Fitness Value of Subjective Information for Living Organisms

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ABSTRACT

Information theory can be used to describe the gain of evolutionary fitness that an organism obtains from sensing, processing, and acting on environmental information. This paper considers the fitness value of subjective information, *i.e.*, the context-dependent value of different kinds of information. A simplified model is given in which the organism requires two essential nutrients, and can prioritize sensing for one or the other. It is shown that a subjective strategy, in which the organism prioritizes a less abundant nutrient for sensing, leads to higher fitness than a balanced strategy, in which total information is maximized and the meaning of the acquired information is disregarded. Using this model, the fitness advantage of subjective information admits an analytical solution, and it is shown that subjective information is more advantageous when the organism's knowledge of the environment is less precise.

CCS CONCEPTS

• **Mathematics of computing** → **Information theory**; • **Computing methodologies** → *Modeling and simulation*; • **Hardware** → **Biology-related information processing**.

KEYWORDS

Fitness-optimal strategies, Chemotaxis, Biological information processing

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1 INTRODUCTION

Organisms must navigate uncertain environments, which contain rewards and dangers. In response to this uncertainty, they gather information about their environments and respond with strategic behavior. For example, in the presence of changing patterns of odors, mice are known to extract information about their environment and change their behavior [1]. Information and uncertainty are connected with the organism's evolutionary fitness: the mouse in the example might smell food, allowing it to thrive and reproduce, or it might smell a predator, allowing it to avoid capture and survive.

The connection between evolutionary fitness, information processing, and behavior has long been of interest. One description of evolutionary fitness, known as Malthusian fitness, holds that an organism's fitness is given by the logarithm of its rate of growth [22]. This form of fitness has an information-theoretic interpretation: the log growth rate of wealth (*e.g.*, of investments) can be related to mutual information [6, 14]. An explicit connection between fitness and information-theoretic quantities has been studied in both the information-theoretic and biological literature [8, 15, 17], and its importance was recognized by Berger, who noted that evolution should guide the choice of a distortion function in a rate-distortion problem [7, Sec. 6.4]. The connection with behavior has also been investigated: the information-fitness relationship has been investigated considering two parts of a cell's overall strategy, sensing and processing, both abstractly [11, 16] and in specific examples such as navigating uncertain chemical gradients [20]. Elucidating the relation between fitness and information processing may also be important for bioengineering and synthetic biology [18].

To date, most work in this area uses classical information theory, in which meaning is disregarded and all bits have the same subjective value; however, for an organism, sensory information can have significantly different value in terms of survival. This observation can be formalized: previous work has shown that information-optimal strategies (*i.e.*, maximizing information without regard to semantics) are not fitness-optimal [2, 5]. Meanwhile, recent developments in information theory also recognize that the semantics of

information can be used to more efficiently accomplish a specific task: for example, much like an animal's sensory system, the sensors on a self-driving car gather an enormous amount of information, only a small fraction of which is relevant to safe operation [13]. Addressing subjective value is thus an important new direction for understanding the fitness value of information.

Building on earlier results [3], the main contribution of this paper is a simplified model for studying the effect of subjective information in biological information processing, and an analytical solution for the subjective information in the model. The new model enables the analytical solution, going beyond previous work. We show that subjective information provides a fitness advantage over a strictly information-maximizing approach. We also show that there are diminishing returns with the total number of receptors, i.e., the advantage of semantic information is higher when the organism's knowledge of the environment becomes less precise.

The remainder of this paper is organized as follows. In Sec. 2, we define the abstract model and express analytical solutions for its fitness and subjective information. In Sec. 3, we discuss relevant numerical results. Finally, we conclude the paper in Sec. 4.

2 ABSTRACT MODEL

In this paper, we define a simple abstract model of a biological organism that includes the essential elements for the subsequent derivations. This organism is thought to be unicellular, capable of sensing environmental concentrations of essential nutrients, and directing its movements accordingly, *i.e.*, chemotaxis. From this model, we can derive two main quantities, namely, the reward and the information metrics.

2.1 Assumptions

The abstract model, as illustrated in Fig. 1, is based on the following simplifying assumptions:

- The unicellular organism is located in a 1D environment.
- The environment contains concentrations of two essential nutrients, 'A' and 'B' molecules, at each location.
- The organism has internal stores of 'A' and 'B' molecules, respectively, which correspond to the internal state of the organism.
- The organism's goal is to maximize the lower value of its 'A' and 'B' stores.
- At each time step, the organism senses the 'A' and 'B' molecule concentrations at its current location through 'A' and 'B' type receptors. It may then direct its movement to an adjacent location according to the sensed concentration and its internal state. The organism then increases its 'A' and 'B' stores proportional to their respective concentrations at this next location.
- The concentration is sensed through a ligand-receptor binding process, which is modeled as a binomial distribution that returns the numbers of bound receptors of type 'A' and 'B' given the 'A' and 'B' concentrations and the total number of 'A' and 'B' receptors, respectively. For simplicity, we assume that the receptor-ligand interaction process is sampled from equilibrium [12, 19].
- The numbers of receptors of type 'A' and 'B' may be set at each time step (*e.g.*, *via* an activator-inhibitor or nutrient-sensitive signaling cascade [9, 10, 23]), and cannot exceed a total sum of 'A' and 'B' receptors. These receptor numbers determine the precision of the organism in sensing the 'A' and 'B' molecule concentrations.
- The organism is also subject to a stress defined as a penalty on the internal stores for each time step, which is a constant cost for living and an additional cost for each receptor regardless of its type.

Env.\Loc.	$l=0$	$l=1$	$l=2$
$\hat{e}=1$	$(3,1)$	$(2,2)$	$(1,3)$
$\hat{e}=2$	$(3,3)$	$(2,1)$	$(1,2)$
$\hat{e}=3$	$(3,2)$	$(2,3)$	$(1,1)$

Table 1: Concentrations of nutrients $(C_A(l,\hat{e}), C_B(l,\hat{e}))$, at different combinations of environment $\hat{e} \in \{1,2,3\}$ and location $l \in \{0,1,2\}$.

2.2 Implementation

In the following, as shown in Fig. 1, we describe an implementation of the model that allows for tractable analytical solutions:

- The environment is finite and periodic (moving right when at the rightmost location would result in the leftmost location) (cf. Tab. 1)
- The environment consists of three possible locations in total for the organism.
- At the start of each time step, the environment and location of the organism are chosen uniformly and at random.
- The 'A' and 'B' molecule concentrations are distributed periodically across these three locations according to the descending (for 'A') and ascending (for 'B') values 3,2,1 and a cyclic permutation of the values 1,2,3, respectively.
- The relative distribution of the 'A' and 'B' molecule concentrations at each time step is chosen at random. This results into three possible different environments shown in Fig. 1.
- The binomial model of the ligand-receptor binding is expressed as the probability of n_m receptors being bound as follows:

$$P_{\mathrm{Bi}}(n_m; N_m, l, \hat{e}) = \binom{N_m}{n_m} p_C(l,\hat{e})^{n_m} \left(1 - p_C(l,\hat{e})\right)^{N_m - n_m}, \tag{1}$$

where N_m and n_m are the total number of receptors and the number of bound receptors, respectively, of type $m \in \{A, B\}$. In the following, we will invariably denote n_A and n_B with a and b, respectively. Here p_C is the probability, as a function of l and $\hat{e}$, that a single receptor is bound in the same time step (whose duration is assumed to be long enough so that the process is in equilibrium [21]), defined as

$$p_C(l,\hat{e}) = \frac{C_m(l,\hat{e})}{C_m(l,\hat{e}) + K_d}, \tag{2}$$

where $C_m(l,\hat{e})$ is the concentration of molecules of type $m \in \{A, B\}$ at the current location l in environment $\hat{e}$, and K_d is the dissociation constant of the binding process.

- The movement of the organism can be either one location to the left or to the right of the current location. The organism may also

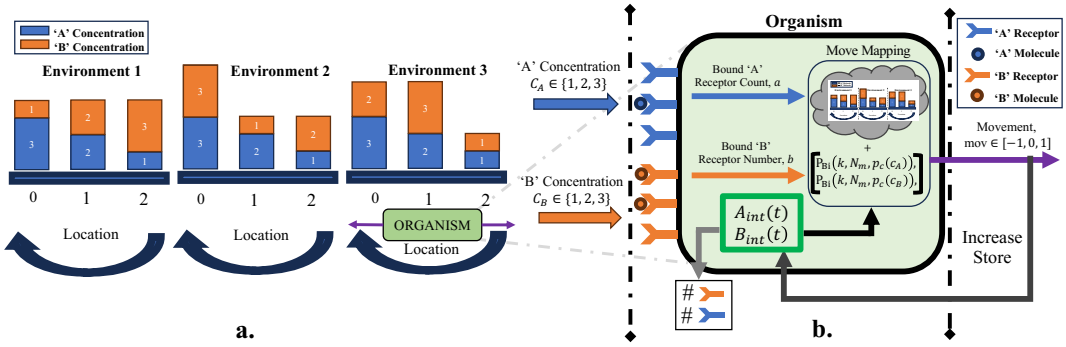

Figure 1: (a) The periodic environmental 'A' and 'B' concentrations at each location where information about one distribution gives no information about the other. (b) The organism senses its surrounding concentration profile based on a binding process and moves based on its internal state and knowledge of the binomial distributions given the environment. The organism can then increase its store of 'A' and 'B' type molecules and choose to reallocate its receptors based on Δ.

not move. The periodicity of the environment ensures that any movement is feasible from/to any location.

- The organism stress is implemented as a penalty as follows:

$$S_{N_{\text{tot}}} = s_1 + s_2 N_{\text{tot}}\,, \tag{3}$$

where s_1 is a constant penalty term, related to the basal energy consumption of the organism, s_2 corresponds to the cost for the organisms to use a receptor, and N_{tot} is the total number of receptors of any type, which is a specific value for each organism.

- The organism's goal, while following a ***strategy***, is to maximize its growth. In general, cells divide and split resources equally between daughter cells. In our model, the organism must attempt to equalize its internal resources to maximize the survival of these daughter organisms. The incremental reward (objective function) is given as

$$\hat{R}(t) = \min[A_{\text{int}}(t), B_{\text{int}}(t)] - \min[A_{\text{int}}(t-1), B_{\text{int}}(t-1)]\,, \tag{4}$$

where $A_{\text{int}}(t)$ and $B_{\text{int}}(t)$ are the values of the internal stores of 'A' and 'B', respectively, at time step t.

- At each time step, the organism combines the probabilities computed via (1) with the *a priori* knowledge of the possible food distributions to move into the location that maximizes $\hat{R}(t)$ (4) for a given **strategy**.

2.3 Analytical Solution

In this paper, the analytical solution to the model described in Sec. 2.1 and 2.2 corresponds to the expression of the expected (asymptotic) incremental reward $\hat{R}_{\text{strategy}}$ that the organism would obtain throughout its life, as follows

$$\begin{aligned}\hat{R}_{\text{strategy}} &= \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \hat{R}(t) \\ &= \mathbb{E}_{\Delta}\left[R(\Delta, N_A(\Delta), N_B(\Delta))\right] - S_{N_{\text{tot}}}\end{aligned} \tag{5}$$

where the expected reward per internal state $R(\Delta, N_A(\Delta), N_B(\Delta))$ is the expected reward at the end of any time step (after the organism may have moved), Δ is a possible internal state of the organism at the beginning of the time step, defined as the difference between the two values of the internal stores, *i.e.*, $\Delta = A_{\text{int}} - B_{\text{int}}$, and $N_A(\Delta), N_B(\Delta)$ are two functions that map Δ to a specific number of receptors of type 'A' and 'B', respectively, according to the organism ***strategy***, as

$$N_m : \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \xrightarrow{\textbf{\textit{strategy}}} \mathbb{N} \tag{6}$$

where m is equal to 'A' or 'B'. Equation (5) holds given the ergodic property of the internal state process.

2.3.1 Expected Reward per Internal State. $R(\Delta, N_A(\Delta), N_B(\Delta))$ is computed as the average of the expected reward $r(a, b, \Delta, N_A(\Delta), N_B(\Delta))$ given a number of bound receptors a and b of type 'A' and 'B', respectively, an internal state Δ and the corresponding number of receptors specified by the adopted ***strategy***:

$$\begin{aligned}R(\Delta, N_A(\Delta), N_B(\Delta)) = \\ \sum_{a=0}^{N_A(\Delta)} \sum_{b=0}^{N_B(\Delta)} \Big[r(a, b, \Delta, N_A(\Delta), N_B(\Delta)) \cdot \\ \cdot P(a; N_A(\Delta)) P(b; N_B(\Delta)) \Big]\,,\end{aligned} \tag{7}$$

where $P(k; N_m)$ is the distribution of k bound receptors when the cell has N_m receptors of type 'm' over all possible concentrations c of 'm' molecules, as follows:

$$P(k; N_m) = \frac{1}{9}\sum_{l}\sum_{\hat{e}} P_{\text{Bi}}(k; N_m, l, \hat{e})\,, \tag{8}$$

where $P_{\text{Bi}}(.)$ is as in (1). $r(a, b, \Delta, N_A(\Delta), N_B(\Delta))$ in (7) is the maximum expected reward (considering all choices of movement) given a number of bound receptors a and b, and a number $N_A(\Delta)$, and $N_B(\Delta)$ of receptors of type 'A' and 'B', respectively, defined as

$$\begin{aligned}r(a, b, \Delta, N_A(\Delta), N_B(\Delta)) = \max\Big[\Big\{m^{(-1)}(a, b, \Delta, N_A(\Delta), N_B(\Delta)), \\ m^{(0)}(a, b, \Delta, N_A(\Delta), N_B(\Delta)), m^{(+1)}(a, b, \Delta, N_A(\Delta), N_B(\Delta))\Big\}\Big]\,,\end{aligned} \tag{9}$$

where $m^{\text{mov}}(a, b, \Delta, N_A(\Delta), N_B(\Delta))$ is the expected reward given a choice of movement mov $\in \{(-1), (0), (1)\}$ for left movement, no

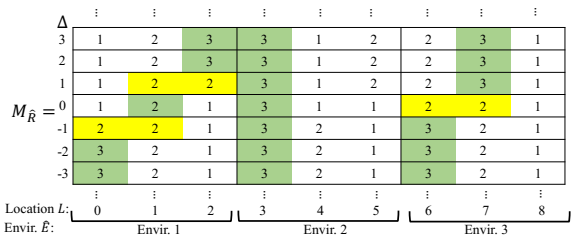

Figure 2: The matrix representing the reward $M_{\hat{R}}$ as function of the starting internal state Δ and location l in the environment $\hat{e}$. The highlighted green cells are for the largest reward in an environment given a Δ. Highlighted yellow cells show where there is more than one largest reward. The rewards remain the same where $\Delta >= 2$ (likewise for $\Delta <= -2$).

movement, and right movement, respectively, expressed as follows:

$$m^{\text{mov}}(a, b, \Delta, N_A(\Delta), N_B(\Delta)) = \sum_{l\in L}\sum_{\hat{e}\in\hat{E}} M_{\hat{R}}(\Delta, ((l+\text{mov})\%3) + 3(\hat{e}-1))\cdot P_{Lc,Ev|\text{State},n_A}(l,\hat{e}|\Delta,a)P_{Lc,Ev|\text{State},n_B}(l,\hat{e}|\Delta,b)\,, \tag{10}$$

where L is the set of all locations, $\hat{E}$ is the set of all environments, and % is the modulus operator. $M_{\hat{R}}(\Delta, i)$ is the reward matrix displayed in Fig. 2, which contains the reward for the organism after movement mov in the environment, as function of the starting internal state Δ as the matrix row index, and matrix column index i, computed in (10) from the location l in the environment $\hat{e}$, *i.e.*, the index i spans all possible locations across all possible environments. The probability $P_{Lc,Ev|\text{state},n_m}(l,\hat{e}|\Delta,k)$ of the organism being at location l and environment $\hat{e}$ given k bound m type receptors with a surrounding m type concentration of $C_m(l,\hat{e})$ is computed as

$$P_{Lc,Ev|\text{State},n_m}(l,\hat{e}|\Delta,k) = \frac{\frac{1}{9}P_{\text{Bi}}(k; N_m(\Delta), l, \hat{e})}{P(k, N_m(\Delta))}, \tag{11}$$

where $P_{\text{Bi}}(.)$ and $P(k, N_m(\Delta))$ are expressed in (1) and (8), respectively. The probability in (11) can be interpreted as the organism's guess about its own location and environment based on information on its own internal state and the number bound receptors. More generally, this ultimately skews the organisms's view of the world based on "subjective" nutritional priorities according to the adopted strategy.

2.3.2 Expected Reward per Strategy $\hat{R}_{strategy}$. If the organism adopts the ***Adaptive*** strategy, it maximizes its reward over all receptor allocations $N_A(\Delta)$, $N_B(\Delta)$ for each internal state Δ. The expected reward for the ***Adaptive*** type strategy is given as,

$$\hat{R}_{\textbf{\textit{Adaptive}}} = \sum_{\Delta} R(N_A^{\max}(\Delta), N_B^{\max}(\Delta))\cdot \Pi(\Delta, N_A^{\max}(\Delta), N_B^{\max}(\Delta)) - S_{N_{\text{tot}}}\,, \tag{12}$$

where $\Pi(\Delta, N_A(\Delta), N_B(\Delta))$ is the stationary probability distributionfor the internal state Δ, defined in Sec. 2.3.3, and $N_m^{\max}(\Delta)$ are the allocations of 'A' or 'B' type receptors that maximize the expected reward $R(\Delta, N_A(\Delta), N_B(\Delta))$ per internal state as

$$(N_A^{\max}(\Delta), N_B^{\max}(\Delta)) = \text{argmax}\,[R(\Delta, N_A(\Delta), N_B(\Delta))] \quad \text{s.t.}\;\; N_A(\Delta) + N_B(\Delta) = N_{\text{tot}}\,, \tag{13}$$

where N_{tot} is the total number of 'A' type plus 'B' type receptors.

If the organism adopts the ***Equivalent*** strategy, the receptor allocations are set to a constant and equal value for type 'A' and 'B', *i.e.*, $N_A(\Delta) = N_B(\Delta) = N_{\text{tot}}/2$ for all values of the internal state Δ. The corresponding expected reward $\hat{R}_{\textbf{\textit{Equivalent}}}$ is expressed similarly to (12) as follows:

$$\hat{R}_{\textbf{\textit{Equivalent}}} = \sum_{\Delta} R\left(\Delta, \frac{N_{\text{tot}}}{2}, \frac{N_{\text{tot}}}{2}\right)\Pi\left(\Delta, \frac{N_{\text{tot}}}{2}, \frac{N_{\text{tot}}}{2}\right) - S_{N_{\text{tot}}}\,. \tag{14}$$

2.3.3 Stationary Probability Distribution. The organism can possibly be in one of three locations within one of three environments for each possible internal state. This combination is defined as a triple that includes the internal state, the location, and the environment, *i.e.*, $(\Delta, l, \hat{e})$. The stationary probability distribution $\Pi(\Delta, N_A(\Delta), N_B(\Delta))$ can be found for each next state as

$$\Pi(\Delta, N_A(\Delta), N_B(\Delta)) = \sum_{l\in L}\sum_{\hat{e}\in\hat{E}} \pi(\Delta, l, \hat{e})\,, \tag{15}$$

where $\pi(\Delta, l, \hat{e})$ is found by solving for the stationary distribution with a relation described as

$$\pi(\Delta, l, \hat{e}) = P_{\delta,\delta'}((\Delta, l, \hat{e}), (\Delta', l', \hat{e}'))\pi(\Delta, l, \hat{e})\,, \tag{16}$$

where, $P_{\delta,\delta'}((\Delta, l, \hat{e}), (\Delta', l', \hat{e}'))$ is the square transition probability matrix from each combination of values of $(\Delta, l, \hat{e})$ to any other new combination $(\Delta', l', \hat{e}')$. Since the organism has an equal chance of being placed into any one location and environment, *i.e.*, 9 different equiprobable positions, this is defined as

$$P_{\delta,\delta'}((\Delta, l, \hat{e}), (\Delta', l', \hat{e}')) = \frac{1}{9}P_{\Delta}(\Delta, l, \hat{e}, \Delta')\,, \tag{17}$$

where $P_{\Delta}(\Delta, l, \hat{e}, \Delta')$ is the transition probability between some state, environment, and location pair $(\Delta, l, \hat{e})$ to any location and environment in the next state Δ', defined as

$$P_{\Delta}(\Delta, l, \hat{e}, \Delta') = P_{(-1)}(\Delta, l, \hat{e}, \Delta') + P_{(0)}(\Delta, l, \hat{e}, \Delta') + P_{(+1)}(\Delta, l, \hat{e}, \Delta')\,, \tag{18}$$

where $P_{\text{mov}}(\Delta, l, \hat{e}, \Delta')$ is the probability of moving into the next internal state Δ' from internal state Δ and location l within environment $\hat{e}$ because of movement mov $\in \{-1, 0, 1\}$, defined as

$$P_{\text{mov}}(\Delta, l, \hat{e}, \Delta') = \sum_{a}^{N_A(\Delta)}\sum_{b}^{N_B(\Delta)} P_{\text{Bi}}(a, N_A(\Delta), l, \hat{e})\, P_{\text{Bi}}(b, N_B(\Delta), l, \hat{e})\mathbb{1}_{M(\Delta,(l+\text{mov})\%3,\hat{e})=\Delta'}\,, \tag{19}$$

where $M(\Delta, l, \hat{e})$ is the matrix defining the state transition from each $(\Delta, l, \hat{e})$ to a next state Δ' (Fig. 3), which contains values different from 0 only where there is a valid transition from $(\Delta, l, \hat{e})$ to Δ', *i.e.*, the change in the internal storage for A and B after the organism movement results into a change in the internal state from Δ to Δ'.

$M =$ Δ									
3	5	3	1	3	4	2	4	2	3
2	4	2	0	2	3	1	3	1	2
1	3	1	-1	1	2	0	2	0	1
0	2	0	-2	0	1	-1	1	-1	0
-1	1	-1	-3	-1	0	-2	0	-2	-1
-2	0	-2	-4	-2	-1	-3	-1	-3	-2
-3	-1	-3	-5	-3	-2	-4	-2	-4	-3
Location L:	0	1	2	3	4	5	6	7	8
Envir. $\hat{E}$:	Envir. 1			Envir. 2			Envir. 3		

Figure 3: State transition matrix $M(\Delta, l, \hat{e})$ contains the value of the next internal state Δ' given the current state Δ, location l and the environment $\hat{e}$.

2.4 Information Metrics for Solution Interpretation

The useful information, I_U is broadly defined as the expected value over the internal state of the organism of the mutual information rate between the organism's input and the organism's response (movement) given its internal state. I_U is defined in this paper as

$$I_{\mathrm{U}}\,|_{strategy} = \mathrm{E}_{\Delta}\left[I\left(C_A^{dist}, C_B^{dist}; \mathrm{mov}(\Delta, strategy)\right)\right], \quad (20)$$

where I is the mutual information of the joint probability distribution $p_j(C_A^{dist}, C_B^{dist}, \mathrm{mov}(\Delta, strategy))$, where $\mathrm{mov}(\Delta, strategy)$ is the random variable representing the movements given the external concentrations (C_A^{dist}, C_B^{dist}) and some state Δ. The *strategy* can either be ***Adaptive*** or ***Equivalent***. The subjective information is defined as:

$$I_{\mathrm{subj}} = I_{\mathrm{U}}\,|_{\boldsymbol{Adaptive}} - I_{\mathrm{U}}\,|_{\boldsymbol{Equivalent}}\,. \quad (21)$$

3 NUMERICAL RESULTS AND DISCUSSION

We present numerical results computed from the analytical solution of the abstract model detailed in Sec. 2, using the following parameter values: a dissociation constant of the binding process $K_d = 2$, a constant penalty term $s_1 = 2$, a cost per receptor $s_2 = 0.001$. We set the largest magnitude of the internal state Δ for calculating the numerical solution to the stationary distribution to 5, which corresponds to the limit absolute value of the row index of the reward matrix $M_{\hat{R}}$ in Fig. 2 and the State transition matrix $M(\Delta, l, \hat{e})$ in Fig. 3. This is because the stationary internal state distribution is light-tailed around the center at $\Delta = 0$ (Fig. 4). The existence of this distribution confirms its ergotic property in the context for the numerical solution presented in the paper. The code implemented to obtain these results is provided in [4].

In Fig. 5 we show the expected reward $R(\Delta, N_A, 500 - N_A)$. When $\Delta \neq 0$, the expected reward is maximized when the organism allocates all of its receptors to 'A' type if $\Delta < 0$ and 'B' type if $\Delta > 0$, respectively. When $\Delta = 0$, the expected reward is maximized when the organism allocates its receptors equally between type 'A' and 'B'. This shows that the cell's expected reward is dependent on how the organism subjectively acquires information, *i.e.*, by setting the number of receptors depending on its internal state. This subjective way of acquiring information, when $\Delta \neq 0$, shows that the reward is maximized when the highest possible number of receptors is set

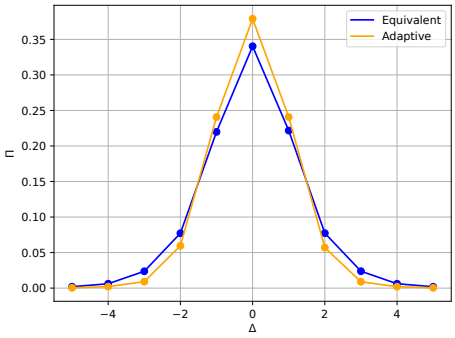

Figure 4: The probability mass function over Δ for the Equivalent and Adaptive strategies for a receptor count of $N_{\mathbf{tot}} = 100$

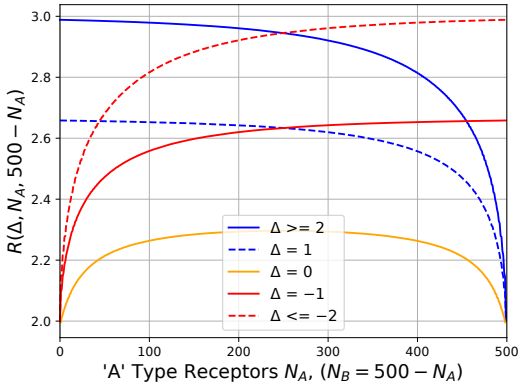

Figure 5: Relationship between the receptor type allocation N_A and $N_B = N_{\mathbf{tot}} - N_A$ and expected reward R for $N_{\mathbf{tot}} = 500$ with no penalty S.

to the type of molecule that is most lacking from the organism's internal storage.

In Fig. 6 we plot the expected reward $\hat{R}_{\mathrm{strategy}}$ against the $I_{\mathrm{U}}\,|_{strategy}$ for both the ***Adaptive*** and the ***Equivalent*** strategies. The colors indicate the total number N_{tot} of receptors corresponding to each data point, displayed on log scale (base 10). The ***Adaptive*** strategy has a higher expected reward than the ***Equivalent*** at their maxima, around $N_{\mathrm{tot}} = 100$, and the trend is the same for all possible N_{tot} values, which is also confirmed later on from Fig. 7. This indicates the higher fitness of the ***Adaptive*** strategy, also correlated in the plot to a larger amount of useful information, which is what we defined as subjective information I_{subj} in (21). This confirms that for a set value of N_{tot}, the ***Adaptive*** strategy can gather a higher amount of information that contributes to wiser choices for the organism's movement, leading to a higher expected reward.

In Fig. 7 we show the ratio of the difference in the expected rewards $\hat{R}_{Adaptive} - \hat{R}_{Equivalent}$ with the subjective information I_{subj}. This represents the gain in the expected reward per bit of I_{subj}. This value appears to be always positive and correlated with I_{subj}, and does not depend on the stress penalty $S_{N_{\mathrm{tot}}}$ in (3). The trend of this gain is monotonically decreasing as the total number

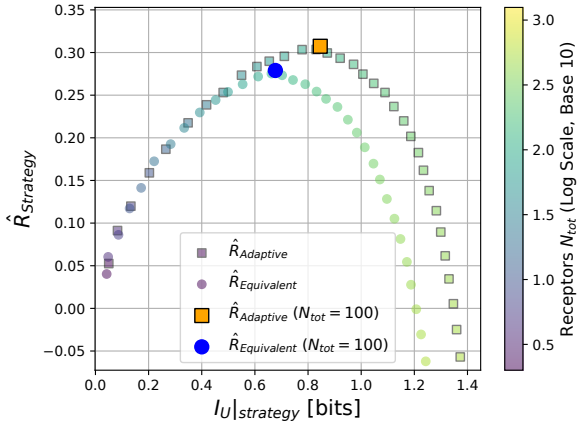

Figure 6: Expected reward $\hat{R}$ plotted against the useful information rate I_U for the *Adaptive* and the *Equivalent* strategies for different values of total receptors $N_{\mathbf{tot}}$.

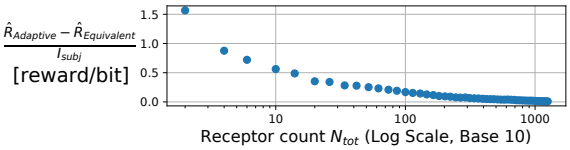

Figure 7: The ratio of the growth difference $\hat{R}_{Adaptive} - \hat{R}_{Equivalent}$ to I_{subj} plotted against the receptor count $N_{\mathbf{tot}}$.

of receptors N_{tot} increases, revealing that as the noise in the information acquisition increases (N_{tot} decreases), the fitness value of the subjective information increases.

4 CONCLUSION

In this paper, motivated by previous work where we defined a new type of information, which we called subjective, and we constructed a computational simulation model for its study, we obtained analytical expressions to derive the same type of information through a novel abstract model. Principles underlying the organization of subcellular components are important for understanding naturally occurring organisms as well as for designing synthetic organisms to accomplish specific purposes. The subjective information is related to how living organisms can increase their fitness by dynamically specializing their sensory apparatus to opportunistically gather information from the environment that is more useful for their growth/survival. By stemming from the obtained analytical expressions, we presented numerical results that reveal not only the emergence of the subjective information, but also its correlation with the organism's fitness. Finally, we present the fitness value (gain) given by the subjective information for varying capabilities for the organism's sensing apparatus, where a higher gain in fitness per useful bit is realized at a higher sensing noise.

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REFERENCES

[1] Tobias Ackels, Andrew Erskine, Debanjan Dasgupta, Alina Cristina Marin, Tom PA Warner, Sina Tootoonian, Izumi Fukunaga, Julia J Harris, and Andreas T Schaefer. 2021. Fast odour dynamics are encoded in the olfactory system and guide behaviour. Nature 593, 7860 (2021), 558–563.
[2] Edward K Agarwala, Hillel J Chiel, and Peter J Thomas. 2012. Pursuit of food versus pursuit of information in a Markovian perception–action loop model of foraging. Journal of theoretical biology 304 (2012), 235–272.
[3] T. Barker, P. J. Thomas, and M. Pierobon. 2023. A Metric to Quantify Subjective Information in Biological Gradient Sensing. In Proc. 2023 IEEE Global Communications Conference (GLOBECOM).
[4] Tyler S Barker. [n. d.]. https://github.com/tbarke/SimpleSubjectiveCell.
[5] Tyler S Barker, Massimiliano Pierobon, and Peter J Thomas. 2022. Subjective Information and Survival in a Simulated Biological System. Entropy 24, 5 (2022), 639.
[6] Andrew R Barron and Thomas M Cover. 1988. A bound on the financial value of information. IEEE Transactions on Information Theory 34, 5 (1988), 1097–1100.
[7] T. Berger. 1971. Rate Distortion Theory: A Mathematical Basis for Data Compression. Prentice-Hall. https://books.google.ca/books?id=-HV1QgAACAAJ
[8] Carl T Bergstrom and Michael Lachmann. 2004. Shannon information and biological fitness. In 2004 IEEE Information Theory Workshop. 50–54.
[9] Lynne Chantranupong, Rachel L Wolfson, and David M Sabatini. 2015. Nutrient-sensing mechanisms across evolution. Cell 161, 1 (2015), 67–83.
[10] Victor Chubukov, Luca Gerosa, Karl Kochanowski, and Uwe Sauer. 2014. Coordination of microbial metabolism. Nature Reviews Microbiology 12, 5 (2014), 327–340.
[11] Matina C Donaldson-Matasci, Carl T Bergstrom, and Michael Lachmann. 2010. The fitness value of information. Oikos 119, 2 (2010), 219–230.
[12] Andrew W Eckford and Peter J Thomas. 2018. The channel capacity of channelrhodopsin and other intensity-driven signal transduction receptors. IEEE Transactions on Molecular, Biological, and Multi-Scale Communications 4, 1 (2018), 27–38.
[13] Deniz Gündüz, Zhijin Qin, Inaki Estella Aguerri, Harpreet S Dhillon, Zhaohui Yang, Aylin Yener, Kai Kit Wong, and Chan-Byoung Chae. 2022. Beyond transmitting bits: Context, semantics, and task-oriented communications. IEEE Journal on Selected Areas in Communications 41, 1 (2022), 5–41.
[14] J. L. Kelly Jr. 1956. A New Interpretation of Information Rate. Bell System Technical Journal 35, 4 (1956), 917–926.
[15] Alexander S Moffett and Andrew W Eckford. 2022. Minimal informational requirements for fitness. Physical Review E 105, 1 (2022), 014403.
[16] Alexander S Moffett, Nigel Wallbridge, Carrol Plummer, and Andrew W Eckford. 2020. Fitness value of information with delayed phenotype switching: Optimal performance with imperfect sensing. Physical Review E 102, 5 (2020), 052403.
[17] Olivier Rivoire and Stanislas Leibler. 2011. The value of information for populations in varying environments. Journal of Statistical Physics 142 (2011), 1124–1166.
[18] R Sarpeshkar. 2014. Analog synthetic biology. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 372, 2012 (2014), 20130110.
[19] Peter J Thomas and Andrew W Eckford. 2016. Capacity of a simple intercellular signal transduction channel. IEEE Transactions on information Theory 62, 12 (2016), 7358–7382.
[20] Massimo Vergassola, Emmanuel Villermaux, and Boris I Shraiman. 2007. 'Infotaxis' as a strategy for searching without gradients. Nature 445, 7126 (2007), 406–409.
[21] R. Wesel, E. Wesel, L. Vandenberghe, C. Komninakis, and M. Medard. 2018. Efficient binomial channel capacity computation with an application to molecular communication. In In Proceedings of the 2018 Information Theory and Applications Workshop (ITA), San Diego, CA, USA. 1–5.
[22] Bin Wu, Chaitanya S Gokhale, Matthijs van Veelen, Long Wang, and Arne Traulsen. 2013. Interpretations arising from Wrightian and Malthusian fitness under strong frequency dependent selection. Ecology and Evolution 3, 5 (2013), 1276–1280.
[23] H. Youk and A van Oudenaarden. 2009. Growth landscape formed by perception and import of glucose in yeast. Nature 462 (2009), 875–879.