

Covert Communications under Environmental Uncertainty on a Continuous-Time Channel

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ABSTRACT

Covert communication is achieved when a transmitter Alice can successfully transmit a message to a receiver Bob without being detected by an attentive and capable adversary Willie. Early results demonstrated the difficulty of the covert communications problem: with AWGN discrete-time channels between all parties, only $O(\sqrt{n})$ bits can be sent in n channel uses. But it was soon recognized that uncertainty about the environment at Willie, for example, uncertainty in his own noise statistics, could allow for a positive rate: $O(n)$ bits can be sent covertly in n channel uses. However, most covert communication results, including this promising positive rate result, have been obtained for a discrete-time communications channel. Here, we demonstrate that the assumption of a discrete-time channel is problematic when trying to exploit Willie's noise uncertainty. In particular, we demonstrate that if Alice transmits $\omega(\sqrt{T})$ bits in a length T interval to Bob on a continuous-time channel, then there exists a detector at Willie that can detect her transmission, as the probability of false alarm and missed detection $P_{MD} + P_{FA} \rightarrow 0$ as $T \rightarrow \infty$. In other words, the communication is not covert, unlike the case of a discrete-time channel.

I. INTRODUCTION

Security and privacy play a major role in contemporary wireless communication systems. Much security research focuses on employing encryption [1] or information-theoretic [2] security approaches to protect the *content* of messages from interception, but there are applications where hiding even the *existence* of a signal can be crucial, for example in hiding the presence of an implanted medical device or in military communication. In covert communication, the transmitter Alice tries to transmit a message reliably to the receiver Bob in the presence of an adversary Willie, who tries to detect the communication.

This work was funded in part by the National Science Foundation under Grant ECCS-2029323.

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Bash et. al. [3] established the fundamental limits of covert communication for the discrete-time additive white Gaussian noise (AWGN) channel. According to the square root law (SRL) of [3], Alice can transmit at most $O(\sqrt{n})$ bits to Bob in n channel uses with low probability of detection by adversary Willie. This work motivated [4] and [5], which established that if Willie has uncertainty in the statistics of the noise impacting his receiver, $O(n)$ bits can be transmitted covertly in n channel uses. Since such uncertainty might not arise naturally in a given system [6], Sobers et. al. [7] employed jamming to achieve $O(n)$ covert throughput in n channel uses.

The results discussed above assume a standard discrete-time communication channel. Researchers generally consider a discrete-time channel for analytical convenience, with the understanding that it is functionally equivalent to the true continuous-time channel. However, these techniques that exploit noise uncertainty or cooperative jamming are vulnerable to detection (i.e. not covert) when the constructions developed for the discrete-time channel are mapped to a true continuous-time channel. Li et. al. [8] showed that the technique of [7] cannot send $O(n)$ bits covertly in n channel uses on a continuous-time channel if Willie employs interference cancellation, even in the presence of a jamming signal as proposed in [7], emphasizing the more realistic continuous-time channel.

Here, we consider the performance of the constructions of [4] and [5] when they are mapped to a continuous-time channel in an obvious manner. By having Willie employ a cyclostationary detector, we show that the performance results established in [4] and [5] are indeed optimistic. By conducting an asymptotic characterization of the cyclostationary detector, we demonstrate that schemes employing standard bauded digital communication approaches (i.e., randomly drawn symbols placed on a standard pulse shape with a fixed symbol interval) are limited to sending at most $O(\sqrt{n})$ covert bits to Bob in n channel uses. Furthermore, we show that Alice would indeed be able to send $O(n)$ bits in n channel uses covertly to Bob on the continuous-time channel if Willie were restricted to a power detector, hence demonstrating that it is the availability of other detectors on the continuous-time channel that leads to the limitation of the throughput of schemes suggested by the design for the discrete-time channel.

Section II presents the natural extension of the constructions of [4] and [5] to the continuous-time channel. Section III presents a sketch of the proofs for the two main results of the paper. Section IV presents the conclusions.

II. SYSTEM MODEL

Consider the scenario presented in Figure 1, where Alice wants to transmit a message to Bob over (continuous-time) interval $[-T/2, T/2]$ while keeping the transmission hidden from adversary Willie, with both channels experiencing additive white Gaussian noise (AWGN). Suppose Alice sends an independent and identically distributed (i.i.d.) sequence of symbols $f_{-m/2}, f_{-m/2+1}, \dots, f_{m/2}$, such that $f_i \in \mathcal{CN}(0, \sigma_a^2)$, using a digital communication signal in the standard format with pulse shape $q(t)$:

$$x_a(t) = \sum_{k=-m/2}^{m/2} f_k q(t - kT_s) \quad (1)$$

where T_s is the symbol period and $m = T/T_s$, hence extending the Lee et. al. [4] discrete-time channel in the natural way to continuous-time channel. We take the pulse shape $q(t)$ to be a square-root raised cosine (SRRC) pulse with bandwidth $(1 + \beta)/2T_s$, where $\beta > 0$ is the excess bandwidth or roll-off factor, but our results are valid for any pulse shape that decays sufficiently fast to zero for $t \rightarrow \infty$. The pulse shape $q(t)$ is normalized such that $\int_{-\infty}^{\infty} |q(t)|^2 dt = 1$.

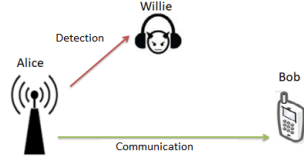


Fig. 1. Covert Communications Model: Transmitter Alice attempts to send information to an intended receiver Bob without detection of the presence of that transmission by a capable and attentive adversary Willie.

Consider H_0 the hypothesis when Alice is not transmitting and H_1 the hypothesis when Alice is transmitting. Then, the received signal $y_w(t)$ at Willie is given under each hypothesis as:

$$H_0 : y_w(t) = n_w(t)$$

$$H_1 : y_w(t) = x_a(t) + n_w(t)$$

where $\{n_w(t)\}_{t=-\infty}^{\infty}$, the noise at Willie, is a zero-mean stationary Gaussian random process with power spectral density $S_w(f) = N_0/2$. Per Section I, we consider the case where N_0 is not perfectly known to Willie [4], [5].

Willie conducts a binary hypothesis test to determine whether Alice is transmitting or not. Let P_{FA} , the probability of false alarm, denote the probability that Willie chooses H_1 when H_0 is true, and P_{MD} , the probability of missed detection, denote the probability that he chooses H_0 when H_1 is true. Using the now standard definition of covertness introduced in [3], a system is said to be covert if $P_{MD} + P_{FA} > 1 - \epsilon$ for any $\epsilon > 0$ as $T \rightarrow \infty$. Conversely, it is apparent that covertness is (severely) compromised if $P_{MD} + P_{FA} \rightarrow 0$, as $T \rightarrow \infty$.

III. MAIN RESULT

Theorem 1: Given the continuous-time channel of Section II, if Alice sends $\omega(\sqrt{T})$ bits over (continuous-time) interval $[-T/2, T/2]$, then Willie can detect her with a low probability of error, $P_{MD} + P_{FA} \rightarrow 0$, as $T \rightarrow \infty$.

Proof: The receiver at Willie is shown in Figure 2. It employs a cyclostationary detector to determine whether Alice is transmitting. After front-end matched filtering to form $y(t) = y_w(t) * q(-t)$, here $y_w(t)$ is the signal received by Willie, the receiver forms

$$z = \frac{1}{T} \int_{-T/2}^{T/2} |y(t)|^2 e^{-j2\pi t/T_s} dt$$

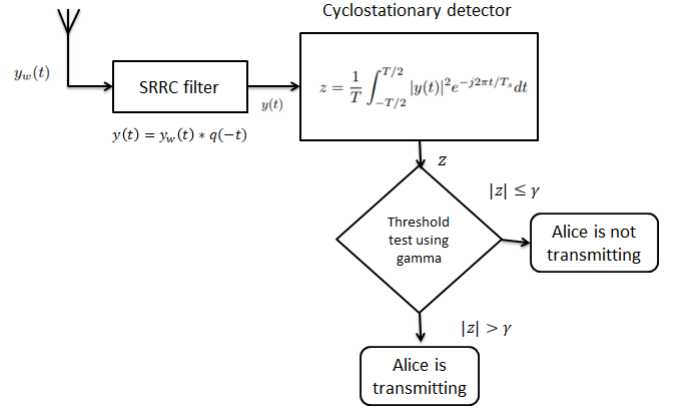


Fig. 2. Receiver construction: cyclostationary detection at Willie

Willie then employs a threshold test on $|z|$ with threshold γ to detect whether Alice is transmitting or not, where γ is a constant that will be determined below. If $|z| > \gamma$, Willie decides that Alice is transmitting; if $|z| \leq \gamma$, Willie decides that Alice is not transmitting.

Statistical Analysis $|z|$ under H_0

We first consider the statistics of $|z|$ when Alice is not transmitting. To do such, consider $E[|z|^2 | H_0]$.

Let the receiver noise be $n(t) = n_w(t) * q(-t)$, the signal obtained after passing the noise $n_w(t)$ on the received signal through the front-end filter at the receiver. Then:

$$\begin{aligned} E[|z|^2 | H_0] &= E \left[\left| \frac{1}{T} \int_{-T/2}^{T/2} |n(t)|^2 e^{-j2\pi t/T_s} dt \right|^2 \right] \\ &= \frac{1}{T^2} \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} E[|n(t)|^2 |n(s)|^2] \\ &\quad e^{-j2\pi t/T_s} e^{j2\pi s/T_s} dt ds. \end{aligned}$$

For jointly Gaussian W, X, Y and Z ,

$$\begin{aligned} E[WXYZ] &= E[WX]E[YZ] + E[WZ]E[XY] \\ &\quad + E[WY]E[XZ]. \end{aligned}$$

Hence:

$$\begin{aligned} E[|n(t)|^2 |n(s)|^2] &= E[n(t)n^*(t)n(s)n^*(s)] \\ &= E[|n(t)|^2] E[|n(s)|^2] + E[n(t)n(s)] E[n^*(t)n^*(s)] \\ &\quad + E[n(t)n^*(s)] E[n^*(t)n(s)] \end{aligned} \quad (2)$$

Consider the behavior of the first term of (2). With

$$E[|n(t)|^2] = \int_{-\infty}^{\infty} S_n(f) df,$$

where $S_n(f)$ is the power spectral density of $n(t) = n_w(t) * q(-t)$. Letting $\mathcal{F}\{q(t)\} = Q(f)$ be the Fourier transform of $q(t)$,

$$\begin{aligned} E[|n(t)|^2] &= \int_{-\infty}^{\infty} |Q(f)|^2 \frac{N_0}{2} df \\ &= \frac{N_0}{2} \int_{-\infty}^{\infty} |Q(f)|^2 df \\ &= \frac{N_0}{2}, \end{aligned} \quad (3)$$

and one can readily show that:

$$\begin{aligned} \frac{1}{T^2} \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} E[|n(t)|^2] E[|n(s)|^2] \\ e^{-j2\pi t/T_s} e^{j2\pi s/T_s} dt ds = O\left(\frac{1}{T^2}\right) \end{aligned} \quad (4)$$

The random phase of the second term of (2) causes the expectation to be 0 for all s and t , so all that remains is to consider the third term, which we will see is the dominant term:

$$\begin{aligned} \frac{1}{T^2} \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} E[n(t)n^*(s)] E[n^*(t)n(s)] \\ e^{-j2\pi t/T_s} e^{j2\pi s/T_s} dt ds \\ = \frac{1}{T^2} \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} R_n^2(t-s) e^{-j2\pi t/T_s} e^{j2\pi s/T_s} dt ds \end{aligned}$$

where $R_n(t-s)$ is the autocorrelation function of $n(t)$.

Letting $v = s + t$ and $u = s - t$ and applying the change of variables to the above equation results in:

$$\frac{2}{T^2} \int_0^T \int_{v-T}^{T-v} R_n^2(u) e^{-j2\pi u/T_s} du dv \quad (5)$$

$$\leq \frac{2}{T^2} \int_0^T \int_{v-T}^{T-v} R_n^2(u) du dv \quad (6)$$

$$\leq \frac{2}{T^2} \int_0^T \int_{-\infty}^{\infty} R_n^2(u) du dv, \quad (7)$$

where we have used $|e^{-j2\pi u/T_s}| \leq 1$ and noted $R_n^2(u) \geq 0$ to get the second line above. Employing Parseval's theorem $\int_{-\infty}^{\infty} R_n^2(u) du = \int_{-\infty}^{\infty} S_n^2(f) df$ motivates the evaluation:

$$\int_{-\infty}^{\infty} S_n^2(f) df = \frac{N_0^2}{4} \int_{-\infty}^{\infty} |Q(f)|^4 df = \frac{N_0^2 K_1}{4} \quad (8)$$

where we have defined the constant $K_1 = \int_{-\infty}^{\infty} |Q(f)|^4 df$. Hence:

$$\lim_{T \rightarrow \infty} E[|z|^2 | H_0] \leq \lim_{T \rightarrow \infty} \frac{2}{T^2} \int_0^T \frac{N_0^2 K_1}{4} dv \quad (9)$$

$$= \lim_{T \rightarrow \infty} \frac{N_0^2 K_1}{2T} \quad (10)$$

Hence, the random variable $|z|$ converges to 0 in the mean squared sense, and the rate of convergence of the mean squared value given in (10) will be important in establishing a tight upper bound on Alice's throughput.

We note that the (probabilistic) convergence of $|z|$ to 0 captures a key feature of the cyclostationary detector relative to a power detector when Willie does not know the noise variance N_0 . For a power detector, the mean is not zero when Alice is not present; rather it depends on N_0 , thus making it difficult for Willie to select a threshold when N_0 is unknown, as shown below in the proof of Theorem 2.

Statistical analysis of $|z|$ under H_1

Under H_1 , the signal at the output of Willie's front-end matched filter is given by:

$$y(t) = \sum_{k=-m/2}^{m/2} f_k p(t - kT_s) + n_w(t)$$

where $p(t) = q(t) * q(-t)$. We desire to lower bound $E[|z| | H_1]$ by a constant so that Willie can set a threshold γ between 0 and that constant; combined with the rates at which the variance under H_0 and H_1 go to zero, this will establish the main result, as shown below.

Given that $E[|z| | H_1] \geq E[z | H_1]$, we consider the more tractable $E[z | H_1]$, which we will show is sufficient. Noting that the noise is zero mean and independent of the signal yields:

$$E[z | H_1] = E\left[\frac{1}{T} \int_{-T/2}^{T/2} |y(t)|^2 e^{-j2\pi t/T_s} dt\right]$$

$$\begin{aligned} &= \frac{1}{T} \int_{-T/2}^{T/2} \left(\sum_{k=-m/2}^{m/2} \sum_{l=-m/2}^{m/2} E[f_l^* f_k] p(t - kT_s) \right. \\ &\quad \left. p(t - lT_s) + E[|n(t)|^2] \right) e^{-j2\pi t/T_s} dt \end{aligned}$$

Using analogous arguments from our work above under H_0 , we know that the second term containing $E[|n(t)|^2] = O\left(\frac{1}{T^2}\right)$. For the first term, using

$$E[f_l^* f_k] = \begin{cases} \sigma_a^2, & l = k, \\ 0, & \text{else} \end{cases}$$

yields

$$\begin{aligned}
E[z|H_1] &= \frac{\sigma_a^2}{T} \int_{-T/2}^{T/2} \sum_{k=-m/2}^{m/2} p^2(t - kT_s) e^{-j2\pi t/T_s} dt \\
&= \frac{\sigma_a^2}{T} \int_{-\infty}^{\infty} \sum_{k=-m/2}^{m/2} p^2(t - kT_s) e^{-j2\pi t/T_s} dt \\
&\quad - \frac{\sigma_a^2}{T} \int_{-\infty}^{-T/2} \sum_{k=-m/2}^{m/2} p^2(t - kT_s) e^{-j2\pi t/T_s} dt \\
&\quad - \frac{\sigma_a^2}{T} \int_{T/2}^{\infty} \sum_{k=-m/2}^{m/2} p^2(t - kT_s) e^{-j2\pi t/T_s} dt \quad (11)
\end{aligned}$$

The second and third terms of (11) will go to zero as $T \rightarrow \infty$, as the tails of the raised cosine pulse are absolutely integrable. If we move the integral through the summation in the first term, we note that the summand is the Fourier transform $\mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi t f} dt = X(f)$ of the (shifted) raised cosine pulse shape squared evaluated at the frequency $f = \frac{1}{T_s}$. Thus,

$$\lim_{T \rightarrow \infty} E[z|H_1] = \lim_{T \rightarrow \infty} \frac{\sigma_a^2}{T} \sum_{k=-m/2}^{m/2} e^{-j2\pi k} P(f) * P(f) \Big|_{f=\frac{1}{T_s}},$$

where $x(\cdot) * y(\cdot)$ is the convolution of $x(\cdot)$ and $y(\cdot)$. Using $m = T/T_s$,

$$\begin{aligned}
\lim_{T \rightarrow \infty} E[z|H_1] &= \lim_{T \rightarrow \infty} \frac{\sigma_a^2(m+1)}{mT_s} P(f) * P(f) \Big|_{f=\frac{1}{T_s}} \\
&= \lim_{T \rightarrow \infty} \left(\frac{\sigma_a^2}{T_s} + \frac{1}{mT_s} \right) P(f) * P(f) \Big|_{f=\frac{1}{T_s}}
\end{aligned}$$

Let the (positive) constant $K_2 = P(f) * P(f) \Big|_{f=\frac{1}{T_s}}$ and noting that as $T \rightarrow \infty$, $m \rightarrow \infty$, yields:

$$\lim_{T \rightarrow \infty} E[z|H_1] = \lim_{m \rightarrow \infty} \left(\frac{\sigma_a^2}{T_s} + \frac{1}{mT_s} \right) K_2 = \frac{\sigma_a^2 K_2}{T_s} \quad (12)$$

Hence, for the cyclostationary detector, as $T \rightarrow \infty$, $E[z|H_1] \rightarrow C$, where $C \neq 0$, and the constant C is not dependent on N_0 . In contrast, we will see below in the proof of Theorem 2 that, for the power detector, $E[z|H_1]$ depends upon N_0 , which is unknown.

The variance of $|z|$ under H_1 is given by:

$$\text{var}(z|H_1) = E[|z|^2|H_1] - (E[z|H_1])^2 \quad (13)$$

$$\leq E[|z|^2|H_1] - (E[z|H_1])^2 \quad (14)$$

$$\begin{aligned}
E[|z|^2|H_1] &= E \left[\frac{1}{T^2} \left[\int_{-T/2}^{T/2} \sum_{k=-m/2}^{m/2} \sum_{l=-m/2}^{m/2} f_l^* f_k \right. \right. \\
&\quad \left. \left. p(t - kT_s) p(t - lT_s) + \sum_{k=-m/2}^{m/2} f_k p(t - kT_s) n^*(t) + \right. \right. \\
&\quad \left. \left. \sum_{l=-m/2}^{m/2} f_l^* p(t - lT_s) n(t) + |n(t)|^2 e^{-j2\pi t/T_s} dt \right] \right] \\
&\quad \left[\int_{-T/2}^{T/2} \sum_{k=-m/2}^{m/2} \sum_{l=-m/2}^{m/2} f_l^* f_k p(s - kT_s) p(s - lT_s) \right. \\
&\quad \left. + \sum_{k=-m/2}^{m/2} f_k p(s - kT_s) n^*(s) + \sum_{l=-m/2}^{m/2} f_l^* p(s - lT_s) n(s) \right. \\
&\quad \left. + |n(s)|^2 e^{-j2\pi s/T_s} ds \right]^* \Big]
\end{aligned}$$

As the product of two expressions, each of which involves four terms, the above expression expands into sixteen terms. After detailed technical evaluation (omitted) the resulting variance has the following bound as $T \rightarrow \infty$:

$$\lim_{T \rightarrow \infty} \text{var}(|z||H_1) \leq \lim_{T \rightarrow \infty} \frac{2\sigma_a^2 K_3 + N_0^2 K_1}{T}, \quad (15)$$

where K_1 is defined above, and K_3 is a (positive) constant that depends on the noise variance and the pulse shaping filter as $K_3 = P(2f) * P(2f) * S_n(f) \Big|_{f=\frac{1}{T_s}}$.

Set the threshold employed by Willie to $\gamma = \frac{\sigma_a^2 K_2}{2T_s}$. The probability of false alarm P_{FA} and the probability of missed detection P_{MD} can then be readily bounded using Chebyshev's inequality. For T large,

$$P_{FA} = P(|z| \geq \gamma | H_0) \leq \frac{\text{var}(|z||H_0)}{\gamma^2} \leq \frac{2N_0^2 K_1 T_s^2}{T(\sigma_a^2 K_2)^2}$$

and

$$\begin{aligned}
P_{MD} &= Pr(|z| < \gamma | H_1) = P \left(\left| |z| - \frac{\sigma_a^2 K_2}{T_s} \right| > \frac{\sigma_a^2 K_2}{T_s} - \gamma \right) \\
&\leq \frac{\text{var}(|z||H_1)}{\left(\frac{\sigma_a^2 K_2}{T_s} - \gamma \right)^2}
\end{aligned}$$

$$P_{MD} \leq \frac{(2\sigma_a^2 K_3 + N_0^2 K_1) 4T_s^2}{T(\sigma_a^2 K_2)^2}$$

Given that T_s , K_1 , K_2 and K_3 are all constants, if $\sigma_a^2 = \omega \left(1/\sqrt{T} \right)$, then $P_{FA} \rightarrow 0$ and $P_{MD} \rightarrow 0$ as $T \rightarrow \infty$. Therefore, following arguments in [3], given that Alice is using a signal power of $\omega \left(1/\sqrt{T} \right)$, indicating that Alice transmits $\omega(\sqrt{T})$ bits in a time interval of $[0, T]$, then $P_{MD} + P_{FA} \rightarrow 0$ as $T \rightarrow \infty$, even with noise uncertainty at Willie. $\square$

The above result demonstrates that a straightforward extension of constructions employed for covert communications on discrete-time channels might not be effective on the continuous-time channel. From [7], it is apparent that a power detector is an optimal receiver for Willie to attempt to detect

Alice's signal for some noise uncertainty models on a discrete-time channel. Next, we demonstrate that this is not true on the continuous-time channel by showing that Alice would be able to transmit at a constant rate to Bob while remaining covert if Willie employs a power detector. Hence, it is the existence of better detectors on the continuous-time channel that exploit the structure of Alice's signal that restricts her ability to communicate covertly on such.

Theorem 2: Consider the model of Section II, with the uncertainty in Willie's noise variance characterized by a uniform random variable: $N_0 \sim U[l, u]$, where l and u are lower and upper bounds on N_0 , respectively. Then, Alice can send $O(T)$ bits covertly to intended receiver Bob over (continuous-time) interval $[-T/2, T/2]$, if Willie employs a power detector.

Proof: A power detector at Willie forms the variable:

$$z_p = \frac{1}{T} \int_{-T/2}^{T/2} |y(t)|^2 dt$$

and compares it to a threshold τ .

The expectation of z_p is given by

$$E[z_p|H_0] = E \left[\frac{1}{T} \int_{-T/2}^{T/2} |n(t)|^2 dt \right] = \frac{N_0}{2}$$

Similarly for H_1 :

$$\begin{aligned} E[z_p|H_1] &= E \left[\frac{1}{T} \int_{-T/2}^{T/2} |y(t)|^2 dt \right] \\ &= \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=-m/2}^{m/2} \sum_{l=-m/2}^{m/2} f_l^* f_k p(t - kT_s) p(t - lT_s) \\ &\quad + \frac{1}{T} \int_{-T/2}^{T/2} |n(t)|^2 dt \\ &= \frac{\sigma_a^2}{T} \int_{-T/2}^{T/2} \sum_{k=-m/2}^{m/2} p^2(t - kT_s) dt + \frac{N_0}{2} \end{aligned}$$

Let $P_x = \frac{1}{T} \sum_{k=-m/2}^{m/2} \int_{-T/2}^{T/2} p^2(t - kT_s) dt$ be the average signal power, which is a constant; then,

$$E[z_p|H_1] = \sigma_a^2 P_x + \frac{N_0}{2} \quad (16)$$

Since we are deriving an achievability result, it is sufficient to demonstrate performance against a detector that upper bounds the performance of Willie. To do such, assume that a genie averages out the variation due to measurement noise on Willie's received signal. Hence, under H_0 , Willie always measures $\frac{N_0}{2}$, and under H_1 he always measures $\sigma_a^2 P_x + \frac{N_0}{2}$. But recall that Willie does not know the value of N_0 and thus struggles to choose the threshold τ . Consider any choice of that threshold τ , and define the event A to be the event that

a "good" threshold is selected such that $A = \{\tau : \frac{N_0}{2} \leq \tau \leq \frac{N_0}{2} + \sigma_a^2 P_x\}$. Now, consider the probability that A occurs:

$$\begin{aligned} P(A) &= P \left(\frac{N_0}{2} \leq \tau \leq \frac{N_0}{2} + \sigma_a^2 P_x \right) \\ &= P(2\tau - 2\sigma_a^2 P_x \leq N_0 \leq 2\tau) \\ &= \frac{2\sigma_a^2 P_x}{(u - l)} \end{aligned} \quad (17)$$

Now note that if A is not true, then $P_{FA} + P_{MD} = 1$. Hence,

$$P_{MD} + P_{FA} \geq P(A^c) = 1 - \frac{2\sigma_a^2 P_x}{(u - l)} \quad (18)$$

and Alice can select a constant power σ_a^2 small enough such that $P_{MD} + P_{FA} \geq 1 - \epsilon$, hence establishing that Alice can employ constant power and transmit $O(n)$ bits covertly in n channel uses when Willie uses a power detector. $\square$

IV. CONCLUSION

If Alice attempts to employ bauded modulation to transmit $\omega(\sqrt{T})$ bits covertly in a time interval of $[0, T]$ to receiver Bob over a continuous-time channel, an attentive adversary Willie can detect that communication as $T \rightarrow \infty$, even in the face of noise uncertainty at Willie, as the bauded signal allows detection by a cyclostationary detector for which the parameters do not depend on the noise variance. In contrast, a power detector, which is the optimal detector in a discrete-time channel under common uncertainty models for the noise variance, has difficulty setting a detection threshold. Hence, the existence of better detectors in continuous-time significantly impacts the covert throughput versus what might be suggested by design for a discrete-time model. This work also helps to demonstrate the challenges of covert communications as we incorporate more faithful representations of the physical layer.

ACKNOWLEDGEMENT

We thank Amir Houmansadr for useful conversations.

REFERENCES

- [1] John Talbot, Dominic Welsh, and Dominic James Anthony Welsh. *Complexity and cryptography: an introduction*, volume 13. Cambridge University Press, 2006.
- [2] Aaron D Wyner. The wire-tap channel. *Bell system technical journal*, 54(8):1355–1387, 1975.
- [3] Boulat A Bash, Dennis Goeckel, and Don Towsley. Limits of reliable communication with low probability of detection on awgn channels. *IEEE Journal on Selected Areas in Communications*, 31(9):1921–1930, 2013.
- [4] Seonwoo Lee and Robert J Baxley. Achieving positive rate with undetectable communication over awgn and rayleigh channels. In *2014 IEEE International Conference on Communications (ICC)*, pages 780–785. IEEE, 2014.
- [5] Seonwoo Lee, Robert J Baxley, Mary Ann Weitnauer, and Brett Walkenhorst. Achieving undetectable communication. *IEEE Journal of Selected Topics in Signal Processing*, 9(7):1195–1205, 2015.
- [6] Dennis Goeckel, Boulat Bash, Saikat Guha, and Don Towsley. Covert communications when the warden does not know the background noise power. *IEEE Communications Letters*, 20(2):236–239, 2015.
- [7] Tamara V Sobers, Boulat A. Bash, Saikat Guha, Don Towsley, and Dennis Goeckel. Covert communication in the presence of an uninformed jammer. *IEEE Transactions on Wireless Communications*, 16(9):6193–6206, 2017.
- [8] Ke Li, Tamara V Sobers, Don Towsley, and Dennis Goeckel. Covert communication in continuous-time systems in the presence of a jammer. *IEEE Transactions on Wireless Communications*, 21(7):4883–4897, 2021.