The Critical Roles of a Mathematics Specialist in Establishing Effective, Coordinated Professional Development Systems

Abstract

For nearly three decades, mathematics research has indicated the need for high quality mathematics instruction including both ambitious and equitable practices, as well as continuous, innovative learning opportunities for educators. While research connects effective professional development (PD) to improved instructional practices, there is need for additional research on the role mathematics specialist's play in developing and sustaining PDs. This qualitative case study examined the multiple roles of a specialist while laying the foundational components for a coordinated PD system to improve and strengthen mathematics teaching and learning in elementary classrooms. The PD efforts included the implementation of a teacher subsystem where a specialist led all components consisting of pull-out PD, mathematics coaching, collaborative time, and teacher networks. This study took place at a rural, pre-K through fourth grade school in a southeastern state. Analysis of the data identified the specialist's key roles in each component of the teacher subsystem. The results indicated that development and implementation of a teacher subsystem impacts the overall effectiveness of PD. Findings serve as a foundation for specialists to design and implement coordinated efforts that can have positive impacts on the teaching and learning of mathematics.

Keywords: mathematic specialist; professional development; case study

Mathematics specialists[1] have emerged to enhance and support mathematics teaching and learning through intentional job-embedded professional learning (Cobb et al., 2018;

[1] Mathematics specialists are referred to as specialists here forward.

Rigelman & Lewis, 2023). Specialists develop and sustain professional learning opportunities to strengthen and enhance instructional practices and increase student learning (Campbell & Malkus, 2011; Cobb et al., 2018; Fennell et al., 2013; National Council of Teachers of Mathematics [NCTM], 2020). The specialist should be included in all components of the PD including the strategic design, implementation, and follow up to guide teachers toward fidelity of the PD efforts (Hjalmarson & Baker, 2020). Defining the roles of the specialist (e.g., Baker et al., 2022; McGatha & Rigelman, 2017) and outlining a framework upon which to build helps those seeking to use a specialist to develop and/or improve PD opportunities (Baker & Knapp, 2019). This qualitative case study examined the multiple roles of a specialist while laying the foundational components for a coordinated PD system to improve and strengthen mathematics teaching and learning in elementary classrooms. The PD efforts included the implementation of a teacher subsystem using a specialist as a vital role in all components consisting of pull-out PD, mathematics coaching, collaborative time, and teacher networks.

Literature Review

It has long been the time for continuous and innovative support for teachers to strengthen their instructional practice and sustain high quality mathematics instruction. Such support must go beyond gaining new understandings and push teachers into critical conversations (Cobb & Jackson, 2021; Horn et al., 2018), reflections (Darling-Hammond & McLaughlin, 2011; Nelson & Hammerman, 1996; Tekir, 2022), and situations where teachers put what they learn into practice more consistently (Sims & Fletcher-Wood, 2021). It is essential that these opportunities result in improvements to instructional practices with intentional actions made by the teacher to influence and improve student learning opportunities (Bay-Williams et al., 2014; Jakopovic, 2021). Effective school improvement efforts must include a focus on assessment, extended time

and opportunities for learning, comprehensiveness, a variety of approaches, consider the learning environment or context, collaborative opportunities, and support and control (Sancar et al., 2021). It is evident that "virtually all school improvement efforts have achieved their gains by changing what is taught, how it is taught, the social climate of the school, and the tools provided to students" (Joyce & Showers, 1996, p. 3). These building blocks are the foundation for job-embedded PD efforts and, when grouped with a common purpose, rationale, and plan, will create a comprehensive system for staff development (Admiraal, et al., 2021; Fullan & Stiegelbauer, 1991; Guskey & Yoon, 2009). Mathematics specialists are gaining importance in the role of enhancing and supporting mathematical teaching and learning since the National Council of Teachers of Mathematics (NCTM) first advocated for specialists in the early 1980s (Dossey, 1984; NCTM, 2014).The mathematics specialist has an important role in developing, scaling, and sustaining PD for elementary teachers.

Mathematics Specialists

Specialists are "dedicated professionals, possessing the necessary knowledge and skill to create opportunities that maximize the learning of mathematics" (NCTM, 2014, p. 112). As such, defining specialist and other related terms in the literature helps to fully understand the role and responsibilities of a specialist (McGatha & Rigelman, 2017; Harbour et al., 2022). Specialists can be found in a variety of models within schools and districts with responsibilities supporting teachers or students or a combination of supporting teachers and students. At the simplest level, a specialist can be a strong mathematics teacher whose primary responsibility is to the K-12 classroom (Webel et al., 2017); a specialist can serve as an interventionist who provides individualized support to students; and a specialist can take on a coaching role where the specialist works alongside teachers and administrators to enhance and support mathematical

teaching and learning. In this study, the latter definition is explored with specific attention on an individual who has expertise in mathematics teaching and learning with responsibility for the leadership of other teachers (McGatha & Rigelman, 2017).

Leading Professional Development

Often one essential role of a specialist is to lead professional learning opportunities. Due to the elaborate and intricate nature of designing mathematics PD (Borko, 2004), it is critical for a specialist to use a systematic approach in deciding the type of learning activities for a district, school, or group of teachers (Baker & Knapp, 2019). A decision-making protocol provides a systematic method for implementing a cycle of goal setting, enactment, and reflection for PD learning opportunities for teachers that consider the intricate nature of teachers' needs, interests, and goals (Caddle et al., 2016; Desimone, 2009; Goos et al., 2007). In order to take into account the teachers' needs, interests, and goals, one option is to include pull-out sessions for enhancing instructional practices, as teachers can be grouped by needs, interests, and/or goals (Jackson et al., 2018). Pull-out PD describes an opportunity for teachers to participate collectively in professional learning during the school day, but not necessarily for the same school or grade level. While Cobb et al. (2018) specifically define this facet as one led by the district and away from the school, research indicates the need for smaller case studies that examine what individual schools do in relation to this large-scale idea (Jackson et al., 2018). Even with the promising initiatives, there remains much to be discovered regarding a specialist's impact on instructional practices as an individual means of PD (Kane et al., 2018).

Additional Specialist Leadership Roles

When supporting teachers with mathematics instruction, a specialist is responsible for leading the improvement of teaching and learning in the school. This is often accomplished

through targeting teachers' understanding of mathematical concepts and instructional practices (Campbell & Malkus, 2014). A specialist interacts with teachers regarding instructional practices and uses this knowledge as foundational pieces upon which to build professional learning. Research is limited on specialists engaging with small groups of educators (Livers, 2019). Gibbons and Cobb (2017) provide a conceptual analysis that were the foundational building blocks for this current study by identifying ways in which specialists can interact with teachers in meaningful ways that include individual contexts as well as group settings. Gibbons and Cobb (2017) identified a small set of group and individual models that the researchers labeled as "potentially productive coaching activities" (p. 414). These activities include "engaging in the discipline, examining student work, analyzing classroom video, engaging in lesson study, co-teaching, and modeling instruction" (p. 415). Additionally, Kane et al. (2018) identify five aspects needed for expert mathematics specialists. Specialists (a) need to be experts in content-specific pedagogies and not just be good teachers who implement good teaching practices, (b) view students as capable of engaging in rigorous mathematics, (c) establish and foster trust and relationships with teachers, (d) have a professional vision and understand how teachers learn, and (e) must be skilled facilitators and have the ability to lead teachers in learning and meaningful discourse (Kane et al., 2018) .

Theoretical Frameworks

This study leverages and integrates two theoretical frameworks. The first is Wenger's (1998) Community of Practice (CoP). The second is the coherent instructional system framework (see Figure 1) developed as a result of the MIST project (Cobb et al., 2018). In the realm of teacher professional development, Wenger's (1998) concept of communities of practice (CoP) emerges as a powerful theoretical framework. Wenger defines a community of practice as a

group of individuals who share a common domain of interest and engage in joint activities, forging a collective learning journey (1998). Applying this framework to teacher professional development can provide a structured and dynamic approach to fostering continuous learning and collaboration among educators.

The CoP model emphasizes the social nature of learning, asserting that knowledge is not solely an individual acquisition but a communal construction (Wenger, 1998). In the context of teacher development, this implies that educators can collectively contribute to and draw from a shared pool of knowledge and experiences. Wenger's framework underscores the significance of relationships and interactions within the community, positing that meaningful engagement and collaboration among teachers are essential for sustained professional growth (1998).

Furthermore, the concept of a community of practice aligns with the evolving nature of education and the need for adaptive teaching strategies. Teachers, as members of a CoP, can collaboratively address challenges, share innovative practices, and collectively adapt to changing educational landscapes. The CoP framework, therefore, serves as a theoretical lens that not only acknowledges the social dimensions of professional development but also highlights the transformative potential of collaborative learning among teachers.

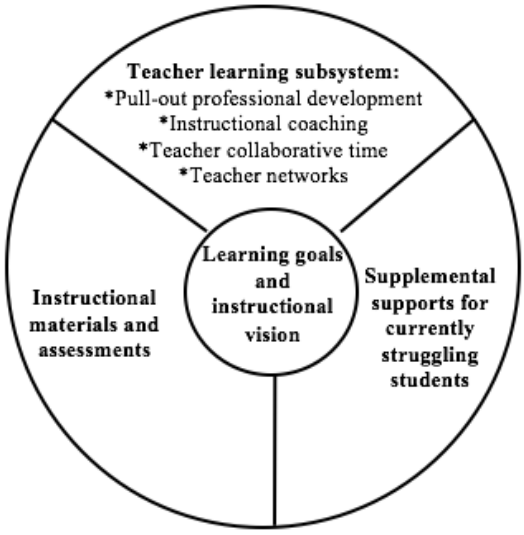
The MIST project allowed researchers to identify what it takes to improve the quality of mathematics teaching and student learning at scale. The coherent instructional system provides a robust theoretical framework for understanding and promoting the professional development of educators within the broader context of instructional improvement. This subsystem recognizes the pivotal role of teachers as learners and underscores the dynamic nature of their ongoing development Cobb et al., 2018). Central to the system is the idea that teacher learning is a multifaceted process that extends beyond traditional professional development sessions. It

encompasses both formal and informal learning experiences, collaborative interactions, and reflective practices (Cobb et al., 2018). The framework emphasizes the importance of creating a supportive environment that encourages continuous learning, experimentation, and adaptation.

Within the subsystem, collaborative inquiry plays a key role. Teachers are encouraged to engage in collaborative learning communities, where they can share experiences, explore new instructional strategies, and collectively reflect on their practice (Cobb et al., 2018). This collaborative aspect fosters a culture of shared expertise, where teachers learn from one another and contribute to the collective knowledge base. The coherent instructional system framework outlines the necessary components of effective mathematical PD reform. It identified pull-out PD, mathematics coaching, teacher collaborative time, and teacher networks as the four types of effective PD. Individually, these four types of PD are insufficient (Cobb et al., 2018). Collectively, however, they create a subsystem of support for teacher growth. The framework provides the foundational components for the PD reform efforts developed and initiated during the study, as well as framing for the analysis.

Figure 1

Coherent Instructional System



Note: Adapted from Systems for Instructional Improvement (p.8), by P. Cobb, K. Jackson, E. Henrick, and M. Smith, 2018: Harvard Education Press.

Methods

Study Context

The case study setting was a Title-1 elementary school in a southeastern state. A notable concern and influence for the study was deficiencies in mathematics learning for students, especially in the fourth grade. The specialist and the administrator identified the fourth-grade team as a focus for the specialist to coordinate PD. To address the targeted areas for improvement, a PD plan was created and implemented by the specialist, collaboratively working alongside the administrator. The goal of the PD was to improve student outcomes through increasing teachers' use of the effective Mathematics Teaching Practices (MTPs; NCTM, 2014) to engage students with the Standards for Mathematical Practice (SMPs) (National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010) . The PD plan included a book study using *Number Talks* (Parrish, 2014; Woods, 2022) as the supplemental resource to support the PD efforts. A grade level team of four fourth-grade teachers, the specialist, and the school principal participated in these coordinated efforts.

The principal was a black female with 43 years of educational experience and a double master's degree in elementary education and educational administration. She completed multiple mathematics certifications for grades K-4.

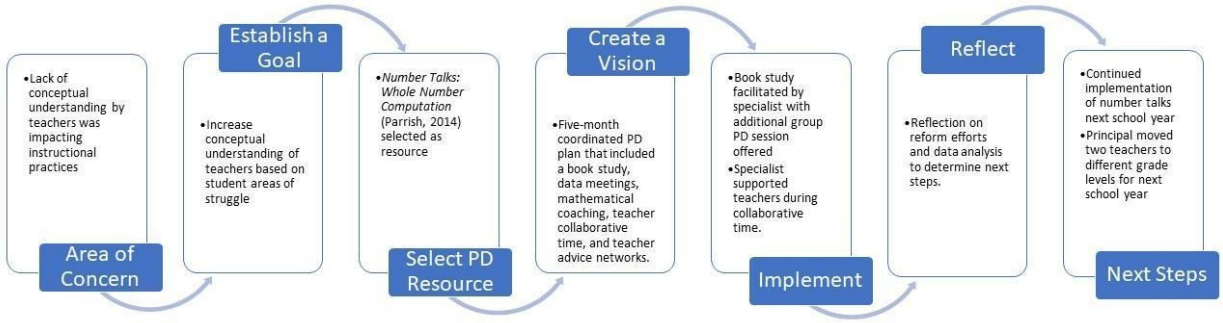
The study's four teachers (Amy, Brittany, Cindy, Dione) were white females, whose experience ranged from 10-24 years with all holding master's degrees and one being a National Board Certified Teacher (NBCT). The teachers had varying experiences of mathematical training, with two having completed fourth-grade mathematics content training provided by state specialists in statewide PD.

The Mathematics Specialist

The specialist, a white female with 16 years as an educator and possessing a master's degree in early childhood education, was employed with the title of "instructional coach" and supported grades pre-K through fourth grade. The specialist worked with both literacy and mathematics efforts and served as a coach in the study's elementary school for the past six years. Through conversations, she classified herself stronger in literacy than mathematics. However, in efforts to enhance her mathematics pedagogy, the specialist completed state level mathematics training (AMSTI) for third grade mathematics. In addition, she also completed Ongoing Assessment Project (OGAP, 2018) training in additive, multiplicative, and fractional reasoning. This training, along with her past teaching experiences, provided the knowledge and expertise needed to support the fourth-grade team of this study. Over the past six years, the specialist spent time and effort fostering trusting relationships with teachers. During her initial interview in this study, she discussed the importance of collaboration and teacher support: "It is important for me to build relationships with the teachers I work with. Over the past six years, I have spent time listening and working alongside these teachers to support their learning and teaching needs." With a keen professional vision of how teachers grow and develop, the specialist learning experiences as part of the PD subsystem's four components that catered to the unique needs, aspirations of the teachers, and specific goals of the PD. Figure 2 presents the roles and actions of the Specialist during the study then followed by descriptions of the four subsystem components implemented in the study.

Figure 2

The Specialist's Roles and Actions



Pull-Out Professional Development

The pull-out PD, as one component of the subsystem observed, included a book study facilitated by the specialist along with an additional whole-day workshop. This book study involved fourth grade teachers who met regularly to discuss and reflect on what they were reading, learning, and applying. Available research on book studies provides compelling arguments on the use of a book study as an effective model of PD for teachers (Arbaugh, 2003; Livers, 2019; Crespo, 2006).

Number Talks by Parrish (2014) was selected as the guiding resource. The specialist worked alongside the principal to plan and conduct the book study PD monthly. The specialist's role was to connect and integrate the SMPs and MTP. She guided discussions during book study sessions to help teachers focus on content along with the SMPs and MTPs while also challenging them to incorporate more standards and practices within their daily lessons. In studying the number talk concept, along with looking for ways to build teachers' focus on their students' conceptual understanding, the specialist developed and facilitated an additional PD workshop. This PD opportunity explored the impact of vocabulary on mathematical discourse. This decision was based on recent studies in which researchers have directed their attention towards vocabulary instruction for the improvements of teacher language in deepening student

understanding and comprehension of mathematics concepts (Herbel-Eisenmann, 2009, Hughes et al., 2016; Kenyon, 2016; Riccomini et al., 2015; Roe et al., 1995). It has long been accepted that language plays a vital role in students' cognitive processing of mathematical ideas (NCTM, 2014; Thompson & Rubenstein, 2000) and ability to engage in meaningful discourse (NCTM, 2014, Shields et al., 2005).

Mathematics Coaching

The specialist supported the teacher participants through a variety of group coaching efforts as part of the subsystem (Campbell & Malkus, 2011, 2014). She facilitated book study sessions with support from the principal implementing dialogic interactions including carefully crafted questions, paraphrasing, and summarizing the essential points (Haneda et al., 2017). She questioned teachers and provided insight to further emphasize main discussion points throughout the book study. Conversations during book study sessions centered on teachers' use of the MTPs while incorporating number talks on a weekly basis. The specialist supported weekly grade-level meetings as an active member of the conversations. Weekly book study meetings offered a time for reflection and additional questions not directly addressed. This was time for the specialist to make connections and help to enhance their instruction of specific content being taught each week. She conversed informally with the teachers whenever they requested, allowing her to provide one-on-one advice and insights regarding teacher concerns and areas of need. She provided an additional whole group PD session on vocabulary and mathematical discourse mid-way through the book study. She visited some classrooms multiple times a month and helped to support the teachers with instructional insights related to the overall goals of the PD efforts, specifically focusing on enhancing teachers' comprehension of mathematics content, operations,

and relationships. The specialist was available for individual coaching sessions at any point throughout the semester.

Teacher Collaborative Time

Teacher collaborative time as part of the subsystem involved teacher discussions related to instructional practices where teachers worked alongside each other to discuss and reflect on personal experiences (Horn et al., 2018; Willis & Valli, 1999). Teacher participants were required to have weekly grade-level meetings. During these meetings, teachers shared ideas on incorporating number talks into their instruction on a daily/weekly basis. Teachers strengthened personal conceptual understanding of content being taught by exploring multiple strategies for teaching each new concept. This collaborative time focused on teachers working together to discuss and reflect on personal experiences in their individual classrooms. The reflection component helped teachers see the importance of incorporating the MTPs on a more regular basis. The specialist was available to attend these weekly meetings and although she was not at every meeting in its entirety, she stopped by often to answer questions and provide advice. The specialist helped to draw connections in the conversation to the overall goal of the PD efforts by posing questions and providing instructional ideas throughout the discussions.

In addition to weekly grade-level meetings, teachers met twice for data meetings. In an effort to have grade-level data meetings be collaborative spaces, teachers individually reviewed their classroom data and identified trends or patterns. When grade-level data meetings began, teachers had already established points to focus the collaborative discussions to improve their understanding of how to engage in meaningful discussions about the trends and patterns. The specialist led the data discussions, posed questions to the teachers, and connected the discussions

back to number talks and the importance of conceptual learning during these strategic conversations.

Teacher Advice Networks

The last component of the subsystem included teacher advice networks to monitor who and with what frequency book study teachers reached out to other coworkers for advice. Teacher networks are a strong mechanism for instructional change (Sun et al., 2014). These involved teachers sought out coworkers for help with teaching practices and personal matters. Some of these conversations differed from the book study collaborative meetings in that the teachers directly sought advice and support for personal issues and concerns (Wilhelm et al., 2018). Furthermore, discussions involved instructional practices and teaching ideas for specific mathematical topics, as it related to the current PD efforts and current instructional math concepts. Although many types of communication were initiated by the individual teachers, the specialist made herself readily available as well. She visited classrooms before and after school multiple times a week to check in on book study participant teachers. As a result, some teachers initiated conversations with the specialist multiple times throughout the study. The focus of these conversations focused on number talks, implementation of the strategies from the book study, and ways to foster conceptual understandings for students. Teachers recorded these interactions in reflection journals, and the frequency of interactions that related specifically to the PD efforts was analyzed by the researcher.

Study Design

A case study was designed to capture the specific work of the mathematics specialist in the design and implementation of the PD and its subsystem components. This case study includes the analysis of PD efforts specifically designed to enhance fourth grade mathematics

instruction. The study used an intensive case study approach examined a real-life bounded setting over time involving multiple sources of information. Case studies have an advantage when trying to answer how and why something occurs and are best suited to provide in-depth understanding of a specific event, activity, program, etc. Qualitative research gives participants a voice, and this case study provided the participants with the opportunity to present their views and perceptions of the coordinated PD efforts (Creswell, 2013; Yin, 2009) in relation to the research questions.

Research Questions

This case study sought to answer the two research questions: Primarily, we ask RQ1: How does a mathematics specialist coordinate PD efforts related to mathematics instruction and the fidelity of implementation of the teacher learning subsystem? Secondarily, we ask RQ-2: How did the teacher participants' mathematics instructional practices change after participating in the coordinated PD efforts and teacher learning subsystem?

Data Collection and Instrumentation

For this case study multiple data sources were collected and analyzed**.** Yin (2009) recommends at least six types of information be collected; therefore, data was collected from (a) pre- and post- qualitative surveys, (b) multiple interviews, (c) classroom observations, (d) book study observations, (e) weekly reflections, and (f) document analyses.

The pre-study survey consisted of demographic and open-ended questions regarding professional development opportunities, the use of a facilitated book study, initial understanding of the eight SMPs and eight MTPs, and teachers' initial understanding of number talks. Participants were asked to complete the pre-survey prior to the end of January when the first book study session was conducted. The post-study survey was administered at the end of the

study. The post-study survey consisted of the same open-ended questions regarding professional development opportunities, the use of a facilitated book study, understanding of the eight SMPs and eight MTPs, and teachers' new understandings of number talks as compared to the pre-book study survey. Although neither survey asked for identification, demographic information and teacher responses made it possible to connect responses to each participant. This was necessary for data analysis after all data was collected. Qualtrics was used for all surveys.

Semi-structured interviews were designed as follow-ups for the surveys. Interviews allowed for clarification about the basic information related to professional development opportunities and teachers' perceptions of their understanding of the SMPs and MTPs. Interviews were conducted at the beginning and end of the study with the four teachers involved in the classroom observations, as well as with the instructional coach and principal. Semi-structured interviews consistently asked participants the same general questions, while allowing for the freedom and flexibility to ask to follow up with in-depth questions as opportunities arose during questioning (Creswell, 2011). Post-study interviews focused on reflection and follow-up questions regarding the book study in its entirety. Data was used to provide supporting evidence to the qualitative data that was collected throughout the study. Interviews were not recorded, per request of the instructional coach prior to the beginning of the study. Participants' responses were transcribed verbatim throughout the interview and additional notes of nonverbal communication were added when possible (Brenner, 2012).

Classroom observations were conducted a minimum of four times with each classroom teacher using the MCOP^2 (Gleason et al., 2015, 2017) with two observations occurring prior to the PD, and two observations at the conclusion of the PD. The MCOP^2 is a mathematics classroom observation instrument was explicitly designed to measure student engagement in the

Standards for Mathematical Practice (SMPs) and implicitly designed to capture teacher facilitation of NCTM's (2014) eight effective Mathematics Teaching Practices (MTPs) grounded in Wenger's (1998) CoP framework (Zelkowski et al., 2020; Gleason et al., 2017). This study focused on the implementation of the SMPs and MTPs that are evident in the MCOP^2. The selection of the MCOP^2 was based on the characteristics of the instrument foci in alignment with PD efforts. The observer (Author-1) was trained, provided resources, and practiced with the appropriate use of the MCOP^2 prior to the study's data collection (Authors-2 & -3 have used the MCOP^2 in prior studies).

Additional semi-structured observations were conducted during book study sessions, data meetings, and any other collaborative meetings. These additional observations were conducted using qualitative methods for data collection. Creswell (2013) distinguishes the role of a qualitative observer into four types that includes "complete participant, participant as observer, nonparticipant/observer as participant, and complete observer" (p. 167). Observations were conducted from the nonparticipant/observer as a participant role. It was important to maintain neutrality and not interject knowledge and experiences to minimize personal researcher bias. Jottings were later used to create full field notes from the observations. These observations served multiple purposes. The first was to provide a depiction of how the PD sessions were facilitated and to identify the expectations that were expressed by the coach and/or administrator. This is important for the professional community to understand how the PD was conducted considering any conclusions that are drawn from the data. Observations also provided an opportunity to collect qualitative evidence for each of the research sub-questions. Jottings focused specifically on what took place during the meeting, while verbal and nonverbal communication between the participants were recorded as it related to each sub-question.

At the beginning of the study participants were asked to answer weekly reflection questions about informal conversations they had related to mathematics instruction. Participants requested that this be done through email. After the first email was sent, one of the teacher participants created a running Google document so that teachers could access the questions at their convenience and write reflections from the week's discussions. Teachers reflected on informal discussions they had, recorded who they spoke with and the frequency of these interactions, and reflected on things that were implemented because of the PD.

Overall, the study specifically observed and analyzed data from surveys, interviews, observations, reflections, and documents (Creswell, 2011). These data sources helped to purposefully answer each of the research questions. Data was collected in three phases: Phase 1, Pre-study surveys, interviews, and classroom observations; Phase 2, During study book study sessions, data meetings, weekly grade-level meetings, and reflection logs; and Phase 3, Post study surveys, interviews, and classroom observations. Figure 3 provides a timeline overview.

Figure 3

Timeline for Study

Month	Activities
One	Administration of Pre-Study Surveys Administration of Pre-Study Interviews Pre-Study Classroom Observations Book Study Session 1
Two	Book Study Session 2 Grade Level Meetings Reflective Feedback 1
Three	Book Study Session 3 Grade Level Meetings Data Meeting 1 Reflective Feedback 2
Four	Book Study Session 4 Grade Level Meetings Reflective Feedback 3
Five	Data Meeting 2 Administration of Post-Study Surveys Administration of Post-Study Interviews Post-Study Classroom Observations Grade Level Meetings Reflective Feedback 4

Data Analysis

Creswell (2013) suggests that researchers follow specific procedures throughout the data analysis phases, from organization to representation of data. For this study, Glesne's (2006) three rounds of analysis procedure was used. Within the first round of data analysis, each data document was converted to an electronic version and organized according to the phase in which it was collected. A preliminary analysis of findings was completed for each phase of data (Glesne, 2006). Eclectic coding involved cycles of coding, initial and versus, and was used to code each data source during each phase (Glaser, 1978; Handwerker, 2015; Saldaña, 2021; Strauss & Corbin, 1998). Figure 4 presents the coding schema used in each of the three phases of the study's data collection and then the analyses.

Figure 4

Coding during the Three Phases of the Study's Data Collection & Analyses

Phase 1 Initial Codes	
Principal	• Has specific purposes for actions • Provides whatever teachers need to teach • Reviews data constantly • Looks for ways to improve instruction
Instructional Coach	• Reviews data constantly • Helps plan professional development opportunities based on need
Teacher Participants	• All teachers have varied experiences and trainings • Teachers reviewed data periodically to identify strengths/weaknesses of students • Discussion of data showed intent to change instruction to meet needs of students (for some teacher participants) • Opinions differ regarding professional development opportunities and the upcoming book study • Teacher shared overall goal to improve overall student performance on formative and summative assessments
Phase 1 Versus Codes	
Interview transcripts	• Principal role vs Coach role • On-grade level training vs Non-grade level training • Critical outlook vs positive outlook
Survey responses	• Critical outlook vs positive outlook • Experience vs training • Book study vs other professional development opportunities
Classroom observations/SMP and MTP comparisons	• Conceptual vs procedural • Standard algorithm vs multiple strategies • Self-ratings vs observation ratings • Instructional strengths vs instructional weaknesses

Phase 2 Initial Codes	
Principal	• Instructional strengths • Organized and focused • Reflections lead actions
Instructional Coach	• Supports principal with instructional strengths • Uses data to drive interactions
Teacher Participants	• View changes throughout book study • Use data to drive instruction • Strong feelings towards personal practices
Instructional Practices	• Strong focus on conceptual understanding • Use of multiple approaches • Use of precise language • Make use of deep connections
Phase 2 Versus Codes	
Book Studies	• Principal role vs Coach role • Critical outlook vs positive outlook • Conceptual focus vs procedural focus • Standard algorithm vs multiple strategies
Data Meetings	• Data performance vs classroom performance • Critical outlook vs positive outlook • Conceptual focus vs procedural focus
Weekly-grade Level Meetings	• Willingness to share vs quiet in book study/data meeting • Standard algorithm vs multiple strategies • Critical outlook vs positive outlook
Reflection Logs	• Beginning vs end • Critical outlook vs positive outlook • Book study vs other professional development opportunities

Phase 3 Initial Codes	
Principal	• Discouraged by teacher actions • Noticed changes in instruction • Purposeful, data-driven plan and focus
Instructional Coach	• Supportive of principal and teachers • Purposeful, data-driven plan and focus
Teacher Participants	• Opinions differ regarding book study resource • Stronger focus on conceptual understanding • Stronger focus on multiple strategies • Stronger focus on student discourse • All showed improvement from pre-study to post-study observations
Phase 3 Versus Codes	
Interview transcripts	• Number talks vs other strategies • Collaboration vs independent
Survey responses	• Useful vs practical • Novice vs experienced
Classroom observations/SMP and MTP comparisons	• Conceptual vs procedural • Standard algorithm vs multiple strategies • Pre-study vs post-study self-ratings • Pre-study vs post-study observations • Instructional strengths vs instructional weaknesses

Note: For purposes of review, the figure appears in portrait, but it is suggested to be landscaped for publishing.

To strengthen the integrity of the analysis, phase three in round one ended with a code landscape and word count analysis that focused on the participants' responses. The second round included data analysis and connecting data back to each research question. The last round focused on a pre/post comparison of participants' responses of the PD efforts.

Results

To answer the primary research question, it is important to look at the role of the specialist coordinating the PD efforts and the fidelity of the teacher learning subsystem which was one of the guiding frameworks. The teacher subsystem was composed of four components that included (a) the pull-out PD, (b) instructional coaching, (c) teacher collaborative time, and (d) teacher advice networks. The results were analyzed and organized within these subsystem components with special attention given to the specialist's role in each component. We then address the second research question addressing how teachers' instructional practice changes because of the PD.

The Specialist's Role in Coordinating Professional Development (RQ1)

It is relatively evident that the specialist had an important role throughout the PD efforts in coordinating all activities and serving as an active member in all discussions. Specifically, the steps depicted in Figure 2 outlined the role of the specialist in developing and scaling up the PD efforts throughout this study (see Figure 3 timeline). We present the findings here organized around the instructional system framework.

Pull-Out Professional Development

Coordinating pull-out PD included duration, content, and facilitation. Teachers spent just under 12 hours in book study sessions plus the amount of time reading the text on their own. Content coordinating was limited but beneficial. During one book study session, the specialist asked the teachers to *do* division using multiple strategies. This provided one of the most insightful and rich conversations regarding teacher content knowledge and the importance of building conceptual foundations. However, teachers *doing* math only took place once and all other times teachers just shared thoughts and ideas. As indicated by the discussion that followed

the division problems, teachers benefited from *doing* the math and instructional practices may have been strengthened had this been incorporated more throughout the sessions.

Choosing the book to use as a catalyst for change had mixed reviews. One pre-survey[2] question asked teachers how they felt about the book study on number talks. Two of the four showed initial interest in the study, while two began the book study feeling like "we already are doing something like it anyway." Post-survey questions specifically addressed the pull-out professional development component to determine teachers' overall thoughts of the book study. Another question asked how beneficial teachers felt the book study was in relation to mathematics instruction. Amy stated she was never a big supporter of the book, so she did not see the book study as beneficial. Instead, Amy contributed her strengths in teaching practices to OGAP and AMSTI training. Dione rated the book study as "about a six." Cindy's response was "somewhat related". Brittany saw the book study as helpful and stated that she "enjoyed listening to other teachers share their experiences with number talks...[and] the videos we watched helped me understand how to implement number talks."

Mathematics Coaching

The specialist facilitated the book study sessions and data meetings, as well as inviting individual coaching sessions for the teacher participants. Even though she was available for individual coaching, no teacher sought this support, so she was not able to coordinate coaching cycles. Lack of one-on-one support meant that teachers did not receive feedback on implementing the number talks or other mathematics instruction. Without feedback, teachers had little knowledge on whether their interpretation and implementation of number talks and mathematics teaching was truly beneficial for their students. This missed opportunity weakened

[2] Authors would be willing to include the pre-post surveys as an appendix if preferred by reviewers and the editorial team.

the mathematical coaching component of the teacher subsystem, thereby weakening the overall subsystem. Group coaching cycles were conducted during teacher collaborative time.

During the last data meeting, the principal stated her intentions of moving both Brittany and Cindy to different grade levels for the upcoming school year. Follow up after the final interview with the instructional coach provided additional insight into this decision. While Brittany requested the move to kindergarten from fourth grade, the principal and instructional coach thought Cindy would be a "better fit" for second grade. The instructional coach stated that discussion with the principal had led her to believe that she would be providing one-on-one math coaching for Cindy at the start of the new school year.

Teacher Collaborative Time

This component of the PD efforts involved coordinating collaboration among the fourth-grade teacher participants and their lower grade colleagues focusing attention on instructional practices and instructional growth (Horn et al., 2018; Willis & Valli, 1999). The collaborative time observed during this study involved data meetings and weekly grade-level meetings.

Post-study interviews revealed how teachers viewed collaborative time. Amy named collaboration with her fourth-grade colleagues as the most valuable component of PD that she participated in. "We share things that work and don't work, just like with the number talks. Our discussions have led us to changes for next year that will hopefully benefit our students," she said. Brittany saw the pull-out component as valuable but said that "discussions with each other allow us to share ideas and talk through things...[this] is a big help with strategies and making sure we understand how to build foundational skills and not just focus on procedures." Cindy specifically identified the weekly meetings as the most beneficial to her but also said that the book study was "good because it made me do something I normally wouldn't do." Like the

others, Dione stated that discussion with coworkers was the most valuable to her because "it gives [us] the chance to talk about what happens in the classroom and share similar experiences."

Survey data revealed that teachers valued collaborative time above pull-out PD and support from the specialist. While teachers valued this time, it was necessary to determine if the time spent was effective and beneficial to the overall goal of the PD efforts. While collaboration was the focus during data meetings, weekly grade level meetings only moved past coordinating lesson plans and engaging in surface-level discussions when the specialist was present according to observation data. Results show discussions moved toward exploratory when the specialist questioned teachers and required them to be reflective of their personal practices. For example, during data meetings, the principal and specialist led teachers in discussions with questions directed at positioning teachers where they need to self-reflect. Weekly meetings of the teachers, without the specialist or administrator present, were mainly expository in nature as observed by Author-1. Teachers spent time sharing and listening to ideas, but they rarely used this time to self-reflect and look for opportunities to enhance teaching practices. Including the specialist in grade level meetings helped to ensure that this time was effective and not wasted on coordination and expository efforts.

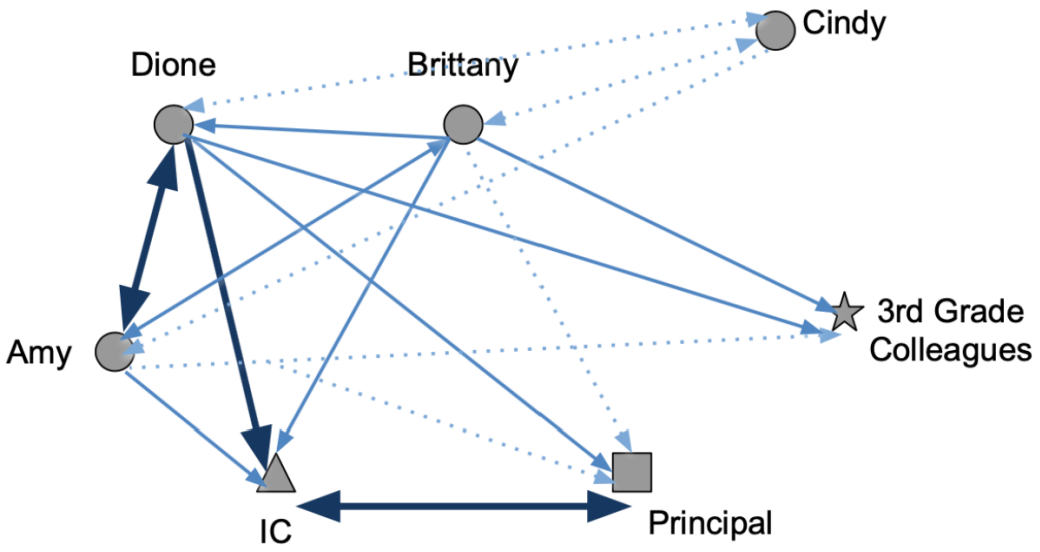
Teacher Advice Networks

Teacher advice networks are the last component of the subsystem and involve teachers seeking out advice from colleagues outside regular collaborative time. Findings revealed that the teacher participants viewed advice networks as the most important component of the subsystem. Teachers were more receptive to these one-on-one interactions with their colleagues, and reflection logs indicated that most of these conversations directly related to strengthening their personal abilities and teaching practices. Specifically, teachers were asked to identify who they

talked to and the frequency of interactions with each person. These frequencies were listed as daily, bi-weekly, weekly, bi-monthly, and monthly. These frequencies were tabulated, organized, and analyzed to create a sociogram – a diagram that provides information on the frequency that participants sought advice from their peers throughout the study (Cobb, et al., 2018). The sociogram below provides a visual of these conversations making it possible to determine the frequency of these interactions for each participant in the study (see Figure 5). The sociogram shows that all teachers, the principal, and the specialist participated in these advice networks, though Cindy barely did so while Dione did a lot. Evidence from this aligns closely with the conclusions provided below for each teacher participant.

Figure 5

Fourth Grade Team Advice Network



Note: Adapted from Systems for Instructional Improvement (p.136), by P. Cobb, K. Jackson, E. Henrick, and M. Smith, 2018, Harvard Education Press. IC=Instructional Coach. Star=3rd grade teacher colleagues. Arrow thickness indicates more/less frequencies of interactions.

Instructional Practice (RQ2)

The second research question examined the change in instructional practices because of the subsystem and the goals/vision that drove the PD efforts. The goal of the PD was to increase teachers' use of the MTPs through increasing students' engagement with the SMPs. To fully understand the impact of the PD book study on instructional practices, classroom observations of instruction were vital. The MCOP^2 lent itself well to this study to assess student engagement and teacher facilitation of number talks and conceptual learning. For each of the indicators on the MCOP^2, teachers were rated from zero to three based on the description for each component. Table 1 provides the means for each teacher. Means are provided for each subcategory, student engagement (SE) and teacher facilitation (TF). An overall change total for pre- and post-study is also provided for each teacher. The use and interpretation of MCOP^2 score-values related to the SMPs and MTPs has been previously discussed (Zelkowski & Gleason, 2016; Zelkowski et al., 2024a; Zelkowski et al., 2024b) as follows: (a) Excellent for scores 2.5 and above, (b) Very good for scores less than 2.5 but at least 2.0, (c) Above average for scores 1.5 to less than 2.0, (d) Below average for scores 1.0 to less than 1.5, and (e) poor for scores less than 1.0.

Table 1

Pre and Post PD MCOP^2 Teacher Observation Mean Scores

	Pre-PD MCOP^2 Means			Post-PD MCOP^2 Means			Pre-Post MCOP^2 Changes		
Teacher	SE	TF	Total	SE	TF	Total	SE	TF	Total
Amy	2.2	2.0	2.1	2.9	2.4	2.5	0.7	0.4	0.4
Brittany	2.6	1.8	2.2	2.9	2.7	2.8	0.3	0.9	0.6
Cindy	0.3	0.5	0.4	1.5	0.8	1.2	1.2	0.3	0.8
Dione	1.9	1.3	1.6	2.3	1.9	2.1	0.4	0.6	0.5

Note: SE=Student Engagement items from MCOP^2. TF=Teacher Facilitation items from MCOP^2. Total=Total mean of all items. Items 4 and 13 contribute to both factors, hence why the total is not a sum or average of SE and TF.

The pre/post scores identify all teachers as having demonstrated increases in practice. Amy and Brittany were identified as stronger implementing teachers of the MTPs and engaging students in the SMPs pre-PD, advancing their practices to excellent levels post-PD. Dione demonstrated comparable increases to Amy and Brittany, though the data indicates her practices were well-behind that of Amy and Brittany pre-PD. The data highlights Cindy as the teacher with the most growth with engaging students in the SMPs but also with the least growth in implementing the MTPs.

Review of the sociogram (Figure 5) shows that Brittany had many interactions with the specialist while Amy and Dione had slightly fewer interactions. Cindy on the other hand had no direct one-on-one interactions with the specialist. By merging the Table 1 observation data with Figure 5 Network observation data, we can see that Brittany's growth and engagement in the network is most ideal, while Cindy's growth while present, likely was as a result of personal choice rather than as a result of engaging in the network. For more in-depth reports on each teacher, Sarrell (2019) provides additional insights.

We examined the MCOP^2 item level for further analysis using the crosswalk (Gleason et al., 2017; Zelkowski et al., 2020) to understand the impact of the PD specifically on some SMPs and MTPs. The SMPs most impacted throughout the study included SMP-1 (Items-1&5), make sense of problems, and persevere in problem solving; SMP-3 (Item-4), construct viable arguments and critique the reasoning of others; SMP-7 (Item-8), look for and make use of structure; and SMP-8 (Item-8), look for and express regularity in repeated reasoning. The MTP most impacted was MTP-4 (Items-11&16), which focus on facilitating meaningful mathematical

discourse. These standards and practices closely align with the overall nature of number talks: student discourse, perseverance, repeated reasonings, and structure.

Qualitatively, teachers were asked on the post-survey what they gained from the book study pull-out professional development and how instruction was impacted. Amy said it was a "great reminder to focus on strategies and have students discuss how they see the numbers." She also stated her students were discussing their thinking more regularly because of her incorporating number talks in math instruction. Dione and Cindy both felt that they did not gain very much from the PD and neither indicated changes in their instructional practices, though the MCOP^2 says differently which could be likely a result of better task choice. Brittany said she "gained knowledge about the purpose of number talks and how to utilize them effectively in the classroom." She said number talks were now a regular part of her instruction. She also said that while students were already accustomed to sharing their thinking and strategies in her classroom, number talks had allowed her students to focus on solving problems mentally.

Teachers were asked to provide any additional information regarding the pull-out professional development and instructional practice reflections. Amy stated that "book studies are a great way to learn new strategies and stay abreast of the latest educational trends. However, this book was not the best book to do that with. It would serve as a great resource for a beginning teacher." Dione said that she "generally enjoys book studies but this was not a favorite for me. I think teachers need more of a voice in what our professional development and book studies could be. Give us some choices!"

In comparing the key components of effective PD to this pull-out book study in the survey data, we found responses indicating the book study was considered mostly redundant (Amy), beneficial (Brittney), forced (Cindy), and an added choice (Dione). The duration was not

long enough for teachers to gain a deep understanding of many mathematical concepts, though it was found through observations which indicated some evidence of content knowledge growth through the MCOP^2 modeling item#7 (see Table 2) which has recently been shown to be correlated to the NBCT content knowledge component, a potential early indication three of the four teachers' content knowledge may have been impacted. Number talks appeared to align with the MTPs and SMPs during the book study; however, implementation of number talks on a regular basis in classroom practices may still not incorporate these to the extent expected.

Table 2

Mathematical Modeling during Classroom Observations of MCOP^2 ITEM#7

Teacher	Observation	Scores	Observations
Amy	Observation #1	– 2	Amy demonstrated a strong understanding pre- and post-study.
	Observation #2	– 3	
	Observation #3	– 3	
	Observation #4	– 3	
Brittany	Observation #1	– 1	Brittany had a small increase from pre- to post-study.
	Observation #2	– 3	
	Observation #3	– 2	
	Observation #4	– 3	
Cindy	Observation #1	– 0	Cindy did not demonstrate growth from pre- to post as no lessons included any evidence of modeling.
	Observation #2	– 0	
	Observation #3	– 0	
	Observation #4	– 0	
Dione	Observation #1	– 0	Dione had the most growth from pre- to post-study, demonstrating lessons with significant modeling.
	Observation #2	– 1	
	Observation #3	– 3	
	Observation #4	– 3	

Note: Observations 1 & 2 are pre-study with observations 3 & 4 post-study. Higher scores on observations demonstrate the potential for higher content knowledge (Zelkowski et al, 2024b, under review).

While the teachers all indicated some knowledge of the SMPs and MTPs at the onset of the book study, classroom observations did not closely align with teachers' self-rating of their daily use. Increases were noticed in post-PD classroom observations, but teachers' self-ratings

did not always align with observation ratings. The captured instructional changes found from pre- to post- observations were certainly observable. It is certainly difficult to directly link number talks directly to the actual implementation of the MTPs and SMPs but it is more likely than not the increases are a result to some degree of PD subsystem components and number talks book use.

Community of Practice

The results reveal elements of structuring a community of practice (CoP) through the formulation of PD subsystems. Through intentional support by the specialist the intent was there to form a community built around improving the teaching and learning of mathematics. As we consider the subsystems the specialist put into place, the teacher collaborative time displayed the most cohesive time where the participants were in a community working together for a common goal. The pull-out PD and the teacher advice networks were promising but fell short in solidifying a strong community as these experiences did not lead to instructional coaching experiences.

Discussion

The analysis of the data and findings generally point towards the notion that the specialist had a greater impact on the three engaged and more receptive teachers with the PD than the fourth teacher. It was found the specialist played a critical role in maximizing the effectiveness of each component of the professional learning initiatives beyond a school's administrative team. While it is acknowledged that one teacher engaged very little with the specialist (see Figure 5), there were still improvements in instructional practices for this teacher (Table 2). The results also indicated the development and implementation of a teacher subsystem impacted the effectiveness of all efforts even with a teacher who was less than participatory with the specialist.

Furthermore, data analysis identified pull-out PD opportunities, mathematics coaching by the specialist with the whole group rather than individually, teacher collaborative time, and teacher advice networks to be evident components of the teacher subsystem. In addition, results showed a slight increase in mathematics instructional practices with data obtained from pre- and post-observations using the MCOP^2 (Gleason et al., 2017) and a crosswalk of the SMPs and MTPs (Zelkowski et al., 2020). Key areas of enhancements in mathematical teaching and learning included making sense and persevering in problem solving, constructing viable arguments, and critiquing reasoning, making use of structure, expressing regularity in repeated reasoning, and facilitating meaningful discourse. The classroom observations of number talks did not always lend themselves to the recommendations laid out by NCTM (2014). For such classroom discussions to be meaningful they must "build on and honor students' thinking; provide students with the opportunity to share ideas, clarify understandings, and develop convincing arguments; and advance the mathematical learning of the whole class" (NCTM, 2014, p. 29). While number talks promote mathematical discussion, it is necessary for teachers to employ equitable practices for equitable student engagement to occur (Bahr & Bahr, 2017). Even with varying degrees of direct participation in their classrooms with the specialist, there were clearly gains in student engagement in the SMPs and teachers' facilitation of the MTPs one could infer came from the book study and PD efforts. Increased interactions between the specialist and individual teachers (Figure 5) directly correlated with higher student engagement in the SMPs and teacher facilitation of the MTPs (Table 2).

It is important to examine and report why the teacher participants in the study did not seek out the specialist for one-on-one coaching for math support at any time over the course of the semester. While the exact reason is unknown, the following list provides possible hypotheses

for the exclusion of one-on-one coaching support: (1) Because the principal did not require it, teachers saw little value in the possibility of using individualized support; (2) The specialist never pushed the support since she knew it was not an expectation from the principal; (3) The teachers were apprehensive of number talks and were not interested in ensuring successful implementation; (4) The teachers felt that their years of experience in the classroom and past mathematical training had helped them already become strong math teachers; and (5) Time became a factor; it was difficult to schedule time for individualized coaching sessions due to other required events taking place (e.g., farm day, field trips, graduation practice). Together, these components help to identify the steps taken throughout the semester to develop and carry out PD efforts designed specifically to impact mathematics instruction. As described, the specialist played a vital role in each step.

The nature of case study lends itself to interpretations of the data by the researcher. The findings were presented in such a way as to remove research bias and tell the case as it was observed in its natural setting. In this case study, however, it was also important to reflect on the study as a whole and provide these reflections for the reader to consider. These reflections help to identify missed opportunities by the participants, provide implications for mathematics specialists, and lastly, we suggest future studies that incorporate number talks should be centered on equitable teaching practices to ensure that the mathematical discussions include equitable participation by all students and promote mathematical understanding for the entire group.

Implications for Consideration

This case study provided evidence on the importance of a coordinated PD initiative that included a specialist as a key component in the development, implementation, and reflection of all professional learning efforts aligning with Hjalmarson and Baker's (2020) findings. Results

indicated that identification of the goals and vision guides all professional learning efforts to be effective. It is critical for such an effort to have buy-in by the teachers and support structures in place that are collaborative in nature within the subsystem (Livers, 2022). The specialist helped to define and clarify this vision and helped to plan learning opportunities that directly correlated with the overall goals. In addition to the careful planning and implementation of the PD activities, research indicates that instructional coaching must include both one-on-one and group methods to build capacity for the entire school (Gibbons & Cobb, 2017; Mangin & Dunsmore, 2015). This study found the need to have administrative support for implementing coaching cycles for teachers. Even though the specialist was available, no teacher sought individualized coaching as previously discussed, though some had interactions with the specialist. This could have been a result of only the specialist and administrator designing the PD without teacher input. We find this as an important implication for future PD efforts and research as several studies have identified the benefits of one-on-one coaching (e.g., Gibbons & Cobb, 2016; Haneda et al., 2017; Russell et al., 2020). Research indicates that PD is more beneficial and successful when teachers have repeated opportunities to practice (Sims & Fletcher-Wood, 2021), buy-in to the PD efforts in what they learn, apply the learning in classroom instruction with coaching cycles, and receive feedback from what they do (Campbell & Malkus, 2011). Lack of one-on-one coaching meant teachers did not receive feedback on implementation of new teaching practices, though there is evidence of improved practices from observations as a likely result of the PD. This study supports the position that it is necessary for the administration to fully understand and support the role of a specialist and set expectations that include required coaching cycles for mathematics support. Teacher advice networks highlighted the importance of multiple interactions between the specialist and the teacher participants, though we further

indicate the needed buy-in by teachers with explicit support of the administrator for improvement. Similar to Sun et al. (2014), we recommend the support and facilitation of creating teacher networks through common planning and creating a culture that develops a community of practice. Findings from this study can serve as a foundation for specialists to design and implement coordinated efforts with administrator support and teacher buy-in that will have positive impacts on mathematical teaching and learning.

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