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Learning through teaching: the development of pedagogical content knowledge among novice mathematics teachers

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ABSTRACT

Pedagogical content knowledge (PCK) has been widely recognised as an important aspect of the expertise for teaching. However, the extent to which teachers' own teaching practice can be a learning resource for them to develop PCK has not been systematically explored. This empirical study aimed to explore the unique contribution of the work of teaching in teachers' PCK growth by concurrently considering other external professional learning opportunities teachers may have on the job. Using longitudinal data from 207 elementary and middle school teachers in the United States, we found that teachers increased their PCK through teaching on their own, albeit at different rates. Our findings were robust when other external learning opportunities teachers had were taken into account. Our findings underscored the importance of teachers' robust knowledge of school mathematics in the development of their PCK through teaching.

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Pedagogical content knowledge; learning through teaching; mathematical knowledge for teaching; novice teachers; longitudinal study; noticing; content knowledge; teacher knowledge

Introduction

The professional growth of novice teachers has been mainly explored through the lens of external professional learning opportunities. A plethora of work has focussed on the role of formal professional development in improving teachers' capacity (Garet et al. 2001; Kennedy 2016; Yoon et al. 2007). Despite the fact that teaching practice stands as a remarkable potential resource for teacher learning (Tzur 2010), limited work has been conducted to investigate the potential of teachers to learn through teaching without external guidance.

Scholars argue that learning through teaching occurs within a cyclical process in which teachers plan, implement, and reflect on their teaching, thereby constructing new knowledge about the subject matter, their teaching, and their students' understanding (Simon 1997; Steinbring 1998; Wilson 1987). Specifically, teachers develop hypothetical students' learning trajectories (i.e. lesson plans), including objectives, tasks, and anticipated student responses (Simon 1997; Steinbring 1998). These trajectories, when applied in real classroom settings, often generate discrepancies between expected and actual student

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learning, particularly in novice teachers' classrooms (Flores 2006; Mintz et al. 2020), prompting reflection and adjustment and fostering new insights for future teaching.

Although theoretically it is possible for teachers to learn through teaching, empirical research on the development of teachers' knowledge in natural, unguided teaching contexts remains limited (Leikin and Zazkis 2010). Most work examining teachers' learning through teaching has involved external interventions. For example, researchers have provided teachers with new curricular materials (e.g. Lewis and Perry 2017) or selected specific teaching segments and prompted teachers to analyse them (Sherin and Van Es 2009). These studies have documented that teaching itself has the potential to enhance teachers' knowledge and skills, although it remains unclear whether teachers could have learnt through their own teaching if they had not received any guidance from experts.

Other research has approached this phenomenon by examining the impact of teachers' years of teaching experience on student outcomes (Kini and Podolsky 2016). These studies imply that novice teachers acquire certain expertise that improves their students' mathematics performance. However, the knowledge or skills teachers gained that led to this improvement have not been identified. Moreover, these studies have not distinctly ascertained whether the knowledge or skills were learnt through teachers' own teaching autonomously or from various supports provided by schools or colleagues (Papay and Kraft 2015).

We aim to address these gaps by investigating whether and how novice elementary and middle school teachers develop pedagogical content knowledge (PCK) through teaching on their own. We focussed on teachers' PCK as one type of content-specific knowledge that is particularly vital for high-quality mathematics instruction and students' mathematics learning (Baumert et al. 2010; Kersting, Sotelo, and Stigler 2010), compared with other general pedagogical knowledge, such as classroom management skills (Kane et al. 2013). We examined learning among novice teachers because learning is more pronounced among teachers in the early years of their teaching career (Kini and Podolsky 2016), which allows us to delineate what can be learnt through teaching more precisely.

Conceptualising teacher learning through teaching

Within the teaching profession, different on-the-job learning opportunities are available, including formal learning opportunities, such as professional development, and informal learning opportunities, such as learning with colleagues and learning on one's own. We focussed on learning through teaching, which we define as the change in teachers' PCK through interactions with students and the curriculum materials around the content without systematic external support for that learning, such as PD programmes or structured mentoring (Kyndt et al. 2016).

Our focus on teachers' development of PCK is both theoretical and empirical. Scholars have underscored the importance of PCK as a necessary domain of knowledge for teaching (Ball, Thames, and Phelps 2008; Copur-Gencturk and Tolar 2022, Shulman 1986). Empirical work has supported its importance by documenting its instrumental role in quality teaching and student learning (Baumert et al. 2010; Copur-Gencturk 2015). Our conceptualisation of PCK focuses on the components that are recognised across national and cross-cultural studies (Copur-Gencturk and Tolar 2022; Schmidt et al. 2007; Tatto et al. 2008) and other subjects (Jordan, Bratsch-Hines, and Vernon-Feagans 2018). Pedagogical content knowledge includes teachers' understanding of content-related issues around students'

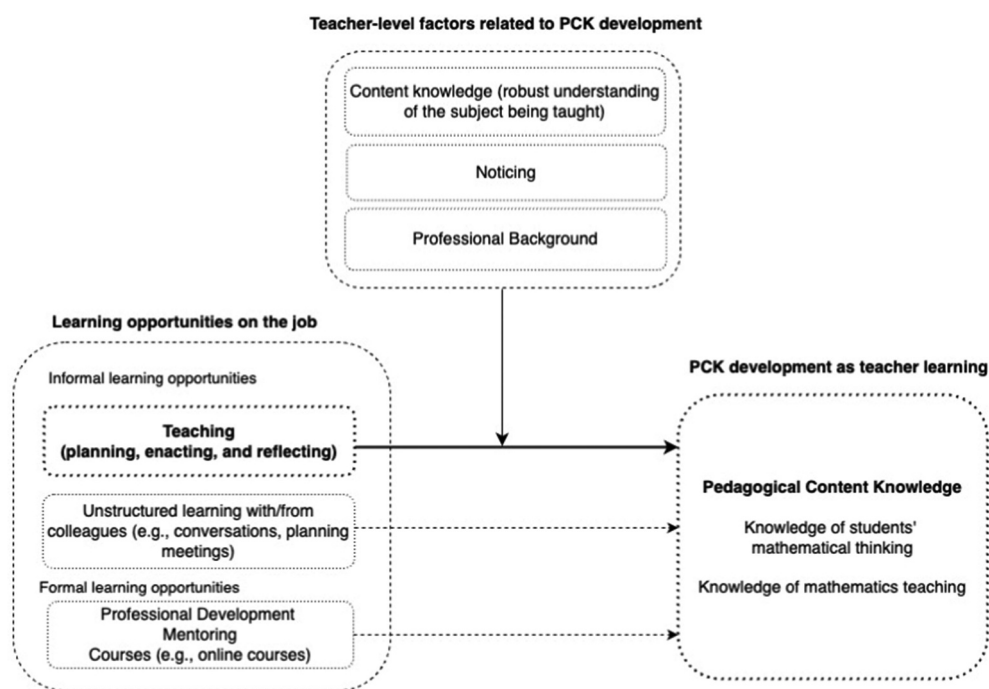


Figure 1. A visual conceptual framework.

learning, such as knowing students' common understanding of certain mathematical concepts and being able to gauge students' understanding based on their responses (Ball, Thames, and Phelps 2008; Copur-Gencturk and Tolar 2022; Tatto et al. 2008). It also encompasses the knowledge and understanding of the affordances and limitations of different representations and tools in fostering students' learning of a particular concept (Copur-Gencturk and Tolar 2022; Tatto et al. 2008). Figure 1 is a visual framework describing our conceptualisation of teachers' PCK development and informing our study design, with the solid line indicating the focus of this study.

Our rationale for anticipating that teaching can foster PCK development is that interactions between a teacher and students around the content theoretically create opportunities for teachers to learn. Specifically, during lesson planning, teachers may learn from curricular materials about instructional strategies and using different representations to solve mathematics problems (Davis and Krajcik 2005). While implementing a lesson, teachers could gain insights into students' mathematical understanding as well as the affordances and limitations of different representations for facilitating students' learning (Remillard and Bryans 2004). Reflection after class could enable teachers to evaluate their teaching practice by comparing their expected teaching outcomes with students' real learning outcomes. Any teacher can experience a discrepancy between expectations and reality, but prior studies have indicated that novices are more likely to experience this (Flores 2006; Mintz et al. 2020).

Prior work on teachers' development of PCK through teaching

Over the past few decades, scholars have devoted their attention to teachers' learning on the job (Kini and Podolsky 2016; Leikin and Zazkis 2010). The first line of research has

focussed on what and how teachers learn from teaching activities based on observations and interviews, primarily using qualitative methods with a small number of in-service or prospective teachers (Lloyd 2008; Remillard and Bryans 2004). These studies suggest novice teachers seemed to enhance their understanding of students' thinking and develop their knowledge of mathematics teaching by engaging in planning and enacting the curriculum, reflecting on students' responses and instructions, and making adaptations to the curriculum design and instruction (e.g. Collopy 2003). However, the changes observed in these studies could have been due to both teachers' self-learning through their teaching practice and other external learning supports the teachers were receiving concurrently.

The other line of research has measured the impact of teaching experience on student achievement gains (i.e. an indicator of teacher effectiveness), using longitudinal data with teacher/student fixed effects (Kini and Podolsky 2016). Kini and Podolsky's (2016) review found studies consistently demonstrated a positive correlation between teaching experience and teacher effectiveness, as reflected in student achievement. Research revealed teacher effectiveness increased the most in early teaching years (Papay and Kraft 2015) and accelerated with accumulated teaching experience in the same grade or subject (Blazar 2015; Ost 2014). The findings suggest teachers gained some content-specific expertise by teaching the same content. However, what expertise teachers gained through teaching that improves students' achievement remains underexplored. Understanding the specific knowledge teachers are able to learn through teaching on their own and the learning opportunities that facilitate it could inform optimal supports for novice teachers.

Knowledge and skills related to the development of PCK

One widely accepted condition for developing PCK is teachers' content knowledge (CK). Content knowledge is the conceptual understanding of school mathematics and the capacity to reason and evaluate different mathematical concepts and situations and solve mathematics problems in school curricula (Copur-Gencturk and Tolar 2022; Tröbst et al. 2018). Prior studies have noted a positive correlation between teachers' CK and PCK (Copur-Gencturk et al. 2019; Copur-Gencturk and Tolar 2022; Kleickmann et al. 2013). In a randomised controlled trial, Tröbst et al. (2018) observed significant PCK gains among prospective teachers following an intervention targeting their CK only. This suggests that teachers' understanding of mathematics might facilitate their PCK development, as it enables them to better understand students' mathematical reasoning and adapt teaching strategies accordingly. Further research is needed to understand how CK contributes to PCK development in unguided teaching scenarios over time.

Teachers' *noticing*, the act of attending to and interpreting classroom events (Sherin and Van Es 2009), plays a role in teacher learning through teaching. Classroom events, whether content-specific (e.g. students' explanations of work) or not (e.g. classroom climate), offer diverse learning resources. Prior research found teachers' noticing of those content-specific classroom events is positively related to their PCK (Copur-Gencturk and Tolar 2022; Franke et al. 2001). Continued attention to students' thinking and effective use of these observations in teaching help teachers develop an understanding of students' mathematical thinking, a key aspect of PCK, even long after completing professional development (Franke et al. 2001). These findings indicate that content-specific noticing may influence the development of PCK. Without external

support, the events teachers notice, and their interpretations of these events may provide varied learning resources, influencing the acquisition of knowledge and skills in teaching mathematics. Thus, further investigation is required to explore whether and how teachers' noticing of content-specific events in an unguided teaching setting help them gain PCK.

Finally, teachers' professional backgrounds, including their credentials and certification pathways, may influence their PCK development (Baumert et al. 2010; Hiebert, Berk, and Miller 2017). Hiebert et al. (2017) found that graduates from a traditional U.S. teacher education programme, which emphasised prospective teachers' skills to learn through teaching, continued to develop their PCK post-graduation. Additionally, teachers with a mathematics teaching credential display higher proficiency in both CK and PCK than do teachers with a general teaching credential (Baumert et al. 2010; Kleickmann et al. 2013).

Present study

This study focussed on the development of PCK and on measuring the PCK as the way teachers use it in teaching. We also considered the external supports provided to teachers, such as formal professional development and informal peer collaboration. By doing so, we were able to distinguish teachers' learning through teaching from the external supports available to teachers. Finally, we investigated how the growth of teachers' PCK was related to other teacher-level factors, such as their CK. Using data collected from more than 200 teachers in three consecutive years, we explored the following research questions:

- (1) To what extent do teachers gain PCK through teaching on their own over time?
- (2) To what extent are teachers' CK and content-specific noticing skills related to the growth in their PCK?
- (3) To what extent are teachers' professional backgrounds (certification path and credential type) related to the development of their PCK?

Methods

Sample

The data used in this study were collected for a multisite research project designed to investigate teachers' content-specific learning through teaching mathematics (Copur-Gencturk and Li 2023; Woods and Copur-Gencturk 2024). The study design and data collection procedures were reviewed and approved by the authors' Institutional Review Board before the study. To increase the generalisability of the study findings, teachers across the United States were invited via email¹ to take part in this study. The email included a brief description of the study and a link to the initial survey. This survey included a consent form that detailed the purpose of the study and the activities participants would be expected to complete as well as the confidential nature, benefits, and potential risks of the study. Data were collected through online surveys only from those who were eligible for the study (i.e. who were teaching mathematics and had less than 3 years of teaching experience at the beginning of the study) and who gave consent. Participation in the study was entirely voluntary, and participants could withdraw from the study at any time with no penalty. Table 1 shows most teachers in the analytic sample

Table 1. Background characteristics of teachers in the present study.

Teacher background characteristic	Sample (%)
Gender	
Female	84.1
Male	15.5
Ethnicity	
White	70.1
Black	8.2
Hispanic	9.7
Other (e.g. Asian, multiracial, and other)	12.1
Professional background	
Credential in mathematics	18.4
Credential in multiple subjects	69.6
Credential in other subjects (e.g. special education)	12.1
Route entering the profession	
Traditional certification ^a	72.0
Alternative certification ^b	28.0

N = 207.

^aTo obtain traditional certification, an individual must first earn a bachelor's degree and complete a teacher preparation programme before they can begin teaching.

^bAlternative certification allows individuals with a bachelor's degree to teach without necessarily having completed a formal teacher preparation programme prior to teaching.

were White (70.1%) and female (84.1%), which were close to national teacher demographics (NCES 2022). No significant differences were found between teachers who completed the 3-year study ($N = 155$) and those who did not ($N = 52$) in terms of race, $\chi^2(3, N = 207) = 1.03, p = .80$, gender, $\chi^2(2, N = 207) = 3.16, p = .21$, and initial PCK level, $t(207) = 0.51, p = 0.61$.

Measures and procedures

PCK measure

Teachers' PCK was measured by having them watch eight video clips of authentic mathematics instructions and respond to open-ended questions about the videos (Kersting 2008). These clips, each lasting between 2–3 minutes, focused on student-teacher interactions around fraction or ratio concepts in Grades 3–7 (i.e. the grade levels teachers were teaching during the study period). Teachers were given context for each video to understand the instructional content shown in the videos. They were then asked to analyse the mathematical understanding of students and provide suggestions to improve the teaching practices shown in the videos to increase students' mathematical understanding. We used the same measure each year. Given the long intervals between measure administrations (one academic year) and no answer keys provided, we believe changes in teachers' responses were not due to the opportunities to practise the tasks. Additionally, a related study (Copur-Gencturk and Orrill 2023) using a repeated measure with items similar to those in our study, shown that retaking the same items did not inflate teachers' scores.

Teachers' responses were evaluated using a 4-point rubric (see Table S1) which captured teachers' ability to analyse students' mathematical thinking (1 = No/incorrect analyses; 4 = Accurate analyses with evidence) and provide ways to improve the teachers'

mathematics teaching practices (1 = instructional strategy irrelevant to mathematical issues; 4 = At least one correct instructional strategy with a rationale). To reduce scoring bias, responses were coded by two raters unaware of the year responses were from. Strong agreement was reached between two raters (Cohen's kappa = 0.92). The measure demonstrated high reliability, with the Cronbach's alpha statistic ranging from 0.79 to 0.84 across the three years. The teachers' total score on the items for each year of administration indicated their PCK for that year (see Table S2 for descriptive statistics).

Time

To determine if and how much teachers' PCK changed during the study period, we created a time variable to denote each data collection point during the study. The variable was scaled from 0 to 2, where 0 marked the initial survey administration (i.e. baseline PCK) and 2 indicated the third survey administration. Thus, the variable reflected the number of academic years elapsed since the study began.

Content knowledge

Fourteen constructed-response items identified from prior literature (Izsák, Jacobson, and Bradshaw 2019; Van de Walle, Karp, and Bay-Williams 2019) were used to capture teachers' CK (for items and the rubric, see Copur-Gencturk and Ölmez 2022; Copur-Gencturk, Baek, and Doleck 2022; Copur-Gencturk and Doleck 2021). Two raters coded the responses to assess both the correctness and accuracy of reasoning in each response. The reliability (i.e. Cronbach's alpha) of this scale was 0.81. The average of the standardised item scores was used in the analyses (see Table S2 for descriptive statistics of the variable).

Content-specific noticing

At the beginning of the study, we captured teachers' content-specific noticing skills by having them watch four video clips of maths instruction and identify the most notable aspect they observed concerning students' mathematics learning and the teachers' instruction of the specific mathematics concept. We evaluated teachers' responses by using a 4-point rubric designed to capture what teachers noticed and how they interpreted it (see Copur-Gencturk and Rodrigues 2021 for rubric). Each video was coded independently, with at least two raters coding 12% of the data. A high Cohen's kappa statistic of .81 demonstrated strong agreement between raters. The reliability (i.e. Cronbach's alpha) of the scale was .66. We used the standardised scores around the mean to indicate teachers' overall noticing skills.

Formal learning opportunities

The formal learning opportunities included PD programmes and mentoring/induction programmes. Teachers were asked to report hours of formal support they had received on mathematics teaching and learning from their schools and districts, using items modified from prior studies (Copur-Gencturk, Plowman, and Bai 2019; Garet et al. 2016) (see Table S2 for descriptive statistics).

Other informal learning opportunities (peer collaboration)

Teachers were asked to report any activities involving both structured and unstructured discussions with colleagues around mathematics teaching and learning, using items derived from earlier research (Garet et al. 1999, 2016). These activities covered regular peer learning meetings with teachers of the same grade or collaborative learning on shared issues with colleagues. Similarly, we created a variable to indicate the intensity of the peer-learning support teachers received on a 3-point scale, according to prior literature (NCTQ 2022).

Teachers' professional background characteristics

We created two binary variables to represent teachers' certification pathways (i.e. alternative certification = 1; traditional certification = 0) and credentials (i.e. having a maths credential = 1; holding a credential in other subjects = 0). We also included a variable indicating years of teaching experience teachers had before participating in our study.

Analytic plan

We employed a linear growth modelling approach to explore growth patterns of teachers' PCK. Growth modelling efficiently manage missing data, allowing the inclusion of samples in the estimation if there is at least one data point for the outcome variable. We applied two-level growth modelling, where Level 1 parameters described PCK development trajectories for each teacher (i.e. years since the study began), and Level 2 parameters identified how the teacher-level factors might account for variations in their PCK growth. A random slope for the time variable was included to allow for variations in the growth patterns across individual teachers. A random intercept was included to allow for variations in teachers' initial level of PCK. See below model specification:

$$\text{Level 1 : } PCK_{it} = \beta_{0i} + \beta_{1i}Time_{it} + e_{it},$$

$$\text{Level 2 : } \beta_{0i} = \gamma_{00} + r_{0i},$$

$$\beta_{1i} = \gamma_{10} + r_{1i},$$

where PCK_{it} refers to the PCK score for teacher i at time t . The intercept, β_{0i} , denotes the initial level of PCK for teacher i when study began (i.e. at time 0), whereas β_{1i} reflects the average yearly increase in the PCK score for teacher i .

To consider the impact of formal or other informal support teachers received during the study period on their PCK growth, we added them as Level 1 covariates because they vary within and between teachers each year. See below model specification:

$$\text{Level 1 : } PCK_{it} = \beta_{0i} + \beta_{1i}Time_{it} + formalsupport_{it} \times [\beta_{2i} + \beta_{3i}Time_{it}] \\ + peersupport_{it} \times [\beta_{4i} + \beta_{5i}Time_{it}] + e_{it},$$

$$\text{Level 2 : } \beta_{0i} = \gamma_{00} + r_{0i},$$

$$\beta_{1i} = \gamma_{10} + r_{1i}.$$

$$\beta_{ji} = \gamma_{j0}, (jis \text{ not equal to } 0 \text{ or } 1) \quad (1)$$

To explore how teachers' CK, noticing skills, and professional background influence their PCK development, we included these time-invariant predictors in Level 2 model separately to estimate their relationship with teachers' initial PCK scores (β_{0i}) and the annual growth of their PCK (β_{1i}).

Results

As shown in Table 2, teachers significantly improved their PCK. Such a gain in PCK seemed to derive from teachers' learning through teaching the subject matter on their own, given that teachers' formal and informal support failed to predict their

Table 2. Estimates of the linear growth models for teachers' PCK.

	Model						
	1	2	3	4	5	6	7
Fixed effect							
Intercept	24.51*** (0.35)	24.63*** (0.37)	24.45*** (0.30)	24.40*** (0.34)	25.75*** (0.72)	24.68*** (0.40)	24.06*** (0.38)
Time (i.e. rate of change)	2.21*** (0.21)	2.09*** (0.34)	2.17*** (0.20)	2.22*** (0.21)	2.16*** (0.43)	2.27*** (0.24)	2.07*** (0.23)
Formal support		0.01 (0.67)					
Formal support $\times$ Time		0.22 (0.49)					
Informal support		-0.96 (0.56)					
Informal support $\times$ Time		0.42 (0.42)					
Effect on intercept							
Content knowledge			5.58*** (0.58)				
Content-specific noticing skills				1.52*** (0.32)			
Entry-level teaching experience					-1.45* (0.69)		
Alternative certification						-0.62 (0.85)	
Math teaching credential							2.42* (0.96)
Effect on slope							
Content knowledge			0.90* (0.36)				
Content-specific noticing skills				-0.15 (0.19)			
Entry-level teaching experience					0.06 (0.43)		
Alternative certification						-0.24 (0.48)	
Math teaching credential							0.70 (0.52)
Random effect (variance)							
Intercept	18.59*** (2.70)	18.91*** (2.72)	11.59*** (2.13)	16.36*** (2.61)	18.05*** (2.72)	18.51*** (2.75)	17.73*** (2.53)
Slope	3.07** (1.09)	3.16*** (1.07)	2.81** (1.09)	3.04** (1.09)	3.07** (1.09)	3.06** (1.09)	3.00** (1.11)
Intercept and slope	1.13 (1.39)	1.01 (1.39)	0.04 (1.25)	1.36 (1.34)	1.16 (1.41)	1.11 (1.39)	0.82 (1.34)
Residual	8.50*** (1.06)	8.20*** (1.04)	8.56*** (1.07)	8.50*** (1.06)	8.49*** (1.06)	8.49*** (1.06)	8.49*** (1.06)

* $p < .05$; ** $p < .01$; *** $p < .001$.

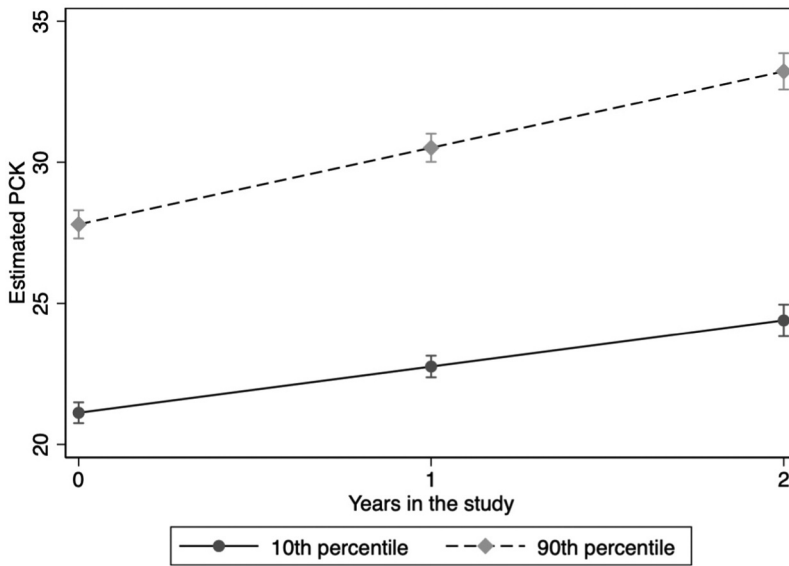


Figure 2. Growth of PCK among teachers with less and more robust mathematics content knowledge at the beginning of the study.

Note: The error bars indicate one standard deviation below and above the average.

PCK (see the results from Model 2). Indeed, the change in teachers' PCK was not associated with the level of formal professional or peer support teachers received ($p = .99$ for formal support and $p = .09$ for peer support). Our findings underscored the importance of CK in the development of PCK. Having a robust understanding of the mathematics being taught was associated with teachers' initial PCK level (an effect size of 0.62; $p < .001$) as well as the growth of their PCK per year (an effect size of 0.24 SD; $p = .013$). As shown in Figure 2, the difference in the growth rate between teachers with robust mathematical knowledge (i.e. 90th percentile) and those with less robust mathematical knowledge (i.e. 10th percentile) is 0.61 SD. Teachers' noticing skills were related to their initial PCK scores (an effect size of 0.35 SD; $p < .001$); however, their noticing skills were not related to the growth in their PCK ($p = .426$; see Model 4).

Lastly, having a mathematics teaching credential was related to teachers' initial level of PCK (see Model 7). Specifically, teachers with a mathematics teaching credential had, on average, a 0.56 SD higher PCK score at the beginning of the study ($p = .012$); however, their rate of PCK development was not statistically different from the rate of those without a mathematics teaching credential ($p = .18$).

Robustness check

To check whether teachers with certain backgrounds received more formal or informal support, which further confounded the role of the support in their PCK growth, we reran the analysis with teachers' professional background indicators included. Still, neither

formal support nor informal support was related to teachers' PCK. We also conducted an additional analysis in which we took into account of the quality of formal and informal support teachers received (for approach, see Copur-Gencturk, Plowman, and Bai 2019). We reran the analysis by the amount of the emphasis given to the practices that were likely to produce changes in teachers' knowledge and instruction based on prior work (Linda, Hyler, and Gardner 2017). The results were similar. Neither formal support nor peer support was relevant to teachers' initial level of PCK ($p = 0.15$ for formal support; $p = 0.34$ for peer support) or PCK growth rate ($p = 0.32$ for formal support; $p = 0.77$ for peer support). These results provide further evidence that the development of PCK observed in our study could mainly be attributed to teaching practice.

Discussion

This study examined the extent to which teachers developed PCK of mathematics through teaching, a type of content-specific knowledge that has been significantly linked to the quality of instruction and students' learning of mathematics (Baumert et al. 2010; Copur-Gencturk 2015). Before we discuss the study findings, we acknowledge the limitations of our study. First, we collected data from a national sample of novice mathematics teachers, but this sample was not nationally representative. Related to this issue, our sample of teachers, all of whom volunteered to participate in the study, might be different from typical novices. Prior work has shown that novice teachers often focus more on classroom management than on teaching and student learning (Berliner 1988), especially in the challenging first year, which often involves many 'reality shocks' (Mintz et al. 2020). Yet the teachers in our study increased their PCK of mathematics, possibly due to the pressure from state-mandated tests that forced them to pay more attention to teaching and learning mathematics. Future studies with novices teaching different subjects and grade levels would provide more insight related to this issue. Second, we did not capture qualitative differences in the professional development opportunities provided to the teachers. Further research is needed to investigate which features of professional support are more effective than others in enhancing novice teachers' continuous learning through teaching. Finally, our study explored only the role of teacher-level factors (e.g. CK, noticing) in teachers' PCK development, leaving many contextual factors unexamined. Future studies could investigate how contextual factors, such as the school environment and administrator support, might facilitate or hinder teachers' learning through their teaching practice.

Our findings indicated that novice mathematics teachers were able to develop PCK of mathematics from teaching, which was robust even after accounting for other learning supports concurrently available to teachers. Prior research has demonstrated that teachers' teaching practice offers rich learning resources for their professional growth (e.g. Leikin and Zazkis 2010; Lloyd 2008). Our study provides additional empirical evidence based on large-scale longitudinal data. The results have implications for research and practice on teachers' learning through teaching. First, given that teaching is teachers' daily work task and that teachers seem to learn from teaching on their own, teacher preparation and professional development programmes should shape the curricula around how they might utilise the task of teaching to enhance teachers' knowledge and skills. For instance, a promising way for professional development programmes to

help teachers learn from their own teaching is by using video clips of teachers' own teaching (e.g. Sherin and Van Es 2009). Additionally, school leaders and policy makers might provide teachers with more time to explore essential aspects of teaching (e.g. reflecting on their own teaching practice), either alone or with peers, ensuring they have sufficient time to enhance knowledge through teaching practice.

We also found that teachers' CK was crucial in their PCK development. Teachers with strong initial level of CK developed PCK through teaching at a faster pace than did their peers with limited CK. This result is not surprising, given that teachers with a robust understanding of school mathematics would be able to analyse their students' mathematical thinking and thus learn from them. Similarly, teachers with a strong understanding of the content being taught could analyse instructional practices and their choice of resources, reflect on the appropriateness of those practices and resources in making the content accessible to their students, and learn from this experience. Teacher education and professional development programmes should devote more time to unpacking the mathematics taught in school so that teachers could develop an understanding of the foundational ideas behind the mathematics taught across grade levels and the conceptual underpinning of the rules and procedures (Copur-Gencturk 2021; Copur-Gencturk and Tolar 2022). Curricular materials, such as the teachers' guide, could provide more conceptual explanations of the content in addition to the pedagogical content to facilitate teachers' understanding of the concepts they need to teach.

In line with prior literature, teachers' noticing skills were related to their PCK (Copur-Gencturk and Tolar 2022). However, our findings also indicated that noticing was not associated with the development of teachers' PCK. Prior work by Franke et al. (2001) showed that noticing played a role in the development of teachers' PCK, but only when teachers consciously viewed the noticing of students' mathematical thinking and their own instruction as learning resources and leveraged what they noticed in the classroom. Thus, noticing alone may not lead to gains in teachers' PCK unless they also consciously reflect on what they notice in class and transform those fleeting moments into action in their practice.

Conclusions

Teaching is a major component of teachers' daily activities; therefore, understanding whether and under which conditions learning occurs through teaching is vital for identifying a mechanism for teachers to continuously improve their capacity. This is particularly crucial for novice teachers, who often need more opportunities to enhance their knowledge and skills. We have documented that novice mathematics teachers generally improved their PCK of mathematics through their teaching practice and that teachers' CK was essential for the development of their PCK. Our findings imply that cultivating a school environment that provides time and support for teachers to focus on often overlooked components of teaching, such as reflecting, could be a cost-effective way for novice teachers to grow professionally. Additionally, teacher preparation and professional development programmes should provide more opportunities for teachers to enhance their understanding of the mathematical concepts taught in school and to equip teachers with the skill to learn from the work of teaching on their own.

Note

1. Teachers were contacted either by the research team through email addresses we obtained from an education research company or by our district or educational organisations on our behalf.

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