Fair-Over-Time Distributed Energy Resource Coordination

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Abstract— **There are numerous strategies to coordinate distributed energy resources (DERs) to provide a variety of services to the power grid. DER coordination can affect resources and participants unequally, for example, by excessively degrading or curtailing particular DERs more than others. However, few DER coordination strategies explicitly take into account fairness, equity, or justice. In this paper, we explore fairness metrics and their applicability to the DER coordination problem. In particular, we investigate metrics from machine learning to identify metrics that could be incorporated into DER coordination problems and we summarize fairness metrics that have been used in the power systems literature. A key challenge is that most existing fairness metrics are static – ensuring fairness at a point in time. DER coordination problems are inherently dynamic and we often care about fairness over time, not at each time. The machine learning literature offers some ways to think about fairness over time and, more generally, how to incorporate fairness into dynamic power systems problems. We use a specific DER coordination problem – the problem of computing dynamic operating envelopes – to demonstrate how incorporation of a fair-over-time metric changes DER coordination solutions, and highlight the trade-offs that arise.**

I. Introduction

As society has come to more fully recognize the disparate impact that electric power systems have had on different populations, the concepts of fairness, equity, and justice have emerged as key objectives in the energy transition [1]. Incorporating fairness, equity, and/or justice metrics within power system problem formulations is one way to steer towards more fair, equitable, and/or just solutions. However, defining fairness, equity, and justice such that they can be included in quantitative problem formulations is challenging. Moreover, some of these concepts, like justice, were developed in the social sciences, and trying to develop quantitative justice metrics may undermine the spirit of the concept.

Fairness is a concept that has appeared in various engineering and technical literatures such as networking, computing, and machine learning (ML) [2]–[4]. For example, fairness has been considered in ML contexts ranging from nondiscriminatory college admissions decisions to recidivism predictions in correction processes to loan lending decisions in banking [5]. Fairness concepts from ML can be leveraged within power systems problem formulations. However, this is not usually trivial – the ML language has to be mapped to the power systems problem and often there is not a clear/clean mapping. One needs to determine what notion of fairness is appropriate, what attribute should be fair (and whether the data exists to quantify this), how to measure fairness, and how to incorporate fairness metrics into the problem formulation, e.g., an optimization or control problem.

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Take, for example, the problem of coordinating distributed energy resources (DERs). DERs can provide a variety of useful services to the power grid. Further, DERs enable prosumers, consumers who also produce, to engage with the power market and reap the benefits of doing so [6]. DER coordination strategies can affect resources and participants unequally, for example, by excessively degrading or curtailing some DERs more than others, affecting energy costs/payments [7], [8]. Despite this, few DER coordination strategies explicitly account for fairness. Identifying and implementing suitable fairness approaches in these settings can be especially challenging since the problems are inherently dynamic. While the ML literature has many examples of static fairness metrics (i.e, metrics that capture fairness at a point in time), there are fewer metrics that capture dynamic fairness, time-varying fairness, or fairness-over-time.

In this paper, we investigate fairness metrics from ML to identify metrics that could be incorporated into DER coordination problems and we summarize fairness metrics that have been used in the DER coordination literature. We find that the ML literature offers some ways to think about fairness over time and, more generally, how to incorporate fairness into dynamic DER coordination problems. We summarize the steps one should take when incorporating fairness into a power systems problem formulation. We then provide a case study that shows how to apply these steps to the dynamic operating envelope problem, which seeks to determine the range of feasible power injections from DERs at each node of the distribution network. We show how a fairness-over-time metric can be incorporated into the problem to spread curtailment across the DERs and across time, achieving a notion of fairness. We also discuss the performance-fairness trade-offs that arise when applying this metric.

Some prior power systems work has discussed fairness. For example, the literature review [9] catalogs 80 works which explore fairness within contexts ranging from demand response to shared/community energy resources, collectively referred to as local energy systems. Fairness in the operating envelope problem has been explored in [10]–[14]; however, none of these papers considered fairness over time, only fairness at each time, which may be unnecessarily constraining.

In summary, the contributions of our paper are fourfold. First, we compare time-varying fairness approaches from the ML literature, and then compare fairness approaches in the DER coordination literature. Second, we provide a principled method to incorporate fairness into power systems problems. Third, we apply this method to the dynamic operating envelope problem, formulate a fair-over-time dynamic operating envelop problem, and discuss the performance-fairness trade-offs that result from its use.

The rest of this paper is organized as follows. Section II describes fairness concepts and metrics in the ML and DER coordination literatures. Section III provides an approach for how to incorporate fairness into power systems problems. Section IV provides a case study on fair-over-time dynamic operating envelopes, and Section V concludes.

II. Fairness Concepts & Metrics

A vast body of work has come out of the ML community to address fairness. Structured approaches to fairness have arisen in the theory, vocabulary, and problem formulations of fair algorithmic decision making [4], [5], [15], [16]. For power systems problems, there is growing interest and research directed toward the incorporation of fairness. We see value in comparing the landscapes of fairness in both areas and exploring what can be gleaned from the fair-ML community. In this section, we first describe common static and time-varying fairness metrics in ML, we then describe existing approaches to fairness in DER coordination, and finally we compare these landscapes of fairness.

A. Fairness in machine learning

1) Static metrics: Many formulations of fairness in ML have emerged to quantify and account for (un)fairness inherent to a problem setting or introduced in the algorithmic decision making process. One example is Demographic Parity (also known as Statistical Parity), which is "the property that the demographics of those receiving positive (or negative) classifications are identical to the demographics of the population as a whole" [17]. In other words, the outcome probability distribution for persons belonging to different groups are the same [18]. For example, this metric can be used to ensure that the proportion of applicants approved for loans from a bank is the same for white and black communities relative to applicant pool demographics [19].

A second example is Disparate Impact [20], which formalizes the notion that, ideally, benefits should not disproportionately accrue among unprotected groups relative to their accrual among protected groups. Ref. [20] conceptualizes Disparate Impact as the condition in which the benefits accrued among the protected group is less that x% of the benefits accrued among the unprotected group.

A third example is Equalized Odds/Opportunity, which formalizes the notion that the performance of a prediction model should be identical for protected groups and unprotected groups, i.e., true positive, true negative, false positive, and false negative outcome prediction rates of the model should be the same for protected and unprotected persons [21]. For more information on common static fairness metrics in ML, including these and others, see [5].

2) Time-varying fairness metrics: The previously mentioned fairness metrics do not explicitly account for the passage of time, which is suitable for many ML contexts characterized by batch or one-shot decision processes. However, there are settings in which decisions are coupled over time, requiring adaptions to these metrics or inviting new approaches that can measure fairness across time or update decision mechanisms in a time varying manner. For example, future decisions can be coupled to past decisions through their effects on group or population characteristics [4]. For example, college admission decisions can have long-term effects on admissions for members of a group, i.e., children of admitted (and now college-educated) parents may be better equipped to apply for college. These kinds of fairness dynamics are often referred to as long-term fairness [22]. Future decisions can also be coupled to past decisions through their effect on the trajectory of a dynamical system. Furthermore, the metric for fairness can be measured, optimized, or constrained over time, coupling past and future decisions. The latter setting is the one we consider in our case study.

The top of Table I compares examples of works that pursue time-varying algorithmic fairness and analyzes them with respect to a set of common elements including the notion of fairness, the metric used to enforce or measure fairness, and the fair decision mechanism.

B. Fairness in DER coordination

The investigation and incorporation of fairness, equity, and justice in power systems problems is an emerging area of interest [1]. One way of coordinating DERs is via electricity rates design, i.e., DER actions can be shaped through their responses to time- (and spatially-) varying electricity prices. Some papers leverage electricity rate design to achieve more equitable or just outcomes, e.g., [29]. However, DER coordination can also be achieved through control- and optimization-based approaches.

For example, aggregations of DERs, such as thermostatically controlled loads (TCLs) or batteries, can be controlled to provide grid balancing services. Control commands are allocated to DERs in real-time based on system needs. This type of control can lead to unequal cycling and degradation of batteries and TCLs, which reduces their lifespan, and unequal temperature excursions (comfort impacts) for TCLs. Ensuring fair impacts is important, and fairness here is usually defined as equality of impact, i.e., the burden of participation should be equally distributed, e.g., [30]. However, mechanisms for achieving this are rarely connected to fairness. For example, in the TCL control literature it is common to assess or penalize TCL switching as a proxy for degradation, but it is uncommon to consider the disparity in the number of additional switching actions per TCL across a population of TCLs in an aggregation.

An example of an optimization-based approach to achieve DER coordination, which has seen a significant focus on fairness recently, is the dynamic operating envelopes problem. DERs have the potential to improve reliability, decrease operating costs, and lower the environmental impacts of power system operations [6], [30]. However, if not properly managed, DERs can cause over- and under-voltages, over-current violations, and transformer overheating [8], [31].

TABLE I

COMPARISON OF TIME-VARYING FAIRNESS APPROACHES FROM THE ML LITERATURE (TOP) AND FAIR OPERATING ENVELOPES APPROACHES FROM THE POWER SYSTEMS LITERATURE (BOTTOM)

Ref.	Problem	Notion of Fairness	Metric	Decision-making Mechanism
[23]	dynamic learn-to-rank	disproportionate allocation of benefits between groups must be limited	exact-k fairness	constrained Markov decision process
[24]	dynamic learn-to-rank	cumulative benefits over time should be proportional to cumulative merit over time	overall disparity	feedback control
[25]	ranked list recommender systems	cumulative benefits over time should be proportional to cumulative merit over time	amortized fairness	integer linear programming
[26]	reinforcement learning in Markov decision process framing	very frequent or severely low quality decisions should be avoided	approximate-choice fairness and approximate-action fairness	Fair-E3 algorithm
[27]	online stochastic bandits	current decisions must be as or more conducive than decisions made for similar individuals in the past	fairness-in-hindsight	Cautious Fair Exploration (CaFE) algorithm
[28]	loan application approval with bank income and applicant credit score dynamics	equal true outcome prediction rates across groups	equality of discovery probability, equal opportunity	constrained Markov decision process
[8]	over-voltage mitigation via DER coordination	burdens should be distributed proportionally	curtailment proportional to potential	Optimization
[10]	dynamic operating envelopes	maximize weighted individual benefit proportionally	weighted proportional fairness	Optimization
[11]	dynamic operating envelopes	benefits should be distributed equally	soft-equitable perspective, constraint: equitable perspective	Optimization
[13]	dynamic operating envelopes	efforts to minimize burdens should prioritize the most burdened	squared curtailment	Optimization
[14]	dynamic operating envelopes	maximize individual benefit proportionally	proportional fairness	Optimization
[12]	over-voltage mitigation via real power caps	burden should be distributed equally	proportionally equal curtailment	Consensus Control

The operating envelope problem determines the net power injection limits for each node in the network and updates them as system conditions change. These limits are known colloquially as dynamic operating envelopes, simply operating envelopes, or dynamic hosting capacity [31], [32]. Such limits can be obtained by solving modified optimal power flow (OPF) problems which maximize a function of power injections/consumption at every node subject to network constraints [33], [34].

Existing literature shows that an objective function which fails to consider fairness can lead to significant discrepancies in limit sizes across the network [13], [35], [36]. This is because power consumption/injections farther from the substation will generally have a larger impact on voltages due to the radial structure of distribution networks and the nonlinearities of power flow [11]. The bottom of Table I compares examples of papers that have formulated fair operating envelope problems. We note that all of these papers formulate single-period optimization problems and impose fairness objectives or constraints at each time, so fairness may change in each time, but is not optimized over time.

C. Comparing fairness landscapes

Table I shows that in both landscapes, notions of fairness are often characterized by the distribution of benefits/burdens and that considering the dimension of time can present new conceptualizations of fairness. The ML time-varying fairness concepts can provide insight on how to extend notions of fairness in DER coordination problems to address their dynamic nature. For example, some notions of fairness include a cumulative component that can address the effects of decisions over time [24], [25] and others introduce a notional of fairness that current decisions should be fair relative to past decisions [27].

III. INCORPORATING FAIRNESS

We aim to assist researchers in integrating fairness into power systems research by offering a straight-forward way to approach fairness and communicate how fairness is considered in one's work. In Section IV, we show the usefulness of this approach through a case study. We note that [9] has also contributed to the structuring of this emerging space by introducing four dimensions – *context*, *scope*, *interpretation of fairness* and *approach* – for characterizing existing approaches to fairness used in the local energy systems literature. In contrast, our work aims to provide a prescriptive approach to the incorporation of fairness into future problem formulations.

Assuming a strong understanding of the problem setting is established, we first recommend a researcher start by identifying qualitative concepts of fairness relevant to one's problem setting. For example, [17] introduces the notion of individual fairness which requires "mapping similar individuals similarly." In the column titled "Notion of Fairness" of Table I, we present notions that describe the approaches to fairness employed in each work, as we understand them.

Secondly, the notion must be formalized mathematically. A good starting point is to identify a measurable quantity relevant the chosen notion of fairness and then define a condition (i.e., a metric) under which the state is (un)fair with respect to that quantity. The broader energy justice community has directed much effort toward generating, identifying, cataloging, and evaluating quantities that capture the role of energy in inequity or injustice [37]–[40]. The power systems community has adopted some of these concepts, for example, [41] uses the distribution of cumulative load shed during public safety power shutoffs used to prevent wildfires and [42] considers its geographic overlap with vulnerable populations, as defined by the social vulnerability index. Other quantities include energy burden (energy expenditures divided by household income) and the energy equity gap [43].

Thirdly, approaches to fairness must identify a fair-decision-making mechanism suitable for the problem. Commonly used mechanisms are an algorithm, control design, or an optimization problem. Communicating each of these steps and the reason for one's choices enables readers to understand and compare approaches across different works.

IV. Case Study

A. Incorporating fairness: the operating envelope problem

We next describe how to leverage the approach in Section III to incorporate fairness into the operating envelope problem described in Section II-B.

1) Notions of fairness: As described earlier, the first step in addressing fairness is to identify what could be considered fair. We will now do this for the operating envelope problem. One notion of fairness is that decisions should aim "to be fair in terms of the intended operation" of each DER [13]. In other words, each DER should be free to exploit their full realizable injection capability, if desired. However, binding network constraints necessitate curtailment of injections to ensure safe operation of the network. Another commonly used notion of fairness is maximizing the minimum benefit received by any user [44]–[46], also known as α-fairness [2] with $\alpha = \infty$. In the context of operating envelopes, this means maximizing the minimum operating envelope that any node receives. There are certainly other notions of fairness that can be applied to the operating envelope problem, but these are the notions we will consider in this paper.

2) Mathematically formalizing fairness: Both of the above notions of fairness have drawbacks in the context of operating envelopes. The first notion, fairness in terms of intended operation, will maximize the total network flexibility and minimize individual curtailment. This motivates minimizing the sum of the squared curtailment at each node in the network. Squaring the curtailment will prioritize allocation of injection limits such that the distance from the limit to the full potential of the DER(s) at each node is similar across the nodes. However, minimizing the sum of the squared curtailment will bias the allocation to nodes with larger capacities. The second notion of fairness, maximizing the minimum, ensures that the base injection limit that every node is allocated is as large as it can be. However, it does not maximize the limits at less constrained nodes, resulting in a missed opportunity for a greater total network flexibility. For these reasons, we will use a linear combination to minimize the squared curtailment at each node and maximize the smallest operating envelope in the network.

3) Identifying the fair decision-making mechanism: Neither of the two notions of fairness considered here are new in the context of operating envelopes. The contribution of this paper is to bring the notion of fairness over time into the context of the operating envelope problem, and the DER coordination problem more generally.

The fair decision-making mechanism best suited for operating envelopes is optimization, where fairness is softly enforced in the objective function. In the literature, operating envelopes are calculated by solving an optimization problem for each time period. In this manner, operating envelope allocations from previous time periods have no impact on the current allocation decisions and fairness is enforced at each time period. Inspired by methods in ML literature, we propose two methods for enforcing fairness over time. The first method is to calculate all operating envelope allocations across a time horizon by solving a single, multi-period optimization problem where allocation decisions in one time period are coupled to decisions of the other time periods through fairness. The second method, inspired by amortized fairness, is to keep track of unfairness incurred from previous allocations and use this history of unfairness to influence current decisions. In the next section, we will detail the mathematical models for these methods in the operating envelope context.

B. Problem formulation

In this section, we detail the formulations of the operating envelope problem used in our case study. We use a nonlinear OPF formulation for calculating operating envelopes, which represent upper limits on the net export at every node. For brevity, we do not consider lower limits. The decision variables are p_i^{OE}, the operating envelope at each node i; p_{ij}, the active power flowing on the line from node i to node j; q_{ij}, the reactive power flowing on the line from node i to node j; $v_i = |V_i|^2$, the squared voltage magnitude at each node i; and $l_{ij} = |I_{ij}|^2$, the squared current magnitude on the line connecting nodes i and j. Operating envelopes p^{OE} can be found without considering fairness by solving

$$\max \sum_{i \in \mathcal{N}} p_i^{\mathrm{OE}} \tag{1a}$$

$$\text{s.t. } 0 \leq p_i^{\mathrm{OE}} \leq p_i^f \quad \forall i \in \mathcal{N} \tag{1b}$$

$$\sum_{i:i\to j} (p_{ij} - r_{ij} l_{ij}) + p_j^{\mathrm{OE}} = \sum_{k:j\to k} p_{jk}, \ \forall j \in \mathcal{N} \tag{1c}$$

$$\sum_{i:i\to j} (q_{ij} - x_{ij} l_{ij}) - q_j^{\mathrm{d}} = \sum_{k:j\to k} q_{jk}, \ \forall j \in \mathcal{N} \tag{1d}$$

$$v_i = v_j + 2(r_{ij} p_{ij} + x_{ij} q_{ij}) - (r_{ij}^2 + x_{ij}^2) l_{ij}, \quad \forall j \in \mathcal{N} \tag{1e}$$

$$p_{ij}^2 + q_{ij}^2 = l_{ij} v_i, \quad \forall ij \in \mathcal{L} \tag{1f}$$

$$p_{ij}^2 + q_{ij}^2 \leq \overline{s}_{ij}^2, \quad \forall ij \in \mathcal{L} \tag{1g}$$

$$\underline{v} \leq v_i \leq \overline{v}, \quad \forall i \in \mathcal{N} \tag{1h}$$

where p_i^{f} is the forecasted power injection at node i. Constraints (1c) and (1d) enforce active and reactive power balance [47], where notation $i : i \to j$ specifies the sum over all lines ij injecting power into j, and $k : j \to k$ specifies the sum over all lines jk consuming power from j. Constraint (1e) defines the voltage drop between bus i and the downstream bus j. Constraint (1f) defines the squared apparent power flowing from bus i to j and (1g) limits it. Finally, constraint (1h) enforces the voltage limits at each bus. This formulation assumes monotonicity, i.e., if p^{OE} defines a valid operating envelope, then any injection less than p^{OE} is feasible. We note that this may not always be true in practice since (1f) is nonconvex.[1] To calculate the operating envelopes across a time horizon, one could solve (1) for each time period. The formulation given in (1) will be henceforth referred to as the Unfair formulation.

To incorporate fairness as it has been previously done, the objective function is changed to align with the notions of fairness introduced in the previous subsection, giving

$$\min \sum_{i\in\mathcal{N}} (1-\lambda)\left(p_i^{\mathrm{f}} - p_i^{\mathrm{OE}}\right)^2 - \lambda\gamma \tag{2a}$$

$$\text{s.t.}(1\text{b}) - (1\text{h})$$

$$\gamma \leq p_i^{\mathrm{OE}}, \forall i \in \mathcal{N} \tag{2b}$$

where λ is a weighting parameter. In this formulation, the second term in the objective function, along with (2b), maximizes the minimum operating envelope across the nodes. Like the Unfair formulation, this formulation is a single-period problem that can be solved for each time period in a time horizon. The formulation given in (2) will be henceforth referred to as the Fair at Each Time (FET) formulation.

The first method we propose to incorporate fairness over time is to calculate all operating envelopes across a time horizon using a single, multi-period optimization problem, such as

$$\min \sum_{i\in\mathcal{N}} (1-\lambda)\left[\sum_{t\in\mathcal{T}} \left(p_{i,t}^{\mathrm{f}} - p_{i,t}^{\mathrm{OE}}\right)^2\right] - \lambda\gamma \tag{3a}$$

$$\text{s.t. } 0 \leq p_{i,t}^{\mathrm{OE}} \leq p_{i,t}^{f} \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \tag{3b}$$

$$\sum_{i:i\to j} (p_{ij,t} - r_{ij} l_{ij,t}) + p_{j,t}^{\mathrm{OE}} = \sum_{k:j\to k} p_{jk,t}, \quad \forall j \in \mathcal{N}, \forall t \in \mathcal{T} \tag{3c}$$

$$\sum_{i:i\to j} (q_{ij,t} - x_{ij} l_{ij,t}) - q_{j,t}^{\mathrm{d}} = \sum_{k:j\to k} q_{jk,t}, \quad \forall j \in \mathcal{N}, \forall t \in \mathcal{T} \tag{3d}$$

$$\begin{aligned} v_{i,t} = v_{j,t} &+ 2(r_{ij} p_{ij,t} + x_{ij} q_{ij,t}) \\ &- (r_{ij}^2 + x_{ij}^2) l_{ij,t}, \qquad \forall j \in \mathcal{N}, \forall t \in \mathcal{T} \end{aligned} \tag{3e}$$

$$p_{ij,t}^2 + q_{ij,t}^2 = l_{ij,t} v_{i,t}, \quad \forall ij \in \mathcal{L}, \forall t \in \mathcal{T} \tag{3f}$$

$$p_{ij,t}^2 + q_{ij,t}^2 \leq \overline{s}_{ij}^2, \quad \forall ij \in \mathcal{L}, \forall t \in \mathcal{T} \tag{3g}$$

$$\underline{v} \leq v_{i,t} \leq \overline{v}, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \tag{3h}$$

$$\gamma \leq \sum_{t\in\mathcal{T}} p_{i,t}^{\mathrm{OE}}, \forall i \in \mathcal{N} \tag{3i}$$

where the second term in the objective function, along with (3i), maximizes the minimum across all nodes of the sum of OE allocations over the time horizon and the variable γ couples the allocation decisions across time periods. The formulation given in (3) will be henceforth referred to as the Fair Over Time (FOT) formulation. In a stochastic setting, this could be implemented with model predictive control (MPC) and/or stochastic optimization.

The second method we propose for enforcing fairness over time is to incorporate the history of unfairness. This method of incorporating fairness can be formulated as either a single-period problem to be solved once for each time period in the time horizon, as with the Unfair and FET formulations. Alternatively, it could be formulated as an MPC problem where a time horizon $1, ..., T$ is divided into smaller time periods $\tau_1, ..., \tau_2$, and a series of multi-period problems could be solved sequentially to obtain allocations for the entire horizon. In this paper, we use the former approach for simplicity. Given a function that captures past unfairness for node i, $H_{i,\tau_1}^{\mathrm{OE}}$, the Fair using History (FUH) formulation is

$$\min \sum_{i\in\mathcal{N}} (1-\lambda)\left[\sum_{t=\tau_1}^{\tau_2} H_{i,\tau_1}^{\mathrm{OE}} \left(p_{i,t}^{\mathrm{f}} - p_{i,t}^{\mathrm{OE}}\right)^2\right] - \lambda\gamma \tag{4a}$$

$$\text{s.t.}(3\text{b}) - (3\text{h}) \text{ with } \mathcal{T} \text{ replacedwith } [\tau_1, \tau_2]$$

$$\gamma \leq \sum_{t\in[\tau_1,\tau_2]} p_{i,t}^{\mathrm{OE}}, \forall i \in \mathcal{N} \tag{4b}$$

Here, we define $H_{i,\tau_1}^{\mathrm{OE}}$ as the weighted moving average of the squared power injection curtailment at node i at time τ, i.e.,

$$H_{i,\tau_1}^{\mathrm{OE}} = \frac{k\left(p_{i,\tau_1-1}^{\mathrm{OE}} - p_{i,\tau_1-1}^{\mathrm{r}}\right)^2 + ... + \left(p_{i,\tau_1-k}^{\mathrm{OE}} - p_{i,\tau_1-k}^{\mathrm{r}}\right)^2}{k + (k-1) + ... + 2 + 1} \tag{5}$$

where $p_{i,\tau}^{\mathrm{r}}$ is the realizable power injection at node i at time τ. In this paper, we assume perfect forecasting such that $p_{i,\tau}^{\mathrm{r}} = p_{i,\tau}^{\mathrm{f}}$, but the formulation is written in a general form.

C. Results

The network used in the case study is the 56-bus balanced distribution network introduced in [48], with no capacitor banks and only a single voltage regulator at the substation. The voltage at the substation is set to 1.02 pu and the bounds on the voltages are $\underline{v} = 0.95$ pu and $\overline{v} = 1.05$ pu. The load in the network varies over time, but every node has the same load at each time. Every node has exactly one DER but the power injection capacity varies randomly across both nodes and time. Table II shows our quantitative results across the four formulations in terms of total network flexibility (performance) and fairness in terms of the smallest

[1] Often a linear approximation of the power flow equations is used for scalability or a convex restriction is used to ensure there are no gaps in the feasible region of the operating envelopes [34]; however, linearizations are inaccurate and convex restrictions can be conservative and/or hard to find.

operating envelope, and the variances of both the operating envelope allocations and total curtailment across the nodes. For the three fair formulations, the results are given for three values of λ. Table II shows that the unfair formulation, (1), results in the largest total network flexibility but in the least fair allocation of operating envelopes over the time horizon based on both notions of fairness. FET results in slightly larger total flexibility than FOT, with the gap between them increasing with increasing λ. However, FOT results in the largest smallest operating envelope and the smallest variance in operating envelopes for each value of λ. The two formulations result in similar curtailment variances. For each value of λ, FUH results in the smallest network flexibility but also the smallest variance for both operating envelopes and curtailment.

Figures 1, 2, and 3 show the results for $\lambda = 0.5$. The total accumulated operating envelope allocation at each node is shown for each formulation in Fig. 1. Figure 1(a) shows that the Unfair formulation allocates large upper limits consistently to some nodes, and very small or even zero allocations to nodes at the feeder's ends. Figure 1(b) shows that FOT gives similar allocations to FET, but is able to increase the allocation of more constrained nodes without sacrificing much allocation from the other nodes. Figure 2 shows similar trends to the accumulated allocations, but shows the percentage of capacity curtailed at each node over the time horizon. This illustrates how the formulations compare with respect to the notion of fairness in intended operation. Under this notion, all the bars in Fig. 2 would be the same height for the fairest outcome.

The amount of injection capacity in the network also impacts the results. In Table III, results for each formulation are shown as the DER capacities are uniformly increased. The first column in the table represents the multiplier, ω, on the nominal capacity value, where a multiplier of 1 implies capacities are at their nominal values. For these test cases, the second term in the objective function for all three fair formulations was multiplied by ω to maintain a similar weight distribution in the objectives. In the case where $\omega = 0.1$, all of the nodes can inject at their full capacities without violating network constraints, resulting in no or negligible curtailment except for under FUH. As the need for curtailment increases with increasing capacity, FOT is able to outperform FET in terms of network flexibility while maintaining better max-min fairness and comparable fairness of intended operation.

Figure 3 illustrates the impact of biasing the current decisions using the historical unfairness. The light blue bars represent the operating envelope allocation in the first time step and the orange bars represent the weighted moving average of historical unfairness at the end of the time horizon. The level of fairness at the end of the time horizon is significantly higher than the level of fairness at the beginning.

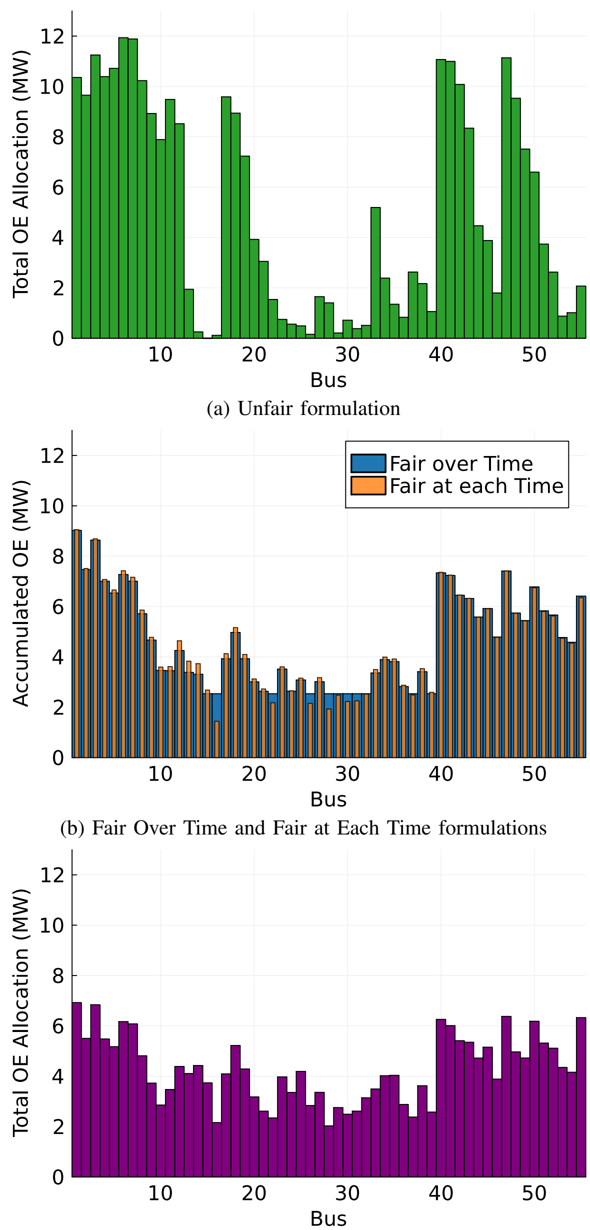
V. Conclusions

In this paper, we discussed time-varying fairness approaches from ML literature and compared them with fair-

TABLE II
PERFORMANCE VS. FAIRNESS

λ	Formulation	Smallest Allocation	OE Variance	Curtailment Variance	Network Flexibility
	Unfair	0	17.76	14.91	276.0
	FOT	2.23	3.63	2.70	253.9
0.25	FET	1.44	3.73	2.71	254.3
	FUH	1.99	2.35	0.93	236.4
	FOT	2.53	3.50	2.68	253.3
0.5	FET	1.44	3.72	2.71	254.2
	FUH	2.03	1.74	0.94	235.7
	FOT	2.93	3.23	2.72	251.9
0.75	FET	1.44	3.73	2.71	254.2
	FUH	1.95	1.99	0.94	236.8

OE stands for Operating Envelope

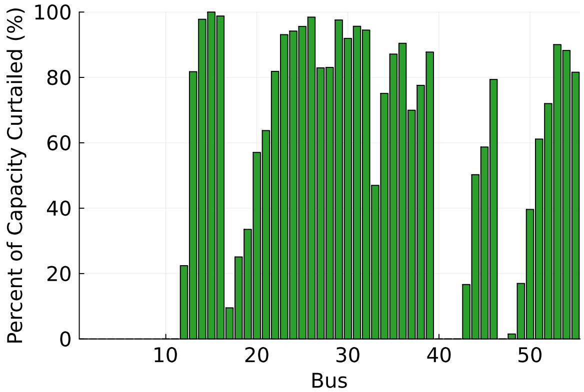


(a) Unfair formulation

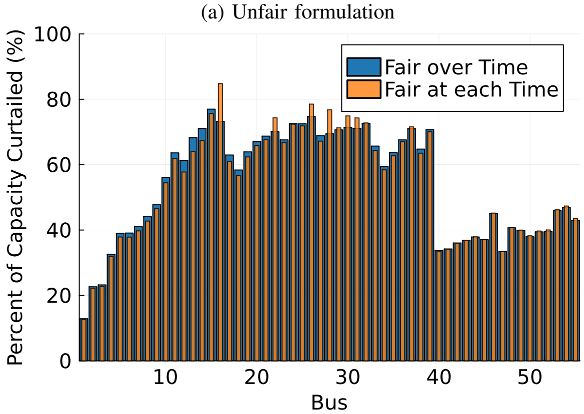
(b) Fair Over Time and Fair at Each Time formulations

(c) Fair Using History Formulation

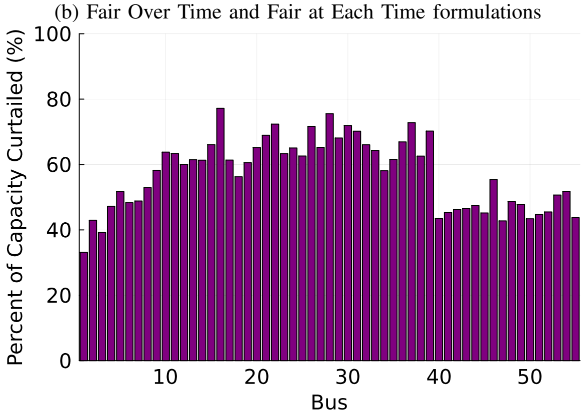
Fig. 1. Accumulated operating envelope allocation at each node in the network



(a) Unfair formulation



(b) Fair Over Time and Fair at Each Time formulations



(c) Fair Using History Formulation

Fig. 2. Total percentage of forecasted capacity curtailed over the time horizon, $\frac{\sum_{t\in\mathcal{T}} p^f_{i,t} - p^{\mathrm{OE}}_{i,t}}{\sum_{t\in\mathcal{T}} p^f_{i,t}}$, at each node in the network

ness approaches that have been previously implemented in DER coordination literature. From this analysis, we outlined a principled method for incorporating fairness into power systems problems: 1) Identify relevant notion(s) of fairness, 2) mathematically formalize the notion(s) of fairness, and 3) identify the fair-decision-making mechanism. By following these steps, authors can help other researchers efficiently assess relevance, compare approaches, and integrate key fairness methods into their own work.

Inspired by time-varying fairness approaches from the ML literature, we proposed incorporating fairness over time and illustrated the concept using the operating envelope problem.

TABLE III
PERFORMANCE VS. FAIRNESS AS DER CAPACITIES INCREASE

ω	Formulation	Smallest Allocation	Variance OE	Variance Curtailment	Network Flexibility
0.1	Unfair	0.789	0.011	0	54.44
	FOT	0.789	0.011	0	54.37
	FET	0.787	0.011	0	54.38
	FUH	0.668	0.010	0	47.00
1	Unfair	0	17.76	14.91	276.0
	FOT	2.53	3.50	2.68	253.4
	FET	1.44	3.72	2.71	254.3
	FUH	2.03	1.99	0.94	235.7
3	Unfair	0	108.11	93.96	331.14
	FOT	1.74	20.48	19.38	287.68
	FET	0.74	20.76	19.36	287.03
	FUH	0.72	9.13	7.10	264.26

ω is the DER capacity multiplier, OE stands for Operating Envelope

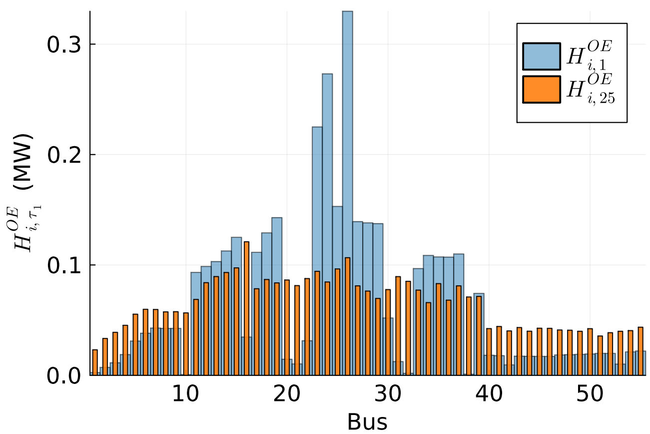


Fig. 3. Comparison between the operating envelope allocation in the first time step, $H^{\mathrm{OE}}_{i,1}$, and the weighted average of historical unfairness at the end of the time horizon, $H^{\mathrm{OE}}_{i,25}$.

Results from the case study suggest that enforcing fairness over time rather than at every time step can lead to fairer outcomes without sacrificing total network flexibility, as compared to existing fair operating envelope implementations.

In future work, we aim to extend the notion of fairness over time to other DER coordination problems, and to explicitly consider fairness with respect to monetary costs/benefits.

REFERENCES

[1] J. Mathieu, "Algorithms for energy justice," in *Women in Power: Research and Development Advances in Electric Power Systems, Springer Women in Engineering and Science Series*, J. S. Tietjen, M. D. Ilic, L. B. Tjernberg, and N. N. Schulz, Eds. Cham, Switzerland: Springer Nature Switzerland AG, 2023.

[2] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Transactions on Networking*, vol. 8, no. 5, pp. 556–567, 2000.

[3] X. Xu and H. Yu, "A game theory approach to fair and efficient resource allocation in cloud computing," *Mathematical Problems in Engineering*, vol. 2014, no. 1, p. 915878, 2014.

[4] X. Zhang and M. Liu, "Fairness in learning-based sequential decision algorithms: A survey," in *Handbook of Reinforcement Learning and Control*. Springer, 2021, pp. 525–555.

[5] D. Pessach and E. Shmueli, "A review on fairness in machine learning," *ACM Computing Surveys (CSUR)*, vol. 55, no. 3, pp. 1–44, 2022.

[6] L. Bird, C. Walker, J. Womble, S. Atcitty, R. Trevizan, L. Concessao, H. Meenawat, and B. Tarekegne, "Distributed energy resources as an equity asset: Lessons learned from deployments in disadvantaged communities," *IEEE Power and Energy Magazine*, vol. 22, no. 4, pp. 64–74, 2024.

[7] M. Sufyan, N. A. Rahim, M. A. Muhammad, C. K. Tan, S. R. S. Raihan, and A. H. A. Bakar, "Charge coordination and battery lifecycle analysis of electric vehicles with V2G implementation," *Electric Power Systems Research*, vol. 184, p. 106307, 2020.

[8] S. Poudel, M. Mukherjee, R. Sadnan, and A. P. Reiman, "Fairness-aware distributed energy coordination for voltage regulation in power distribution systems," *IEEE Transactions on Sustainable Energy*, vol. 14, no. 3, pp. 1866–1880, 2023.

[9] J. Soares, F. Lezama, R. Faia, S. Limmer, M. Dietrich, T. Rodemann, S. Ramos, and Z. Vale, "Review on fairness in local energy systems," *Applied Energy*, vol. 374, p. 123933, 2024.

[10] K. Petrou, M. Z. Liu, A. T. Procopiou, L. F. Ochoa, J. Theunissen, and J. Harding, "Operating envelopes for prosumers in LV networks: A weighted proportional fairness approach," in *IEEE PES Innovative Smart Grid Technologies Europe*, 2020, pp. 579–583.

[11] M. R. Alam, P. T. Nguyen, L. Naranpanawe, T. K. Saha, and G. Lankeshwara, "Allocation of dynamic operating envelopes in distribution networks: Technical and equitable perspectives," *IEEE Transactions on Sustainable Energy*, vol. 15, no. 1, pp. 173–186, 2023.

[12] S. Alyami, Y. Wang, C. Wang, J. Zhao, and B. Zhao, "Adaptive real power capping method for fair overvoltage regulation of distribution networks with high penetration of PV systems," *IEEE Transactions on Smart Grid*, vol. 5, no. 6, pp. 2729–2738, 2014.

[13] K. Petrou, A. T. Procopiou, L. Gutierrez-Lagos, M. Z. Liu, L. F. Ochoa, T. Langstaff, and J. M. Theunissen, "Ensuring distribution network integrity using dynamic operating limits for prosumers," *IEEE Transactions on Smart Grid*, vol. 12, no. 5, pp. 3877–3888, 2021.

[14] B. Liu and J. H. Braslavsky, "Robust dynamic operating envelopes for DER integration in unbalanced distribution networks," *IEEE Transactions on Power Systems*, 2023.

[15] S. Caton and C. Haas, "Fairness in machine learning: A survey," *ACM Computing Surveys*, vol. 56, no. 7, pp. 1–38, 2024.

[16] Z. Tang, J. Zhang, and K. Zhang, "What-is and how-to for fairness in machine learning: A survey, reflection, and perspective," *ACM Computing Surveys*, vol. 55, no. 13s, pp. 1–37, 2023.

[17] C. Dwork, M. Hardt, T. Pitassi, O. Reingold, and R. Zemel, "Fairness through awareness," in *Innovations in Theoretical Computer Science Conference*, 2012, pp. 214–226.

[18] N. Mehrabi, F. Morstatter, N. Saxena, K. Lerman, and A. Galstyan, "A survey on bias and fairness in machine learning," *ACM Computing Surveys (CSUR)*, vol. 54, no. 6, pp. 1–35, 2021.

[19] M. Lee, S. Ah, and L. Floridi, "Algorithmic fairness in mortgage lending: from absolute conditions to relational trade-offs," *Minds and Machines*, vol. 31, no. 1, pp. 165–191, 2021.

[20] M. Feldman, S. A. Friedler, J. Moeller, C. Scheidegger, and S. Venkatasubramanian, "Certifying and removing disparate impact," in *ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 2015, pp. 259–268.

[21] M. Hardt, E. Price, and N. Srebro, "Equality of opportunity in supervised learning," *Advances in Neural Information Processing Systems*, vol. 29, 2016.

[22] Y. Hu and L. Zhang, "Achieving long-term fairness in sequential decision making," in *AAAI Conference on Artificial Intelligence*, vol. 36, no. 9, 2022, pp. 9549–9557.

[23] Y. Ge, S. Liu, R. Gao, Y. Xian, Y. Li, X. Zhao, C. Pei, F. Sun, J. Ge, W. Ou, *et al.*, "Towards long-term fairness in recommendation," in *ACM International Conference on Web Search and Data Mining*, 2021, pp. 445–453.

[24] M. Morik, A. Singh, J. Hong, and T. Joachims, "Controlling fairness and bias in dynamic learning-to-rank," in *ACM SIGIR Conference on Research and Development in Information Retrieval*, 2020, pp. 429–438.

[25] A. J. Biega, K. P. Gummadi, and G. Weikum, "Equity of attention: Amortizing individual fairness in rankings," in *ACM SIGIR Conference on Research and Development in Information Retrieval*, 2018, p. 405–414.

[26] S. Jabbari, M. Joseph, M. Kearns, J. Morgenstern, and A. Roth, "Fairness in reinforcement learning," in *International Conference on Machine Learning*. PMLR, 2017, pp. 1617–1626.

[27] S. Gupta and V. Kamble, "Individual fairness in hindsight," *Journal of Machine Learning Research*, vol. 22, no. 144, pp. 1–35, 2021.

[28] A. D'Amour, H. Srinivasan, J. Atwood, P. Baljekar, D. Sculley, and Y. Halpern, "Fairness is not static: deeper understanding of long term fairness via simulation studies," in *Conference on Fairness, Accountability, and Transparency*, 2020, pp. 525–534.

[29] H. A. U. Khan, B. Ünel, and Y. Dvorkin, "Electricity tariff design via lens of energy justice," *Omega*, vol. 117, p. 102822, 2023.

[30] L. Li, M. Dong, D. Song, J. Yang, and Q. Wang, "Distributed and real-time economic dispatch strategy for an islanded microgrid with fair participation of thermostatically controlled loads," *Energy*, vol. 261, p. 125294, 2022.

[31] J. S. Russell, P. Scott, and J. Iria, "Network-secure aggregator operating regions with flexible dispatch envelopes in unbalanced systems," *Electric Power Systems Research*, vol. 235, p. 110728, 2024.

[32] N. Nazir and M. Almassalkhi, "Convex inner approximation of the feeder hosting capacity limits on dispatchable demand," in *IEEE Conference on Decision and Control*, 2019, pp. 4858–4864.

[33] M. Z. Liu, L. F. Ochoa, P. K. C. Wong, and J. Theunissen, "Using OPF-based operating envelopes to facilitate residential DER services," *IEEE Transactions on Smart Grid*, vol. 13, no. 6, pp. 4494–4504, 2022.

[34] N. Nazir and M. Almassalkhi, "Grid-aware aggregation and realtime disaggregation of distributed energy resources in radial networks," *IEEE Transactions on Power Systems*, vol. 37, no. 3, pp. 1706–1717, 2022.

[35] Y. Yi and G. Verbič, "Fair operating envelopes under uncertainty using chance constrained optimal power flow," *Electric Power Systems Research*, vol. 213, p. 108465, 2022.

[36] H. Moring, S. Jang, N. Ozay, and J. L. Mathieu, "Nodal operating envelopes vs. network-wide constraints: What is better for network-safe coordination of DERs?" *Electric Power Systems Research*, vol. 234, p. 110702, 2024.

[37] N. L. Hanus, J. Barlow, A. Satchwell, and P. Cappers, "Assessing the current state of US energy equity regulation and legislation," Lawrence Berkeley National Laboratory. Retrieved from https://escholarship.org/uc/item/5170t55k, 2023.

[38] J. Barlow, R. Tapio, and B. Tarekegne, "Advancing the state of energy equity metrics," *The Electricity Journal*, vol. 35, no. 10, p. 107208, 2022.

[39] T. Lanckton and S. DeVar, "Justice in 100 metrics," Initiative for Energy Justice. Retrieved from https://iejusa.org/wp-content/uploads/2021/03/Justice-in-100-Metrics-2021.pdf, 2021.

[40] D. Nock, A. J. Jones, S. Bouzarovski, H. Thomson, and D. J. Bednar, "Reducing energy burden in the power sector: Metrics for assessing energy poverty," *IEEE Power and Energy Magazine*, vol. 22, no. 4, pp. 26–37, 2024.

[41] A. Kody, A. West, and D. K. Molzahn, "Sharing the load: Considering fairness in de-energization scheduling to mitigate wildfire ignition risk using rolling optimization," in *IEEE Conference on Decision and Control*, 2022, pp. 5705–5712.

[42] S. Taylor, G. Setyawan, B. Cui, A. Zamzam, and L. A. Roald, "Managing wildfire risk and promoting equity through optimal configuration of networked microgrids," in *Proceedings of the 14th ACM International Conference on Future Energy Systems*, 2023, pp. 189–199.

[43] S. Cong, D. Nock, Y. L. Qiu, and B. Xing, "Unveiling hidden energy poverty using the energy equity gap," *Nature Communications*, vol. 13, no. 1, p. 2456, 2022.

[44] C. Zhang and J. A. Shah, "Fairness in multi-agent sequential decision-making," *Advances in Neural Information Processing Systems*, vol. 27, 2014.

[45] I. Bistritz, T. Baharav, A. Leshem, and N. Bambos, "My fair bandit: Distributed learning of max-min fairness with multi-player bandits," in *International Conference on Machine Learning*, 2020, pp. 930–940.

[46] P. Lahoti, A. Beutel, J. Chen, K. Lee, F. Prost, N. Thain, X. Wang, and E. Chi, "Fairness without demographics through adversarially reweighted learning," *Advances in Neural Information Processing Systems*, vol. 33, pp. 728–740, 2020.

[47] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification—part I," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2554–2564, 2013.

[48] S. Bolognani and S. Zampieri, "On the existence and linear approximation of the power flow solution in power distribution networks," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 163–172, 2016.