigoplus z_r : z_l \in \mathcal{S}_l(w^*, j), z_r \in \mathcal{S}_r(j)\}$.
8: $\quad$ **end for**
9: $\quad$ **for** each $z^n \in \mathcal{S}(w^*)$ **do**
10: $\quad\quad$ **if** $C(|y^n - z^n|) = 1$ **then**
11: $\quad\quad\quad$ **return** $c^{n,*} = |y^n - z^n|$
12: $\quad\quad$ **end if**
13: $\quad$ **end for**
14: **end for**

A demonstration of how the noise sequences with given Total Weight $W^* = w^*$ are generated is presented: for $w^* = 7$ under the situation where the parameters $n = 5$, $\theta = 4$, $c = 0$, the goal is to generate the set $\mathcal{S}(7) = \{z^n : W^*(z^n) = 7\}$. Since $\theta = 4, n = 5$, each noise sequence is split into two parts, $z_l^3 = (z_1, z_2, z_3)$ and $z_r^2 = (z_4, z_5)$. For each different j, $\mathcal{S}_r(j)$ contains the candidates of $z_r^2 = (z_4, z_5)$ while $\mathcal{S}_l(7, j)$ contains the candidate of z_l^3. Since $\lfloor w^*/(\theta + c) \rfloor = 1$, only the cases for $j = 0, 1$ needs to be considered. In the first iteration where $j = 0$, the only possible z_r^2 that has $W_H(z_r^2) = 0$ is $(0, 0)$ and therefore $\mathcal{S}_r(0) = \{(0, 0)\}$. However, $\mathcal{S}_l(7, 0) = \emptyset$ because there is no z_l^3 such that $W_L(z_l^3) + cW_H(z_l^3) = 7$. Therefore in the first iteration, $\mathcal{S}(7) = \emptyset$. After that, the next iteration with $j = 1$ is performed, two sequences $z_r^2 = (1, 0)$ or $(0, 1)$ are found to have Hamming Weight $W_H(z_r^2) = 1$ and therefore $\mathcal{S}_r(1) = \{(1, 0), (0, 1)\}$. Then all possible sequences z_l^3 such that $W_L(z_l^3) + 0 \cdot W_H(z_l^3) = 7 - 4 = 3$ are generated, which results in $\mathcal{S}_l(7, 1) = \{(0, 0, 1), (1, 1, 0)\}$. Each element in $\mathcal{S}_l(7, 1)$ and $\mathcal{S}_r(1)$ are then appended to each other to produce the set $\mathcal{S}_l(7, 1) \bigotimes \mathcal{S}_r(1) = \{(0, 0, 1, 1, 0), (1, 1, 0, 1, 0), (0, 0, 1, 0, 1), (1, 1, 0, 0, 1)\}$. The elements in the set $\mathcal{S}_l(7, 1) \bigotimes \mathcal{S}_r(1)$ are then added to $\mathcal{S}(7) = \emptyset$, which updates the latter set to be $\mathcal{S}(7) = \{(0, 0, 1, 1, 0), (1, 1, 0, 1, 0), (0, 0, 1, 0, 1), (1, 1, 0, 0, 1)\}$. As $j = 1$ is the last iteration for $w^* = 7$, the set $\mathcal{S}(7)$ is finalized and will be used for membership testing in line 9 through 13.

LORBGRAND generates all possible noise patterns iteratively with increasing value of Total Weight, i.e., noise patterns are generated from highest likelihood to lowest on the fly. The query order is illustrated in Fig. 4.

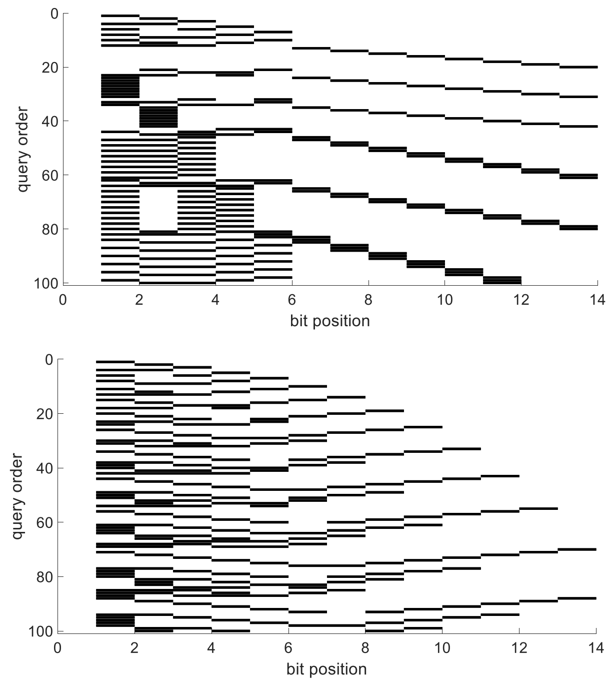

Fig. 4. First 100 noise queries are shown for LORBGRAND in upper figure and for ORBGRAND1 in lower figure. In both cases we let $n = 13$. Each row is a noise sequence with white being no bit flip and black corresponding to a bit flip. The change point θ in the simulation of LORBGRAND is 6. One of the main differences between the two algorithms is that ORBGRAND1 gradually includes less reliable bits, while LOBRGRAND quickly considers all latter bits as they are equally unreliable.

IV. Performance Evaluation

If not otherwise specified, solid lines indicate that the receiver and the decoder assume AWLN, and thus LLR and reliability are calculated using (4) and (5). In contrast, dashed lines are used when the receiver and decoder assume AWGN, and thus (2) and (3) are used, even though the true noise distribution is Laplacian.

Error Rate Performance. We first present simulation results for decoders LORBGRAND and ORBGRAND1 [21] in CRC codes. While CRC code are ubiquitously used for error detection, GRAND decoders are capable of using CRC codes for error correction and they have been established to be good codes [27]. The results of the ORBGRAND1 (ORB) decoder are represented by a red line, while those of the LORBGRAND (LORB) decoder are represented by a blue line. While ORB provides near optimal decoding in AWGN, the resulting performance improvement in having a matched Laplace-specific decoder in LORB is of the order of ∼1dB as can be seen in Fig. 5 and 6.

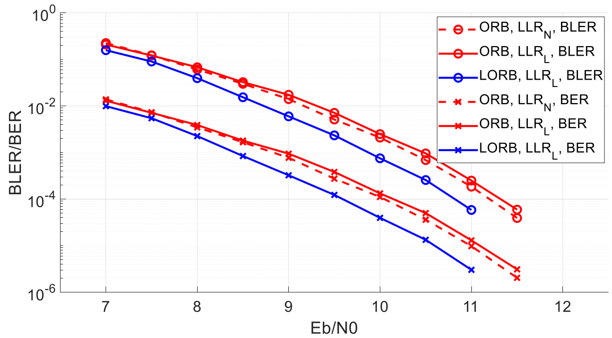

Fig. 5. AWLN channel employing a $[128, 110]$ CRC code. Circles mark BLER while crosses mark BER. As depicted in the figure, LORBGRAND shows an improvement of ~ 0.5 dB in Eb/N0 compared to ORBGRAND1.

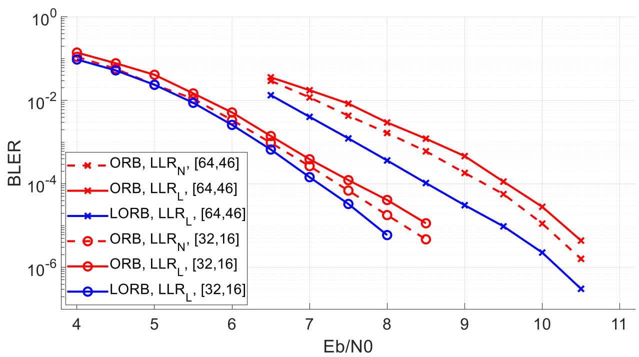

Fig. 6. AWLN channel employing CRC code with different dimensions. In both cases when $[n, k] = [64, 46]$ or $[32, 16]$, which are represented by cross marks and circle marks respectively, LORBGRAND exhibits consistent improvement of the order ~ 1 dB in Eb/N0.

While CRC codes are non-traditional, as GRAND algorithms can decode any code, we also consider the non-systematic CA-Polar code used for control channel communications in the 5G New Radio (NR) standard. CA-Polar codes have an established decoder in the form of CRC Assisted Successive Cancellation List (CA-SCL) decoding [39]. The CA-Polar code has dimensions $n = 64$ and $k = 40$, which has 24 parity bits and uses the $11-$bit CRC specified for 5G NR up-link control channels. Instead of setting the list size of CA-SCL to 8 as normally recommended for computational feasibility, we set it to 16 to increase its accuracy. For LORBGRAND, it is set to abandon the search and record a block error if no codeword is found within 2^{26} queries. Note that all GRAND algorithms identify an erroneous decoding after approximately geometrically distributed number of codebook queries with mean 2^{n-k} [19], [36]. Hence setting the abandoning threshold to be $2^{26} > 2^{24}$ where 24 is the number of parity bits, is sufficiently to ensure that LORBGRAND rarely abandons. Fig. 7 provides a comparison of BLER between CA-SCL and LORBGRAND and demonstrates the advantage of GRAND decoders, which fully utilize the CRC bits for error correction [27] unlike CA-SCL, extends to AWLN.

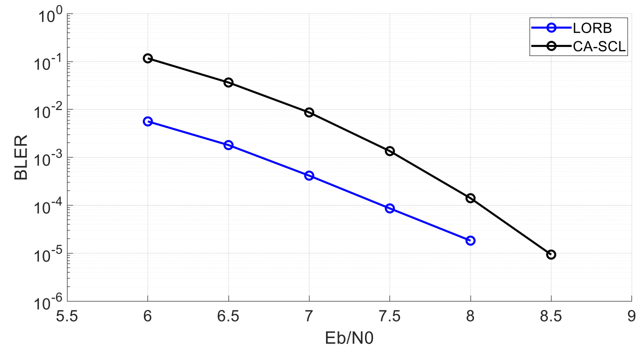

Fig. 7. The simulation is conducted using $[64, 40]$ CA-SCL under AWLN channel and all decoders are fed with correct LLR. The figure records the comparison of BLER where Blue line represents LORBGRAND while Black line represents CA-SCL [39]. As depicted in the plot, LORBGRAND shows at least 0.5 dB improvement down to 10^{-5} for BLER.

Soft Output Accuracy. As recently derived in [36], [37], soft input GRAND algorithm can provide soft output in the form of an *a posteriori* probability that the decoding is correct. The algorithm for generating soft output [36] can be applied to any GRAND algorithm as long as the input LLRs are correct as these are used to evaluate the likelihood of each queried noise effect sequence via (6). We use the soft output algorithm in LORBGRAND to provide an estimate of the probability of correct decoding. To evaluate the robustness of the approach in AWLN channel, we conducted simulations whose results are presented in Fig. 8. The accuracy of the soft output that was previously reported for AWGN channels is sustained in the AWLN setting. Using this soft output, for example, LORBGRAND can use CRC codes for both error correction and error detection simultaneously, where the latter is done by tagging for erasure based on the soft output.

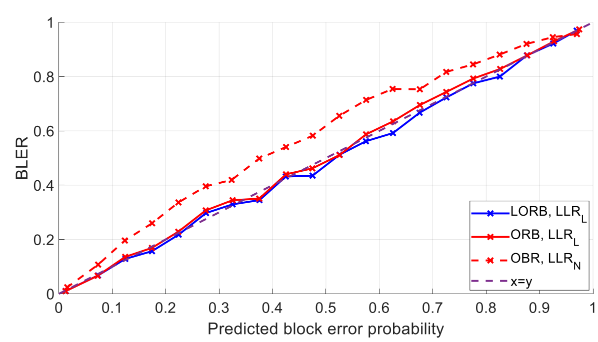

Fig. 8. Simulation result of soft output accuracy in AWLN channel. The code is a $[64, 46]$ CRC with Eb/N0$\in [2, 5]$dB. The soft output provides the predicted block error probability plotted against the empirical BLER conditioned on the value of the soft output. For a given value of $\hat{p}$ denoting the predicted block error probability, the corresponding conditioned BLER is calculated by the BLER of the samples with soft output approximately equal to $\hat{p}$. Dashed line indicates that the LLR is calculate using mismatched formula while solid line indicates correct calculation of LLR. As depicted in the graph, when ORB and LORB decoders are fed with correct LLR, their plots are almost identical to the $x = y$ line, indicating the predicted block error probability coincides with the actual BLER. Conversely, misusing LLR calculated with (2) leads to plot that is above from the $y = x$ line, meaning that incorrect LLR provides inaccurate estimates.

V. Conclusion

In this work, we introduced a GRAND decoder called Laplacian-ORBGRAND (LORBGRAND), that is specifically designed for decoding in the presence of AWLN. The decoder generates noise effect sequences in a fashion that has elements of both ORBGRAND, known for its precision in Gaussian noise, and hard detection GRAND for binary symmetric channels. By matching the decoder to the channel, the LORBGRAND decoder exhibits improved BLER and BER performance over the original ORBGRAND while maintain the same computational efficiency. Through simulation, we found that LORBGRAND consistently provides an improvement of order $\sim$ 1dB for different codes. Furthermore, we extended the investigation to include the calculation of soft output, a critical aspect for upgrading error detection [36] and enabling iterative decoding [37] in AWLN channels.

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