Enhanced Power System State Estimation With Overhead Line Sensors

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***Abstract*--This letter addresses the incorporation of transmission line sensor measurements into power system state estimation (SE). These measurements are obtained from various locations along a line and are different from conventional RTU measurements at line terminals (i.e., substations). We propose a measurement function that can relate the current measurements at arbitrary positions along a line to the existing state variables at substation buses using line parameters and sensor locations. Hence, these measurements can be easily incorporated into SE without altering its state vector or system model. Simulation results on the IEEE 14-bus system and the NPCC 140-bus system demonstrate that the incorporation of line sensor measurements can significantly enhance SE's noise filtering and bad data processing performances.**

***Index Terms*-- State estimation, transmission line monitoring, line sensor, current measurement.**

I. INTRODUCTION

OVERHEAD line sensors have been gaining popularity in improving the situational awareness of power systems in recent years [1]-[3]. They are typically installed along the span of a transmission line to monitor its detailed conditions in support of *line-level* applications such as dynamic line rating (DLR) [4] and fault detection and location [5], etc. However, this new source of information has not been incorporated into *system-level* applications, such as state estimation (SE). The incorporation of such measurements can enhance the information redundancy that contributes to the noise filtering and bad data processing performance of SE [6]; It also expands the set of use cases of line sensor assets and increases returns on investments.

As conventional SEs only use the measurements at the terminals of transmission lines (RTUs at substations), they typically employ the lumped π-equivalent model of the lines [8]. In contrast, overhead line sensors are usually equipped along the span of a line, with varying current measurements at different locations due to the distributed shunt capacitance along the line. Hence, the measured current cannot be easily related to the existing state variables at substation buses (bus voltage phasors). A straightforward idea is to segment the transmission line into multiple π-models and add virtual nodes at the locations of the sensors; however, this will change the system models used by conventional SEs and introduce unnecessary states at the locations of sensor installations, which are not of interest to SE.

To address this issue, this letter proposes a novel method for incorporating the current measurements of the sensors installed along a transmission line into SE *without* changing the conventional SE model (i.e., without introducing virtual nodes and states). Specifically, current measurements collected by overhead line sensors are related to the state variables of the two terminal buses via the sensor location as well as the line parameters. Simulation results show that with its ease of implementation, the proposed method can fully utilize sensor measurements to enhance SE's noise filtering and bad data processing performance to a significant extent.

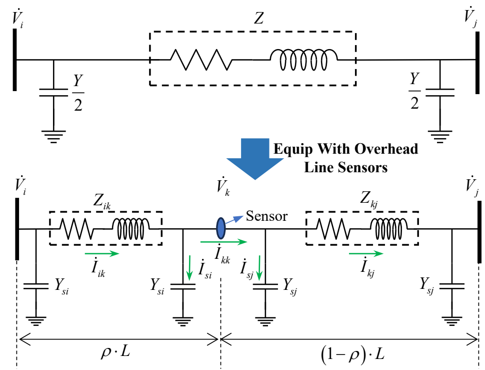

Fig. 1. π-equivalent models with and without overhead line sensors.

It should be noted that the primary objective in this letter is to develop a novel approach to enhance SE by measurements from existing line sensors primarily purposed for other applications, such as DLR and fault detection/location. Hence, the optimization of sensor placement for SE performance enhancement will not be considered. Moreover, regarding measurement uncertainty modeling and advanced SE methods for countering the uncertainties, please refer to our recent work [7].

II. ENHANCED POWER SYSTEM STATE ESTIMATION WITH OVERHEAD LINE SENSORS

A. *Proposed Measurement Functions for Line Sensor Current*

Overhead line sensors are typically equipped along the transmission line to monitor diverse quantities such as current and conductor temperature [1]-[3]. As line sensors are installed along the span of (as opposed to at the terminal of) a line, the representation of their current measurements at least requires a split of the π-equivalent model into two, as illustrated in Fig. 1. However, this will change power system models formulated by conventional SE, and introduce new state variables at the virtual node, $\dot{V}_k$, which is not of interest to SE. In this section, we will derive a novel measurement function for incorporating line sensor currents without expressing $\dot{V}_k$. Based on the π-equivalent model of the line between bus *i* and sensor *k*, the current phasor measurement can be expressed as follows,

$$\dot{I}_{ik} = \left(\dot{V}_i - \dot{V}_k\right) / Z_{ik}, \quad (1)$$

$$\dot{I}_{si} = \dot{V}_k \cdot Y_{si}, \quad (2)$$

$$\dot{I}_{kk} = \dot{I}_{ik} - \dot{I}_{si}, \tag{3}$$

where $\dot{V}_i$ and $\dot{V}_k$ represent voltage phasors at bus i and sensor k, respectively; Z_{ik} and Y_{si} are the series impedance and half shunt admittance of the line segment between bus i and sensor k; $\dot{I}_{ik}$ and $\dot{I}_{si}$ represent the current phasors through Z_{ik} and Y_{si}, respectively; and $\dot{I}_{kk}$ represents the current phasor at sensor k.

Substituting (1) and (2) into (3), the current phasor at sensor k can be expressed as,

$$\dot{I}_{kk} = \left(\dot{V}_i - \dot{V}_k\right)/Z_{ik} - \dot{V}_k \cdot Y_{si}. \tag{4}$$

Similarly, based on the π-equivalent model of the line between sensor k and bus j, the current phasor can alternatively be expressed as follows,

$$\dot{I}_{kj} = \left(\dot{V}_k - \dot{V}_j\right)/Z_{kj}, \tag{5}$$

$$\dot{I}_{sj} = \dot{V}_k \cdot Y_{sj}, \tag{6}$$

$$\dot{I}_{kk} = \dot{I}_{kj} + \dot{I}_{sj}, \tag{7}$$

where $\dot{V}_j$ is the voltage phasor at bus j; Z_{kj} and Y_{sj} are the series impedance and half shunt admittance of the line segment between sensor k and bus j; $\dot{I}_{kj}$ and $\dot{I}_{sj}$ are the current phasors through Z_{kj} and Y_{sj}, respectively.

Substituting (5) and (6) into (7), the current phasor at sensor k can also be expressed as,

$$\dot{I}_{kk} = \left(\dot{V}_k - \dot{V}_j\right)/Z_{kj} + \dot{V}_k \cdot Y_{sj}. \tag{8}$$

To eliminate $\dot{V}_k$, (4) and (8) can be transformed as follows,

$$\frac{Z_{ik}}{1 + Y_{si} \cdot Z_{ik}} \cdot \dot{I}_{kk} = \frac{\dot{V}_i}{1 + Y_{si} \cdot Z_{ik}} - \dot{V}_k, \tag{9}$$

$$\frac{Z_{kj}}{1 + Y_{sj} \cdot Z_{kj}} \cdot \dot{I}_{kk} = -\frac{\dot{V}_j}{1 + Y_{sj} \cdot Z_{kj}} + \dot{V}_k. \tag{10}$$

By adding (9) and (10), $\dot{V}_k$ will be eliminated, allowing the expression of the current phasor at sensor k, i.e., $\dot{I}_{kk}$, by using the state variables at the terminals of the line, $\dot{V}_i$ and $\dot{V}_j$, and network parameters,

$$\left(\frac{Z_{ik}}{1 + Y_{si} \cdot Z_{ik}} + \frac{Z_{kj}}{1 + Y_{sj} \cdot Z_{kj}}\right) \cdot \dot{I}_{kk} = \frac{\dot{V}_i}{1 + Y_{si} \cdot Z_{ik}} - \frac{\dot{V}_j}{1 + Y_{sj} \cdot Z_{kj}}. \tag{11}$$

Finally, the current phasor at sensor k can be expressed as:

$$\dot{I}_{kk} = A \cdot \dot{V}_i - B \cdot \dot{V}_j, \tag{12}$$

where

$$A = \frac{1 + Y_{sj} \cdot Z_{kj}}{Z_{ik} \cdot \left(1 + Y_{sj} \cdot Z_{kj}\right) + Z_{kj} \cdot \left(1 + Y_{si} \cdot Z_{ik}\right)} = \frac{1 + (1-\rho)^2 \cdot Z \cdot Y/2}{Z + \rho(1-\rho) \cdot Z^2 \cdot Y/2}; \tag{13}$$

$$B = \frac{1 + Y_{si} \cdot Z_{ik}}{Z_{ik} \cdot \left(1 + Y_{sj} \cdot Z_{kj}\right) + Z_{kj} \cdot \left(1 + Y_{si} \cdot Z_{ik}\right)} = \frac{1 + \rho^2 \cdot Z \cdot Y/2}{Z + \rho(1-\rho) \cdot Z^2 \cdot Y/2}; \tag{14}$$

Z and Y are the series impedance and shunt admittance of the transmission line between bus i to bus j, respectively; ρ is the ratio between the length of the line segment between bus i and sensor k and the total length of the line between bus i and bus j.

To obtain the current magnitude from (12), the law of cosines can be adopted. First, the two terms in (12) can be expressed by using the polar coordinate,

$$A \cdot \dot{V}_i = |A| \cdot V_i \angle\left(\theta_i + \varphi_A\right), \tag{15}$$

$$B \cdot \dot{V}_j = |B| \cdot V_j \angle\left(\theta_j + \varphi_B\right), \tag{16}$$

where θ_i and θ_j represent voltage phase angles of buses i and j, respectively; $|A|$ and $|B|$ represent the magnitudes of variables A and B, respectively; and φ_A and φ_B represent phase angles of variables A and B, respectively. Then, the measurement function of the current magnitude measured by sensor k is as follows,

$$I_{kk} = \sqrt{|A|^2 \cdot V_i^2 + |B|^2 \cdot V_j^2 - 2|A| \cdot |B| \cdot V_i \cdot V_j \cdot \cos\left(\theta_i - \theta_j + \varphi_A - \varphi_B\right)}. \tag{17}$$

B. State Estimation Incorporating Line Sensor Current

Consider the set of measurements that includes current magnitude measurements collected by overhead line sensors:

$$\tilde{z} = \tilde{\boldsymbol{h}}(\boldsymbol{x}) + \tilde{\boldsymbol{e}}, \tag{18}$$

where $\tilde{z} \in \mathbb{R}^{(m+s)\times 1} = \left[\boldsymbol{z}^T, \boldsymbol{z}_I^T\right]^T$ is the measurement vector, which consists of the conventional measurements $\boldsymbol{z} \in \mathbb{R}^{m\times 1}$ and the current magnitude measurements collected by overhead line sensors $\boldsymbol{z}_I \in \mathbb{R}^{s\times 1}$; m and s are the numbers of measurements $\boldsymbol{z}$ and $\boldsymbol{z}_I$, respectively; $\tilde{\boldsymbol{h}} \in \mathbb{R}^{(m+s)\times 1} = \left[\boldsymbol{h}^T, \boldsymbol{h}_I^T\right]^T$ is the measurement function vector, which consists of the conventional measurement functions $\boldsymbol{h} \in \mathbb{R}^{m\times 1}$ and the proposed line sensor current magnitude measurement function $\boldsymbol{h}_I \in \mathbb{R}^{s\times 1}$, i.e., (17); $\boldsymbol{x} \in \mathbb{R}^{n\times 1}$ is the same state vector as in conventional SE; n is the number of state variables; $\tilde{\boldsymbol{e}} \in \mathbb{R}^{(m+s)\times 1}$ is the measurement error vector.

The weighted least squares (WLS) state estimator aims to minimize the following objective function:

$$\min_{\boldsymbol{x}} \quad J(\boldsymbol{x}) = \left[\tilde{z} - \tilde{\boldsymbol{h}}(\boldsymbol{x})\right]^T \cdot \tilde{\boldsymbol{R}}^{-1} \cdot \left[\tilde{z} - \tilde{\boldsymbol{h}}(\boldsymbol{x})\right], \tag{19}$$

where $\tilde{\boldsymbol{R}} \in \mathbb{R}^{(m+s)\times(m+s)}$ is the measurement error covariance.

By exploring the Gauss-Newton method [8], the non-linear WLS SE can be solved via an iterative solution scheme as:

$$\Delta\hat{\boldsymbol{x}}^{l+1} = \left[\tilde{\boldsymbol{H}}\left(\hat{\boldsymbol{x}}^l\right) \tilde{\boldsymbol{R}}^{-1} \tilde{\boldsymbol{H}}\left(\hat{\boldsymbol{x}}^l\right)\right]^{-1} \cdot \tilde{\boldsymbol{R}}^{-1} \cdot \left[\tilde{z} - \tilde{\boldsymbol{h}}\left(\hat{\boldsymbol{x}}^l\right)\right], \tag{20}$$

$$\hat{\boldsymbol{x}}^{l+1} = \hat{\boldsymbol{x}}^l + \Delta\hat{\boldsymbol{x}}^{l+1}, \tag{21}$$

where $\hat{\boldsymbol{x}}$ is the state estimate vector; l is the iteration index; and $\tilde{\boldsymbol{H}} \in \mathbb{R}^{(m+s)\times n}$ is the Jacobian matrix, which is given as follows,

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \partial\boldsymbol{h}/\partial\boldsymbol{\theta} & \partial\boldsymbol{h}/\partial\boldsymbol{V} \\ \partial\boldsymbol{h}_I/\partial\boldsymbol{\theta} & \partial\boldsymbol{h}_I/\partial\boldsymbol{V} \end{bmatrix}, \tag{22}$$

where $\boldsymbol{\theta}$ and $\boldsymbol{V}$ represent voltage phase angle and magnitude state variables, respectively. Note that although the new line sensor measurements are introduced, the state vector and the functions of the conventional measurements (i.e., substation RTUs) remain unaltered, owing to the elimination of the virtual node voltage $\dot{V}_k$, making the formulation easily implementable. To address the flat start issue, state estimates obtained from SE using the measurement set excluding current magnitude measurements (i.e., SCADA only) are used as the initial values. In addition, as these overhead line sensor measurements are all redundant measurements to a set of SCADA power measurements, solution multiplicity is not an issue in this problem.

III. Case Study

The IEEE 14-bus and NPCC 140-bus systems are used to verify the proposed method. For the IEEE 14-bus test system, the conventional measurements include 1 voltage magnitude, 3 pairs of real and reactive bus power injection, and 13 pairs of real and reactive branch flow measurements. For the NPCC 140-bus test system, there are 20 voltage magnitude, 50 pairs of bus power injection, and 139 pairs of branch flow measurements. All measurement errors are assumed to follow a zero-mean Gaussian distribution with a 0.01 p.u. standard deviation.

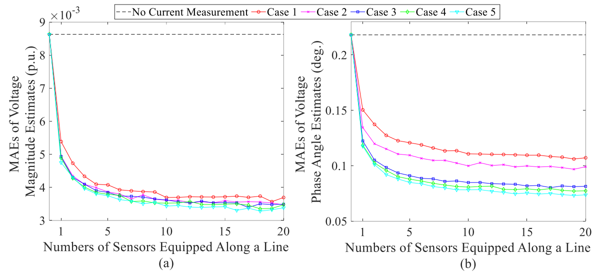

Fig. 2. MAEs of State Estimates of the IEEE 14-bus System Under Different Cases. (a) Voltage Magnitudes and (b) Voltage Phase Angles.

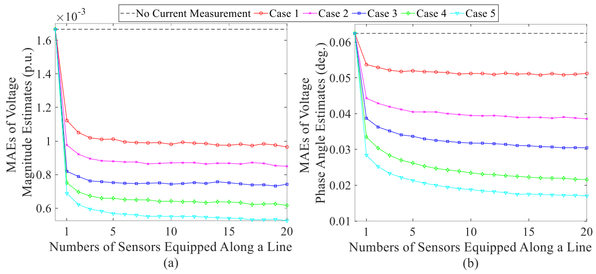

Fig. 3. MAEs of State Estimates of the NPCC 140-bus System Under Different Cases. (a) Voltage Magnitudes and (b) Voltage Phase Angles.

TABLE I

NORMALIZED RESIDUALS OF CMS OF IEEE 14-BUS SYSTEM (**UD**: UNDEFINED, INDICATING UNDETECTABLE BAD DATA)

	No Current	Case 1	Case 2	Case 3	Case 4	Case 5
$P_{2\text{-}3}$	UD	UD	6.56	7.64	7.67	7.66
$Q_{2\text{-}3}$	UD	UD	0.79	2.32	2.33	2.33
$P_{4\text{-}7}$	UD	UD	UD	4.28	4.71	4.83
$Q_{4\text{-}7}$	UD	UD	UD	0.42	0.42	0.42
$P_{6\text{-}12}$	UD	UD	UD	UD	UD	6.76
$Q_{6\text{-}12}$	UD	UD	UD	UD	UD	4.51
$P_{6\text{-}13}$	UD	UD	UD	UD	UD	6.45
$Q_{6\text{-}13}$	UD	UD	UD	UD	UD	3.76
$P_{7\text{-}8}$	UD	UD	UD	1.87	1.86	1.85
$Q_{7\text{-}8}$	UD	UD	UD	6.85	6.80	6.79
$P_{9\text{-}14}$	UD	UD	UD	UD	6.62	6.73
$Q_{9\text{-}14}$	UD	UD	UD	UD	4.46	4.45

TABLE II

NUMBERS OF CMS AND CPS OF IEEE 14-BUS AND NPCC 140-BUS SYSTEMS UNDER DIFFERENT CASES

Test Systems	CM /CP	No Current	Case 1	Case 2	Case 3	Case 4	Case 5
IEEE 14-bus	CMs	12	12	10	6	4	0
	CPs	0	0	1	1	2	4
NPCC 140-bus	CMs	24	20	18	14	10	10
	CPs	36	32	21	19	17	8

All simulations are repeated 3,000 times to report the average. 5 different cases (Cases 1∼5) are designed to have 10%, 30%, 50%, 70%, and 90% of lines equipped with sensors, respectively. In each case, the number of sensors installed along each line is varied from 1 to 20; the sensors are assumed to be evenly distributed along the span of a line. In real-world power systems, the number of overhead line sensors deployed along a line is impacted by different factors but are typically large in support of line-level applications such as DLR. The mean absolute error (MAE) is used to evaluate the performance of SE.

A. Enhancement of State Estimation Under Regular Noise

The MAEs of voltage magnitude and phase angle estimates of the two test systems are illustrated in Figs. 2-a (2-b) and 3-a (3-b), respectively. The results demonstrate that the accuracy of

TABLE III

BAD DATA DETECTION RATES (%) OF THE IEEE 14-BUS SYSTEM BASED ON THE **PROPOSED METHOD (I)** AND THE **VIRTUAL NODE METHOD (II)** UNDER DIFFERENT CASES AND GROSS ERRORS ($\not\surd$ REPRESENTS **DETECTABLE BUT UNIDENTIFIABLE**; √ REPRESENTS **DETECTABLE AND IDENTIFIABLE**).

	10σ				30σ				50σ			
	$\not\surd$		√		$\not\surd$		√		$\not\surd$		√	
	I	II	I	II	I	II	I	II	I	II	I	II
No C.	0	0	42	42	9	9	55	55	16	16	48	48
Case 1	3	15	46	36	9	23	57	41	14	26	52	43
Case 2	8	21	47	35	16	34	58	41	16	35	58	44
Case 3	0	45	70	33	24	50	60	32	26	55	60	35
Case 4	7	49	71	33	20	51	69	36	25	58	67	36
Case 5	17	59	71	30	21	59	77	34	23	71	78	29

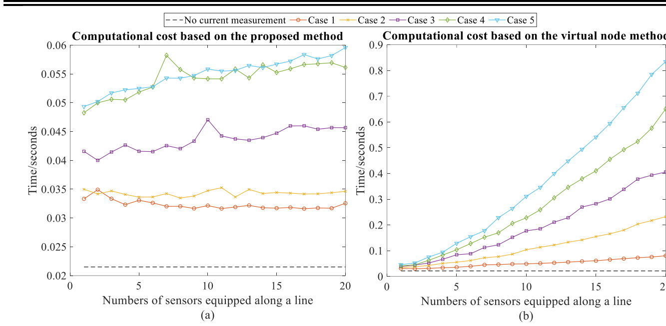

Fig. 4. Computational costs of the IEEE 14-bus system under different cases. (a) based on the **proposed method**; and (b) based on the **virtual node method**.

SE is significantly improved by incorporating line sensor measurements. For instance, even when only 10% of lines are equipped with a single sensor, the performance of SE can be improved by 37% (31%) and 35% (14%) for the voltage magnitude (phase angle) estimates in the IEEE 14-bus (NPCC 140-bus) system, respectively. Moreover, the improvement increases with the number of lines equipped with sensors and the number of sensors per line. It is generally saturated when the number of sensors per line is over 5; typically, sensors installed for monitoring each line is greater than 5 to fulfill the requirements of other applications such as DLR; therefore, it can be concluded that typical line sensor configurations in practice have remarkable benefits to SE. As the number of sensors per line increases, the MAE improvement finally reaches the range of 60-70% when 70% of lines are equipped with line sensors.

B. Enhancement of Bad Data Detection and Identification

To evaluate the performance of bad data detection and identification, we identify critical measurements (CMs) of the system before line sensor measurements are introduced. These are the measurements whose bad data cannot be detected in conventional SE, reflected by the fact that their normalized residuals are *undefined* (please refer to Chapter 5 of [8] for detailed information). Then, the CMs are corrupted with 0.1 p.u. gross error, and we attempt to compute the normalized residuals after line sensors are incorporated. Results in the IEEE 14-bus system are summarized in Table I. The normalized residuals of CMs become computable when line sensor measurements are incorporated into SE, implying that CMs become redundant measurements whose bad data can be detected. Moreover, the numbers of CMs and critical pairs (CPs) of both test systems are shown in Tables II. Here, a CP is a pair of measurements whose bad data are detectable but non-distinguishable from each other [8]. These results indicate the following: 1) the number of CMs will be decreased as the number of lines equipped with sensors increases; 2) the number of CPs may be increased or decreased; the cases with increased CPs are due to the fact that CMs are converted into CPs. Notably, all the new CPs are pairs of real and reactive power measurements at the same location, indicating that the source of errors can at least be narrowed down to the P and Q measurements derived from the same potential transformer (PT) and current transformer (CT). This is a significant improvement compared with CMs, since bad data in CMs are not detectable at all.

In addition, the bad data detection rates of the IEEE 14-bus system based on the proposed method and the virtual node methods under different cases and gross errors are presented in Table III. These results demonstrate that the virtual node method leads to a larger number of measurements with detectable but unidentifiable errors, while the proposed method leads to a larger number of measurements with detectable and identifiable errors. This is because the virtual node method introduces a large number of new state variables into the SE problem and does not enjoy the benefit of increased measurement redundancy as the proposed method. The computation times of the two methods shown in Fig. 4 also demonstrate the benefit of the proposed method: as it does not introduce new state variables, the computation cost stays almost constant with the increase of the number of line sensors.

IV. Conclusion

In this letter, we propose a method to enhance SE by incorporating the current measurements of sensors deployed along the span of transmission lines. The developed measurement function relates these sensor measurements to conventional state variables at line terminals and does not introduce additional state variables, making it implementation-friendly. Simulation results verify the feasibility of the proposed method and demonstrate clear values of line sensor assets to SE in terms of noise filtering and bad data processing. In the future, the proposed method can be evaluated on real-world power systems and complex measurement environments.

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