

Wholesale Market Participation of DERAs: DSO-DEIRA-ISO Coordination

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Abstract—Distributed energy resource aggregators (DERAs) must share the distribution network together with the distribution utility in order to participate in the wholesale electricity markets that are operated by independent system operators (ISOs). We propose a forward auction that a distribution system operator (DSO) can utilize to allocate distribution network access limits to DERAs. As long as the DERAs operate within their acquired limits, these limits define operating envelopes that guarantee distribution network security, thus defining a mechanism that requires no real-time intervention from the DSOs for DERAs to participate in the wholesale markets. Our auctions take the form of robust and risk-sensitive markets with bids/offers from DERAs and utility's operational costs. Properties of the proposed auction, e.g., resulting surpluses of DSO and the DERAs, and the auction prices, along with empirical performance studies, are presented.

Index Terms—DERA and DER aggregation, behind-the-meter distributed generation, network access allocation mechanism.

I. INTRODUCTION

THE landmark ruling of the Federal Energy Regulatory Commission (FERC) Order 2222 in [2] aims to remove barriers to the direct participation of distributed energy resource aggregators (DERAs) in the wholesale market operated by independent system operators (ISOs) (or regional transmission operators, RTO). Since distributed energy resources (DERs) originate in a distribution network, aggregated DERs must pass through the distribution grid managed by a distribution system operator (DSO) that can be the distribution utility or an independent entity. A coordination mechanism among the DSO,

ISO, and DERAs is necessary to ensure system reliability and open access to all DERAs. FERC Order 2222 recognizes the significance of DSO-DEIRA-ISO coordination while leaving the specifics of the coordination design to the regulators, market operators, and stakeholders.

DSO-DEIRA-ISO coordination poses significant theoretical and practical challenges. Net power injections from DERAs will likely depend on wholesale market conditions such as wholesale locational marginal prices (LMPs), real-time regulation service needs, and available behind-the-meter DERs in the distribution system. Notwithstanding these uncertainties, the DSO must ensure the reliable operation of the distribution grid, both in delivering services to all customers and allowing DERAs to offer services to the wholesale market. Moreover, any coordination mechanism must provide open and nondiscriminatory access to multiple competing DERAs operating over the same distribution network.

DSO-DEIRA-ISO coordination has been actively debated since the release of FERC Order 2222. In [3], coordination models have been classified into four categories, ranging from the least to the most DSO involvement. Type I models assume no DSO control (see, e.g., [4], [5]), because installed DER capacities are deemed to lie within the network's hosting capacity limits. One approach is to impose strict net injection limits [6], [7] on individual prosumers so that the system's reliability is ensured as long as the limits are respected. In Type II models, e.g., [8], [9], [10], the DSO strives to prevent constraint violations, considering the randomness of power injections from DERs. Type III models involve coordination among DERAs, DSO, and ISO, where DERAs can provide distribution grid services in addition to delivering wholesale market products (see, e.g., [11], [12]). Type IV models require DERA aggregation through the DSO, with the DSO performing all reliability functions and participating in the wholesale market on behalf of the DERAs as in [13], [14], [15], [16].

In this paper, we develop a Type II DSO-DEIRA-ISO coordination mechanism aimed at achieving efficient and reliable multi-DEIRA aggregations without significant deviations from existing DSO and ISO/RTO interaction models. Fig. 1 illustrates the power, financial, and control interactions among DSO, DEIRA, and ISO. Our coordination approach decouples the complex DSO-DEIRA-ISO interactions into nearly independent, pairwise interactions. In particular, we propose a *forward auction* run by the DSO that allows DERAs to acquire *network access limits*—the right to inject or withdraw any amount of power within those

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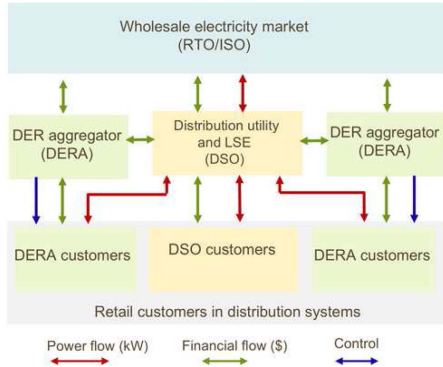


Fig. 1. Power flow, financial flow, and control interactions in the DSO-DERA-ISO coordination model.

limits over which the auction outcomes stand. These limits are auctioned off in a way that *all* power transactions from DERAs within these limits will satisfy distribution system operation constraints. Thus, DERA's wholesale market interaction with the ISO in real-time can be agnostic to these operational constraints, and ISOs can dispatch DERs without direct visibility into the distribution network. The DERA-ISO interaction can include models such as demand response, virtual storage, etc., and *any* way they choose to operate the DERs will not violate distribution network security as long as they remain within the network access limits they acquire from the DSO-operated auction. Thus, our design removes the need for DSO's interventions in ISO's real-time dispatch and DERAs' aggregation actions under normal system operating conditions.

We consider two network access allocation mechanisms. Section III presents a robust optimization-based market clearing formulation for network access, which guarantees satisfaction of network constraints when DERAs aggregate within their acquired access limits, thus removing the need for DSO to participate in real-time DERA-ISO interactions. DERAs do not even need knowledge of the underlying physical network when its aggregation strategy respects said limits.

The robust access allocation can be conservative in that it assumes the worst-case aggregation and network operating conditions. We present a risk-based stochastic allocation mechanism in Section IV that allows DSO to share the common network resources subject to an acceptable risk constraint on network security violations. For both the robust and stochastic access limit auctions, we provide theoretical analyses and empirical studies to characterize and quantify access allocation properties, including the nonnegative surpluses for DSO, the benefits of participating in the access limit auction for DERAs, the behavior of access allocation prices, and a comparison of the two formulations.

Our DSO-DERA coordination mechanism via access limits shares both parallels and important differences with two distant relatives. One is the transmission right allocation problem for participants with bilateral contracts considered in the early years of wholesale market deregulation. Allocating physical transmission rights was deemed impractical and unnecessary

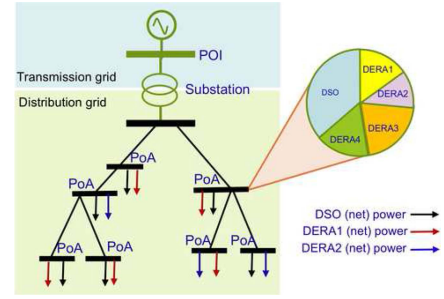


Fig. 2. Distribution system and DER resources.

(see [17]) with the “loop-flow” problem in meshed transmission networks. Ultimately, wholesale markets evolved to adopt a centrally coordinated economic dispatch run by the ISO, where bilateral transactions are protected from the risks of LMP fluctuations using derivatives such as financial transmission rights. In our proposed coordination mechanisms, no central authority coordinates DERAs in real-time. Making access limit allocation *physical* allows DERAs to inject or withdraw any amount of power within their purchased access limits. The loop-flow problem does not affect our design, thanks to the radial nature of distribution networks. Also, we deliberately separate access limit allocation from real-time decisions; we envision the forward auction for access limits to run once a day or a week. We posit that tight coordination of dispatch decisions via a centralized market mechanism, matched to that operated by the ISO, may be impractical in the near-term and possibly unnecessary, owing to smaller trade volumes and less stringent system constraints in the distribution grid.

Second, our mechanism is reminiscent of the notion of operating envelopes defined by DSO-imposed net nodal DER injection and withdrawal limits. For instance, an Australian market imposes 5 kW net-injection limits on residential customers with DERs [6]. Instead of using pre-determining dynamic operating envelopes as in [7], [9], [10], we auction off these envelopes among competing DERAs based on each DERA's aggregation needs and aggregation strategy.

The paper is organized as follows. We begin with the preliminaries of the network and the DSO-DERA-ISO coordination models in Section II. Then, we present the robust and the stochastic network access allocation problems and their properties in Sections III and IV, respectively. A simple yet illustrative example is discussed in Section V. Then, we provide a numerical case study on a 141-bus distribution network in Section VI. Theoretical claims are proven in the appendix.

II. NETWORK AND COORDINATION MODELS

Consider K DERAs operating over a radial distribution network across N buses shown in Fig. 2, where bus 1 is the reference bus. Let the power injection profile from DERA k across the network be given by $\mathbf{p}_k \in \mathbb{R}^N$, and the total DERA net injection to the system is given by $\sum_{k=1}^K \mathbf{p}_k$. Let the power injection profile from the DSO's customers be given by $\mathbf{p}_0 \in \mathbb{R}^N$. Assuming a uniform power factor for all power injections, the reactive power injection profile is given by $\alpha(\sum_{k=1}^K \mathbf{p}_k + \mathbf{p}_0)$.

TABLE I
MAJOR SYMBOLS

$\underline{p}_0, \bar{p}_0, \underline{p}_0$	power injections and network accesses of DSO
$\underline{p}_k, \bar{C}_k, \underline{C}_k$	power injections and network accesses of DERA k
$\underline{C}_k^{\min}, \underline{C}_k^{\min}$	minimum injection/withdrawal access of DERA k
$\underline{P}^{\max}, \underline{P}^{\max}$	injection/withdrawal access limits of the network
φ_k	utility of DERA k , induced by its bid
J	operational cost of DSO
\mathbf{A}	network parameters for linearized power flow
\bar{b}, \underline{b}	limits on network voltage/power flows
K, S	total number of DERAs and scenarios

This assumption simplifies our presentation but is not crucial to the design of our auctions. These real and reactive power injections then induce power flows and voltage magnitudes over the distribution network that are related via the power flow equations. In this work, we adopt a linear distribution power flow (LinDistFlow) model [18] adopted from [19]. Voltage and line capacity/thermal limits define security constraints. As explained in Appendix A, these constraints take the form,

$$\underline{b} \leq \mathbf{A} \left(\sum_{k=1}^K \underline{p}_k + \underline{p}_0 \right) \leq \bar{b}. \quad (1)$$

Inequalities are interpreted element-wise.

With the linearized network model, we propose forward auctions for DSO-DERA coordination. In these auction models, the DERAs bid for injection/withdrawal *access* at various buses of the distribution network, where the DERA commands DERs from its customers; see Fig. 2. In Sections III and IV, we describe, respectively the robust and the stochastic, optimization problems to clear the forward auctions and the settlements for the DERAs. The auctions determine the *range* of injection and withdrawal access for each DERA at each bus of the network, and the DERAs' payments to acquire those access limits. We thus design the auction of an operating envelope, where the DERA can inject/withdraw *any* amount of power from the DERs they command within this envelope that they purchase from the DSO. Thus, the real-time power of DERA stays within the limits/envelopes they purchase at the points of aggregation (PoAs) from the DSO through the proposed forward auctions. Our network model contains buses at the PoAs, but we do not model individual customers/DERs downstream from the PoAs. Transactions between a DERA and the ISO occur at the point of interconnection (PoI).

Our auctions for network access are hosted ahead of real-time operations—it can be a few hours to a week in advance of power delivery. To participate, all DERAs must submit their bids/offers to this forward auction and they are cleared simultaneously. Through this auction, DERAs get access to limits/capacities that they must obey during real-time operations. Since the auction outcome binds over multiple real-time interactions, we account for the range of possible operating conditions through robust and stochastic programming based auction-clearing formulations.

III. THE ROBUST AUCTION MODEL

We now design an auction for network access that accounts for a variety of real-time operating conditions through a robust optimization formulation. Specifically, let $\underline{C}_k \in \mathbb{R}^N$ and $\bar{C}_k \in \mathbb{R}^N$ denote the vectors of (real) power withdrawal and injection capacity limits acquired by DERA k . Then, all power injections from assets controlled by DERA k must respect $\underline{p}_k \in [-\underline{C}_k, \bar{C}_k]$. Let the DSO's own customers have net power injections \underline{p}_0 that take values in the set $[-\underline{p}_0, \bar{p}_0]$. Given these ranges of the various power injections, the DSO solves,

$$\underset{\underline{C}, \bar{C}, \underline{P}, \underline{P}}{\text{maximize}} \quad \sum_{k=1}^K \varphi_k(\underline{C}_k, \bar{C}_k) - J(\bar{P}, \underline{P}), \quad (2a)$$

$$\text{subject to} \quad \bar{C}_k^{\min} \leq \bar{C}_k, \quad \underline{C}_k^{\min} \leq \underline{C}_k, \quad (2b)$$

$$\bar{P} \leq \bar{P}^{\max}, \quad \underline{P} \leq \underline{P}^{\max}, \quad (2c)$$

$$\bar{P} = \sum_{k=1}^K \bar{C}_k + \bar{p}_0, \quad (2d)$$

$$\underline{P} = \sum_{k=1}^K \underline{C}_k + \underline{p}_0, \quad (2e)$$

$$\underline{b} \leq \mathbf{A} \left(\sum_{k=1}^K \underline{p}_k + \underline{p}_0 \right) \leq \bar{b}, \quad (2f)$$

$$\text{for all } \underline{p}_k \in [-\underline{C}_k, \bar{C}_k], \underline{p}_0 \in [-\underline{p}_0, \bar{p}_0],$$

$$\text{for } k = 1, \dots, K. \quad (2g)$$

Here, DERA k provides the bid $\varphi_k : \mathbb{R}^{2N} \rightarrow \mathbb{R}$ to the DSO, where $\varphi_k(\underline{C}_k, \bar{C}_k)$ represents DERA k 's willingness to pay for power transactions. Bid/offer construction for φ_k depends on the DERA's aggregation strategy; we refer to our work in [20] for a candidate construction. Let $\bar{P} \in \mathbb{R}^N$ and $\underline{P} \in \mathbb{R}^N$ represent the vectors of the total injection and withdrawal capacities, respectively. Also, define $\bar{C} := (\bar{C}_k)$, $\underline{C} := (\underline{C}_k)$ as the matrices that collect the access limits across the K DERAs. The operational cost of the DSO is encoded in $J : \mathbb{R}^{2N} \rightarrow \mathbb{R}$ that is assumed to be convex and non-decreasing. Being a regulated monopoly running the distribution grid, we anticipate that J will include the cost of reactive power support, network maintenance, line losses, etc. required to maintain quality of service to existing retail customers. See [21] for example cost component constructions. The objective function then represents the induced social surplus from DERAs' bids and DSO's costs. Let φ_k be concave and non-decreasing for each k . Additionally, $(\bar{C}_k^{\min}, \underline{C}_k^{\min})$ are the vectors of minimum injection and withdrawal capacities that DERA k is willing to acquire across the network, and $(\bar{P}^{\max}, \underline{P}^{\max})$ are the vectors of the maximum injection and withdrawal capacities across the network. All capacity limits are assumed nonnegative. Equations (2b) and (2c) encode the DERAs' minimum access requirements and the DSO's maximum access limits, respectively. Equations (2d) and (2e) define the total injection and withdrawal accesses in terms of DSO's access limits and those sold to individual DERAs. With the linearized network model

in (1), the relations in (2f) and (2g) enforce the engineering constraints of the grid for *every possible* power injection profile from the DSO's customers and those of all DERAs within their acquired capacities.

The formulation in (2) contains robust constraint enforcement in (2f)-(2g), but it admits a reformulation as a classic convex program that we present next. The result requires additional notation. For any scalar z , define $z_+ := \max\{z, 0\}$ and $z_- := z_+ - z$. Define the same for a matrix/vector, where the operations are carried element-wise.

Lemma 1: Problem (2) is equivalent to

$$\begin{aligned} \underset{\underline{C}, \underline{\bar{C}}, \underline{P}, \underline{P}}{\text{maximize}} \quad & \sum_{k=1}^K \varphi_k(\underline{C}_k, \underline{\bar{C}}_k) - J(\underline{P}, \underline{P}), \end{aligned} \quad (3a)$$

$$\text{subject to} \quad \text{for } k = 1, \dots, K,$$

$$\bar{\eta}, \underline{\eta} : \quad \underline{\bar{C}}_k^{\min} \leq \underline{\bar{C}}_k, \quad \underline{C}_k^{\min} \leq \underline{C}_k, \quad (3b)$$

$$\bar{\omega}, \underline{\omega} : \quad \underline{P} \leq \underline{P}^{\max}, \quad \underline{P} \leq \underline{P}^{\max}, \quad (3c)$$

$$\bar{\lambda} : \quad \underline{P} = \sum_{k=1}^K \underline{\bar{C}}_k + \underline{p}_0, \quad (3d)$$

$$\underline{\lambda} : \quad \underline{P} = \sum_{k=1}^K \underline{C}_k + \underline{p}_0, \quad (3e)$$

$$\bar{\mu} : \quad \underline{A}_+ \underline{P} + \underline{A}_- \underline{P} \leq \underline{b}, \quad (3f)$$

$$\underline{\mu} : \quad \underline{b} \leq -\underline{A}_+ \underline{P} - \underline{A}_- \underline{P}. \quad (3g)$$

We remark that for the linear power flow model developed in Appendix A, $\underline{A} = \underline{A}_+$ and $\underline{A}_- = \mathbf{0}$. Our auction mechanism and its properties hold more generally for any linear approximation to power flow equations, and hence, we present our results with a more general \underline{A} . Associate Lagrange multipliers with the various constraints in (3) as shown. We now introduce the prices that will define the settlements with the DERAs using the *optimal* Lagrange multipliers (indicated with \star) for (3).

Definition 1 (LMAP-R): For the robust access allocation problem (3), the locational marginal access price (LMAP) for access limits to the distribution network is defined by the vector of optimal dual solutions $\bar{\lambda}^\star \in \mathbb{R}^N, \underline{\lambda}^\star \in \mathbb{R}^N$ of (3).

Specifically, the injection and the withdrawal access price at bus i are given by $\bar{\lambda}^{(i)}$ and $\underline{\lambda}^{(i)}$, respectively. With these prices, for obtaining injection and withdrawal access of $\underline{\bar{C}}_k$ and \underline{C}_k , respectively, DERA k pays

$$\mathcal{P}_k(\underline{\bar{C}}_k^\star, \underline{C}_k^\star) = \bar{\lambda}^\star \top \underline{\bar{C}}_k^\star + \underline{\lambda}^\star \top \underline{C}_k^\star \quad (4)$$

to the DSO. Having derived the prices from an auction that is reminiscent of the economic dispatch problem solved by RTO/ISO in wholesale electricity markets, LMAP-R shares strong parallels with locational marginal prices (LMPs). For example, LMAP-R is nodally uniform. That is, all DERAs pay the same injection and withdrawal access price at a specific distribution bus. These prices, however, may differ across locations in the distribution network.

The second parallel between LMAP-R and LMP comes from the interpretations of these prices as sensitivities of the optimal objective function value of the market clearing problem to nodal parameters. For LMPs, the price of a bus equals the sensitivity of the optimal power procurement costs to nodal demands. For LMAP-R, it is the sensitivity of the induced social welfare \mathcal{W} to DSO's own access limits $\underline{\bar{p}}_0, \underline{p}_0$. More precisely, define the optimal social welfare from the optimal value of (3) as $\mathcal{W}^\star(\underline{\bar{p}}_0, \underline{p}_0)$. Then, envelope theorem, per [22, Chapter 7], states that when \mathcal{W}^\star is differentiable,

$$\bar{\lambda}^\star = -\nabla_{\underline{\bar{p}}_0} \mathcal{W}^\star(\underline{\bar{p}}_0, \underline{p}_0), \quad \underline{\lambda}^\star = -\nabla_{\underline{p}_0} \mathcal{W}^\star(\underline{\bar{p}}_0, \underline{p}_0), \quad (5)$$

which represents the marginal decrease in social welfare when supporting network access by DSO rather than selling access to DERAs. We further shed light on the contributions of network parameters to LMAP-Rs in our next result.

Proposition 1: LMAP-R satisfies

$$\begin{aligned} \bar{\lambda}^\star &= \nabla_{\underline{\bar{P}}} J(\underline{\bar{P}}^\star, \underline{P}^\star) + \underline{A}_+^\top \bar{\mu}^\star + \underline{A}_-^\top \underline{\mu}^\star + \bar{\omega}^\star, \\ \underline{\lambda}^\star &= \nabla_{\underline{P}} J(\underline{\bar{P}}^\star, \underline{P}^\star) + \underline{A}_+^\top \bar{\mu}^\star + \underline{A}_-^\top \underline{\mu}^\star + \underline{\omega}^\star. \end{aligned} \quad (6)$$

The first term in (6) equals the DSO's marginal cost for disseminating access limits at the optimum of (3). Our result then characterizes the price markup in LMAP-R above said marginal cost. Specifically, if voltage and line constraints are non-binding, and the access allocations do not reach the injection limits, the remaining terms in $\bar{\lambda}^\star, \underline{\lambda}^\star$ vanish. In addition, if DSO's operational cost structure is additive and uniform across the buses of the distribution network, then LMAP-Rs become equal at all buses. The tighter the constraints on total access limits being auctioned (encoded in the entries of $\underline{\bar{P}}^{\max}, \underline{P}^{\max}$) are, and the more stringent the network constraints (represented in the entries of $\underline{b}, \underline{b}$) are, we expect LMAP-Rs to differ from DSO's marginal costs.

Next, we investigate DSO's and the DERAs' surplus at an optimal robust access allocation. Define DSO's surplus as

$$\Pi^{\text{DSO}} := \sum_{k=1}^K \mathcal{P}_k(\underline{\bar{C}}_k^\star, \underline{C}_k^\star) - \left(J(\underline{\bar{P}}^\star, \underline{P}^\star) - J(\underline{\bar{p}}_0, \underline{p}_0) \right). \quad (7)$$

The first term equals the total rent that the DSO collects from the DERAs. The second summand equals the cost that the DSO affords when allowing DERAs to operate over the distribution network. Specifically, $J(\underline{\bar{P}}^\star, \underline{P}^\star)$ equals the cost of the net injection and withdrawal access $\underline{\bar{P}}^\star, \underline{P}^\star$ when the DSO provides the network accesses for DERAs and itself, and $J(\underline{\bar{p}}_0, \underline{p}_0)$ equals the operational cost for access required to support the DSO's customers alone. Thus, the last summand in (7) measures the cost of the DSO when supporting the network accesses to the DERAs. Next, define DERA k 's surplus as

$$\Pi_k^{\text{DERA}} := \varphi_k(\underline{C}_k, \underline{\bar{C}}_k) - \mathcal{P}_k(\underline{\bar{C}}_k^\star, \underline{C}_k^\star), \quad (8)$$

which equals the induced utility (inferred from the bid) minus the payment to the DSO.

Proposition 2: (i) $\Pi^{\text{DSO}} \geq 0$, (ii) $\Pi_k^{\text{DERA}} \geq 0$, if $\varphi_k(\mathbf{0}, \mathbf{0}) \geq 0$ and one of the following two conditions holds: (a) $\bar{\mathcal{C}}_k^{\min} = 0$, $\underline{\mathcal{C}}_k^{\min} = 0$, or (b) the constraints in (3b) are non-binding at optimality, i.e., $\bar{\mathcal{C}}_k^* > \bar{\mathcal{C}}_k^{\min}$, $\underline{\mathcal{C}}_k^* > \underline{\mathcal{C}}_k^{\min}$.

The last result suggests that DSO always gains from running the auction in that its surplus is nonnegative. For DERAs, nonnegative surplus is assured under certain sufficient conditions. Among these conditions, $\varphi_k(\mathbf{0}, \mathbf{0}) \geq 0$ is natural as one expects the inferred utility of DERA to be nonnegative for any nonnegative value of the access limits.

The condition $\bar{\mathcal{C}}_k = \underline{\mathcal{C}}_k = 0$ indicates that DERA k may obtain zero network access limits at some buses. Such a condition is violated when the DERA may require a minimum demand to be met or net injection cannot be curtailed beyond a threshold. In such an event, the DERA may need side-payments to make the auction outcome favorable for its participation. The design and discussion of side-payments are relegated to a future effort. We remark that this phenomenon is reminiscent of the challenge in wholesale markets where minimum generation constraints can negate a generator's rationale for market participation. Additionally, we expect the lower bounds $\bar{\mathcal{C}}_k^{\min}$, $\underline{\mathcal{C}}_k^{\min}$ to be non-binding for DERAs that are *marginal* to the auction, i.e., when their marginal implied costs define the prices.

Our next result sheds light on how LMAP-Rs behave along the distribution feeder. Unlike our last two results, the next one specifically utilizes the power flow model presented in Appendix A. To present the result, we need additional notation. We say bus n is an *ancestor* of bus m in the distribution network if n lies on the unique (undirected) path from the reference bus to bus m .

Proposition 3: When J is linear and homogeneous across buses ($J(\bar{\mathbf{P}}, \underline{\mathbf{P}}) = \sum_{i=1}^N \bar{J} \cdot \bar{P}^{(i)} + \sum_{i=1}^N \underline{J} \cdot \underline{P}^{(i)}$) and the constraints in (3c) are non-binding at all buses, then $\bar{\lambda}^{(n)*} \geq \bar{\lambda}^{(m)*}$ and $\underline{\lambda}^{(n)*} \geq \underline{\lambda}^{(m)*}$, if bus n is an ancestor of bus m .

Said differently, LMAP-Rs do not decrease along the network away from the substation under certain conditions. Our numerical experiments reveal that these conditions are only sufficient for price monotonicity; the monotonic trend holds even when these conditions are not met. Such a price monotonicity reveals that it is costlier to guarantee the voltage limits and line capacity limits for the leaf buses, compared to those closer to the substation. The non-binding nature of the constraints is only *sufficient* for the conclusion to hold; our simulations will show that they are *not* necessary.

IV. THE STOCHASTIC AUCTION DESIGN

The robust access allocation requires the network constraints to be satisfied for *all possible* injections from DERAs' and the DSO's customers. Naturally, the resulting allocations are dictated by the worst-case power injection/withdrawal combinations during the forward auction and fully ignore the statistics of network usage. Such an approach inherently limits the DERAs' collective network access. We present in this section a risk-based

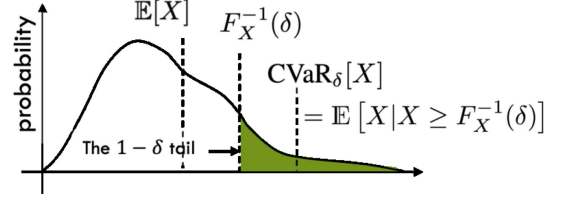


Fig. 3. CVaR of a random variable X .

stochastic allocation mechanism that allows *controlled violation* of the network constraints in the access allocation auction in a way that accounts for the statistics of power transactions by DSO's customers. Stochastic resource allocation with controlled violation is common in many areas, especially in computer systems and communication networks, where the resources are constrained and usage patterns are random, especially over time. Such an allocation builds on the premise that random usage patterns often do not overlap, and this time-multiplexing allows higher access limits to be accommodated than the robust counterpart. Notice that we allow for possible controlled violations only in the forward auction. Said violations in the auction do not amount to actual violations in real time. In practice, DSO can implement mechanisms to curtail access to enforce reliability constraints in real-time; we side-step such considerations and purely focus on the merits of a stochastic forward auction.

When the power transactions by DSO's customers are considered random, the power flows and the bus voltages become random quantities. In the forward auction, we limit the *risk* of constraint violations, where we measure this risk via the conditional value at risk (CVaR) measure. CVaR, analyzed and popularized by [23], [24], has recently been advocated in power system planning, e.g., in [25]. For a random variable (think loss) X with smooth cumulative distribution function F_X , CVaR at level $\delta \in [0, 1)$ equals the conditional mean of X over the $(1 - \delta)$ -tail of the distribution (see Fig. 3).

For power flows over a certain distribution line, CVaR_δ measures the average value among the top $1 - \delta$ fraction of the largest power flow values. Thus, by constraining the $\text{CVaR}_{0.99}$ value of the line flow below a threshold \bar{b} , for example, implies that not only 99% of the power flows are below \bar{b} , but also the average power flow among the worst (largest) 1% is below \bar{b} . In other words, the CVaR-based constraint restricts both the probability and the extent of the line limit violation. Following [24], CVaR is defined as

$$\text{CVaR}_\delta[X] := \underset{t \in \mathbb{R}}{\text{minimize}} \left\{ t + \frac{1}{1 - \delta} \mathbb{E}[X - t]_+ \right\} \quad (9)$$

for random variables X with general distributions.

Given the distribution of \mathbf{p}_0 , the risk-aware access limit auction becomes,

$$\underset{\underline{\mathcal{C}}, \bar{\mathcal{C}}}{\text{maximize}} \quad \sum_{k=1}^K \varphi_k(\underline{\mathcal{C}}_k, \bar{\mathcal{C}}_k) - \mathbb{E}[J(\bar{\mathbf{P}}, \underline{\mathbf{P}})], \quad (10a)$$

subject to (2b)–(2e),

$$\text{CVaR}_\delta \left[\mathbf{A} \left(\sum_{k=1}^K \mathbf{p}_k + \mathbf{p}_0 \right) \right] \leq \bar{\mathbf{b}}, \quad (10b)$$

$$\text{CVaR}_\delta \left[-\mathbf{A} \left(\sum_{k=1}^K \mathbf{p}_k + \mathbf{p}_0 \right) \right] \leq -\underline{\mathbf{b}},$$

$$\text{for all } \mathbf{p}_k \in [-\underline{\mathbf{C}}_k, \bar{\mathbf{C}}_k], \text{ for } k = 1, \dots, K. \quad (10c)$$

Compared to the robust network model in (2), the risk-constrained mechanism differs in three aspects. First, $\bar{\mathbf{P}}, \underline{\mathbf{P}}$ in this formulation is random, and hence, (2c) imposes the upper bounds for all possible values of \mathbf{p}_0 . Second, we consider the DSO's *expected* operational cost to serve the random $\bar{\mathbf{P}}, \underline{\mathbf{P}}$ in the objective function. In effect, we maximize the average induced social surplus. Third, and most importantly, we impose CVaR constraints on network limit violations in (10b) and (10c) for all possible values of injections/withdrawals from assets controlled by DERAs within their acquired access limits. These constraints are such that for *any* power transactions by DERAs within their access limits, they limit network constraint violation probabilities below $1 - \delta$ and ensure that the average magnitude of those risky power flows/voltages remains within the specified limits. By imposing the constraint for all $\mathbf{p}_k \in [-\underline{\mathbf{C}}_k, \bar{\mathbf{C}}_k]$, this formulation inherits the benefits of decoupled DERA-DSO operations from the robust formulation. Specifically, any real-time DERA-ISO contract within the DERA's acquired limit imposes at most a pre-defined level of security risk to the distribution network. Said risk stems from the uncertainty in power transactions from DSO's customers alone for which they typically have statistics from historical data.

Next, we present a scenario-approach to approximate (10). Consider S independent and identically distributed samples $\mathbf{p}_0[1], \dots, \mathbf{p}_0[S]$ for the injection of DSO's customers \mathbf{p}_0 . Using (9) and replacing all expectations with empirical means over S samples, we arrive at the following optimization program for stochastic access allocation. The detailed derivation is relegated to Appendix F.

$$\begin{aligned} & \underset{\substack{\bar{\mathbf{C}}, \underline{\mathbf{C}}, \bar{\mathbf{P}}[s], \underline{\mathbf{P}}[s], \\ \bar{\mathbf{t}}, \underline{\mathbf{t}}, \bar{\boldsymbol{\gamma}}[s], \underline{\boldsymbol{\gamma}}[s]}}{\text{maximize}} & \sum_{k=1}^K \varphi_k(\underline{\mathbf{C}}_k, \bar{\mathbf{C}}_k) - \frac{1}{S} \sum_{s=1}^S J(\bar{\mathbf{P}}[s], \underline{\mathbf{P}}[s]), \end{aligned} \quad (11a)$$

$$\text{such that} \quad \text{for } k = 1, \dots, K, \quad s = 1, \dots, S,$$

$$\bar{\boldsymbol{\eta}}, \underline{\boldsymbol{\eta}} : \quad \bar{\mathbf{C}}_k^{\min} \leq \bar{\mathbf{C}}_k, \quad \underline{\mathbf{C}}_k^{\min} \leq \underline{\mathbf{C}}_k, \quad (11b)$$

$$\bar{\boldsymbol{\omega}}[s], \underline{\boldsymbol{\omega}}[s] : \quad \bar{\mathbf{P}}[s] \leq \bar{\mathbf{P}}^{\max}, \quad \underline{\mathbf{P}}[s] \leq \underline{\mathbf{P}}^{\max}, \quad (11c)$$

$$\bar{\boldsymbol{\lambda}}[s] : \quad \bar{\mathbf{P}}[s] = \sum_{k=1}^K \bar{\mathbf{C}}_k + \mathbf{p}_0[s], \quad (11d)$$

$$\underline{\boldsymbol{\lambda}}[s] : \quad \underline{\mathbf{P}}[s] = \sum_{k=1}^K \underline{\mathbf{C}}_k - \mathbf{p}_0[s], \quad (11e)$$

$$\bar{\boldsymbol{\beta}}[s] : \quad \mathbf{A}_+ \bar{\mathbf{P}}[s] + \mathbf{A}_- \underline{\mathbf{P}}[s] - \bar{\mathbf{b}} - \bar{\mathbf{t}} \leq \bar{\boldsymbol{\gamma}}[s], \quad (11f)$$

$$\bar{\boldsymbol{\alpha}}[s] : \quad \mathbf{0} \leq \bar{\boldsymbol{\gamma}}[s], \quad (11g)$$

$$\bar{\boldsymbol{\mu}} : \quad \bar{\mathbf{t}} + \frac{1}{1-\delta} \frac{1}{S} \sum_{s=1}^S \bar{\boldsymbol{\gamma}}[s] \leq \mathbf{0}, \quad (11h)$$

$$\underline{\boldsymbol{\beta}}[s] : \quad \mathbf{A}_- \bar{\mathbf{P}}[s] + \mathbf{A}_+ \underline{\mathbf{P}}[s] + \underline{\mathbf{b}} - \underline{\mathbf{t}} \leq \underline{\boldsymbol{\gamma}}[s], \quad (11i)$$

$$\underline{\boldsymbol{\alpha}}[s] : \quad \mathbf{0} \leq \underline{\boldsymbol{\gamma}}[s], \quad (11j)$$

$$\underline{\boldsymbol{\mu}} : \quad \underline{\mathbf{t}} + \frac{1}{1-\delta} \frac{1}{S} \sum_{s=1}^S \underline{\boldsymbol{\gamma}}[s] \leq \mathbf{0}. \quad (11k)$$

Sample average approximations for such stochastic programs are known to become more accurate with growing number of samples, as [26] suggests. Thus, the solution of (11) should approach that of (10) as $S \rightarrow \infty$. One expects that when $\delta \uparrow 1$, the stochastic model constraints (11f)–(11k) roughly approximate network security constraints (3f)–(3g) from the robust auction model. Per our experiments, even with $\delta = 0.99$, the resulting social welfare can be significantly higher than from the robust auction ($\sim 20\%$), upon tolerating only 1% network security constraint violation risk at a few buses.

To define the settlement mechanism from the above auction, associate Lagrange multipliers with the constraints in (11) as shown and denote optimal values of variables with (\star) .

Definition 2 (LMAP-S): For the stochastic allocation mechanism, the locational marginal access prices for injection and withdrawal access limits to the distribution network are defined by the vectors $\bar{\boldsymbol{\Lambda}}^\star := \sum_{s=1}^S \bar{\boldsymbol{\lambda}}^\star[s] \in \mathbb{R}^N$ and $\underline{\boldsymbol{\Lambda}}^\star := \sum_{s=1}^S \underline{\boldsymbol{\lambda}}^\star[s] \in \mathbb{R}^N$, respectively, obtained from the optimal Lagrange multipliers $\bar{\boldsymbol{\lambda}}^\star[s], \underline{\boldsymbol{\lambda}}^\star[s]$ of (11).

With LMAP-S, DERA k 's payment to the DSO is given by

$$\tilde{\mathcal{P}}_k(\bar{\mathbf{C}}_k^\star, \underline{\mathbf{C}}_k^\star) = \bar{\boldsymbol{\Lambda}}^{\star\top} \bar{\mathbf{C}}_k^\star + \underline{\boldsymbol{\Lambda}}^{\star\top} \underline{\mathbf{C}}_k^\star. \quad (12)$$

These locational prices are dependent on the forecast scenarios but are uniform at each distribution bus. DERA k 's induced surplus in the stochastic auction equals

$$\tilde{\Pi}_k^{\text{DERA}} = \varphi_k(\underline{\mathbf{C}}_k^\star, \bar{\mathbf{C}}_k^\star) - \tilde{\mathcal{P}}_k(\bar{\mathbf{C}}_k^\star, \underline{\mathbf{C}}_k^\star) \quad (13)$$

and the DSO's sample average surplus equals

$$\begin{aligned} \tilde{\Pi}^{\text{DSO}} := & \sum_{k=1}^K \mathcal{P}_k(\bar{\mathbf{C}}_k^\star, \underline{\mathbf{C}}_k^\star) \\ & - \frac{1}{S} \sum_{s=1}^S \left(J(\bar{\mathbf{P}}^\star[s], \underline{\mathbf{P}}^\star[s]) - J(\bar{\mathbf{p}}_0[s], \underline{\mathbf{p}}_0[s]) \right). \end{aligned} \quad (14)$$

With this notation, we now present the properties of the stochastic allocation in the next result.

$$\begin{aligned} \bar{\boldsymbol{\Lambda}}^\star &= \sum_{s=1}^S \frac{1}{S} \nabla_{\bar{\mathbf{P}}[s]} J(\bar{\mathbf{P}}^\star[s], \underline{\mathbf{P}}^\star[s]) \\ &+ \sum_{s=1}^S \left(\mathbf{A}_+^\top \bar{\boldsymbol{\beta}}^\star[s] + \mathbf{A}_-^\top \underline{\boldsymbol{\beta}}^\star[s] + \bar{\boldsymbol{\omega}}^\star[s] \right), \\ \underline{\boldsymbol{\Lambda}}^\star &= \sum_{s=1}^S \frac{1}{S} \nabla_{\underline{\mathbf{P}}[s]} J(\bar{\mathbf{P}}^\star[s], \underline{\mathbf{P}}^\star[s]) \end{aligned}$$

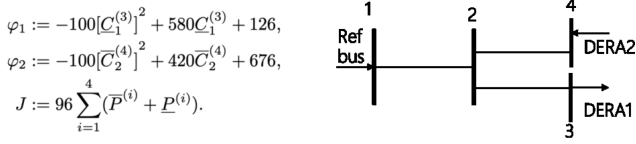


Fig. 4. 4-bus distribution network example. Left: Bid-in functions of DERAs and DSO; right: network topology.

$$+ \sum_{s=1}^S \left(\mathbf{A}_+^\top \underline{\beta}^*[s] + \mathbf{A}^\top \overline{\beta}^*[s] + \underline{\omega}^*[s] \right). \quad (15)$$

Proposition 4: The following statements hold for the stochastic network access allocation (11):

- 1) LMAP-S satisfies (15).
- 2) $\tilde{\Pi}^{\text{DSO}} \geq 0$.
- 3) $\tilde{\Pi}_k^{\text{DERA}} \geq 0$, if $\varphi_k(\mathbf{0}, \mathbf{0}) \geq 0$ and one of the following two conditions holds: (a) $\overline{C}_k^{\min} = \mathbf{0}$, $\underline{C}_k^{\min} = \mathbf{0}$, or (b) the constraints in (11b) are non-binding at optimality, i.e., $\overline{C}_k^* > \overline{C}_k^{\min}$, $\underline{C}_k^* > \underline{C}_k^{\min}$.
- 4) When J is linear and homogeneous across buses ($J(\overline{\mathbf{P}}[s], \underline{\mathbf{P}}[s]) = \sum_{i=1}^N \overline{J} \cdot \overline{P}^{(i)}[s] + \sum_{i=1}^N \underline{J} \cdot \underline{P}^{(i)}[s]$) and the constraints in (11c) are non-binding at all buses and scenarios, then $\overline{\Lambda}^{(n)*} \geq \overline{\Lambda}^{(m)*}$ and $\underline{\Lambda}^{(n)*} \geq \underline{\Lambda}^{(m)*}$, if bus n is an ancestor of bus m .

Overall, this result shows that the stochastic auction outcome behaves similarly to the robust counterpart recorded in Propositions 1-3. Specifically, LMAP-S admits a sensitivity interpretation and is monotonic along the distribution feeder under similar sufficient conditions as LMAP-R. The resulting settlement covers DSO's operating cost (on average) and DERAs have nonnegative surpluses under similar sufficient conditions as the robust auction.

V. AN ILLUSTRATIVE EXAMPLE

We illustrate the properties of our access allocation mechanisms via a 4-bus network example with two different DERAs operating at two of the buses as shown in the right of Fig. 4. The bids of the DERAs and the DSO's cost are shown in the left of Fig. 4. Capacity limits for lines 2-3, 2-4, and 1-2 are taken as 1, 1, 2 (p.u.), respectively. The minimum access requirements of DERAs are zero, i.e., $\overline{C}_k^{\min} = \mathbf{0}$, $\underline{C}_k^{\min} = \mathbf{0}$, for $k = 1, 2$, and the maximum access available is $\overline{P}^{\max} = 1$ p.u., $\underline{P}^{\max} = 1$ p.u. over all buses. DSO's customers have injections ranging over $[-0.15, 0.15]$ p.u. at all buses. We ignore voltage constraints for simplicity. For the stochastic model, we set $\delta = 0.9$ and draw 2000 scenarios for p_0 for which each entry is i.i.d. truncated normal distributions with mean zero and standard deviation (σ) of 0.05 over $[-0.15, 0.15]$, all in per units.

Results from the robust and stochastic auction mechanisms are included in Tables II and III. As Table II reveals, LMAP-R and LMAP-S along the bus-sequences 1-2-3 and 1-2-4 increase, aligned with the conclusions of Propositions 3 and 4. The surpluses of the DERAs and the DSO in Table III are non-negative and hence, align with the results of Propositions 2 and 4. Notice that compared to the robust allocation model, the stochastic

TABLE II
ACCESS ALLOCATION RESULT FOR THE 4-BUS EXAMPLE

Allocation	Parameter	Bus i			
		1	2	3	4
Robust	$\underline{C}_1^{(i)*}$	0	0	0.85	0
	$\underline{\Lambda}^{(i)*}$	96	96	410	96
	$\overline{C}_2^{(i)*}$	0	0	0	0.85
	$\overline{\Lambda}^{(i)*}$	96	96	96	250
Stoch ($\delta = 0.9$)	$\underline{C}_1^{(i)*}$	0	0	0.91	0
	$\underline{\Lambda}^{(i)*}$	96	96	397.18	96
	$\overline{C}_2^{(i)*}$	0	0	0	0.91
	$\overline{\Lambda}^{(i)*}$	96	96	96	237.16

TABLE III
SURPLUS DISTRIBUTION FOR THE 4-BUS EXAMPLE

Allocation	DERA ₁	DSO	DERA ₂	Social Surplus
Robust	198.25	282.6	748.25	1229.1
Stoch ($\delta = 0.90$)	209.51	404.36	759.61	1373.48

allocation yields lower access prices, higher allocations to DERAs, and higher surpluses for the DERAs and the DSO. In effect, tolerating a small security risk yields less conservative allocations supported by lower prices and higher social surplus. In practice, one should set the risk tolerances based on exhaustive simulations for which observed real-time violations of security constraints are deemed acceptable.

VI. CASE STUDY ON A 141-BUS NETWORK

We consider a 141-bus radial distribution network from [27] with 4 DERAs that aggregate households at different buses with different levels of behind-the-meter distributed generation (BTMDG). Simulations were performed on a personal computer with Intel(R) Xeon(R) Gold 6230R CPU @ 2.10 GHz and 256 GB RAM. We used YALMIP [28] with Gurobi 10.0.0 [29] on MATLAB R2021a to solve the optimization problems. The robust auction took < 1 s; the stochastic model with 500 scenarios took < 5 min and with 1500 scenarios took < 1 hr.

Network parameters for this system, including the resistance, reactance, topology, and base values, were adopted from [27], [30]. The base value for voltage was 12.47 kV and for power was 10MW. We assumed a fixed power factor of 0.98 lagging across all buses and set the line flow limits to be 20MW for all branches. The BTM DG for households under DERAs 1, 2, 3, and 4 were set to 0.2 kW, 1.2 kW, 3.2 kW, and 4.2 kW, respectively. DERAs 1-3 aggregated resources from all buses and DERA 4 only aggregated over buses 118-134. The bid-in utility function φ_k of DERA _{k} is assumed to equal the sum of $\varphi_k^{(i)}$ for each bus i , where DERA _{k} operates; $\varphi_k^{(i)}$'s are reported in Table IV. The derivation of $\varphi_k^{(i)}$ is relegated to [20]; a short explanation is included in Appendix H. Minimal access limits for the DERAs were assumed uniform across all buses, values for which are in Table IV. DSO's operational cost was assumed to be the sum of quadratics, $\frac{1}{2}bx^2 + ax$ with $a = \$0.009/\text{kWh}$, $b =$

TABLE IV
BID-IN UTILITY FUNCTION AND MINIMUM NETWORK WITHDRAWAL/INJECTION
LIMITS FOR DERAS AT EACH BUS

DERA k	$\varphi_k^{(i)}$	\underline{C}_k^{\min} (kW)	\overline{C}_k^{\min} (kW)
1	$-0.1[\underline{C}^{(i)}]^2 + 2.8\underline{C}^{(i)} - 1.655$	4.1	0
2	$-0.1[\underline{C}^{(i)}]^2 + 1.8\underline{C}^{(i)} + 1.513$	0	0
3	$-0.1[\overline{C}^{(i)}]^2 + 0.2\overline{C}^{(i)} + 7.393$	0	0
4	$-0.1[\overline{C}^{(i)}]^2 + 1.2\overline{C}^{(i)} + 2.833$	0	0

TABLE V
VARIATION OF DERA SURPLUS WITH σ

DERA	σ (MW)			
	0	0.004	0.006	0.008
1	599.54	488.00	431.20	369.41
2	324.07	291.43	277.58	265.01
3	1043.85	1042.54	1042.41	1042.41
4	80.18	76.74	75.09	73.49

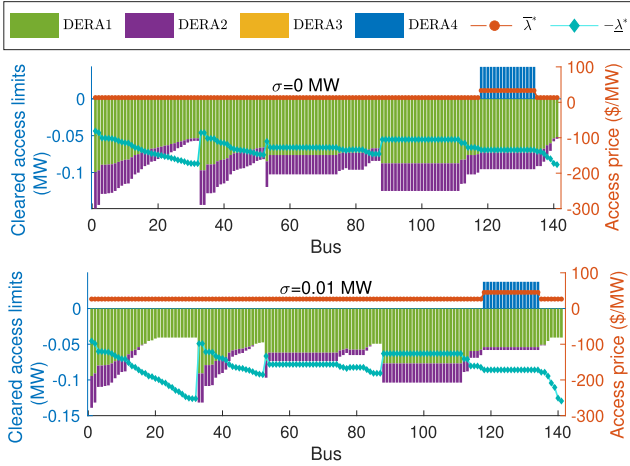


Fig. 5. Auction results over the 141-bus distribution network. Left y-axis: cleared access limits, right y-axis: access clearing prices.

$\$0.0005/(\text{kWh})^2$ for both the injection and withdrawal access at all buses.

Power injection $p_0^{(i)}$ from the DSO's customer at each bus i was sampled from independent truncated normal distributions with mean $\mu = 5$ kW, standard deviation σ , truncated to $[p_0^{(i)}, \overline{p}_0^{(i)}] = [\mu - 3\sigma, \mu + 3\sigma]$. We used these intervals for the robust allocation model, but utilized 1500 random samples within said intervals for the stochastic allocation results.

A. Running the Robust Access Allocation

The access allocation results for the 4 DERAs under different σ are illustrated in Fig. 5. The positive and negative segments of the left y-axis respectively represent the allocated injection and withdrawal ranges. The right y-axis shows the injection and withdrawal access prices. We show the injection access price over the positive segment of the right y-axis, and the negative range of the right y-axis shows the opposite number for the withdrawal access price. The plots reveal that cleared access limits for DERAs were smaller with higher σ . Such a trend is expected as higher σ increases the burden from distribution utility's customers on the distribution network, implying a lesser share of the pie available to the DERAs. This burden makes network access more expensive. This manifested in higher locational access prices when σ was larger.

DERAs 1 and 2 had lower BTM DG compared to DERAs 3 and 4. Thus, DERAs 1 and 2 behaved as net consumers, who bid for and received withdrawal access, as shown in Fig. 5. DERAs

3 and 4, on the other hand, largely acted as net power producers, and they purchased injection access through the auction at buses they operated. We remark that DERA 3 has a small access limit across all buses with $\sigma = 0$ MW, and it vanishes with $\sigma = 0.01$ MW. DERA 3 commands less BTM DG than DERA 4, and the surplus it can generate for its customers is lower. As a result, it bids a higher induced utility, compared to DERA 4. Consequently, its cleared access remains lower than that of DERA 4. With the highest BTM DG, DERA 4 has the largest incentive to acquire injection access at the buses it operates at, i.e., buses 118 - 134.

The access prices varied by location. As Proposition 1 reveals, prices must be uniform unless either network constraints or maximum injection/withdrawal limits are binding. Indeed, with both values of σ , voltage constraints at buses 52 and 141 were binding, leading to locationally varying access prices. Moreover, the figures suggest that prices are monotonic only over certain segments of the distribution network. However, as the network structure in [27] reveals, the price is monotonic along paths away from the substation. Notice that price monotonicity is only guaranteed by Proposition 3 with linear cost structures for the DSO. Our numerical results are derived with quadratic DSO costs, and yet, the conclusion of Proposition 3 holds, implying that the conditions for the result as stated are only sufficient, but not necessary.

The surpluses of the various DERAs are reported in Table V. As evident, the surpluses reduce with higher σ . The larger the σ , the lower the DERAs' access to the network and consequently, their surpluses.

B. Comparing the Robust and Stochastic Auctions

For the stochastic access allocation models, we considered three different risk levels— $\delta = 0.99, 0.9, 0.8$. The allocation results for the withdrawal access and price are shown at the top of Fig. 6. Negative values indicate withdrawal access and positive values encode injection access. Compared to the robust allocation mechanism, the stochastic model had larger withdrawal accesses cleared and lower prices for those access limits. The difference in the outcomes from the robust and the stochastic models are substantial, even with a high risk parameter of $\delta = 0.99$. With more uncertainty (larger σ), the cleared withdrawal accesses were lesser and allocation prices were higher—a trend we already observed with the robust allocation model.

The DSO's surplus on the top left of Fig. 7 was nonnegative for all experiments. The same holds for the DERAs' surpluses, the aggregate value for which is plotted on the top right of Fig. 7.

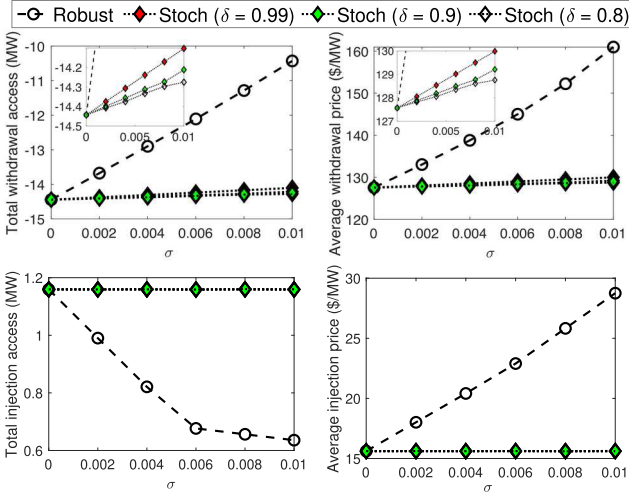


Fig. 6. Top left: injection access, top right: injection price, bottom left: withdrawal access, bottom right: withdrawal price.

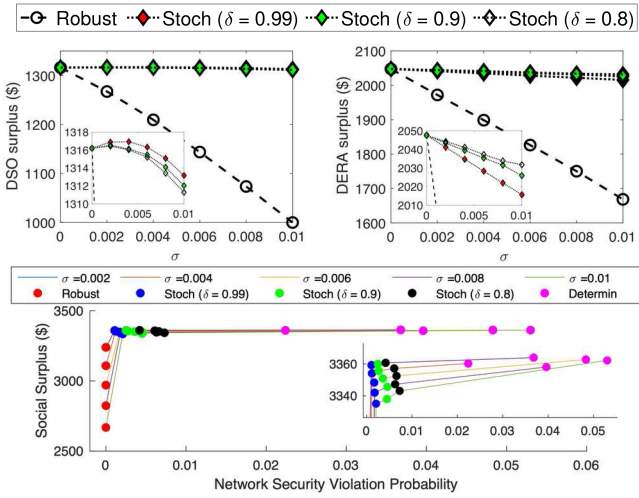


Fig. 7. Top left: DSO surplus. Top right: DERA surplus. Bottom: The network security violation probability and social surplus.

These verify Propositions 2-3 and 4. The surpluses are higher under the stochastic model, compared to the robust counterpart. Correspondingly, the social surplus (the sum of DERAs' and DSO's surplus) at the bottom of Fig. 7, is higher in the stochastic model. In fact, the conservative robust allocation had DSO and DERA surpluses around 20% lower than in the stochastic setting when $\sigma = 0.01$ MW. This observation suggests that even a small $\sim 1\%$ risk tolerance can drastically improve the surpluses of the auction participants.

The variation of DERA surpluses with σ is expected—largely, the DERAs' surpluses reduced with higher σ that burdens the distribution network with higher possible injections from the DSO's customers. As a result, DERAs got lesser access limits with lower surpluses.

We also compared our results with a *deterministic* model that computed the access against the average of all sampled scenarios by running (11) with that average scenario. As such an auction model ignores the uncertainty in the power injection/withdrawal scenarios, network security constraint violation probabilities

were 7-30 times higher than that of the stochastic auction, as Fig. 7 reveals. Recall that uncertainties in operating conditions result from two sources—the first is the natural variation in the real-time DERA-ISO interactions over time, and the second is the aleatory uncertainty in possible real-time system conditions as visible at the forward auction stage. Undoubtedly, any uncertainty model that a DSO adopts, will affect the outcomes of our auctions. From practical use, a DSO must calibrate the uncertainty model, the resulting auction outcomes, and its implications for real-time DERA-ISO transactions through exhaustive simulations.

VII. CONCLUSION

We have proposed a DSO-DERA-ISO coordination mechanism for multi-DERA participation in the wholesale market. Through a forward auction and welfare-maximizing robust and risk-sensitive market clearings, the proposed mechanism allocates network access limits to DERAs based on their willingness to pay for the access of the DSO-operated network. The key advantage of our proposed coordination mechanism is that it decouples DSO-DERA-ISO operations—the DSO-DERA forward auction in our model computes operating envelopes for real-time DERA-ISO transactions. This salient feature makes the proposed solution compatible with existing jurisdictional boundaries of DSOs and ISOs and yet, guarantees distribution network security with minimal coordination.

Several relevant issues remain outside the scope of this work, requiring further investigation. We have not considered possible topology changes of the distribution network in our auction designs. We anticipate that a security-constrained version of our auction can be designed similarly to the security-constrained unit commitment and economic dispatch problem for wholesale market clearing. In this work, we have not addressed the question of bid/offer formation for DERAs. The overall market efficiency of our designs rests on this bid/offer formation process with price-taking and price-making DERAs. See our parallel effort in [20] for a step towards such an analysis. We have not delved into the details of the DSO's bid-in operational costs. Defining the scientific basis for a regulatory framework to calculate such costs, building on insights from [21] remains an interesting direction for future research. Admittedly, our simulations are limited in scope, designed primarily to study the properties of our design. As we have repeatedly pointed out, more realistic empirical analyses on larger systems are required to validate the practical efficacy of our designs.

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