

Universal dual-port grid-forming control: bridging the gap between grid-forming and grid-following control

Irina Subotić and Dominic Groß

Abstract—We study a dual-port grid-forming (GFM) control for power systems containing ac and dc transmission, converter-interfaced generation and energy storage, and legacy generation. To operate such a system and provide standard services, state-of-the-art control architectures i) require assigning grid-following (GFL) and GFM controls to different converters, and ii) result in highly complex system dynamics. In contrast, dual-port GFM control i) subsumes standard functions of GFM and GFL controls in a simple controller, ii) can be applied to a wide range of emerging technologies independently of the network configuration, and iii) significantly reduces system complexity. In this work, we provide i) an end-to-end modeling framework that allows to model complex topologies through composition of reduced-order device models, ii) an in-depth discussion of universal dual-port GFM control for emerging power systems, and iii) end-to-end stability conditions that cover a wide range of network topologies, emerging technologies, and legacy technologies. Finally, we validate our findings in a detailed case study.

I. INTRODUCTION

Electrical power systems are transitioning from fuel-based legacy synchronous generators (SGs), whose physical properties (e.g., rotational inertia) and controls (e.g., speed governor) form the foundation of today's system operation and analysis, to converter-interfaced resources such as renewable generation, energy storage systems, and high voltage direct current (HVDC) transmission. This large-scale integration of converter-interfaced resources results in significantly different system dynamics that jeopardize stability and reliability [1].

Today, the majority of converter-interfaced resources uses GFL control to, e.g., maximize the energy yield of renewables or minimize HVDC transmission losses. This approach typically relies on a phase-locked loop (PLL) for synchronization and assumes that a stable ac voltage waveform (i.e., frequency and magnitude) at the point of interconnection is guaranteed by, e.g., the presence of SGs. While GFL power converters can provide typical ancillary services (e.g., primary frequency control), dynamic stability of power system can rapidly deteriorate as the share of GFL resources increases [1], [2].

To resolve this issue, GFM converters that impose a stable ac voltage at their terminal and self-synchronize are envisioned to be the cornerstone of future power systems [2]. While prevalent GFM controls such as active power - frequency ($P_{ac} - f$) droop control [3], virtual synchronous machine control [4], and (dispatchable) virtual oscillator control [5], [6], provide fast and reliable grid support [2], they may destabilize the system if the resource interfaced by the converter reach their power generation limits [7].

While the classification into GFM and GFL is commonly applied to the point of connection with the ac grid (i.e., ac-GFM and ac-GFL) it is also useful to characterize the dc terminal behavior of dc/ac voltage source converters (VSCs) for

renewable integration and HVDC transmission, i.e., dc-GFL assumes a stable dc voltage while dc-GFM stabilizes the VSC dc terminal [8]–[10]. Most of the existing literature focuses on ac-GFM/dc-GFL and dc-GFM/ac-GFL control and treats these concepts as mutually exclusive. From this perspective, ac-GFM and dc-GFM are complementary, i.e., operating emerging power systems requires assigning ac-GFL/dc-GFM and ac-GFM/dc-GFL controls to different converters to support, e.g., maximum power point tracking (MPPT) [9], high-voltage ac (HVAC) and reliably operate HVDC transmission [10]. This approach results in complex, heterogeneous system dynamics that introduces significant challenges in system operation, e.g., assigning GFM/GFL roles to VSCs is non-trivial and no control configuration may be stable for all relevant operating points [10] and/or available reserves [7]. Finally, to the best of our knowledge, no analytical stability conditions are available for systems containing ac-GFM/dc-GFL VSCs, ac-GFL/dc-GFM VSCs, and legacy devices.

Most of the existing works on stability analysis of ac power systems with ac-GFM VSCs neglect the dc terminal and can be categorized into numerical [11]–[14] and analytical [5], [6] approaches. Only a few numerical works consider i) renewable generation with limited controllability [14] or ii) a mix of ac-GFL VSCs, ac-GFM VSCs, and SGs [12]. In contrast, dc voltage - frequency ($v_{dc} - f$) droop inspired by machine emulation control [15] is analyzed in [7], [16], [17] assuming proportional dc voltage control through its dc source.

An often overlooked feature of $v_{dc} - f$ droop control is its ability to bridge the gap between ac-GFM/dc-GFL and ac-GFL/dc-GFM control by simultaneously controlling ac frequency and dc voltage (ac-GFM/dc-GFM) [8], [9], [18]. This dual-port GFM concept unifies standard functions of ac-GFL/dc-GFM (e.g., MPPT) and ac-GFM/dc-GFL (e.g., primary frequency control) in a GFM control [18] and reduces complexity by using the same control independently of the network configuration [8] and VSC power source [18]. In contrast to standard ac-GFM/dc-GFL and ac-GFL/dc-GFM control, dual-port GFM control provides bidirectional grid support functions whose direction across the dc/ac interface autonomously adapts to the system topology, operating point, and control reserves. For instance, for photovoltaics (PV) operating at their maximum power point (MPP), the dc voltage is stabilized through controlling the ac power injection. In contrast, curtailed PV stabilizes the dc voltage which in turn stabilizes the ac frequency. Finally, combining $v_{dc} - f$ and $P_{ac} - f$ droop control results in so-called power-balancing dual-port GFM control and insightful analytical stability conditions for systems containing ac and dc transmission, renewables, SGs, and synchronous condensers (SCs) are available [18].

Despite these appealing features, power-balancing dual-port

GFM control has several conceptual drawbacks. Specifically, significant oscillations may occur during transients because the transient and steady-state response cannot be tuned separately [8], post-disturbance steady-state frequencies are generally not synchronous for ac systems interconnected through HVDC [18], and $P_{ac}-f$ droop may result in a counter-intuitive response when post-contingency power flows significantly differ from the VSC power set-point [8]. Moreover, for some system topologies, stability conditions still hinge on the presence of controllable power sources on VSC dc terminals [18].

Instead, the main contribution of this is a rigorous analysis of a universal dual-port GFM control based on proportional-derivative (PD) $v_{dc}-f$ droop control [8], [9] that retains all features of power-balancing dual-port GFM control while overcoming its conceptual limitations. In particular, we show that the transient and steady-state response can be tuned separately and that, for typical HVDC systems, ac systems interconnected through HVDC admit a quasi-synchronous steady-state. Moreover, after disturbances and contingencies, the universal dual-port GFM control rebalances the system irrespective of the deviation from the scheduled power flow [8] while the nominal operating point can still be fully dispatched through generator power setpoints and VSC dc voltage setpoints. Simplified stability conditions for universal dual-port GFM control are available for a single VSCs [8] and a single back-to-back connected wind turbine [9].

Our main contributions can be summarized as follows. The graph-based modeling framework from [18] is extended to more realistic models of wind turbines (WT) and PV and discuss various real-world examples. We develop insightful analytical stability conditions for universal dual-port GFM control in systems containing ac and dc transmission, renewables, SGs, and synchronous condensers (SCs) on the control gains and system topology that do not hinge on controllable dc power sources on VSC dc terminals. Our steady-state analysis provides conditions under which a quasi-synchronous steady-state exists in hybrid dc/ac power systems, and we show that universal dual-port GFM control can be tuned to meet standard steady-state specifications. We illustrate that universal dual-port GFM control provides bidirectional grid support (i.e., primary frequency control) through asynchronous interconnections (e.g., HVDC, low frequency AC (LFAC)). Finally, a case study is used to illustrate the analytical results.

Notation: Given a matrix A , $A \succcurlyeq 0$ ($A \succ 0$) denotes that A is symmetric and positive semidefinite (definite). The identity matrix of dimension n is denoted by I_n . Matrices of zeros of dimension $n \times m$ are denoted by $\mathbb{0}_{n \times m}$. Column vectors of zeros and ones of length n are denoted by $\mathbb{0}_n$ and $\mathbb{1}_n$. Given $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, we define $(x, y) := [x^\top y^\top]^\top \in \mathbb{R}^{n+m}$. The cardinality of a set $\mathcal{X} \subset \mathbb{N}$ is denoted by $|\mathcal{X}|$. Finally, $(\cdot)_\delta$ denotes deviations of a signal from its operating point $(\cdot)^*$.

II. POWER SYSTEM MODELING

This section briefly summarizes linearized device and network models and illustrates how complex power systems can be modeled by combining individual device and network models through a graph-based modeling framework.

A. Network model

To formalize the modeling of arbitrary networks, we briefly review the graph representation of the overall hybrid ac/dc system [18]. A connected, undirected graph $\mathcal{G}_N = (\mathcal{N}_N, \mathcal{E}_N)$ with nodes $\mathcal{N}_N := \mathcal{N}_{ac} \cup \mathcal{N}_{dc} \cup \mathcal{N}_c$, and edges $\mathcal{E}_N := \mathcal{E}_{ac} \cup \mathcal{E}_{dc}$ is used to model the overall systems. Nodes encompass energy conversion devices (synchronous machines \mathcal{N}_{ac} and dc/ac power converters \mathcal{N}_c) and dc nodes \mathcal{N}_{dc} . Each ac node corresponds to an ac voltage phase angle $\theta_l \in \mathbb{R}$ and magnitude $V_l \in \mathbb{R}_{\geq 0}$ while each dc node corresponds to a dc voltage $v_l \in \mathbb{R}_{\geq 0}$. We distinguish between ac $\mathcal{E}_{ac} \subseteq (\mathcal{N}_{ac} \cup \mathcal{N}_c) \times (\mathcal{N}_{ac} \cup \mathcal{N}_c)$ that model ac power flows $P_{ac,l} \in \mathbb{R}$ and dc edges $\mathcal{E}_{dc} \subseteq (\mathcal{N}_{dc} \cup \mathcal{N}_c) \times (\mathcal{N}_{dc} \cup \mathcal{N}_c)$ that model dc power flows $P_{dc,l} \in \mathbb{R}$. Linearizing the quasi-steady-state power flows and dc bus dynamics at $V_l^* = V_k^* = 1$ p.u., $v_l^* = v_k^* = 1$ p.u and $\theta_l^* = \theta_k^*$ results in

$$P_{\delta,ac,l} = \sum_{(k,l) \in \mathcal{E}_{ac}} b_{kl}^{ac}(\theta_{\delta,l} - \theta_{\delta,k}) + P_{d_{ac},l}, \quad (1a)$$

$$P_{\delta,dc,l} = \sum_{(k,l) \in \mathcal{E}_{dc}} g_{kl}^{dc}(v_{\delta,l} - v_{\delta,k}) + P_{d_{dc},l}, \quad (1b)$$

$$C_l v_l^* \frac{d}{dt} v_{\delta,l} = P_{\delta,l} - P_{\delta,dc,l}, \quad (1c)$$

where $b_{kl}^{ac} \in \mathbb{R}_{\geq 0}$ and $g_{kl}^{dc} \in \mathbb{R}_{\geq 0}$ denotes the line ac susceptance and dc line conductance. Moreover, $P_{\delta,l} \in \mathbb{R}$ denotes power injections by a dc power source, and $P_{d_{ac},l} \in \mathbb{R}$ and $P_{d_{dc},l} \in \mathbb{R}$ model perturbations in load/generation.

B. Power conversion

Besides dc buses, the nodes of the graph \mathcal{G}_N model power conversion devices, i.e., machines and power converters.

1) *Synchronous machine (SM)*: are modeled via the standard swing equation model

$$\frac{d}{dt} \theta_{\delta,l} = \omega_{\delta,l}, \quad J_l \omega_l^* \frac{d}{dt} \omega_{\delta,l} = P_{\delta,l} - P_{\delta,ac,l} \quad (2)$$

with frequency $\omega_{\delta,l} \in \mathbb{R}_{>0}$, machine inertia $J_l \in \mathbb{R}_{>0}$, and mechanical power applied to the rotor $P_{\delta,l} \in \mathbb{R}$. If a SM is used as an energy storage element (e.g., flywheel) or synchronous condenser (SC), then $P_{\delta,l} = 0$.

2) *dc/ac VSC*: For brevity of presentation, we consider an averaged model of a two-level VSC¹. Using standard time-scale separation arguments, the VSC output filter dynamics and inner control loops that track references for the ac voltage magnitude $V_{\delta,l} \in \mathbb{R}_{\geq 0}$ and phase angle $\theta_{\delta,l} \in \mathbb{R}$ are neglected [6]. Thus, $V_{\delta,l}$ and $\theta_{\delta,l}$ are the remaining control inputs used by the GFM control introduced in see Sec. IV and we obtain the dc-link capacitor charge dynamics

$$C_l v_l^* \frac{d}{dt} v_{\delta,l} = -P_{\delta,ac,l} - P_{\delta,dc,l} \quad (3)$$

with capacitance $C_l \in \mathbb{R}_{>0}$, dc power $P_{\delta,dc,l}$ flowing into the dc-link and ac power $P_{\delta,ac,l}$ flowing out of the VSC.

C. Power sources

Finally, we abstractly model common power generation technologies through mechanical power sources (e.g., steam turbine, wind turbine) and dc power sources (e.g., PV).

¹Our control and analysis are applicable to more complex converter topologies, such as modular multilevel converters (MMCs) [8].

1) *Turbine/governor*: The prevalent first-order model

$$T_g \frac{d}{dt} P_{\delta,l} = -P_{\delta,l} - k_g \omega_{\delta,l} \quad (4)$$

with turbine time constant $T_g \in \mathbb{R}_{\geq 0}$, governor gain (i.e., sensitivity to frequency) $k_g \in \mathbb{R}_{\geq 0}$, and turbine power $P_{\delta,l} \in \mathbb{R}$ is used to model legacy generation (e.g., steam turbine).

2) *Controllable dc power sources*: Within their limits, and on the time scales of interest, controllable DC sources (e.g., battery storage or two-stage PV system) can be modeled by

$$T_{g,l} \frac{d}{dt} P_{\delta,l} = -P_{\delta,l} - k_{g,l} v_{\delta,l}, \quad (5)$$

with time constant $T_{g,l} \in \mathbb{R}_{> 0}$ and sensitivity to dc voltage deviations $k_{g,l} \in \mathbb{R}_{\geq 0}$. DC sources that track power setpoints or their MPP are modeled by $k_{g,l} = 0$.

3) *Wind turbine (WT)*: The mechanical power generated by a wind turbine is characterized by (cf. [9])

$$T_{g,l} \frac{d}{dt} \beta_l = -\beta_l + u_l^\beta, \quad P_l = \frac{1}{2} \rho_l \pi R_l^2 C_{p,l}(\lambda_l, \beta_l) v_{w,l}^3, \quad (6)$$

with air density $\rho_l \in \mathbb{R}_{> 0}$, rotor radius $R_l \in \mathbb{R}_{> 0}$, and wind speed $v_{w,l} \in \mathbb{R}_{> 0}$. The function $C_{p,l} \in \mathbb{R}_{> 0} \times \mathbb{R}_{> 0} \rightarrow \mathbb{R}_{> 0}$ models the captured wind power as a function of the blade pitch angle $\beta_l \in \mathbb{R}_{\geq 0}$, tip speed ratio $\lambda_l = R_l \omega_l / v_{w,l}$, and WT rotor speed $\omega_l \in \mathbb{R}$. The time constant and control input of the pitch motor are denoted by $T_{g,l} \in \mathbb{R}_{> 0}$ and $u_l^\beta \in \mathbb{R}$. Figure 1 illustrates the WT power generation as a function of rotor speed ω_l . To analyze (6) in our framework, we linearize P_l around a nominal operating point $(v_w^*, \omega_l^*, \beta_l^*)$ to obtain $P_{\delta,l} = -k_{w,l} \omega_{\delta,l} - k_{\beta,l} \beta_{\delta,l}$ with sensitivities

$$k_{w,l} := -\frac{\partial P_l}{\partial \omega_l} \Big|_{(\omega_l, v_{w,l}, \beta_l) = (\omega_l^*, v_{w,l}^*, \beta_l^*)} \in \mathbb{R}_{\geq 0}, \quad (7a)$$

$$k_{\beta,l} := -\frac{\partial P_l}{\partial \beta_l} \Big|_{(\omega_l, v_{w,l}, \beta_l) = (\omega_l^*, v_{w,l}^*, \beta_l^*)} \in \mathbb{R}_{\geq 0}, \quad (7b)$$

to changes in rotor speed and blade pitch angle. Combining $P_{\delta,l}$ with the (linear) pitch angle dynamics and applying the change of coordinates $P_{\delta,\beta,l} = k_{\beta,l} \beta_{\delta,l}$ we obtain

$$P_{\delta,l} = -k_{w,l} \omega_{\delta,l} + P_{\delta,\beta,l}, \quad T_{g,l} \frac{d}{dt} P_{\delta,\beta,l} = -P_{\delta,\beta,l} - k_{\beta,l} u_{\delta,l}^\beta. \quad (8)$$

4) *Solar photovoltaics (PV)*: The current i_l generated by a PV panel is modeled by [19, Fig. 4]

$$i_l = i_{L,l} - i_{0,l} \left(\exp\left(\frac{v_l + R_{s,l} i_l}{v_{t,l} \alpha_l}\right) - 1 \right) - \frac{v_l + R_{s,l} i_l}{R_{p,l}}, \quad (9)$$

where $v_l \in \mathbb{R}_{\geq 0}$, $i_{L,l} \in \mathbb{R}_{\geq 0}$, and $i_{0,l} \in \mathbb{R}_{\geq 0}$ denote the dc voltage, photovoltaic current, and saturation current. The thermal voltage of the PV's cell array is $v_{t,l} \in \mathbb{R}_{> 0}$, while $R_{s,l} \in \mathbb{R}_{> 0}$ and $R_{p,l} \in \mathbb{R}_{> 0}$ are series and parallel resistances. Figure 1 illustrates the PV power as a function of dc voltage.

For analysis purposes, we linearize $P_l = v_l i_l$ at the nominal operating point (i.e., dc voltage) and obtain $P_{\delta,l} = -k_{pv,l} v_{\delta,l}$ with sensitivity $k_{pv,l} := \frac{dP_l}{dv_l} \Big|_{v_l = v_l^*}$ to dc voltage deviations. In the remainder of the manuscript we assume that the nominal operating point of renewable generation changes slowly compared to the time-scales of interest in this work and is periodically updated (see, e.g., [20] and [9] for further details).

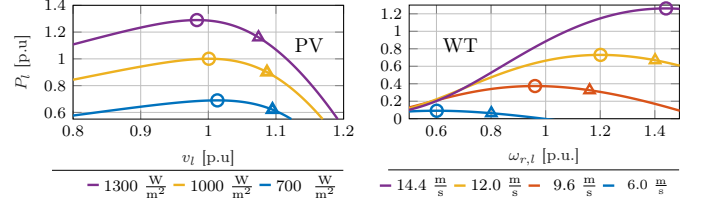


Fig. 1. Nonlinear power characteristic of a PV (left) as a function of dc voltage and irradiation (constant temperature of 25° C) and WT (right) as a function of rotor speed and wind speed (zero blade pitch angle). The MPP and a (stable) operating point at 90% MPP are denoted by circles and triangles.

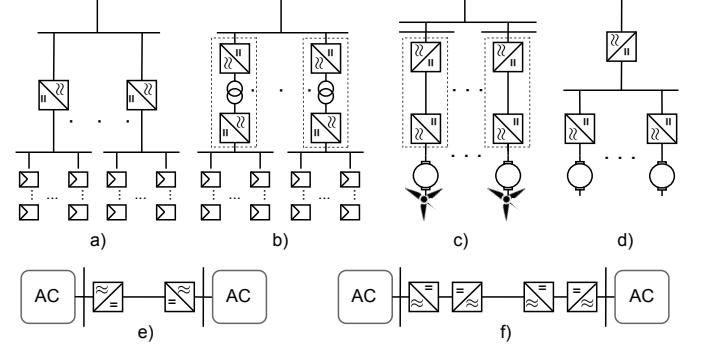


Fig. 2. a) PV plant and dc/ac power converters b) PV plant dc collector network with dual active bridge converter, c) wind farm with PMSG WTs d) flywheel energy storage system and with dc network e) high voltage dc (HVDC) link, and f) low frequency ac (LFAC) connection.

D. Modeling complex power systems

A wide range of emerging transmission, generation, and power conversion technologies can be readily modeled through composition of models of power generation (see Sec. II-C), power conversion (see Sec. II-B), and power transmission through the graph-based network model (see Sec. II-A).

For instance, fuel-based legacy SGs are modeled as a combination of a synchronous machine and turbine/governor system. PV plants with standard single-stage solar PV systems are modeled as interconnection of a dc/ac VSC and a PV module through a dc edge as shown in Fig. 2 a) while advanced configurations with dc collector networks and, e.g., dual active bridge (DAB) converters [21] can be modeled by connecting ac terminals of dc/ac VSCs as in Fig. 2 b). Permanent magnet synchronous generator (PMSG) WTs are modeled by combining two dc/ac VSCs with an SM and the WT aerodynamics as shown in Fig. 2 c). Moreover, emerging transmission technologies can be readily modeled. For example, high voltage dc (HVDC) transmission [8] can be modeled by connecting two dc/ac VSCs via a dc line as in Fig. 2 e) and low-frequency ac (LFAC) transmission [22] is modeled by four ac/dc power converters as in Fig. 2 f). By combining these steps one can readily model an offshore wind farm as a combination of PMSG WTs and, e.g., a point-to-point HVDC link to shore. Large-scale flywheel storage system that connect multiple flywheels through a dc collector network can be modeled using SMs (without generation), dc/ac VSCs, and a dc network as in Fig. 2 d). Finally, we use the system in Fig. 3 to illustrate modeling of the emerging hybrid ac/dc networks containing renewable generation and

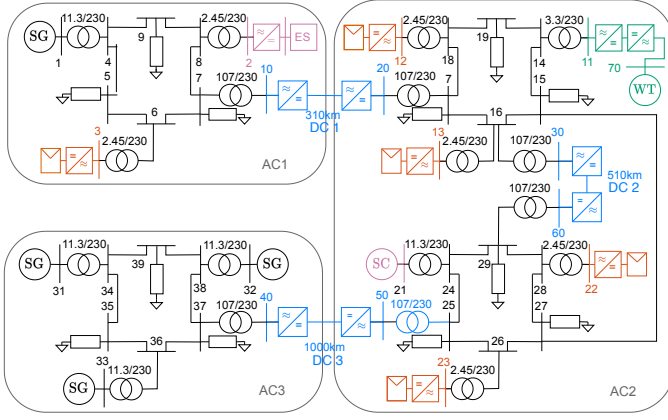


Fig. 3. Hybrid ac/dc power grid with renewable and traditional generation, HVDC links, battery storage and synchronous condenser.

HVDC transmission. The system consists of three bulk ac systems, each based on IEEE 9-bus systems. The system AC 1 contains a single-stage PV plant (orange), a battery energy storage system (pink), and legacy synchronous generation. AC 2 consists of two systems interconnected through ac and dc transmission and that connect four single-stage PV plants (orange), a synchronous condenser (pink), and a PMSG WT (green). The system AC 3 only contains legacy synchronous generation. Finally, three HVDC links (blue) connect the AC systems AC 1, AC 2, and AC 3. We emphasize that the ac and dc connections internal to the PMSG WT and PV plants are also modeled through ac and dc networks. The graph representation of the system in Fig. 3 is shown in Fig. 4.

III. CURTAILMENT & LOCAL CONTROL OF RENEWABLES

Independently of the control strategy used, the ability of renewable generation to provide grid support strongly depends on their operating point and curtailment strategy. Thus, before presenting our control strategy and analysis, we briefly review basic curtailment strategies and connect them with the linearized models and sensitivities derived in Sec. II-C. In particular, if a renewable source is operated below its MPP (i.e., $P_l^* < P_l^{\text{mpp}}$), its power output can be adjusted to contribute to frequency/dc voltage stabilization [18].

1) *Solar PV*: Curtailment of the PV is achieved by operating at a dc voltage above the MPP voltage, i.e., $v_l^* > v_l^{\text{mpp}}$ (see Fig. 1). At this operating point, the PV sensitivity k_{pv} corresponds to the slope of the curve at the operating point (P_l^*, v_l^*). Thus, if $v_l^* > v_l^{\text{mpp}}$, then $k_{pv,l} \in \mathbb{R}_{>0}$, and if $v_l^* = v_l^{\text{mpp}}$, $k_{pv,l} = 0$. In other words, if $v_l^* > v_l^{\text{mpp}}$, PV provides proportional dc voltage control. In contrast, at the MPP (i.e., $v_l^* = v_l^{\text{mpp}}$), the PV does not respond to dc voltage deviations. Moreover, if $v_l^* < v_l^{\text{mpp}}$, the physics of PV are inherently unstable (i.e., $k_{pv,l} \in \mathbb{R}_{<0}$) [20]. Because there is no significant advantage to operating in the unstable region, we only consider operating points in the stable region.

2) *Wind generation*: Curtailment of WTs can be achieved by increasing the rotor speed beyond its optimal speed ω_l^{mpp} (i.e., $\omega_l^* > \omega_l^{\text{mpp}}$, see Fig. 1) and/or by increasing the blade pitch angle (i.e., $\beta_l^* \in \mathbb{R}_{>0}$) (cf. [9], [23]). In particular, rotor speed-based curtailment (i.e., $\omega_l^* > \omega_l^{\text{mpp}}$) results in

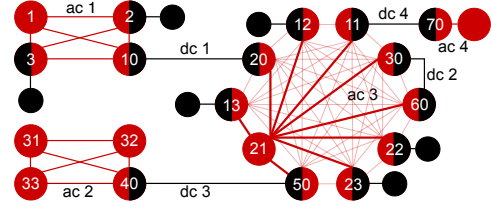


Fig. 4. Graph of the system in Fig. 3 with Kron reduced ac subnetworks.

$k_{w,l} \in \mathbb{R}_{>0}$ and can be interpreted as proportional WT speed control. Moreover, due to the large inertia constant J_l of the WT, rotor speed-based curtailment provides a significant amount of kinetic energy storage. While our analysis framework allows to model both rotor speed-based and pitch-angle based curtailment, our case studies will prioritize rotor speed-based curtailment over pitch angle-based curtailment to leverage the significant kinetic energy storage of WTs. Finally, we assume that $\omega_l^* \geq \omega_l^{\text{mpp}}$ to exclude the inherently unstable operating region $\omega_l^* > \omega_l^{\text{mpp}}$, i.e., $k_{w,l} \in \mathbb{R}_{<0}$ [9].

To implement MPPT, commercial WTs often rely on proportional-integral (PI) control of the rotor speed through the pitch angle. In contrast, to provide grid-support functions, we use the proportional rotor speed control

$$u_l^\beta := \beta_l^* + k_{bp,l}(\omega_l - \omega_l^*) \quad (10)$$

with control gain $k_{bp,l} \in \mathbb{R}_{\geq 0}$. In other words, if $\beta_l^* > 0$, $k_{bp,l} > 0$, the pitch angle is adjusted to respond to rotor speed deviations (i.e., power imbalances). Substituting $u_{\delta,l}^\beta = u_l^\beta - \beta_l^*$ into (6), we can express the pitch angle-based response in the form of the turbine/governor system (4) with WT pitch angle power generation sensitivity $k_{g,l} := k_{\beta,l}k_{bp,l} \in \mathbb{R}_{\geq 0}$.

IV. UNIVERSAL DUAL-PORT GFM CONTROL

The ac voltage phase angle dynamics of each dc/ac VSC, are prescribed by the universal dual-port GFM control

$$\frac{d}{dt}\theta_{\delta,l} = \omega_l - \omega_l^* = k_{p,l}\frac{d}{dt}v_{\delta,l} + k_{w,l}v_{\delta,l} \quad (11)$$

with proportional-derivative (PD) $v_\delta - f$ droop with gains $k_{w,l} \in \mathbb{R}_{>0}$ and $k_{p,l} \in \mathbb{R}_{>0}$. While we will use (11) for analysis and interpretation, the derivative-free implementation

$$\theta_{\delta,l} = k_{p,l}v_{\delta,l} + k_{w,l}\gamma_l, \quad \frac{d}{dt}\gamma_l = v_{\delta,l} \quad (12)$$

is more appropriate in practice. Finally, the ac voltage magnitude $V_{\delta,l}$ is obtained using reactive power droop control $V_{\delta,l} = -m_{q,l}Q_{\delta,l}$ with reactive power droop gain $m_{q,l} \in \mathbb{R}_{>0}$.

The controller (11) resembles machine emulation control [15], [16], but adds a dc voltage setpoint (i.e., $v_{\delta,l} = v_l - v_l^*$) and derivative feedback. The dc voltage setpoint is crucial to dispatch the nominal operating point of, e.g., HVDC [8]. The derivative feedback is crucial for our stability analysis in Sec. V, to tune the transient response, and to ensure stability of network circuit dynamics [24]. Moreover, (11) exhibits similar properties (e.g., bidirectional grid support, unifies GFL and GFM) to power-balancing dual-port GFM control [18]

$$\frac{d}{dt}\theta_{\delta,l} = \omega_l - \omega_l^* = m_{p,l}P_{\delta,ac,l} + k_{w,l}v_{\delta,l} \quad (13)$$

with active power droop coefficient $m_{p,l} \in \mathbb{R}_{>0}$. In particular, under the lossless averaged VSC model (3), (11) becomes

$$\frac{d}{dt}\theta_{\delta,l} = \omega_l - \omega_l^* = \frac{k_{p,l}}{C_l}(P_{\delta,ac,l} - P_{\delta,dc,l}) + k_{\omega,l}v_{\delta,l} \quad (14)$$

In other words, (11) does not require a power setpoint and instead balances the power difference between ac and dc terminals through the $\frac{d}{dt}v_{\delta,l} - f$ droop term. While it may seem counter-intuitive to omit $P_{ac} - f$ droop from the control, we highlight that the objective is to stabilize the VSCs ac and dc terminal voltages and balance the system, not to operate close to a prescribed power transfer between dc and ac terminals. In particular, the $P_{ac} - f$ and $v_{\delta} - f$ droop terms in (13) represent conflicting objectives (i.e., operating close to a power setpoint vs. balancing ac and dc systems) and counteract each other. This may result in significant oscillations during transients and counter-intuitive responses when post-contingency power flows significantly differ from the VSC power set-point [8].

Instead, (11), maps the signals indicating power imbalance (i.e., ac frequency and dc voltage deviation) between the VSC terminals without requiring measurements or estimates of the ac grid voltage frequency. In other words by decreasing or increasing the ac frequency and dc voltage, (11) passes on power imbalances to power generation in the ac and dc networks that respond according to their sensitivities established in Sec. II. Thus, as shown in Sec. V, the overall system is stable under mild assumptions on the network topology and number of devices with non-zero sensitivities. Moreover, the steady-state response of (11) is uniquely characterized by $k_{\omega,l}$ and $k_{p,l}$ can be used to adjust, e.g., oscillation damping.

V. END-TO-END STABILITY AND STEADY-STATE ANALYSIS

This section presents stability conditions for hybrid dc/ac power systems and analyzes their steady-state response.

A. Partitioning of the node and edge sets

To model the overall power system, we define the ac subgraph $\mathcal{G}_{ac} = \bigcup_{i=1}^{N_{ac}} \mathcal{G}_{ac}^i$ using its N_{ac} connected ac components $\mathcal{G}_{ac}^i = (\mathcal{N}_{ac}^i \cup \mathcal{N}_{c,ac}^i, \mathcal{E}_{ac}^i)$ with edge set \mathcal{E}_{ac}^i , ac nodes \mathcal{N}_{ac}^i , and dc/ac nodes $\mathcal{N}_{c,ac}^i$ (i.e., VSC nodes \mathcal{N}_c that are part of the i th ac graph) modeling ac subnetworks. Analogously, we define the dc subgraph $\mathcal{G}_{dc} = \bigcup_{i=1}^{N_{dc}} \mathcal{G}_{dc}^i$ using its N_{dc} connected dc components $\mathcal{G}_{dc}^i = (\mathcal{N}_{dc}^i \cup \mathcal{N}_{c,dc}^i, \mathcal{E}_{dc}^i)$ with edge set \mathcal{E}_{dc}^i , dc nodes \mathcal{N}_{dc}^i , and dc/ac nodes $\mathcal{N}_{c,dc}^i$ (i.e., VSC nodes \mathcal{N}_c that are part of the i th dc graph) modeling dc subnetworks. Moreover, we partition the ac and dc buses/nodes $N_{ac} \cup N_{dc}$ into four broad groups: 1) nodes $\mathcal{N}_r \subseteq \mathcal{N}_{ac} \cup \mathcal{N}_{dc}$ connected to generation (4) or (5) that responds to frequency/dc voltage deviations (i.e., $k_{g,l} > 0$), ii) nodes $\mathcal{N}_{zs} \subseteq \mathcal{N}_{ac} \cup \mathcal{N}_{dc}$ connected to generation (4) or (5) that does not respond to frequency/dc voltage deviations (i.e., $k_{g,l} = 0$), iii) nodes $\mathcal{N}_{pv} \subseteq \mathcal{N}_{dc}$ connected to curtailed PV (i.e., $k_{pv,l} > 0$) and iv) nodes $\mathcal{N}_w \subseteq \mathcal{N}_{ac}$ connected to WTs using rotor speed-based curtailment (i.e., $k_{w,l} > 0$).

B. Overall linearized model

To simplify the analysis, we change coordinates from phase angle deviations θ_{δ} to phase angle differences, i.e., $\eta_{\delta} := B_{ac}^T \theta_{\delta}$ [16, cf. Sec. III], where $B_{ac} \in \{-1, 0, 1\}^{|\mathcal{N}_{ac} \cup \mathcal{N}_{c,ac}| \times |\mathcal{E}_{ac}|}$ is the oriented incidence matrix of the ac graph \mathcal{G}_{ac} . Moreover, we define the graph Laplacian L_{dc} of the dc graph \mathcal{G}_{dc} with edge weights g_{lk}^{ac} and the matrix of ac line susceptances $\mathcal{W}_{ac} := \{b_{lk}^{ac}\}_{(l,k) \in \mathcal{E}_{ac}}$ (see [18]). The matrix $T := \text{blkdiag}\{I_{|\mathcal{N}_{ac} \cup \mathcal{N}_{c,ac}|}, J, C, T_r, T_{zs}\}$ collects i) machine inertia constants $J := \text{diag}\{J_l \omega_l^*\}_{l=1}^{|\mathcal{N}_{ac}|} \succ 0$, ii) dc capacitances $C := \text{diag}\{C_l\}_{l=1}^{|\mathcal{N}_{c,ac} \cup \mathcal{N}_{dc}|} \succ 0$, iii) time constants of the power generation providing frequency response $T_r := \{T_{g,l}\}_{l=1}^{|\mathcal{N}_r|} \succ 0$, and iv) the time constants of the generation with zero sensitivity $T_{zs} := \{T_{g,l}\}_{l=1}^{|\mathcal{N}_{zs}|} \succ 0$. Additionally, $K_w := \text{diag}\{k_{w,l}\}_{l=1}^{|\mathcal{N}_w|} \succ 0$, $K_{pv} := \text{diag}\{k_{pv,l}\}_{l=1}^{|\mathcal{N}_{pv}|} \succ 0$, and $K_g = \text{diag}\{k_{g,l}\}_{l=1}^{|\mathcal{N}_r|} \succ 0$, collect the sensitivities of WTs using rotor speed-based curtailment, curtailed PV, and generation (4) or (5) providing frequency or dc voltage control. Finally, the interconnection matrices $\mathcal{I}_{c,ac}$, $\mathcal{I}_{c,dc}$, \mathcal{I}_{ac} , \mathcal{I}_{dc} , $\mathcal{I}_{r,ac}$, $\mathcal{I}_{r,dc}$, $\mathcal{I}_{zs,ac}$, $\mathcal{I}_{zs,dc}$, \mathcal{I}_w and \mathcal{I}_{pv} model the interconnection of devices to buses and are defined in App. A. For instance, the entry (i, j) of the interconnection matrix $\mathcal{I}_w \in \{0, 1\}^{|\mathcal{N}_w| \times |\mathcal{N}_{ac}|}$ is 1 if the j th ac node is connected to the i th WT. Then, combining the models from Sec. II and (11), the overall system model is given by $P_d := (P_{dac}, P_{ddc}) \in \mathbb{R}^{n_d}$, $n_d := |\mathcal{N}_{ac}| + |\mathcal{N}_{dc}| + 2|\mathcal{N}_c|$, $B_{\eta} := -(\mathcal{I}_{c,ac} B_{ac})^T K_p \mathcal{I}_{c,dc} C^{-1}$, and (15).

C. Stability conditions on the VSC control gains

To simplify the exposition, we assume identical VSC control gains $k_{\omega,l}$ on every dc subnetwork.

Condition 1 [18, Cond. 1] For all $i \in \mathbb{N}_{[1, N_{dc}]}$ and all $(n, l) \in \mathcal{N}_{c,dc}^i \times \mathcal{N}_{c,dc}^i$, $k_{\omega,n} = k_{\omega,l} := k_{\omega}^i$ holds.

Cond. 1 ensures a consistent steady-state mapping between VSC frequency deviations and DC voltage deviations and is required to eliminate circulating power flows between ac subnetworks that are interfaced through multiple dc subnetworks (cf. [18, Ex. 2], [8, Sec. V-D]). Notably, this restriction can be lifted for ac subnetworks that are only connected through one dc subnetwork such as PMSG WTs (cf. [9, Thm. 1]) or two ac systems connected through a point-to-point HVDC link.

The following condition on the dc line conductance $g_{l,k}^{dc}$, dc bus capacitance c_l , and control gains ensures stability of (15).

Condition 2 For all $l \in \mathcal{N}_{c,dc}^i$ and all $i \in \mathbb{N}_{[1, N_{dc}]}$, the gains $k_{p,l} \in \mathbb{R}_{>0}$ and $k_{\omega}^i \in \mathbb{R}_{>0}$ satisfy $\sum_{(l,k) \in \mathcal{E}_{dc}^i} \frac{g_{l,k}^{dc}}{c_l} < 2 \frac{k_{\omega}^i}{k_{p,l}}$.

Notably, k_{ω}^i is typically given by steady-state specifications (see Sec. V-F) and $k_{p,l}$ can be used to satisfy Cond. 2. In particular, decreasing dc line losses or dc bus capacitance both require decreasing $k_{p,l}$, i.e., $k_{p,l} \rightarrow 0$ as $g_{l,k}^{dc} \rightarrow \infty$ or $c_l \rightarrow 0$.

D. Stability conditions on the ac network topology

We emphasize that, in the general setting considered in this work, the system is not asymptotically stable for all network topologies and line parameters (cf. [18, Ex. 1]). Thus, in

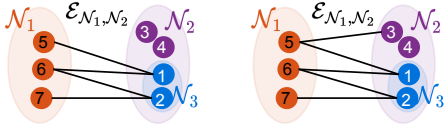


Fig. 5. Example of topologies that do (left) and do not (right) satisfy Cond. 4.

addition to Cond. 1 and Cond. 2, the system topology needs to be restricted. First, at least one power source is required to respond to frequency or dc voltage deviations.

Condition 3 [18, Assump. 2] *There exists $l \in \mathcal{N}_N$ such that $k_{g,l} > 0$ or $k_{pv,l} > 0$ or $k_{w,l} > 0$ (i.e., $\mathcal{N}_r \cup \mathcal{N}_{pv} \cup \mathcal{N}_w \neq \emptyset$).*

While there are no restrictions on dc network topologies, we impose topological stability conditions that can be verified independently for each ac subnetwork \mathcal{G}_{ac}^i and do not require knowledge of the line parameters. We note that conditions for a wide range of ac subnetwork topologies and, e.g., $N - 1$ stability, can be obtained by leveraging [18, Alg. 1]. Instead, in this work, we apply the following simplified condition that covers the vast majority of practically relevant topologies.

Condition 4 *Given node sets \mathcal{N}_1 and $\mathcal{N}_2 \supseteq \mathcal{N}_3$, we define the graph $\mathcal{E}_{\mathcal{N}_1, \mathcal{N}_2} := \mathcal{E}_{ac}^i \cap (\mathcal{N}_1 \times \mathcal{N}_2)$. For every $k \in \mathcal{N}_3$, there exists $(l, k) \in \mathcal{E}_{\mathcal{N}_1, \mathcal{N}_2}$ such that $l \in \mathcal{N}_1$ has no other edge in $\mathcal{E}_{\mathcal{N}_1, \mathcal{N}_2}$.*

The graph on the left in Fig. 5 satisfies Cond. 4. In contrast, the graph on the right does not satisfy Cond. 4 because node #1 is only connected to nodes from \mathcal{N}_1 that have other outgoing edges. In other words, Cond. 4 requires the existence of edges from nodes in \mathcal{N}_3 to nodes in \mathcal{N}_1 that have no other edge in $\mathcal{E}_{\mathcal{N}_1, \mathcal{N}_2}$. To apply Cond. 4 to the ac subnetworks \mathcal{G}_{ac}^i , we distinguish between i) ac nodes \mathcal{N}_{ac}^i with frequency control (i.e., $k_{g,l} > 0$ or $k_{w,l} > 0$) and ii) dc/ac nodes \mathcal{N}_{ac}^i connected to a dc network with at least one node providing dc voltage control (i.e., $k_{g,l} > 0$, $k_{pv,l} > 0$). Finally, $\mathcal{N}_{ac}^i := \mathcal{N}_{ac}^i \setminus \mathcal{N}_{ac}^i$ collects nodes corresponding to SCs and $\mathcal{N}_{ac}^i := \mathcal{N}_{ac}^i \setminus \mathcal{N}_{ac}^i$ collects the remaining ac/dc nodes (e.g., HVDC VSCs).

Condition 5 *Every ac subnetwork $i \in \mathbb{N}_{[1, N_{ac}]}$ satisfies Cond. 4 with either*

- i) $\mathcal{N}_1 = \mathcal{N}_{ac}^i \cup \mathcal{N}_{ac}^i$, $\mathcal{N}_2 = \mathcal{N}_{ac}^i \cup \mathcal{N}_{ac}^i$, and $\mathcal{N}_3 = \mathcal{N}_{ac}^i$, or
- ii) $\mathcal{N}_1 = \mathcal{N}_{ac}^i$, $\mathcal{N}_2 = \mathcal{N}_3 = \mathcal{N}_{ac}^i$.

Cond. 5 i) and Cond. 5 ii) are not mutually exclusive. However, broadly speaking, Cond. 5 i) is directly applicable to ac subnetworks with *few SMs without frequency control* and Cond. 5 ii) is directly applicable to *converter dominated* ac subnetworks. In particular, Cond. 5 i) immediately holds for ac subnetworks in which all SGs provide frequency control (i.e., $\mathcal{N}_{ac}^i = \emptyset$). Moreover, Cond. 5 ii) immediately holds for ac subnetworks containing only power converters (i.e., $\mathcal{N}_{ac}^i = \emptyset$).

In other words, well-known topology independent stability conditions can be recovered as special cases of Cond. 5.

E. Small signal stability

Finally, to prove nominal stability (i.e., $P_d = \mathbb{0}_{n_d}$) of (15), we first define $x_\delta := (\eta_\delta, \omega_\delta, v_\delta, P_\delta) \in \mathbb{R}^n$ with $n := |\mathcal{E}_{ac}| + |\mathcal{N}_{ac}| + 2|\mathcal{N}_c| + |\mathcal{N}_{dc}| + |\mathcal{N}_r|$, restrict (15) to $\bar{P}_\delta = \mathbb{0}_{|\mathcal{N}_{zs}|}$. Next, we define the LaSalle function $V := x_\delta^\top \mathcal{M} x_\delta$ with $\mathcal{M} := \frac{1}{2} \text{blkdiag}\{\mathcal{W}_{ac}, M, \tilde{K}_\omega C, \tilde{K} T_r\}$, $\tilde{K} := (\mathcal{I}_{r,ac} \mathcal{I}_{r,ac}^\top + \mathcal{I}_{r,dc} \mathcal{I}_{dc}^\top \tilde{K}_\omega \mathcal{I}_{dc} \mathcal{I}_{dc}^\top) K_g^{-1}$, and $\tilde{K}_\omega := \text{diag}\{k_\omega^i I_{|\mathcal{N}_c^i \cup \mathcal{N}_{dc}^i|}\}_{i=1}^{N_{dc}}$ with k_ω^i from Cond. 1. We first bound $\frac{d}{dt} V$ under Cond. 1-2.

Proposition 1 *Under Cond. 1-2 the function V is positive definite and for $P_d = \mathbb{0}_{n_d}$ its time derivative along the trajectories of (15) restricted to $\bar{P}_\delta = \mathbb{0}_{|\mathcal{N}_{zs}|}$ satisfies $\frac{d}{dt} V = -\tilde{x}_\delta^\top \mathcal{V} \tilde{x}_\delta - \frac{1}{2} v_\delta^\top (\tilde{K}_\omega \Xi + \Xi \tilde{K}_\omega) v_\delta - \omega_\delta^\top \mathcal{I}_w^\top K_w \mathcal{I}_w \omega_\delta - P_\delta^\top \tilde{K} P_\delta \leq 0$, with $\tilde{x}_\delta := (\mathcal{I}_{ac} B_{ac} \mathcal{W}_{ac} \eta_\delta, v_\delta)$, $\Xi := \mathcal{I}_{dc}^\top \mathcal{I}_{pv}^\top K_{pv} \mathcal{I}_{pv} \mathcal{I}_{dc}$, and*

$$\mathcal{V} := \begin{bmatrix} K_p \mathcal{I}_{dc} C^{-1} \mathcal{I}_{dc}^\top & \frac{1}{2} K_p \mathcal{I}_{dc} C^{-1} L_{dc} \\ * & \frac{1}{2} (\tilde{K}_\omega L_{dc} + L_{dc} \tilde{K}_\omega) \end{bmatrix}.$$

A proof is provided in App. B. To characterize the set of asymptotically stable states, the following proposition characterizes the largest invariant set $\bar{\mathcal{S}}$ (i.e., $x_\delta(0) \in \bar{\mathcal{S}} \Rightarrow x_\delta(t) \in \bar{\mathcal{S}}$ for all $t \geq 0$) contained in $\mathcal{S} := \{x_\delta \in \mathbb{R}^n | \frac{d}{dt} V(x_\delta(t)) = 0\}$.

Proposition 2 *Consider the dynamics (15) with $P_d = \mathbb{0}_{n_d}$ and restricted to $\bar{P}_\delta = \mathbb{0}_{|\mathcal{N}_{zs}|}$. If Cond. 1-3 and Cond. 5 hold, then the origin is the largest invariant set contained in \mathcal{S} .*

A proof sketch is provided in App. B. Finally, we are ready to state our main stability result.

Theorem 1 *If Cond. 1-3 and Cond. 5 hold, then (15) with $P_d = \mathbb{0}_{n_d}$ is asymptotically stable with respect to the origin.*

The theorem follows from Prop. 1, Prop. 2, and the same steps used in the proof of [18, Thm. 1]. Linearity and asymptotic stability of (15) directly implies the following corollary.

Corollary 1 *Under Cond. 1-3, Cond. 5, and $\frac{d}{dt} P_d = 0$, (15) is exponentially stable with respect to $x^{ss} = -A^{-1} B P_d$.*

Next, we analyze the steady state frequency for constant P_d .

F. Steady-state frequency analysis

In steady-state (i.e., $\frac{d}{dt} x_{\delta,l} = 0$) the derivative term in the universal dual-port GFM control (11) vanishes and the steady-state satisfies $\omega_{\delta,l}^{ss} = k_{\omega,l} v_{\delta,l}^{ss}$. We recall that the droop coefficient $\kappa_{p,l} := 1/k_{g,l}$ of SGs describes its steady-state frequency deviation $\omega_{\delta,l}^{ss}$ as a function of the steady-state power injection $P_{\delta,l}^{ss}$ (i.e., $\omega_{\delta,l}^{ss} = -\kappa_{p,l} P_{\delta,l}^{ss}$). Analogously, we use $\omega_{\delta,l}^{ss} = -\kappa_{P,l} P_{\delta,l}^{ss}$ to define the *effective droop coefficient*

$$T \frac{d}{dt} \begin{bmatrix} \eta_\delta \\ \omega_\delta \\ v_\delta \\ P_\delta \\ \bar{P}_\delta \end{bmatrix} = \underbrace{\begin{bmatrix} B_\eta \mathcal{I}_{c,ac}^\top \mathcal{I}_{c,ac} B_{ac} \mathcal{W}_{ac} & (\mathcal{I}_{ac} B_{ac})^\top & (\mathcal{I}_{c,ac} B_{ac})^\top K_\omega \mathcal{I}_{c,dc} - B_\eta L_{dc} & \mathbb{0}_{|\mathcal{E}_{ac}| \times |\mathcal{N}_r|} & \mathbb{0}_{|\mathcal{E}_{ac}| \times |\mathcal{N}_{zs}|} \\ -\mathcal{I}_{ac} B_{ac} \mathcal{W}_{ac} & -\mathcal{I}_w^\top K_w \mathcal{I}_w & \mathbb{0}_{|\mathcal{N}_c| \times |\mathcal{N}_c \cup \mathcal{N}_{dc}|} & \mathcal{I}_{r,ac}^\top & \mathcal{I}_{zs,ac}^\top \\ -\mathcal{I}_{c,dc}^\top \mathcal{I}_{c,ac} B_{ac} \mathcal{W}_{ac} & \mathbb{0}_{|\mathcal{N}_c \cup \mathcal{N}_{ac}| \times |\mathcal{N}_{ac}|} & -L_{dc} - (\mathcal{I}_{pv} \mathcal{I}_{dc})^\top K_{pv} \mathcal{I}_{pv} \mathcal{I}_{dc} & (\mathcal{I}_{r,dc} \mathcal{I}_{dc})^\top & (\mathcal{I}_{zs,dc} \mathcal{I}_{dc})^\top \\ \mathbb{0}_{|\mathcal{N}_r| \times |\mathcal{E}_{ac}|} & -K_g \mathcal{I}_{r,ac} & -K_g \mathcal{I}_{r,dc} \mathcal{I}_{dc} & -I_{|\mathcal{N}_r|} & \mathbb{0}_{|\mathcal{N}_r| \times |\mathcal{N}_{zs}|} \\ \mathbb{0}_{|\mathcal{N}_{zs}| \times |\mathcal{E}_{ac}|} & \mathbb{0}_{|\mathcal{N}_{zs}| \times |\mathcal{N}_{ac}|} & \mathbb{0}_{|\mathcal{N}_{zs}| \times |\mathcal{N}_c \cup \mathcal{N}_{dc}|} & \mathbb{0}_{|\mathcal{N}_{zs}| \times |\mathcal{N}_r|} & -I_{|\mathcal{N}_{zs}|} \end{bmatrix}}_{=:A} \begin{bmatrix} \eta_\delta \\ \omega_\delta \\ v_\delta \\ P_\delta \\ \bar{P}_\delta \end{bmatrix} + \underbrace{\begin{bmatrix} B_\eta & B_\eta \\ I_{n_d} \\ \mathbb{0}_{|\mathcal{N}_r \cup \mathcal{N}_{zs}| \times n_d} \end{bmatrix}}_{=:B} P_d \quad (15)$$

$\kappa_{P,l} \in \mathbb{R}_{>0}$ of renewable generation provided in Table I. Notably, the droop coefficient $\kappa_{P,l}$ only depends on $k_{\omega,l}$ and

TABLE I

Effective droop	PV	DC sources	PMSG WT [9]
$\kappa_{P,l}$	$\frac{k_{\omega,l}}{k_{pv,l}}$	$\frac{k_{\omega,l}}{k_{g,l}}$	$\frac{k_{\omega,k}}{k_{\omega,l}} \frac{1}{(k_{g,l} + k_{w,l})}$

$k_{p,l}$ can be used to shape the transient response. Moreover, as discussed in Sec. V-C, the restrictions imposed on $k_{\omega,l}$ by Cond. 1 can be relaxed for standard renewable generation such as PMSG WTs (see, e.g., [9]) to achieve droop coefficients $\kappa_{P,l}$ prescribed by, e.g., a system operator and/or grid code.

Next, under mild assumptions, the steady-state frequency of the overall system is determined by the droop coefficients $\kappa_{P,l}$ and the generation/load mismatch P_d .

Proposition 3 Assume that $g_{lk}^{dc} \rightarrow \infty$ for all $(l, k) \in \mathcal{E}_{dc}$. Then, under Cond. 1 and Cond. 5 it holds that

$$\omega_{\delta,l}^{ss} = -\frac{1}{\sum_{l \in \mathcal{N}_r \cup \mathcal{N}_{pv} \cup \mathcal{N}_w} \kappa_{P,l}^{-1}} \mathbb{1}_{n_d}^T P_d \quad \forall l \in \mathcal{N}_{ac} \cup \mathcal{N}_c.$$

A proof is provided in App. B. A key feature of the universal dual-port GFM control (11) is that the system admits a quasi-synchronous steady-state with identical frequency deviations if dc line losses are negligible (i.e., $g_{lk}^{dc} \rightarrow \infty$). Therefore, the signals that indicate power imbalances (i.e., $\omega_{\delta,l}$ and $v_{\delta,l}$) synchronize up to scaling by $k_{\omega,l}$ and the overall system response can be easily predicted based on the droop coefficients $\kappa_{P,l}$ of each power source. This feature can also be beneficial for system planning purposes. Moreover, this result highlights the ability of the universal dual-port GFM control (11) to induce a system-wide primary frequency response that rebalances short-term energy storage elements (e.g., SG and WT kinetic energy) and their corresponding frequencies/dc voltages after a load disturbance. Despite the lack of VSC power setpoints, the nominal operating point of the overall system is fully determined through VSC frequency and dc voltage setpoints and setpoints of the power generation.

In contrast, using power-balancing dual-port GFM control (13) the steady-state frequencies of different ac subnetworks exhibit a complicated dependence on the power setpoints and droop coefficients of both the VSCs and power sources and do not admit a quasi-synchronous steady state. This can result in counter-intuitive responses when post-contingency power flows significantly differ from the VSC power set-point [8].

VI. ILLUSTRATION OF STABILITY CONDITIONS AND CONTROL TUNING

In this section, we illustrate our stability conditions in the context of the application examples presented in Sec. II-D and discuss control tuning to meet steady-state specifications.

A. Illustration of the topological stability conditions

We first illustrate our topological stability conditions using the system in Fig 3. In particular, Cond. 3 trivially holds since the legacy SGs in AC 1 provide primary frequency control

(i.e., $k_{g,l} > 0$). Next, we show that the ac subnetworks (AC 1-AC 4) satisfy Cond. 5 for their kron-reduced graphs in Fig. 4. Moreover, Cond. 5 i) trivially holds for AC 1, AC 3 and AC 4 because they only contain SMs with non-zero sensitivity. Notably, while AC 4 only satisfies Cond. 5 i) if the WT provides frequency response, Cond. 5 ii) always holds (i.e., AC 4 only contains SM and a VSC) regardless of the WT's operating point. Lastly, AC 2 is converter dominated, and we show that Cond. 5 ii) holds. To verify this, we only consider edges (highlighted in Fig. 4) between SCs ($\mathcal{N}_2 = \mathcal{N}_3$) and VSCs (\mathcal{N}_1). Since all VSCs have no other edge in $\mathcal{E}_{\mathcal{N}_1, \mathcal{N}_2}$ other than the edge to the SC, Cond. 5 ii) holds.

B. Discussion of the steady-state & illustration of Cond. 2

First, we discuss common network topologies (see Fig. 2), i.e., HVDC, LFAC, and converter (i.e., VSC and DAB) interfaced renewable generation (i.e., PV and WT).

1) *HVDC*: The frequencies of HVDC VSCs interconnecting ac networks satisfy $\omega_{\delta,l}^{ss}/\omega_{\delta,k}^{ss} = k_{\omega,l}/k_{\omega,k}$ if HVDC line losses are negligible. In other words, Cond. 1 is necessary if the ac areas are synchronous. For point-to-point HVDCs, Cond. 1 can be relaxed, allowing for different $k_{\omega,l}$ gains, if the ac terminals of the HVDC link are connected to asynchronous ac areas. Additionally, for a point-to-point HVDC, Cond. 2 simplifies to $k_{p,l}/k_{\omega,l} < 2c_l/g_{lk}^{dc}$, i.e., decreased losses and dc-link capacitance require smaller $k_{p,l}$ because $k_{\omega,l}$ is typically fixed by steady-state specifications. Moreover, the nominal power across HVDC can be scheduled through v_l^* and v_k^* [8].

2) *LFAC*: LFAC systems are modeled by combining VSCs with back-to-back dc connection. In steady state, the LFAC system in Fig. 2 f) satisfies $\omega_{\delta,l}^{ss}/\omega_{\delta,k}^{ss} = k_{\omega,l}/k_{\omega,k}$ for the VSCs l and k at the ends of the LFAC link. In this case, it can be shown that Cond. 2 can be replaced by $k_{p,l} > 0$ [9].

3) *DAB*: Dual active bridge DC/DC converters are modeled by two VSCs in back-to-back ac connection. In steady state, the l th and k th VSC dc voltages (i.e., terminals of the DAB) satisfy $v_{\delta,l}^{ss}/v_{\delta,k}^{ss} = k_{\omega,k}/k_{\omega,l}$ and we require Cond. 2, i.e., the bound on the control gains scale with the dc network losses and dc-link capacitor size.

4) *Converter interfaced renewable generation*: Through (11), curtailed converter-interfaced renewable generation responds to the power imbalance in the grid providing grid support and conventional ac-GFM/dc-GFL functions with effective droop coefficients given in Table I. In contrast, if the generation is operated at the MPP, due to the negligible sensitivity of the renewable sources (Fig. 1), the proposed control does not respond to the power imbalances, providing conventional ac-GFL/dc-GFM functions. In the following, we examine bounds on the derivative gain of renewable generation conditioned on the operating point (i.e., curtailed vs. MPP).

Regardless of the operating point of *PMSG WTs*, it can be shown that Cond. 2 can be replaced by $k_{p,l} > 0$ [9]. On the other hand, for *solar PV* and *controllable DC sources* (e.g., battery energy storage), Cond. 2 simplifies to $(k_{p,l}g_{lk}^{dc})/(k_{\omega,l}c_l) < 2$ where k denotes the dc bus the generation is connected to. If a power source is directly connected to the VSC dc terminal, Cond. 2 can be replaced by $k_{p,l} > 0$ for PV at MPP and $k_{p,l} < 4c_l\kappa_{p,l}$ for curtailed PV.

C. Tuning of the control gains

The transient response of VSCs is dominated by the gain $k_{p,l}$ while the steady state is determined by the gain $k_{\omega,l}$ (see Sec. IV). For curtailed renewable generation and controllable dc sources providing dc voltage response, the effective droop gain κ_P (see Table I) is determined by steady-state specifications, e.g., provided by the system operator or determined through market mechanisms. For renewables κ_P depends on the sensitivities (i.e., curtailment), and $k_{\omega,l}$. Consequently, given κ_P , Table I can be used to select $k_{\omega,l}$ to meet steady-state droop specifications. As discussed in [9], Cond. 1 can be relaxed for PMSG WTs and $k_{\omega,l}/k_{\omega,k}$ can be used to match a given κ_P and reduce the stress on the WT rotor.

In contrast, if renewables are operated at the MPP, we obtain the upper bound

$$k_{\omega,l} \leq \Delta\omega_l^{\max} / \Delta v_l^{\max}. \quad (16)$$

on the proportional gain $k_{\omega,l}$ based on the largest expected frequency deviation (e.g., the boundary of the nominal operating range) $\Delta\omega_l^{\max}$ and the largest acceptable dc voltage deviation Δv_l^{\max} . In case of a PMSG WT, the steady-state relationships between the VSC frequencies, dc voltages, and physical limitations of the WT have to be considered when selecting $k_{\omega,l}$ (for details see [9]).

The derivative gain $k_{p,l}$ can be used to adjust oscillation damping and compensate circuit dynamics to ensure small-signal stability [24]. In particular, Cond. 2 provides an upper bound of the derivative gain $k_{p,l}$ that depends on the time constant (i.e., c_l) of the dc-link capacitor and losses of the outgoing dc lines. In other words, a significant amount of the energy stored in the VSC (larger capacitance c_l) and/or larger steady-state gain $k_{\omega,l}$ allow for higher oscillation damping and slower transients. In contrast, a larger number of outgoing dc lines requires slower control of ac transients (i.e., smaller $k_{p,l}$).

VII. CASE STUDY

An electromagnetic transient simulation of the system in Fig. 3 is used to illustrate the results. Notably, this system combines various legacy and emerging technologies. Reliable operation of this system can require up to six different conventional VSC control strategies that may need to be switched online depending on the operating point of renewables. In contrast, the proposed control can be applied on all VSCs irrespective of the specific technology and operating point.

A. Case study model & parameters

First, we define the base power $S_b = 100$ MW and frequency $f_b = 50$ Hz, while the nominal values of different voltage levels can be found in [18, Table I].

The SGs/SCs are modeled using the 8th order model with the exciter and automatic voltage regulator. Moreover, SGs use a delta-omega multiband power system stabilizer and a first-order turbine model with 5% speed droop (see [7, Table I] for parameters). The WT is modeled by aggregating 5 MW PMSG WTs (for WT and its VSC parameters see [9]). Solar PV is modeled (for parameters see Table II) by aggregating AUO PM060MBR modules, while the electrical storage system

(ESS) is modeled as a controllable dc voltage source with the nominal active power set-point (see [7] for parameters). Moreover, the ESS and PV systems are interfaced by two-level VSCs with RLC filter (see [7] for parameters) and cascaded inner loops (see Table II). Additionally, we use dynamical models of the low-voltage/high-voltage, medium-voltage/high-voltage [7, Table I] and high-voltage/high-voltage [18] transformers. The ac lines and dc cables are modeled using the standard π -line dynamics with parameters as in [18]. The parameters of the HVDC VSCs (with RLC filters) can be found in [18]. The MPP of the renewable generation, their nominal operating points, control gains, and effective droop coefficients are given in Table II. While the reactive power of WT is controlled to be zero, all converters interfacing PVs, EES and HVDC use standard GFM $Q - V$ droop [4] of 1%.

TABLE II
MPP, NOMINAL OPERATING POINTS & CONTROL GAINS

PV₃ (1200 parallel strings of 90 modules) [p.u.]					
$P_3^{\text{mpp}} \approx 0.33$,	$P_3^* = P_3^{\text{mpp}}$,	$k_{g,3} = 0$,	$k_{p,3} = 0.001$,	$k_{\omega,3} = 0.2$,	$\kappa_{p,3} = /$
PV₁₂ (5000 parallel strings of 90 modules) [p.u.]					
$P_{12}^{\text{mpp}} \approx 1.37$,	$P_{12}^* = 94.4\%P_{12}^{\text{mpp}}$,	$k_{g,12} \approx 2.5$,	$k_{p,12} = 0.001$,	$k_{\omega,12} \approx 0.12$,	$\kappa_{p,12} = 5\%$
PV₁₃ (3000 parallel strings of 100 modules) [p.u.]					
$P_{13}^{\text{mpp}} \approx 0.92$,	$P_{13}^* = P_{13}^{\text{mpp}}$,	$k_{g,13} = 0$,	$k_{p,12} = 0.001$,	$k_{\omega,13} = 0.05$,	$\kappa_{p,13} = /$
PV₂₂ (5500 parallel strings of 95 modules) [p.u.]					
$P_{22}^{\text{mpp}} \approx 1.6$,	$P_{22}^* = P_{22}^{\text{mpp}}$,	$k_{g,22} = 0$,	$k_{p,22} = 0.001$,	$k_{\omega,22} = 0.2$,	$\kappa_{p,22} = /$
PV₂₃ (5700 parallel strings of 100 modules) [p.u.]					
$P_{23}^{\text{mpp}} \approx 1.6$,	$P_{23}^* = P_{23}^{\text{mpp}}$,	$k_{g,23} = 0$,	$k_{p,23} = 0.001$,	$k_{\omega,23} = 0.2$,	$\kappa_{p,23} = /$
WT₇₀ ($v_w = 12$ m/s, $\beta^* = 0$) & DC-B2B [p.u.]					
$P_{70}^{\text{mpp}} = 0.75$,	$P_{70}^* = 90\%P_{70}^{\text{mpp}}$,	$k_{g,70} = 0.6$,	$k_{p,70} = 0.015$,	$k_{\omega,70} = 5$,	$\kappa_p = 3.33\%$
			$k_{p,11} = 0.015$,	$k_{\omega,11} = 0.1$,	
ESS₂ (rated power 1.4 p.u.) [p.u.]					
$P_2^* = 1.1$,		$k_{g,2} = 4$,	$k_{p,2} = 0.001$,	$k_{\omega,2} \approx 0.026$,	$\kappa_{p,2} = 5\%$
SG₁ (rated power 1.5 p.u.), SG₃₁ , SG₃₂ , SG₃₃ (rated power 1 p.u.) [p.u.]					
$P_1^* \approx 0.82$,	$P_{31}^* \approx 0.77$,	$P_{32}^* \approx 0.57$,	$P_{33}^* \approx 0.37$,	$k_{g,l} = 20$,	$\kappa_{p,l} = 5\%$
HVDC links: DC 1 (310 km), DC 2 (510 km)					
			$k_{p,l} = 0.001$	$k_{\omega,l} = 0.2$	
HVDC link DC 3 (1000 km) [p.u.]					
	$k_{p,40} = k_{p,40} = 0.001$	$k_{\omega,40} = 0.2$		$k_{\omega,50} = 0.35$	
VSC_l , $l \in \{2, 3, 11, 12, 13, 22, 23, 19, 20, 30, 60, 40, 50\}$ [p.u.]					
Current PI: $k_{p,l}^i = 2.1$, $k_{i,l}^i = 0.79$ S			Voltage PI: $k_{p,l}^i = 0.15$, $k_{i,l}^i = 0.69$		

B. Sequence of events

To verify the behavior of the proposed universal dual-port GFM control, we simulate events in Table III. The dispatch points and control gains are given in Table II and Table IV.

For the simulated sequence, Fig. 6 shows frequency, active power and dc voltage in the areas AC 1-AC 3, while Fig. 7 shows PMSG WT frequency, dc voltage of the WT VSCs, and mechanical power of the WT (i.e., AC 4). Moreover, we emphasize that the 0.25 p.u. load step is very large and pushes the system to the boundary of the normal operating range. Nonetheless, the system dynamics stay well behaved. Moreover, as expected, the power imbalance propagates to all ac and dc subnetworks, and the curtailed power sources provide primary frequency response according to their effective droop coefficients. After the load step at $t = 5$ s, the PV systems operated at the MPP do not change their power output, even

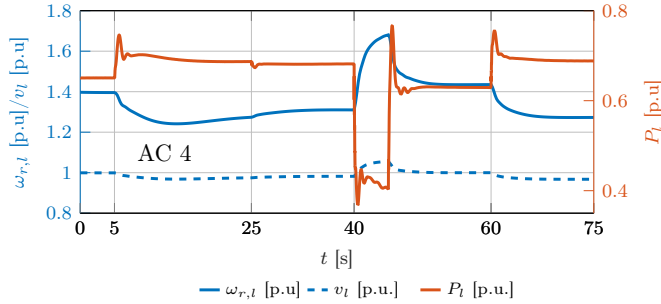
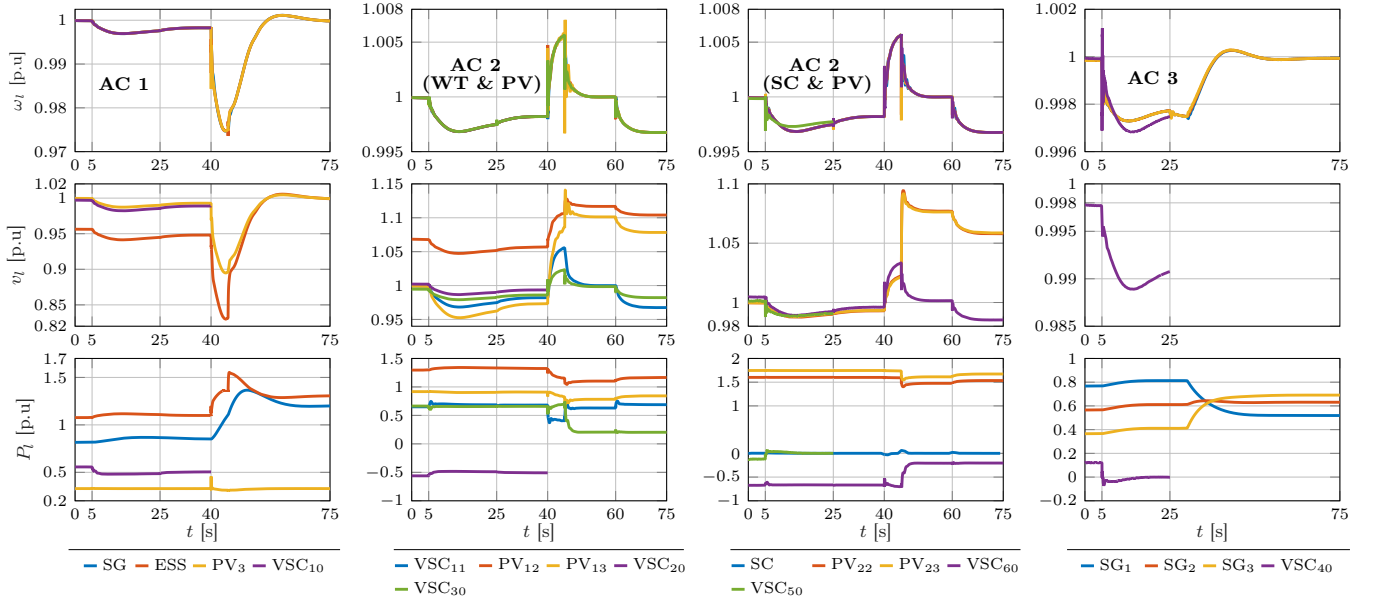


Fig. 7. PMSG rotor speed ω_l , dc voltage v_l of the WT VSCs, and generated power P_l .

though the dc voltage slightly varies around the MPP voltage. This verifies that the sensitivity of the PV power generation to dc voltage variations is approximately zero around the MPP (see Fig. 1). Hence, we show that proposed GFM control (11) can exhibit conventional ac-GFL/dc-GFM functions without switching control modes. Additionally, we stress that the instances in which $v_l < v_l^{\text{mpp}}$ can be prevented by including a dc voltage limiter similar to [14, Fig. 11] and [9, Fig. 4], that becomes active when $v_l < v_l^{\text{mpp}}$.

TABLE III

t	Sequence of Events						
	0 s	5 s	25 s	30 s	40 s	45 s	60 s
event	nominal	0.25 p.u. b37	DC 3 disconnect	AC 3 redispatch	DC 1 disconnect	AC 2 redispatch	0.125 p.u. b16 & b29
Different scenarios of load-steps and their locations							
S #	1	2	3	4	5	6	7
b7	0.25	/	/	/	/	0.0625	0.125
b15	/	0.25	/	/	0.083	0.0625	0.1
b29	/	/	0.25	/	0.167	0.0625	/
b39	/	/	/	0.25	/	0.1	/

Moreover, we use a loss of two HVDC lines as an extreme and unlikely contingency to illustrate that the system stays well-behaved even during those events. After the loss of DC 3,

TABLE IV
REDISPATCHED SETPOINTS [P.U.]

AC 1		$P_1^r = 1.2$, $P_2^r \approx 1.35$, $k_{g,2}^r \approx 1.16$, $k_{\omega,2}^r \approx 0.23$, $\kappa_{P,l} = 5\%$,			
AC 2	$P_{12}^r \approx 86.9\% P_{12}^{\text{mpp}}$,	$k_{g,12}^r \approx 4.2$,	$k_{\omega,12}^r \approx 0.21$,	$\kappa_{P,l} = 5\%$,	
	$P_{13}^r \approx 98.1\% P_{13}^{\text{mpp}}$,	$k_{g,13}^r = 2.1$,	$k_{\omega,13}^r \approx 0.11$,		
	$P_{22}^r \approx 89.2\% P_{22}^{\text{mpp}}$,	$k_{g,22}^r \approx 2.7$,	$k_{\omega,22}^r \approx 0.13$,		
	$P_{23}^r \approx 87.7\% P_{23}^{\text{mpp}}$,	$k_{g,23}^r \approx 2.8$,	$k_{\omega,23}^r \approx 0.14$,		
	$P_{70}^r \approx 89.1\% P_{70}^{\text{mpp}}$,	$k_{g,70}^r = 0.67$,	/		
AC 3		$P_{31}^r = 0.52$,	$P_{32}^r = 0.63$,	$P_{33}^r = 0.69$	

the frequency in the AC 1 and AC 2 rises, and the frequency in AC 3 drops since AC 3 was importing power. However, there are enough power reserves in AC 3 to support its load. Hence, after redispatching the power generation in AC 3, the frequency settles to its nominal value. On the other hand, after the loss of DC 1, the frequency in AC 1 significantly drops since power was imported via the HVDC link. Hence, in real-world scenarios, under frequency load shedding would be initiated in AC 1. However, for simulation purposes, we allow for very large frequency deviations and redispatch the system such that the SGs and ESS supply the load. Moreover, in contrast to the classical PLL-based ac-GFL/dc-GFM MPPT control, despite the large frequency deviation, the proposed control strategy reliably keeps the PV system in area AC 1 at the MPP. Finally, after both HVDC links are disconnected, the response of the converter-dominated system AC 2 is still well-behaved. After the loss of the DC 1, due to the relatively large mismatch of power production and consumption in AC 2, the frequency in AC 2 rises significantly. Moreover, the proposed control responds to the power imbalance by reducing the renewable generation, i.e., exhibits conventional ac-GFM/dc-GFL grid support functions. Hence, to operate the system at the nominal frequency we redispatch ($t = 45$ s) the renewable generation

in AC 2 to a heavily curtailed operating point, nominal operation with significant reserves. The occurring at $t = 60$ s further test the system control renewable generation, HVDC, and SC (AC 2). In this case, the system dynamics stay well-behaved. power imbalances propagate throughout the system power sources share the additional load according to effective droop coefficients κ_P .

C. Frequency RoCoF and Nadir

To compactly illustrate the system response to various load steps, we simulate seven different scenarios for the system in Fig 3. The locations and values of the load steps occurring at $t = 25$ s are given in Table III. Figure 8 shows RoCoF (calculated as the largest frequency change in the time window of 300 ms) and frequency nadir. We stress that the load steps are large and that they drive the system on the boundary of the nominal operating range. We note that the largest RoCoF occurs at bus 40 during the event #4 (i.e., disturbance at bus 37) and the HVDC link (DC 3) is importing power from other areas. Notably, bus 40 corresponds to the HVDC VSC for which large RoCoF is not a concern. In contrast, during the same event, the RoCoF of the SG at bus 33 (i.e., close to the disturbance) stays within the RoCoF limits of SGs. Finally, as discussed in Sec. VI-B, Cond. 1 can be relaxed for point-to-point HVDC. To illustrate this aspect, the VSCs in DC 3 have significantly different gains $k_{\omega,l}$ and, as a result, the frequency nadir of devices in AC 3 and AC 1 and AC 2 differ significantly.

VIII. CONCLUSION AND OUTLOOK

In this paper, we investigate a unified dual-port GFM control paradigm for dc/ac VSCs. We provide stability conditions and prove small-signal stability results for generic hybrid ac/dc systems containing different types of renewable generation, ac networks, and dc networks. Next, we interpret the analytical stability conditions in the context of typical application scenarios and illustrate how to tune control gains to meet steady-state specifications and tune the transient response. Moreover, we illustrate how the proposed modeling and control framework can be used to obtain any complex network topology. Finally, we use a system that includes standard legacy technologies and emerging technologies to illustrate theoretical results and claims. While these results are encouraging, further work is needed to understand aspects such as current limiting and fault ride through that is well understood for GFL control but requires further study for dual-port GFM control.

APPENDIX

A. Interconnection matrices

The interconnection of SMs and power sources with $k_{g,l} > 0$ are modeled by $\mathcal{I}_{r,ac} \in \{0,1\}^{|\mathcal{N}_r| \times |\mathcal{N}_{ac}|}$, i.e., $\{\mathcal{I}_{r,ac}\}_{(i,j)} = 1$ if the power source $i \in \mathcal{N}_r$ is connected to the SM $j \in \mathcal{N}_{ac}$ and $\{\mathcal{I}_{r,ac}\}_{(i,j)} = 0$ otherwise. Similarly, $\mathcal{I}_{r,dc} \in \mathbb{R}^{|\mathcal{N}_r| \times |\mathcal{N}_{dc}|}$ models the connection of dc nodes to dc power source with $k_{g,l} > 0$, i.e., $\{\mathcal{I}_{r,dc}\}_{(i,j)} = 1$ if $j \in \mathcal{N}_r$ is connected to $i \in \mathcal{N}_{dc}$

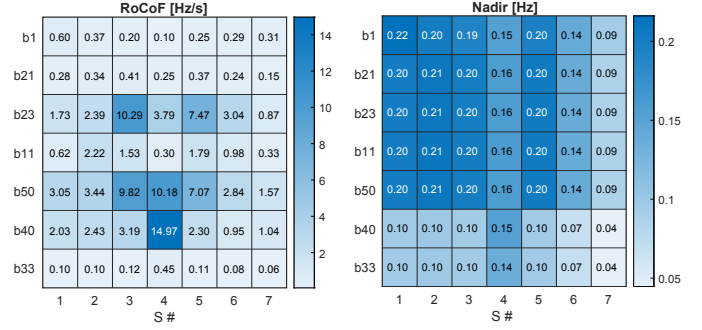


Fig. 8. Frequency RoCoF a), and Nadir b) during scenarios as in Table III.

and $\{\mathcal{I}_{r,dc}\}_{(i,j)} = 0$ otherwise. Analogously, the matrices $\mathcal{I}_{zs,ac} \in \{0,1\}^{|\mathcal{N}_{zs}| \times |\mathcal{N}_{ac}|}$ and $\mathcal{I}_{zs,dc} \in \{0,1\}^{|\mathcal{N}_{zs}| \times |\mathcal{N}_{dc}|}$ model the interconnection between SMs and power sources \mathcal{N}_{zs} and dc nodes and power sources \mathcal{N}_{zs} . Finally, the matrix $\mathcal{I}_{pv} \in \{0,1\}^{|\mathcal{N}_{pv}| \times |\mathcal{N}_{dc}|}$ (resp. $\mathcal{I}_w \in \{0,1\}^{|\mathcal{N}_w| \times |\mathcal{N}_{ac}|}$) models connections between dc buses and PV (resp. SMs and WTs). We also define $\mathcal{I}_{ac} \in \{0,1\}^{|\mathcal{N}_{ac}| \times |\mathcal{N}_{ac} \cup \mathcal{N}_{dc}|}$ and $\mathcal{I}_{dc} \in \{0,1\}^{|\mathcal{N}_{dc}| \times |\mathcal{N}_{dc} \cup \mathcal{N}_{ac}|}$ to extract machine and converter angles from the phase angle vector θ , e.g., $\mathcal{I}_{ac}\theta$ is the vector of all machine angles and $\mathcal{I}_{dc}\theta$ collects all converter phase angles. Similarly, $\mathcal{I}_{c,dc} \in \{0,1\}^{|\mathcal{N}_{dc}| \times |\mathcal{N}_{dc} \cup \mathcal{N}_{ac}|}$ and $\mathcal{I}_{dc} \in \{0,1\}^{|\mathcal{N}_{dc}| \times |\mathcal{N}_{dc} \cup \mathcal{N}_{ac}|}$ extract the dc voltages of VSCs and dc nodes from the vector v .

B. Proofs

Lemma 1 For all $i \in \mathbb{N}_{[1,N_{dc}]}$ and $l \in \mathcal{N}_{c,dc}^i$, Cond. 2 implies that $\sum_{(l,k) \in \mathcal{E}_{dc}^i} g_{lk}^{dc,i} e_l + \sum_{(l,k) \in \mathcal{E}_{dc}^i \cap (\mathcal{N}_{c,dc}^i \times \mathcal{N}_{dc}^i)} g_{lk}^{dc,i} \sqrt{e_l e_k} < 4k_{\omega}^i$ holds with $e_l := k_{p,l}/c_l \in \mathbb{R}_{>0}$.

Proof of Lemma 1: It directly follows from Cond. 2 that $2 \sum_{(l,k) \in \mathcal{E}_{dc}^i} g_{lk}^{dc,i} / (4k_{\omega}^i) < c_l / k_{p,l} = e_l^{-1} > 0$ for all $l \in \mathcal{N}_{c,dc}^i$, i.e., $\max_{l \in \mathcal{N}_{c,dc}^i} e_l < 4k_{\omega}^i / (2 \sum_{(l,k) \in \mathcal{E}_{dc}^i} g_{lk}^{dc,i})$. The Lemma directly follows from $\max_{\xi \in \mathcal{N}_{c,dc}^i} e_{\xi} (\sum_{(l,k) \in \mathcal{E}_{dc}^i} g_{lk}^{dc,i} + \sum_{k \in \mathcal{N}_{c,dc}^i} g_{lk}^{dc,i}) \leq 2 \max_{\xi \in \mathcal{N}_{c,dc}^i} e_{\xi} \sum_{(l,k) \in \mathcal{E}_{dc}^i} g_{lk}^{dc,i} < 4k_{\omega}^i$. ■

Proof of Prop. 1 It can be verified that $\frac{d}{dt}V = \frac{1}{2}x_{\delta}^T(\mathcal{M}A + A^T\mathcal{M})x_{\delta}$. By substituting \mathcal{M} and A and performing elementary algebraic manipulations, it follows that $\frac{d}{dt}V = -\tilde{x}_{\delta}^T \mathcal{V} \tilde{x}_{\delta} - (\mathcal{I}_w \omega_{\delta})^T K_w \mathcal{I}_w \omega_{\delta} - \frac{1}{2}v_{\delta}^T (\tilde{K}_w \Xi + \Xi \tilde{K}_w) v_{\delta}$. By definition, $K_w \succ 0$, $K_{pv} \succ 0$ and $\tilde{K}_w \succ 0$. Then, using the Schur complement $\mathcal{V} \succeq 0$ holds if and only if $B_{dc}^i \mathcal{W}_{dc}^{i \frac{1}{2}} (k_{\omega}^i I_{|\mathcal{E}_{dc}^i|} - \frac{1}{4} E^i \mathcal{I}_{dc}^i E^i) \mathcal{W}_{dc}^{i \frac{1}{2}} B_{dc}^{i \top} \succ 0$ for all $i \in \mathbb{N}_{[1,N_{dc}]}$ and $E^i = \text{diag}\{\sqrt{e_l}\}_{l=1}^{|\mathcal{N}_{c,dc}^i|} \mathcal{I}_{dc}^i B_{dc}^i \mathcal{W}_{dc}^{i \frac{1}{2}}$. Applying Gershgorin's theorem we obtain $\lambda_{\max}(E^i \mathcal{I}_{dc}^i E^i) \leq \max_{l \in \mathcal{N}_{c,dc}^i} \sum_{(l,k) \in \mathcal{E}_{dc}^i} g_{lk}^{dc,i} e_l + \sum_{k \in \mathcal{N}_{c,dc}^i} g_{lk}^{dc,i} \sqrt{e_l e_k}$. Finally, using Lemma 1 it can be verified that Cond. 2 ensures $\tilde{\mathcal{V}} \succeq 0$ and the proposition follows. ■

Proof sketch of Prop. 2: Comparing (14) and (13) and with $m_{p,l} = \frac{k_{p,l}}{C_l}$, the difference between universal dual-port GFM control (11) applied to the VSC model (3) and the power-balancing dual-port GFM control in [18] is the term $-\frac{k_{p,l}}{C_l} P_{\delta,dc,l}$. This results in the term $K_p \mathcal{I}_{dc} C^{-1} L_{dc} v_{\delta}$ in (15). Using the same steps as in the proof of [18, Prop. 2] this term vanishes when x_{δ} is restricted to the invariant set $\tilde{\mathcal{S}}$ and the proof of Prop. 2 follows from the proof of [18, Prop. 2]. ■

Proof of Prop. 3: Letting $\frac{d}{dt}v_{\delta,l} = 0$ and combining (3), (1c), and (5), results in $(\mathcal{I}_{\text{cac}}P_{\text{ac}}, 0_{|\mathcal{N}_{\text{dc}}|}) = P_{\text{dc}} + (L_{\text{dc}} + K'_{\text{g,dc}})(\mathcal{I}_{\text{c}_{\text{dc}}}v_{\delta}, \mathcal{I}_{\text{dc}}v_{\delta})$ with diagonal matrix $K'_{\text{g,dc}} := \mathcal{I}_{\text{dc}}^{\top}(\mathcal{I}_{\text{pv}}K_{\text{pv}}\mathcal{I}_{\text{pv}} + \mathcal{I}_{\text{r,dc}}K_{\text{g}}\mathcal{I}_{\text{r,dc}})\mathcal{I}_{\text{dc}}$. Using $\mathbf{g} := \max_{(l,k) \in \mathcal{E}_{\text{dc}}} g_{l,k}^{\text{dc}}$, we introduce the normalized dc network Laplacian $\bar{L}_{\text{dc}} := \mathbf{g}^{-1}L_{\text{dc}}$. Next, we use Kron reduction [25] to eliminate all dc nodes to obtain $\mathcal{I}_{\text{cac}}P_{\text{ac},\delta} = -(\mathbf{g}\bar{L}_{\text{dc}} + \Delta_{\text{dc}})\mathcal{I}_{\text{c}_{\text{dc}}}v_{\delta} - \bar{D}_{\text{cac}}P_{\text{dc}}$. From [25, Thm. III.6-3]) the matrices $\mathbf{g}\bar{L}_{\text{dc}}$, Δ_{dc} (as a function of \mathbf{g}^{-1}), and \bar{D}_{cac} correspond to the loop-less Laplacian, self-loops, and mapping of P_{dc} to the VSCs. In steady-state, (11) implies $\mathcal{I}_{\text{cac}}\omega_{\delta} = K_{\omega}\mathcal{I}_{\text{c}_{\text{dc}}}v_{\delta}$, i.e.,

$$\mathcal{I}_{\text{cac}}P_{\text{ac},\delta} = -(\mathbf{g}\bar{L}_{\text{dc}} + \Delta_{\text{dc}})K_{\omega}^{-1}\mathcal{I}_{\text{c}_{\text{ac}}}\omega_{\delta} - \bar{D}_{\text{cac}}P_{\text{dc}} \quad (17)$$

Using the same arguments as in the proof of Prop. 2 and [18, Prop. 8] with $\frac{d}{dt}P_d = 0_{n_d}$, Cond. 5 ensures synchronization to a synchronous frequency Ω_i for each ac subgrid $i \in \{1, \dots, N_{\text{ac}}\}$. Substituting (17) and $L := B_{\text{ac}}\mathcal{W}_{\text{ac}}B_{\text{ac}}^{\top}$ into the ac and ac/dc node steady-state equations results in

$$\begin{bmatrix} (\mathcal{I}_{\text{w}}^{\top}K_{\text{w}}\mathcal{I}_{\text{w}} + \mathcal{I}_{\text{r,ac}}K_{\text{g}}\mathcal{I}_{\text{r,ac}})\mathcal{I}_{\text{ac}} \\ (\mathbf{g}\bar{L}_{\text{dc}} + \Delta_{\text{dc}})K_{\omega}^{-1}\mathcal{I}_{\text{c}_{\text{ac}}} \end{bmatrix} \omega_{\delta} = -L\theta_{\delta} - P_{\text{d,ac}} - \mathcal{I}_{\text{c}_{\text{ac}}}^{\top}\bar{D}_{\text{cac}}P_{\text{dc}}.$$

Summing over the rows for each ac subnetwork results in $(\Delta + \mathbf{g}\bar{L}'_{\text{dc}})\Omega_{\delta} = -\bar{P}_d$ where the vector Ω_{δ} collect all $\Omega_{\delta,i}$. Next, $\{\mathcal{B}_{\omega}\}_{i,j \in \{0,1\}^{|\mathcal{N}_{\text{c}}| \times N_{\text{ac}}}}$ models how VSCs are connected to the ac subnetwork (i.e., $\{\mathcal{B}_{\omega}\}_{i,j} = 1$ if $i \in \mathcal{N}_{\text{c}_{\text{ac}}}^j$). The matrix $\bar{L}'_{\text{dc}} := B_{\omega}\bar{L}_{\text{dc}}K_{\omega}^{-1}B_{\omega}^{\top}$ corresponds to the (loop-less) Laplacian matrix of a graph where nodes correspond to ac subnetworks and edges correspond to their interconnection through dc networks. The matrix $\Delta := \text{diag}\{\sum_{l \in \mathcal{N}_{\text{ac}}^i} \kappa_{P,l}^{-1}\}_{i=1}^{N_{\text{ac}}} + B_{\omega}\Delta_{\text{dc}}K_{\omega}^{-1}B_{\omega}^{\top}$ corresponds to the self-loops, and $\bar{P}_d := B_{\omega}\mathcal{I}_{\text{c}_{\text{ac}}}\bar{D}_{\text{cac}}P_{\text{dc}} + \text{diag}_{i=1}^{N_{\text{ac}}}\{\sum_{l \in \mathcal{N}_{\text{ac}}^i \cup \mathcal{N}_{\text{ac}}^i} P_{\text{dc},l}\}\mathbb{1}_{N_{\text{ac}}}$ is a vector of total disturbances in each ac subnetwork. We use $(\cdot)_{\text{r}}$ and $(\cdot)_{\text{zs}}$ to differentiate between variables corresponding to the nodes with and without self-loops. Without loss of generality, we write $\text{blkdiag}\{\tilde{\Delta}, 0_{|\Omega_{\text{zs},\delta}|}\}(\Omega_{\text{r},\delta}, \Omega_{\text{zs},\delta}) = -\mathbf{g}\bar{L}'_{\text{dc}}(\Omega_{\text{r},\delta}, \Omega_{\text{zs},\delta}) - (\bar{P}_{\text{d,r}}, \bar{P}_{\text{d,zs}})$. By Cond. 3 $\tilde{\Delta} \succ 0$ is non-empty and collects all non-zero self loops from Δ . By applying Kron reduction² to remove variables $\Omega_{\text{zs},\delta}$, we obtain $(\tilde{\Delta} + \mathbf{g}\bar{L})\Omega_{\text{r},\delta} = -(\bar{P}_{\text{d,r}} + \bar{D}_d\bar{P}_{\text{d,zs}})$. From [25, Thm. III.6-3]), the matrices $\mathbf{g}\bar{L}$ and \bar{D}_d correspond to the loop-less Laplacian, and mapping of $P_{\text{d,zs}}$ to the nodes with self-loops. Solving the previous equation for $\Omega_{\text{r},\delta}$ and multiplying it with \tilde{B}^{\top} (the incidence matrix of \tilde{L}) we have $\tilde{B}^{\top}\Omega_{\text{r},\delta} = -\tilde{B}^{\top}(\tilde{\Delta} + \mathbf{g}\bar{L})^{-1}(\bar{P}_d + \bar{D}_d\bar{P}_{\text{d,zs}})$. Applying the Woodbury matrix identity to $(\tilde{\Delta} + \mathbf{g}\bar{L})^{-1}$ and letting $\mathbf{g} \rightarrow \infty$, we obtain $\lim_{\mathbf{g} \rightarrow \infty} \tilde{B}^{\top}\Omega_{\text{r},\delta} = 0_{|\Omega_{\text{r},\delta}|}$, i.e., $\Omega_{\text{r},\delta,\infty} := \lim_{\mathbf{g} \rightarrow \infty} \Omega_{\text{r},\delta} \in \text{null}\{\tilde{B}^{\top}\} = \delta_{\omega}\mathbb{1}_{|\Omega_{\text{r},\delta}|}$, with $\delta_{\omega} \in \mathbb{R}$. Using $\lim_{\mathbf{g} \rightarrow \infty} \mathbb{1}_{|\Omega_{\text{r},\delta}|}^{\top}(\tilde{\Delta} + \mathbf{g}\bar{L})^{-1} = \mathbb{1}_{|\Omega_{\text{r},\delta}|}^{\top}$ we have $\delta_{\omega} = -(\mathbb{1}_{n_d}^{\top}P_d)/(\sum_{l \in \mathcal{N}_{\text{r}} \cup \mathcal{N}_{\text{pv}} \cup \mathcal{N}_{\text{w}}} \kappa_{P,l}^{-1})$. Moreover, it can be verified that $\Omega_{\text{zs},\delta,\infty} = \mathbb{1}_{|\Omega_{\text{zs},\delta}|}\delta_{\omega}$. The proof directly follows from $\omega_{\delta,l}^{\text{ss}} = \Omega_{\delta,i}$, for all $l \in \mathcal{N}_{\text{ac}}^i \cup \mathcal{N}_{\text{c}_{\text{ac}}}^i$ with $i \in \{1, \dots, N_{\text{ac}}\}$. ■

REFERENCES

[1] F. Milano, F. Dörfler, G. Hug, D. J. Hill, and G. Verbič, “Foundations and challenges of low-inertia systems (invited paper),” in *Power Systems Computation Conference*, 2018.

²If $|\tilde{\Delta}| = 1$, then $\tilde{B} = \tilde{B}^{\top} := \mathbb{1}$, $\tilde{W} := \tilde{L}$. Otherwise, \tilde{B} , \tilde{W} correspond to the incidence and edge weight matrices of the strictly-loopless Laplacian \tilde{L} .

[2] J. Matevosyan, B. Badrzadeh, T. Prevost, E. Quitmann, D. Ramasubramanian, H. Urdal, S. Achilles, J. MacDowell, S. H. Huang, V. Vital, J. O’Sullivan, and R. Quint, “Grid-forming inverters: Are they the key for high renewable penetration?” *IEEE Power Energy Mag.*, vol. 17, no. 6, pp. 89–98, 2019.

[3] M. Chandorkar, D. Divan, and R. Adapa, “Control of parallel connected inverters in standalone ac supply systems,” *IEEE Trans. Ind. Appl.*, vol. 29, no. 1, pp. 136–143, 1993.

[4] S. D’Arco, J. A. Suul, and O. B. Fosso, “A virtual synchronous machine implementation for distributed control of power converters in smartgrids,” *Electr. Pow. Sys. Res.*, vol. 122, pp. 180–197, 2015.

[5] D. Groß, M. Colombino, B. Jean-Sébastien, and F. Dörfler, “The effect of transmission-line dynamics on grid-forming dispatchable virtual oscillator control,” *IEEE Trans. Control Netw. Syst.*, vol. 6, no. 3, pp. 1148–1160, 2019.

[6] I. Subotić, D. Groß, M. Colombino, and F. Dörfler, “A Lyapunov framework for nested dynamical systems on multiple time scales with application to converter-based power systems,” *IEEE Trans. Autom. Control*, vol. 66, no. 12, pp. 5909–5924, 2021.

[7] A. Tayyebi, D. Groß, A. Anta, F. Kupzog, and F. Dörfler, “Frequency stability of synchronous machines and grid-forming power converters,” *IEEE Trans. Emerg. Sel. Topics Power Electron.*, vol. 8, no. 2, pp. 1004–1018, 2020.

[8] D. Groß, Sánchez-Sánchez, E. Prieto-Araujo, and O. Gomis-Bellmunt, “Dual-port grid-forming control of MMCs and its applications to grids of grids,” *IEEE Trans. Power Del.*, vol. 37, no. 6, pp. 4721–4735, 2022.

[9] X. Lyu, I. Subotić, and D. Groß, “Unified grid-forming control of pms wind turbines for fast frequency response and MPPT,” in *Bulk Power Systems Dynamics and Control Symposium*, 2022.

[10] J. Arévalo Soler, D. Groß, E. P. Araujo, and O. G. Bellmunt, “Interconnecting power converter control role assignment in grids with multiple AC and DC subgrids,” *IEEE Trans. Power Del.*, 2023.

[11] N. Pogaku, M. Prodanovic, and T. C. Green, “Modeling, analysis and testing of autonomous operation of an inverter-based microgrid,” *IEEE Trans. Power Electron.*, vol. 22, no. 2, pp. 613–625, 2007.

[12] U. Marković, O. Stanojev, E. Vrettos, P. Aristidou, and G. Hug, “Understanding small-signal stability of low-inertia systems,” *IEEE Trans. Power Syst.*, 2021.

[13] P. Vorobev, P.-H. Huang, M. Al Hosani, J. L. Kirtley, and K. Turitsyn, “High-fidelity model order reduction for microgrids stability assessment,” *IEEE Trans. Power Syst.*, vol. 33, no. 1, pp. 874–887, 2018.

[14] R. H. Lasseter, Z. Chen, and D. Pattabiraman, “Grid-forming inverters: A critical asset for the power grid,” *IEEE Trans. Emerg. Sel. Topics Power Electron.*, vol. 8, no. 2, pp. 925–935, 2020.

[15] I. Cvetkovic, D. Boroyevich, R. Burgos, C. Li, and P. Mattavelli, “Modeling and control of grid-connected voltage-source converters emulating isotropic and anisotropic synchronous machines,” in *Workshop on Contr. and Modeling for Power Electr.*, 2015.

[16] P. Monshizadeh, C. De Persis, T. Stegink, N. Monshizadeh, and A. van der Schaft, “Stability and frequency regulation of inverters with capacitive inertia,” in *IEEE Conf. on Dec. and Contr.*, 2017, pp. 5696–5701.

[17] A. Tayyebi, A. Anta, and F. Dörfler, “Grid-forming hybrid angle control and almost global stability of the dc-ac power converter,” *IEEE Trans. Autom. Control*, 2022.

[18] I. Subotić and D. Groß, “Power-balancing dual-port grid-forming power converter control for renewable integration and hybrid ac/dc power systems,” *IEEE Trans. Control Netw. Syst.*, vol. 9, no. 4, pp. 1949–1961, 2022.

[19] M. G. Villalva, J. R. Gazoli, and E. R. Filho, “Comprehensive approach to modeling and simulation of photovoltaic arrays,” *IEEE Trans. Power Electron.*, vol. 24, no. 5, pp. 1198–1208, 2009.

[20] A. F. Hoke, M. Shirazi, S. Chakraborty, E. Muljadi, and D. Maksimović, “Rapid active power control of photovoltaic systems for grid frequency support,” *IEEE Trans. Emerg. Sel. Topics Power Electron.*, vol. 5, no. 3, pp. 1154–1163, 2017.

[21] H. A. B. Siddique and R. W. De Doncker, “Evaluation of DC collector-grid configurations for large photovoltaic parks,” *IEEE Trans. Power Del.*, vol. 33, no. 1, pp. 311–320, 2018.

[22] D. Sehloff and L. A. Roald, “Low frequency AC transmission upgrades with optimal frequency selection,” *IEEE Trans. Power Syst.*, vol. 37, no. 2, pp. 1437–1448, 2022.

[23] J. Aho, P. Fleming, and L. Y. Pao, “Active power control of wind turbines for ancillary services: A comparison of pitch and torque control methodologies,” in *American Control Conference*, 2016, pp. 1407–1412.

[24] D. Groß, “Compensating network dynamics in grid-forming control,” in *Allerton Conference on Communication, Control, and Computing*, 2022.

- [25] F. Dörfler and F. Bullo, “Kron reduction of graphs with applications to electrical networks,” *IEEE Trans. Circuits Syst. I*, vol. 60, no. 1, pp. 150–163, 2013.