Time-Shift Coding for Uncoordinated MACs

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***Abstract*—In this work, we discuss time-shift coding and GRAND ZigZag decoding for uncoordinated multiple access channels (MACs). Time-shift coding relies on the delay difference of the received packets to prevent fully overlapped collisions. Users transmit with different time-shifts in designated time slots until the receiver successfully decodes all messages. At the receiver side, ZigZag decoding is employed to separate packets with linear complexity in noiseless scenarios. For cases with noise, a guessing random additive noise decoding (GRAND)-based algorithm is utilized to identify the most probable noise vector in conjunction with ZigZag decoding. Simulation results demonstrate that time-shift coding significantly reduces collision probabilities, thereby enhancing system throughput in noiseless scenarios. In the context of Gaussian MAC, the GRAND ZigZag decoding method outperforms successive interference cancellation (SIC)-based schemes in high signal-to-noise ratio (SNR) regimes.**

***Index Terms*—Time-shift coding, Uncoordinated multiple access, GRAND**

I. Introduction

A non-orthogonal multiple access channel (MAC) is a commonly used communication mode. In the MAC model, users share the same channel resources and attempt to transmit packets to the same destination simultaneously. The successive interference cancellation (SIC)-based approach was first introduced in [1], [2], aiming to resolve both interference and noise concurrently. The receiver attempts to decode a packet from one user and treats the interference (packets from other users) as noise. The ALOHA protocol [3] was originally proposed to avoid collisions. Coded slotted ALOHA [4] considers SIC in ALOHA-like systems to improve throughput. ZigZag decoding [5] is capable of separating packets in noiseless MAC scenarios. An improvement over the original ZigZag approach involves exploiting soft message passing on a sparse graph, known as SigSag [6]. In [7], the authors demonstrate that guessing random additive noise decoding (GRAND) is a maximum-a-posteriori (MAP) decoding technique for noisy MACs.

In this work, we discuss some details of the algorithm proposed in [7]. We compare time-shift coding and GRAND ZigZag decoding with existing schemes for noiseless and Gaussian uncoordinated MAC. Simulation results for the noiseless MAC scenario demonstrate that time-shift coding effectively reduces collision probabilities through the incorporation of a guard interval. Furthermore, the simulation results for the Gaussian MAC scenario highlight the potential of the GRAND ZigZag decoder to outperform SIC-based MAC systems by effectively treating noise and interference separately.

This paper is organized as follows. Section II gives background on the problem. Time-shift coding and GRAND ZigZag decoding are discussed in Section III. Simulation results of two study cases are shown in Section IV. Section V concludes the paper.

II. Preliminaries

In this paper, vectors are represented as $x^n = (x_1, x_2, \ldots, x_n)$. The i-th entry of x^n is denoted as x_i. The all-zeros matrix with dimensions $n \times k$ is represented as $0^{n\times k}$. I_n refers to an $n \times n$ identity matrix. A random variable (RV) is denoted by an uppercase letter, such as X. A specific realization of X is indicated by the corresponding lowercase letter x. A vector of random variables is expressed as $X^n = (X_1, X_2, \ldots, X_n)$. The probability density function (PDF) of a continuous RV and the probability mass function (PMF) of a discrete RV evaluated at x are denoted as $f_X(x)$. $F_X(x)$ represents the cumulative distribution function (CDF) of the RV X. The bitwise exclusive OR operation is denoted by $\oplus$. For functions and operations initially defined with scalar inputs, we extend them to vector inputs as their element-wise counterparts, i.e.,

$$f(x^n) = (f(x_1), f(x_2), \ldots, f(x_n)),$$
$$x^n \oplus y^n = (x_1 \oplus y_1, x_2 \oplus y_2, \ldots, x_n \oplus y_n).$$

A. Uncoordinated Multiple Access

In this work, we consider a slotted MAC model in which K users share the same channel resources and endeavor to transmit m message bits to a common destination. We make the following assumptions:

- the receiver and users are slot-synchronous,
- the users are uncoordinated and are unaware of the number of active users K,
- the receiver is aware of the number of active users and possesses knowledge of all channel states,
- The (coded) packet size is the same for each user.

B. GRAND

All GRAND [8] algorithms seek to identify the noise effect

$$z^n \triangleq c^n \oplus \tilde{c}^n, \text{ where } \tilde{c}^n \text{ is the hard decision} \tag{1}$$

Algorithm 1: GRAND

Input : hard decisions $\tilde{c}^n$, weight w^n
Output: estimates $\hat{c}^n$, decoding state ϕ

$\phi \leftarrow$ false, $e^n = 0^n$
if $\tilde{c}^n \in \mathcal{C}$ **then**
  $\hat{c}^n \leftarrow \tilde{c}^n, \phi \leftarrow$ true
  return
while $\phi =$ false **do**
  $e^n \leftarrow$ next error pattern with least score $S(e^n)$
  if $\tilde{c}^n \oplus e^n \in \mathcal{C}$ **then**
    $\hat{c}^n \leftarrow \tilde{c}^n \oplus e^n, \phi \leftarrow$ true
    return
return

that has impacted the transmission, from which the decoded codeword is deduced. GRAND creates binary error patterns e^n rank-ordered by score $S(e^n)$ to find a valid codeword, i.e., $\hat{c}^n = \tilde{c}^n \oplus e^n \in \mathcal{C}$, i.e.,

$$\hat{c}^n = \tilde{c}^n \oplus \hat{z}^n, \text{ where } \hat{z}^n = \underset{e^n : \tilde{c}^n \oplus e^n \in \mathcal{C}}{\arg\min} S(e^n) \tag{2}$$

where the score of an error pattern e^n is defined by

$$S(e^n) \triangleq \sum_{i=1}^{n} e_i \cdot w_i \tag{3}$$

where $w_i > 0$ denotes the weight of the hard decision $\tilde{c}_i$, which could be also considered as the cost to flip $\tilde{c}_i$. Pseudo-code for GRAND is shown in Algorithm 1.

In a hard decision BSC, GRAND [9], [8] doesn't have any reliability information and thus uses weight $w_i = 1$ for $i = 1, \ldots, n$, i.e., the score of error pattern e^n is equal to its Hamming weight. If a bursty statistical channel characterization is available at the receiver, a Markov-informed order can be used to generate error patterns [10], [11].

Soft GRAND (SGRAND) uses the non-quantized reliability as the weight [12], i.e.,

$$w_i = \ell_i, \quad i = 1, \ldots, n \tag{4}$$

where ℓ_i is the reliability of the hard decision $\tilde{c}_i$, which is the absolute value of the the symbol-wise log-likelihood ratio (LLR) of c_i based on the corresponding channel observation.

SGRAND provides maximum-likelihood (ML) decisions. However, SGRAND requires a dynamic data structure to generate the error patterns rank-ordered by the score depending upon the real-valued reliabilities ℓ^n.

Since the rank-ordered reliabilities are increasing almost linearly at low to moderate signal-to-noise ratio (SNR) regime, the basic version of ordered reliability bits GRAND (ORBGRAND) [13] considers the received bits rank-ordered in increasing reliability and their weights are increasing linearly, i.e.,

$$w_i = r, \quad \ell_i \text{ is the } r\text{-th smallest element in } \ell^n. \tag{5}$$

ORBGRAND sorts the reliabilities ℓ^n and set the weights w^n to its rank orders $r \in \{1, 2, \ldots, n\}$, i.e., ORBGRAND uses a $\lceil \log_2(n) \rceil$ bits input-related statistical model-based quantizer. Then error pattern generation could be solved by determining distinct integer partitions. ORBGRAND's advantage is that, once ranking is complete, pattern generation can be done on the fly with essentially no memory. ORBGRAND provides near-ML decoding for block error rates (BLERs) greater than 10^{-4}, but it is less precise at high SNR. To overcome this problem, a multi-line ORBGRAND with a more sophisticated statistical model is proposed in [14].

III. Time-Shift Coding and ZigZag Decoding

A. Time-Shift Coding

As mentioned in Sec. II-A, each user attempts to transmit a packet of length n to a receiver. Collisions occur due to the lack of coordination between users and the receiver. To resolve these collisions, users send their packets with random time-shifts in synchronized slots, ensuring the packets do not overlap completely.

The proposed scheme is depicted in Fig. 1. In each individual time slot, every user independently decides whether to transmit during that slot or not, based on a preset transmission probability P_t. The transmission delay τ is then randomly selected from the set $\{0, 1, \ldots, \tau_{\max}\}$. Each user continues transmitting until they receive a broadcast acknowledgment sent by the receiver once all messages are successfully decoded.

Let $x_{[k]}^n$ denote the packet from user k and $y_{(t)}^{n+\tau_{\max}}$ the received symbols in slot t. We have

$$y^N = x^{nK}\Lambda + z^N, \tag{6}$$

where z^N is the noise vector and

$$N = T \cdot (n + \tau_{\max}) \tag{7}$$

$$y^N = \left(y_{(1)}^{n+\tau_{\max}}, y_{(2)}^{n+\tau_{\max}}, \ldots, y_{(T)}^{n+\tau_{\max}}\right) \tag{8}$$

$$x^{nK} = \left(x_{[1]}^n, x_{[2]}^n, \ldots, x_{[K]}^n\right) \tag{9}$$

$$\Lambda = \begin{bmatrix} 0^{n\times\tau_{1,1}} & I_n & 0^{n\times(\tau_{\max}-\tau_{1,1})} & \cdots \\ & \vdots & & \ddots \end{bmatrix}. \tag{10}$$

Let $\gamma^N = x^{nK}\Lambda$ denote the noiseless version of received symbols. A mini example of Λ is shown in Fig. 2.

B. ZigZag Decoding and GRAND

On the receiver side, we try to find the most likely estimate of transmitted symbols, i.e.,

$$\hat{x}^{nK} = \underset{x^{nK}:\ x_{[k]}^n \in \mathcal{C},\ k=1,\ldots,K}{\arg\max} f_{Y^N|\Gamma^N}\left(y^N \middle| x^{nK}\Lambda\right). \tag{11}$$

Equivalently, our objective is to find the most probable estimate $\hat{\gamma}^N$ that satisfies the two following constraints,

1) A unique solution $\hat{x}^{nK}$ exists in linear simultaneous equations $\hat{x}^{nK}\Lambda = \hat{\gamma}^N$.

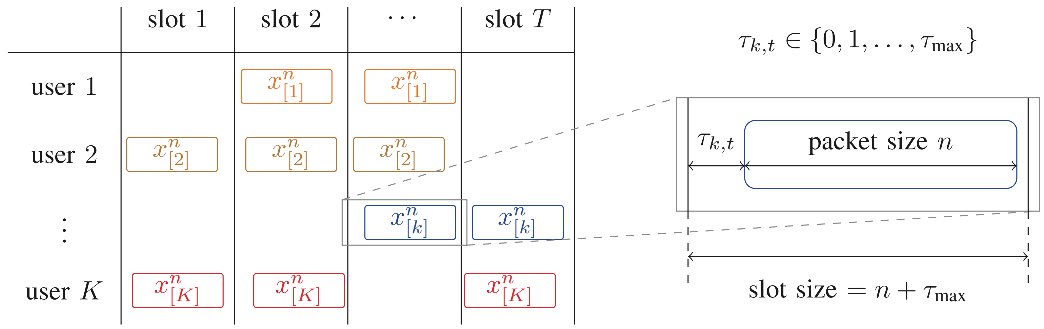


Fig. 1: The proposed system model involves time-shift coding, where each user continuously transmits with a predetermined transmission probability and an independently chosen random time-shift. Transmission continues until a broadcast acknowledgment is received from the receiver upon successful decoding of all messages.

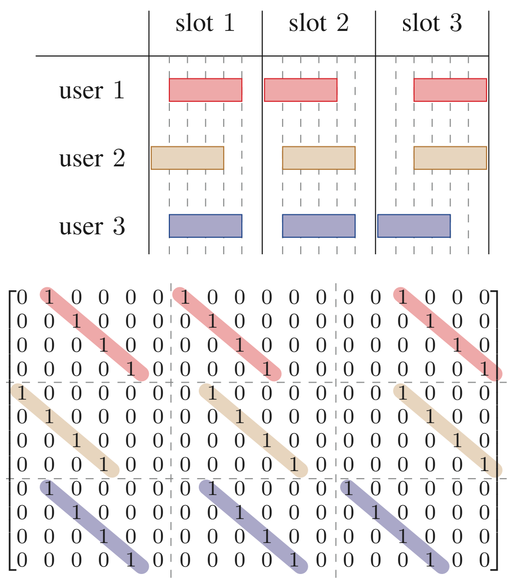


Fig. 2: Matrix representation Λ for $n = 4,\ \tau_{\max} = 2,\ K = 3,\ T = 3,$ $\tau_{1,\{1,2,3\}} = \{1, 0, 2\},\ \tau_{2,\{1,2,3\}} = \{0, 1, 2\},\ \tau_{3,\{1,2,3\}} = \{1, 1, 0\},$ $N = 18,\ \mathrm{rank}(\Lambda) = 12.$

2) $\hat{x}^n_{[1]}, \hat{x}^n_{[2]}, \ldots, \hat{x}^n_{[K]}$ are all valid codewords.

To find a *unique* solution, the matrix Λ must be full rank, i.e., $\mathrm{rank}(\Lambda) = nK$. We employ a GRAND algorithm to decode the messages. Given a full-rank Λ, the receiver initiates the generation of the error pattern e^N, which is ranked by the score $S\left(e^N\right)$, to identify a vector γ^N that satisfies the aforementioned constraints. It's worth noting that the error pattern e^N is not binary. The approach proposed in [15] is utilized to generate higher-order error patterns. Further details are provided in Algorithm 2.

Solving a system of nK linear equations has a complexity ranging from at least $\mathcal{O}\left((nK)^2\right)$ to at most $\mathcal{O}\left((nK)^3\right)$. The most efficient algorithm known to date was developed by Don Coppersmith and Shmuel Winograd in 1990, with a complexity of $(nK)^{2.376}$ [16]. By capitalizing on the time-shift-based structure outlined in Fig.1, we employ a ZigZag decoder [5] to reduce this complexity. We elucidate the ZigZag decoding process using a small example illustrated in Fig. 3 and the following steps:

1) Identify a weight-one column (isolated symbol) j in Λ and locate the unique non-zero element $\Lambda_{i,j}$. Discover all non-zero elements in row i, denoted as $\Lambda_{i,s}$ for $s \in \mathcal{S}$. If $\Lambda_{i,s}$ is the solitary non-zero element in column s, it is added to the set $\mathcal{S}'$. For instance, in the initial iteration in Fig. 3a, we identify $\Lambda_{5,1}$ (denoted by the red circle), leading to $\mathcal{S} = 1, 8, 15$ and $\mathcal{S}' = 1$.

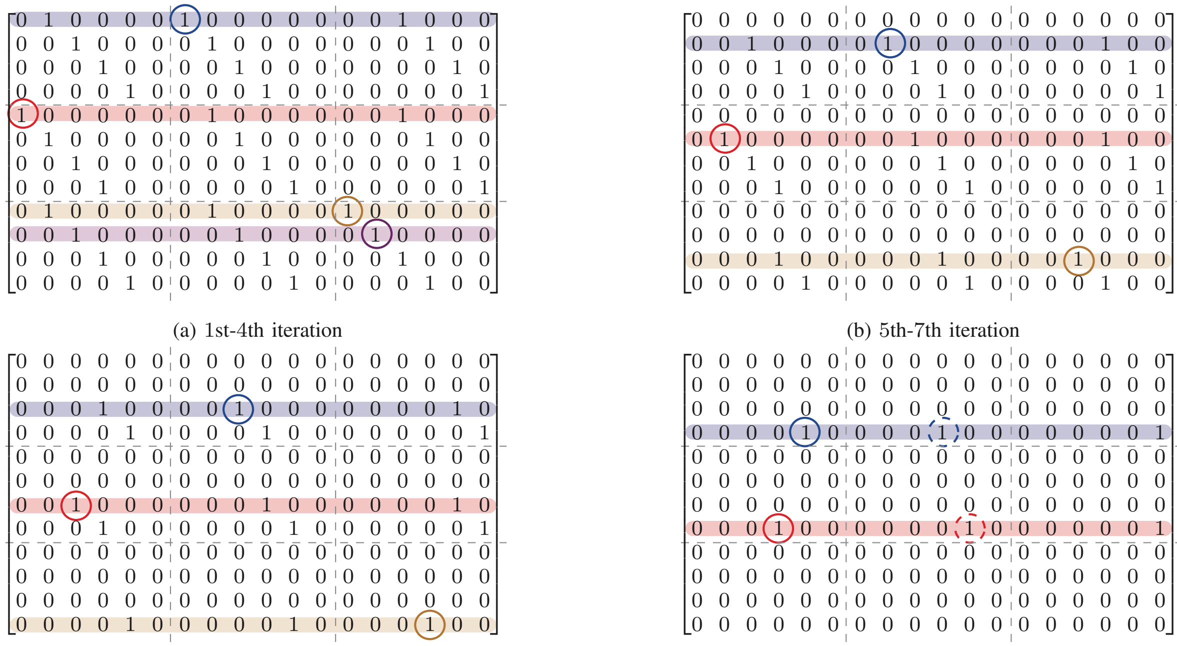
2) Upon finding $\Lambda_{i,j} = 1$, we deduce $\tilde{x}i = \tilde{\gamma}j$. For instance, in the initial iteration in Fig.3a, we obtain $\tilde{x}5 = \tilde{\gamma}1$. If an inconsistent pair is detected (i.e., $\tilde{\gamma}s_1 \neq \tilde{\gamma}s_2$ for $s_1, s_2 \in \mathcal{S}'$), the ZigZag decoding process is promptly terminated and a decoding failure (ψ = false) is reported. For example, in the 11th iteration in Fig.3d, we find $\Lambda 8, 4 = 1$ (marked by the red circle), resulting in $\mathcal{S} = 4, 11, 16$ and $\mathcal{S}' = 4, 11$. If $\tilde{\gamma}4 \neq \tilde{\gamma}_{11}$, the ZigZag decoding fails.

3) Reset row i to all-zero, i.e., $\Lambda_{i,s} \leftarrow 0$ for $s \in \mathcal{S}$. Extract the decoded coded symbols from vector $\tilde{\gamma}^N$, i.e., $\tilde{\gamma}_s \leftarrow \tilde{\gamma}_s - \tilde{x}_i$ for $s \in \mathcal{S}$.

4) If all nK coded symbols are successfully decoded, a decoding success (ψ = true) is returned along with the estimates. If not, return to step 1).

Obviously, we decode one coded symbol in every single iteration and thus need nK iterations to complete the ZigZag decoding. In each iteration, we extract the decoded symbol from $\tilde{\gamma}^N$, which requires a maximum of T operations since each symbol is transmitted at most T times. Therefore, the complexity of ZigZag decoding is upper-bounded by nKS, i.e., the number of non-zero elements in Λ.

Remark 1. Note that $\hat{x}^{nK}\Lambda = \hat{\gamma}^N$ represents an nK-dimensional system with N equations. Given that $N = T \cdot (n + \tau_{\max})$, it is likely that N is greater than nK. A fully-rank coefficient matrix Λ^{T} guarantees the system to have a unique solution or no solution. If the rank of the augmented matrix $\left[\Lambda^{\mathrm{T}} | \tilde{\gamma}^T\right]$ is greater than nK, the system



(a) 1st-4th iteration

(b) 5th-7th iteration

(c) 8th-10th iteration

(d) 11st and 12nd iteration

Fig. 3: An example of the ZigZag decoder. The system parameters are the same as those described in Fig. 2. The circled "1"s denote the unique non-zero elements in the columns. Only one coded symbol is decoded in each iteration, although it's possible that more than one iteration could be processed in parallel. E.g., we finished $4, 3, 3, 2$ iterations parallelly in Fig. 3a,3b,3c,3d, respectively.

is inconsistent, as stated in [17]. Hence, it is prudent to check whether

$$\text{rank}\left(\left[\Lambda^{\mathrm{T}}|\tilde{\gamma}^{T}\right]\right) = \text{rank}\left(\Lambda\right) = nK \tag{12}$$

before commencing the ZigZag decoding, rather than checking during the decoding process as described in step 2.

IV. Simulation results

In this section, we provide simulation results for two study cases,

- the throughput of the systems with different $\tau_{\max}$ in noiseless MACs,
- the error correction performance of ZigZag decoding and SIC-based decoder in Gaussian MACs with a fixed spectral efficiency.

A. Study case 1:

In this study case, we consider the system throughput in noiseless MACs with different numbers of users K. The throughput is defined by

$$\text{TP} = \frac{nK}{\left(n+\tau_{\max}\right)T}. \tag{13}$$

Note that the receiver is not capable of sending acknowledgements to specific users.[1] The receiver broadcasts an acknowledgement to all users when all messages are decoded.

[1]If the receiver were able to send acknowledgements to specific users whose messages are already decoded, the throughput would always be 1 by setting $P_t = 1$ and $\tau_{\max} = 0$.

Algorithm 2: GRAND ZigZag for MAC

Input : received vector $y^N = \left(y_{(1)}^{n+\tau_{\max}}, \ldots, y_{(T)}^{n+\tau_{\max}}\right)$

Output: estimates $\hat{x}_{[k]}^n,\ k \in \{1, \ldots, K\}$

$\phi \leftarrow$ false, $e^N \leftarrow 0^N$, $\tilde{x}^N \leftarrow$ hard decision $\left(y^N\right)$
while $\phi =$ false **do**
  $\tilde{\gamma}^N = \tilde{x}^N - e^N$
  $\left(\tilde{x}_{[1]}^n, \ldots, \tilde{x}_{[K]}^n, \psi\right) \leftarrow$ ZigZag decoder $\left(\tilde{\gamma}^N, \Lambda\right)$
  if $\psi =$ true **then**
    if $\tilde{x}_{[k]}^n \in \mathcal{C},\ k \in \{1, \ldots, K\}$ **then**
      $\hat{x}_{[k]}^n \leftarrow \tilde{x}_{[k]}^n,\ k \in \{1, \ldots, K\}$
      $\phi \leftarrow$ true
      return
  $e^N \leftarrow$ next error pattern with least score $S(e^N)$
return

The simulation results are depicted in Fig.4. The slot size is fixed at $n + \tau_{\max} = 50$, requiring a trade-off between efficiency $n/\left(n+\tau_{\max}\right)$ and collision probability by adjusting $\tau_{\max}$. The transmission probability is optimized

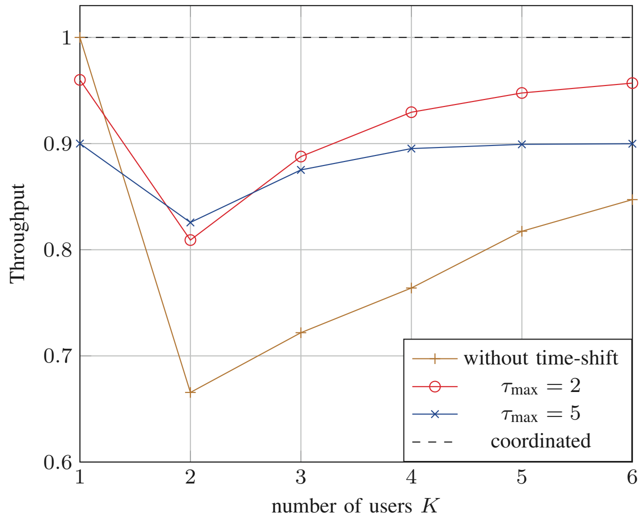


Fig. 4: Throughput of time-shift coding with ZigZag decoding for different values of K and $\tau_{\max}$ is shown. The slot size is fixed at 50, i.e., $n+\tau_{\max} = 50$. The transmission probability is optimized for each operating point in the case without time-shift, while it is set to $1/(1+\tau_{\max})$ for the case with time-shifts.

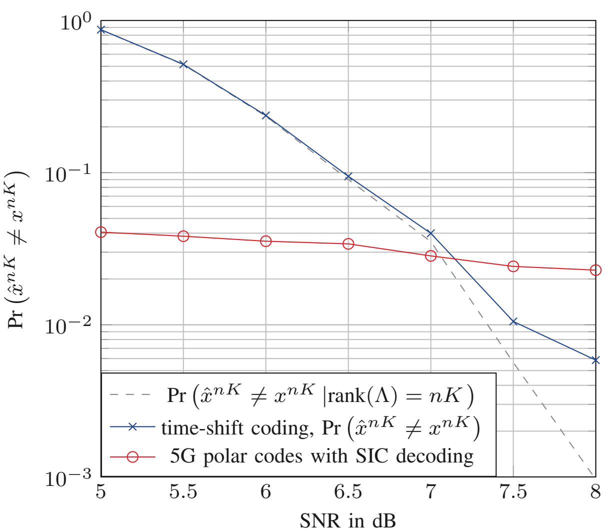


Fig. 5: Error rate $\Pr\left(\hat{x}^{nK} \neq x^{nK}\right)$ vs. SNR for a Gaussian MAC, $K = 4,\ m = 24,\ N = 128$.

for each operating point in the case without time-shifts, and set to $1/(1+\tau_{\max})$ for the case with time-shifts. We observe that random time-shifts significantly reduce the probability of fully overlapped collisions.[2] Furthermore, they enhance the system throughput for the case of $K \geq 2$.

B. Study case 2:

In this study case, we examine the error correction performance of the time-shift coded system with a fixed transmission rate, i.e., fixed K and T, in Gaussian MACs.

[2]This behavior is analogous to random linear network coding (RLNC)[18] with coefficients of different orders.

To simplify the system model, we assume that each user employs the same transmission power and sends packets modulated with binary phase-shift keying (BPSK). The SNR is defined as the ratio of the single-user power to the noise variance.

The simulation results are shown in Fig. 5. We consider a system where $K = 4$ users share $N = 128$ channel uses and each attempts to transmit $m = 24$ message bits. For time-shift coding, we use uncoded packets, i.e., $n = m = 24$. The slot size is set to 32 ($\tau_{\max} = 8$) and $P_t = 1$. With above mentioned setting, the receiver gets a matrix Λ with rank less than nK after $T = 4$ slots with probability

$$\Pr\left(\mathrm{rank}(\Lambda) \neq nK\right) = 0.0049. \tag{14}$$

If the matrix Λ is not full rank, the transmission fails. Otherwise, we initiate Algorithm 2 to find the most probable estimate of the vector γ^N. Here, we utilize ORBGRAND to generate error patterns e^N with a maximum of $Q_{\max} = 10^5$ guesses. As we have more than nK columns (constraints) in Λ, the time-shift code itself exhibits some error correction capability. The dashed curve in Fig. 5 displays the error rate of GRAND ZigZag decoding when Λ is full rank. The total error rate of time-shift coding with GRAND ZigZag decoding is shown by the blue curve and we have

$$\begin{aligned}&\Pr\left(\hat{x}^{nK} \neq x^{nK}\right) = \\ &\Pr\left(\hat{x}^{nK} \neq x^{nK} \,|\mathrm{rank}(\Lambda) = nK\right) \cdot \Pr\left(\mathrm{rank}(\Lambda) = nK\right).\end{aligned} \tag{15}$$

For reference, we also present the performance of an SIC-based system. In this setup, each user encodes the message bits using a $(128, 24)$ 5G polar code, and the coded symbols are rearranged using individual interleavers. The receiver attempts to reverse the shuffling of the signal y^N using $K = 4$ individual deinterleavers and then decodes it using an successive cancellation list (SCL) decoder [19] ($L = 16$), treating the other 3 packets as noise. If at least one decoding is successful, the receiver extracts the decoded packet from y^N and repeats the SIC procedure for $K = 3$, until all packets are decoded. As depicted in Fig. 5, the error rate of the SIC-based scheme does not notably decrease with increasing SNR, as the packets from other users are regarded as noise and the signal-to-interference-plus-noise ratio (SINR) remains consistently lower than $1/3$ for the first iteration of SIC decoding. It is noteworthy that time-shift coding with uncoded packets outperforms the SIC-based system in the high SNR regime, as it treats interference and noise separately.

V. Conclusions and Future Works

In this work, we discussed time-shift coding and GRAND ZigZag decoding for noiseless and Gaussian uncoordinated MAC. Due to space limitations, we present only two illustrative study cases. The simulation results for the noiseless MAC demonstrate that time-shift coding reduces collision probabilities by introducing a guard interval of size $\tau_{\max}$.

Furthermore, the simulation results for the Gaussian MAC highlight that the GRAND ZigZag decoder shows promise in outperforming the SIC-based MAC system by handling noise and interference independently.

For the extended version of this work and future research, we plan to explore the following topics:

- In scenarios where the receiver obtains a full-rank Λ but has very noisy, non-decodable observations, we should investigate improved decoding schemes to minimize the guessing required when dealing with noisier symbols.
- Analyze the minimum Euclidean distance of time-shift codes for a given Λ, either directly or in an average sense.
- While the current work assumes that all time-shifts and thus Λ are known to the receiver, the extended version will consider joint time-shift and channel estimation.
- Conduct a comparison between time-shift coding and (coded) slotted ALOHA-based systems, as well as SigSag decoding [6].

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