Assessment of Direct Position Estimation Performance in Multipath Channels

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BIOGRAPHY

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ABSTRACT

Multipath signal is the satellite signal that is delayed due to the reflection or diffraction off various obstacles or surfaces and thus endangers the leverage of signal under Line-of-Sight model. Multipath error is one of the most important error sources in Global navigation satellite system and many advanced tracking, filtering and classifying techniques are developed in past decades to better understand and mitigate the multipath effect. Direct Position Estimation (DPE), which is a novel positioning technique, has its potential to resist the multipath due to its early combination of the satellite information and operation in the navigation domain without estimating intermediate variables. This paper analyzes and assesses the DPE positioning performance under multipath environments. The multipath error envelope, as an important tool to understand the multipath error functioning, is computed and plotted to appeal the relation between the multipath error and positioning error. The multipath channel model, developed by the German Aerospace Center (DLR) in the ITU-R P.681 recommendation, is used to simulate realistic multipath scenarios and evaluate the DPE positioning performance. It is shown that in all simulations of multipath environments, the DPE approach outperforms the traditional two-step method in terms of precision.

I. INTRODUCTION

Global navigation satellite system (GNSS) provides positioning, navigation and timing solutions by utilizing transmitted signals from satellites. The traditional method of GNSS positioning is usually recognized as a two-step (2SP) method since the GNSS observables, such as pseudorange, pseudorange rate and carrier phase, are first estimated by the receiver during acquisition and tracking stages. Then those observables will be employed by weighted least square (WLS) or Kalman Filter (KF) to solve the unknown position, velocity and time (PVT). With the growing demand for high accuracy and high precision, many techniques have been developed in past decades for hardware (e.g. receiver design) and software (e.g. position algorithm) to improve positioning performance [Kaplan and Hegarty, 2017, Hofmann-Wellenhof et al., 2007, Borre et al., 2007, Groves, 2013, Tsui, 2005]. Among those developments, mitigating the influence of a variety of error sources, such as atmosphere error, multipath propagation and interference, is one of the most intriguing techniques.

Many approaches have been proposed in order to mitigate those effects on standard tracking loops in the first stage of the 2SP method, as depicted in Fig. 1 [Li et al., 2022]. The existing multipath mitigation studies mainly focus on: 1) enhanced discriminator functions, such that the delay/phase locked loops (DLL/PLL) are less sensitive to multipath effects. For instance, the Narrow Correlator [Van Dierendonck et al., 1992], the Pulse Aperture Correlator (PAC) [Jones et al., 2004], the Double

Delta Correlator [McGraw, 1999], the Early1/ Early2 (E1/E2) tracking technique [Van Dierendonck, 1997] or the Multipath Elimination Technology (MET) [Townsend and Fenton, 1994] are classical discriminator designs in this area; or 2) advanced receiver architectures that jointly estimate the line-of-sight and multipath components for each satellite link, in a usually computationally complex process. For instance, the Multipath Estimating Delay Lock Loop (MEDLL) [Van Nee et al., 1994], the Vision Correlator [Fenton and Jones, 2005], the Multipath Mitigation Technique (MMT) [Weill, 2002], or the use of particle filtering to discriminate among propagation paths [Closas et al., 2009a]. These methods yield to robust correlation and tracking results, however, it is worth noting that they heavily rely on accurate physics-based models and deviations from those models might cause dramatic degradation in performance. Besides, with the popularity of machine learning (ML) algorithms in the GNSS community, varieties of data-driven approaches have emerged recently, especially for estimation and classification tasks. For example, a learning algorithm based on support vector machine (SVM) [Hsu, 2017] and a convolutional neural network (CNN) [Suzuki et al., 2020] are designed for multipath detection. A decision-trees learning-based algorithm is developed for line-of-sight (LOS) and multipath signal classification [Guermah et al., 2018]. A long short-term memory (LSTM) neural network (NN) is trained for GNSS measurement uncertainty prediction under multipath effect. A feed-forward multilayer perceptron (MLP) is designed for code phase estimation in multipath environments [Orabi et al., 2020] and a deep neural network (DNN) is trained to perform correlation and disentangle multipath signals from LOS signals [Li et al., 2022].

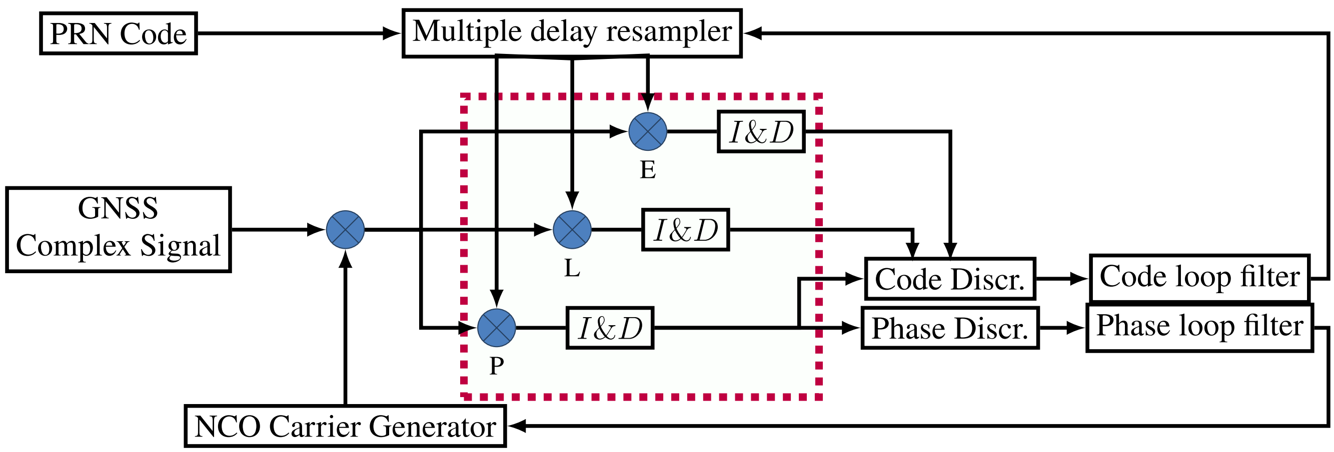


Figure 1: Diagram of a code/carrier tracking loop scheme with standard correlation method using multipliers and Integrate and Dump (I&D) operators[Li et al., 2022].

Recently, the interest in direct position estimation (DPE) arises rapidly since the high-sensitivity (HS) property of the DPE approach enables better positioning performance in harsh environments. Unlike the traditional 2SP method, DPE solves for the PVT from the raw satellite signal without estimating intermediate quantities, such as pseudorange and carrier phase. In the GNSS context, DPE was first proposed in [Closas et al., 2007], and recently revisited in [Closas and Gusi-Amigo, 2017] and [Morton et al., 2021, Ch. 21]. From both theoretical analysis [Closas et al., 2009b, Gusi-Amigó et al., 2014, Gusi-Amigó et al., 2018] and practical experiments [Dampf et al., 2019, Tang et al., 2023], it has been shown that the DPE approach has a better performance than the traditional method in terms of precision and robustness under unfavorable environments.

In general, the aforementioned discriminator and lock loop filter techniques are unavailable for the DPE approach, considering that the tracking stage is excluded and the intermediate observable estimation is unnecessary in DPE. Thanks to the ability of fusing the satellite signal in an early stage and operating in the navigation domain, the DPE approach naturally has its potential to enhance positioning performance under multipath environments. It has been shown that in real urban [Vicenzo et al., 2023] or dense multipath scenarios [Bialer et al., 2012], the DPE achieves more precise positioning results compared to the 2SP approaches. This paper will focus on explaining the multipath tolerance of the DPE approach from jointly combining the satellite information at an earlier stage. We then provide the particular multipath error envelope (MEE) of the given user location and satellite positions in DPE context. The MEEs are computed based on parametric single multipath signal and compared with the ones of 2SP approach. We also construct the multipath scenario using the Land Mobile Satellite Channel Model (LMSCM) in the ITU-R P.681 recommendation [ITU-R P.2145-2, 2017] developed by the German Aerospace Center (DLR) [Steingass Alexander and Lehner Andreas , 2019]. This setup provides a controlled environment for assessing the performance of the DPE approach under multipath influence.

The rest of the paper is organized as following:

- Section II explains how DPE achieves multipath mitigation by jointly utilizing the satellite information.
- Section III implements the DPE approach and compares its positioning performance with 2SP method under controlled

multipath environments. It provides the positioning performance assessment, including the parameterized MEEs and position estimation under realistic multipath channel generated by LMSCM.

- Section IV concludes the multipath tolerance ability of the DPE approach based on the previous assessment of the positioning performance.

II. DPE AND ITS MULTIPATH TOLERANCE

In DPE literature, the signal model is the same with the one we usually defined in 2SP context [Kaplan and Hegarty, 2017], which is parameterized by the the observables of each satellite, such as the delay τ_i and Doppler-shift $f_{d,i}$. However, those observables are further considered as the functions of the variables of interest, such as the position $\boldsymbol{\theta}$, velocity $\boldsymbol{v}$, clock offset δt, and clock drift $\dot{\delta t}$ of the receiver. Concatenating all variables of interest together in a vector, e.g., $\boldsymbol{\kappa} = (\boldsymbol{\theta}^\top, \delta t, \boldsymbol{v}^\top, \dot{\delta t})^\top$, the delay of GNSS signal propagation is given by

$$\tau_i(\boldsymbol{\kappa}) = \frac{1}{c}||\boldsymbol{\theta} - \boldsymbol{r}_i|| + (\delta t - \delta t_i), \tag{1}$$

where c is speed of light, $\boldsymbol{\theta} = (\theta_x, \theta_y, \theta_z)^\top$ denotes the 3-dimension position of the receiver, $\boldsymbol{r}_i$ denotes the position of the i-th satellite and $||\cdot||$ denotes the ℓ_2-norm of the vector. δt and δt_i denote the unsolved receiver clock bias and the i-th satellite clock bias with respect to GNSS time. Doppler shift is given by

$$f_{d,i}(\boldsymbol{\kappa}) = -(\boldsymbol{v}_i - \boldsymbol{v})^\top \boldsymbol{u}_i(\boldsymbol{\theta})\frac{f_c}{c} + \dot{\delta t}_i - \dot{\delta t}, \tag{2}$$

where $\boldsymbol{v}_i$ denotes the velocity of the i-th satellite, $\boldsymbol{v}$ denotes the velocity of the receiver, $\dot{\delta t}_i$ is the satellite clock shift and $\dot{\delta t}$ is the receiver clock shift. $\boldsymbol{u}_i = \frac{\boldsymbol{r}_i - \boldsymbol{\theta}}{||\boldsymbol{r}_i - \boldsymbol{\theta}||}$ denotes the unit vector between the receiver and the i-th satellite. f_c denotes the carrier frequency of the satellite signal. The baseband received signal model is then given by [Closas and Gusi-Amigo, 2017]

$$x(t) = \sum_{i=1}^{M} a_i s_i(t - \tau_i(\boldsymbol{\kappa})) \exp(j2\pi f_{d,i}(\boldsymbol{\kappa})t) + n(t), \tag{3}$$

where the subindex $i \in \{1, 2, \ldots, M\}$ denotes each satellite and M is the number of satellites the receiver has in view. $s(t)$ denotes the navigation signal spread through PRN code. $\boldsymbol{a} = [a_1, \ldots, a_M]$ is the complex amplitude and $n(t)$ denotes a complex additive white Gaussian noise (AWGN).

Assuming the sampling rate of the receiver is $f_s = 1/T_s$, the discrete received signal at the receiver is $x_k = x(kT_s)$. Considering that K samples are available in a given observation window, the samples form the snapshot vector $\boldsymbol{x} = [x_1, x_2, \ldots, x_K]^\top \in \mathbb{C}^{K\times 1}$. The maximum likelihood estimation of $\boldsymbol{\kappa}$ with the above model leads to a solution which has the similar form with the cross ambiguity function (CAF) in 2SP approach. The maximum likelihood estimator in DPE context is given by [Morton et al., 2021, Ch. 21]

$$\hat{\boldsymbol{\kappa}} = \arg\max_{\boldsymbol{\kappa}} \underbrace{\left\{ \sum_{i=1}^{M} |\boldsymbol{x}^H \boldsymbol{c}_i(\boldsymbol{\kappa})|^2 \right\}}_{\Lambda_{\text{DPE}}(\boldsymbol{a},\boldsymbol{\kappa})}, \tag{4}$$

where $\boldsymbol{c}_i$ is the local replica sequence of K samples

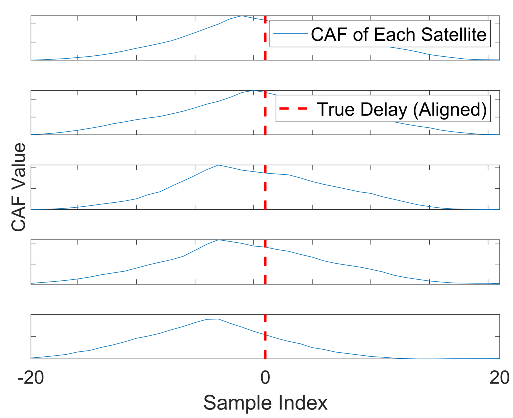
$$\boldsymbol{c}_i = \begin{bmatrix} s_i(T_s - \tau_i(\boldsymbol{\kappa})) \exp(-j2\pi f_{d,i}(\boldsymbol{\kappa})T_s) \\ s_i(2T_s - \tau_i(\boldsymbol{\kappa})) \exp(-j2\pi f_{d,i}(\boldsymbol{\kappa})2T_s) \\ \vdots \\ s_i(KT_s - \tau_i(\boldsymbol{\kappa})) \exp(-j2\pi f_{d,i}(\boldsymbol{\kappa})KT_s) \end{bmatrix}. \tag{5}$$

The estimator can be interpreted as looking for the variable $\boldsymbol{\kappa}$ to optimize the cost function, which is the combination of CAFs from the M satellites. The resulting optimization problem (4) comes with several challenges, such as the non-convexity of the cost function, a multivariate estimation and the goal of finding the solution in a finite time. Due to these issues, grid-based search approaches or gradient-like methods might not be suitable for the task [Closas et al., 2009a]. In this paper, the Accelerated Random Search (ARS) [Appel et al., 2004] is employed to implement DPE solutions in the simulations. How ARS works for DPE is illustrated in Fig. 4 in Section III.

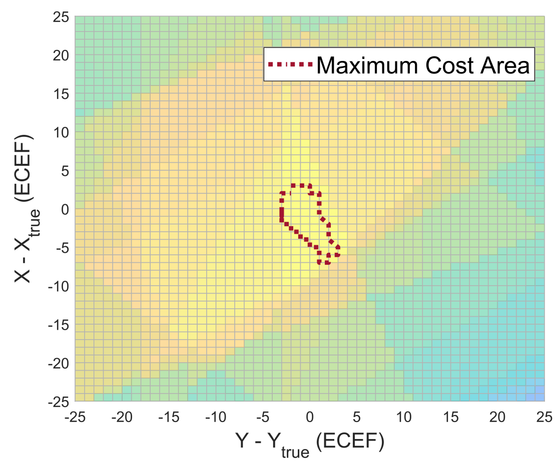
It was shown that the DPE approach outperforms the 2SP approach [Closas et al., 2009b]. Additionally, the derivation of theoretical estimation bounds show that the DPE approach features a lower variance of position estimation error, e.g. CRB[Gusi-Amigó et al., 2014] and Ziv-Zakai bound (ZZB) [Gusi-Amigó et al., 2018].

[Bialer et al., 2012, Vicenzo et al., 2023] state that the DPE has advantages in multipath tolerance compared with 2SP. The multipath on 2SP disturbs the tracking stage and produces errors in the delay/pseudorange estimation. Those contaminated measurements from all satellites will be jointly used for position estimation. However, in DPE, the information of each satellite is combined early by computing the cost function with respect to parameter candidates (e.g. the position of the receiver) as a summation of the CAFs of satellites. Considering the fact that CAF in DPE is a function of position, the CAFs of LOS signal from different satellites reach their maximum at the same position. As a result, the summation of CAFs accumulates their amplitudes of peaks. On the other hand, the CAFs of multipath signal from different satellites maximize at different positions. That says, the DPE is increasing Signal to Multipath Ratio (SMR), reducing multipath influence on positioning performance.

The following figures demonstrate the deviation of estimation caused by multipath in 2SP and DPE. The 2SP tracking correlation and DPE direct correlation result based on the same multipath-contaminated signal sampled at 20.46MHz are plotted in Fig. 2. Fig. 2a shows the CAFs computed from 5 detected satellites. For sake of convenience, their true delays are aligned at the same position in the figure to compare. One can find that the peak of each CAF deviates 1-5 samples away from the true delay, which may bring $15 - 75$m error for pseudorange measurement. On the other side, Fig. 2b shows the summation of the CAFs of 5 satellites as a function of the position as defined in Eq. (4). Under the multipath influence, the peak of the cost becomes flat, but only gives an deviation of around 10m. This happens because the CAF caused by multipath from each satellite usually locates at different position. Only the amplitude of CAFs brought by LOS signal can be accumulated in the cost function.



(a) Delay estimation errors of 5 satellites according to the peaks of CAF in 2SP approach. The true delays are aligned at the same position for convenient comparison.



(b) Position estimation error according to the peak of the cost function in DPE. Varying colors denotes different cost value. The axes represent the X and Y error in ECEF coordinates. The origin denotes the true position.

Figure 2: CAFs in the 2SP tracking stage and cost function in the DPE maximum likelihood estimator based on a multipath-contaminated signal.

III. SIMULATIONS AND EXPERIMENTS

1. Simulation Settings

The simulations in this section include the exploration of the MEE in DPE context and the implementation of DPE in a controlled realistic multipath environment. The extent of multipath interference can be different in each scenario according to amplitudes, number and errors of the multipath signals. The tracking discriminator performance in 2SP also depends on the urban density or receiver dynamics. However, the basis of the simulation is set as following. The user receives the LOS and multipath signal from at most 8 satellites, which are GPS $\{8, 10, 11, 16, 23, 25, 26, 27\}$ from L1 band, at 20.46MHz sampling rate. Since the influence of positioning performance by the multipath depends on the geometry of the satellites, the satellite sky plot is shown in Fig. 3.

The positioning and tracking results from the 2SP were provided as a benchmark to compare with the DPE approach. The

GNSS-SDR [Fernandez-Prades et al., 2011, Pany et al., 2024] were employed to implement 2SP method from I&Q sample level. The only adjustment was that the satellite information was provided instead of decoding the navigation bits. It is noted that the GNSS-SDR is using weighted least square (WLS) method to solve the position from the delay measurement. Besides, since the GNSS-SDR only ensures the difference of the delays from the satellites but not the absolute value, the height estimation will be biased. To ensure a fair comparison, the following positioning errors in both 2SP and DPE are computed based on 2-dimensional position at north and east axes in East, North, Up coordinates.

As mentioned before, the optimization problem of DPE (4) could be time-consuming due to its multivariate searching and difficulty to computing the gradient of the cost function. We utilized a stochastic optimization algorithms, namely ARS, to speed up the searching process of the solution. Fig. 4 visualizes a searching path of ARS searching process in 2-dimension, where X_{true} and Y_{true} represents the true position of the receiver in ECEF coordinate and $\{X, Y\}$ are the variables we want to estimate. The y-axis represents the cost value computed from Eq. (4). It is shown that the ARS is able to optimize the cost function in a few searching steps.

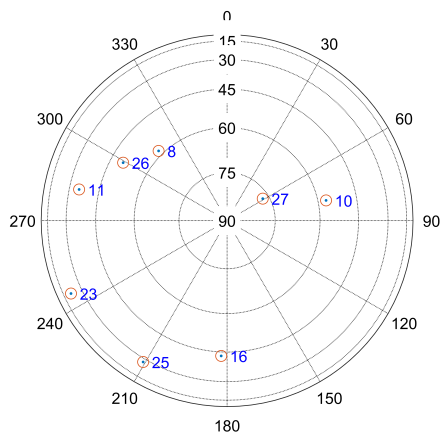


Figure 3: Sky plot of 8 available satellites in the simulations.

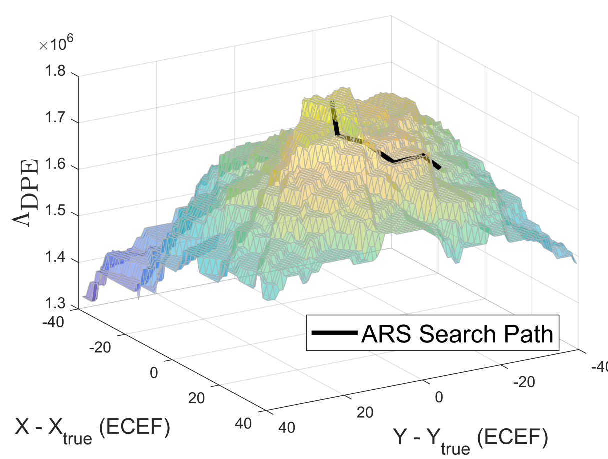


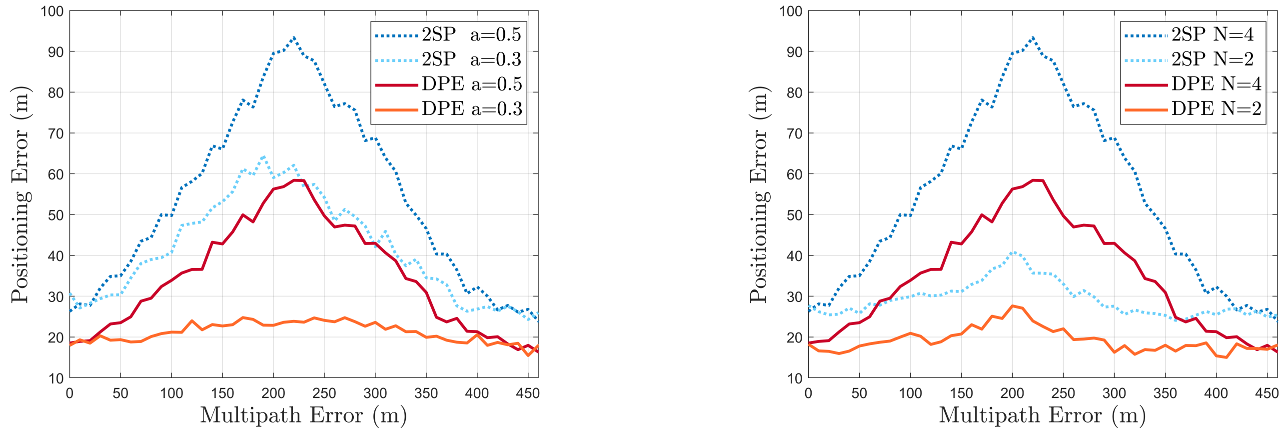
Figure 4: 2-dimensional ARS search path on cost function. The origin is the true position.

2. Multipath Error Envelope

The traditional MEE usually appeals the range/delay error caused by the multipath error for one satellite. Typically, when multipath error is so large that the CAF peak of multipath signal deviates from the one of the LOS signal by more than 0.5 chip, the multipath error will have no influence to discriminator. However, since the DPE approach skips the tracking stage, our MEE reflects the positioning error caused by multipath error on multiple satellites. Fig. 5 plots two sets of MEEs when parameters of multipath signal varies. For each parameter setting, a 6 sec I&Q record of 8 satellites was sampled from a static receiver. For each one millisecond, we implemented non-coherent 2SP and DPE approach, which gave 6000 solutions for every record. Each data point on Fig. 5 represents a root-mean square error (RMSE) computed from those 6000 solutions.

Fig. 5a compares the MEEs of DPE and 2SP when real-valued amplitude a of the multipath signal changes. The amplitude of the LOS signal is always $a = 1$. Notice that there are 4 out of 8 satellites are set to suffer from multipath interference. Since the number of the satellites which produces the multipath propagation will influence the positioning performance, we also demonstrates the MEEs according to the number of multipath-suffering satellites in Fig. 5b, where the amplitude of the multipath signal is fixed at $a = 0.5$.

It is shown that under the aforementioned simulation environment, the estimation of the position reaches its maximum error when multipath error is around $200 - 250$m. However, the DPE approach achieves more precise positioning than the 2SP does and the maximum positioning error of DPE is much less than the one of the 2SP under all scenarios. Despite the fact that the increasing amplitude of the multipath and the increasing number of satellites which experiences multipath effect may degrade the positioning performance, the DPE approach always outperforms the 2SP method in terms of the precision. Particularly, when the multipath interference is not too huge, e.g. amplitude $a = 0.3$ or only 2 out of 8 satellites have multipath propagation, the DPE approach is almost able to keep its precision as zero multipath error.



(a) MEEs under the varying amplitude of NLOS signal. The amplitude of the LOS signal is always 1.

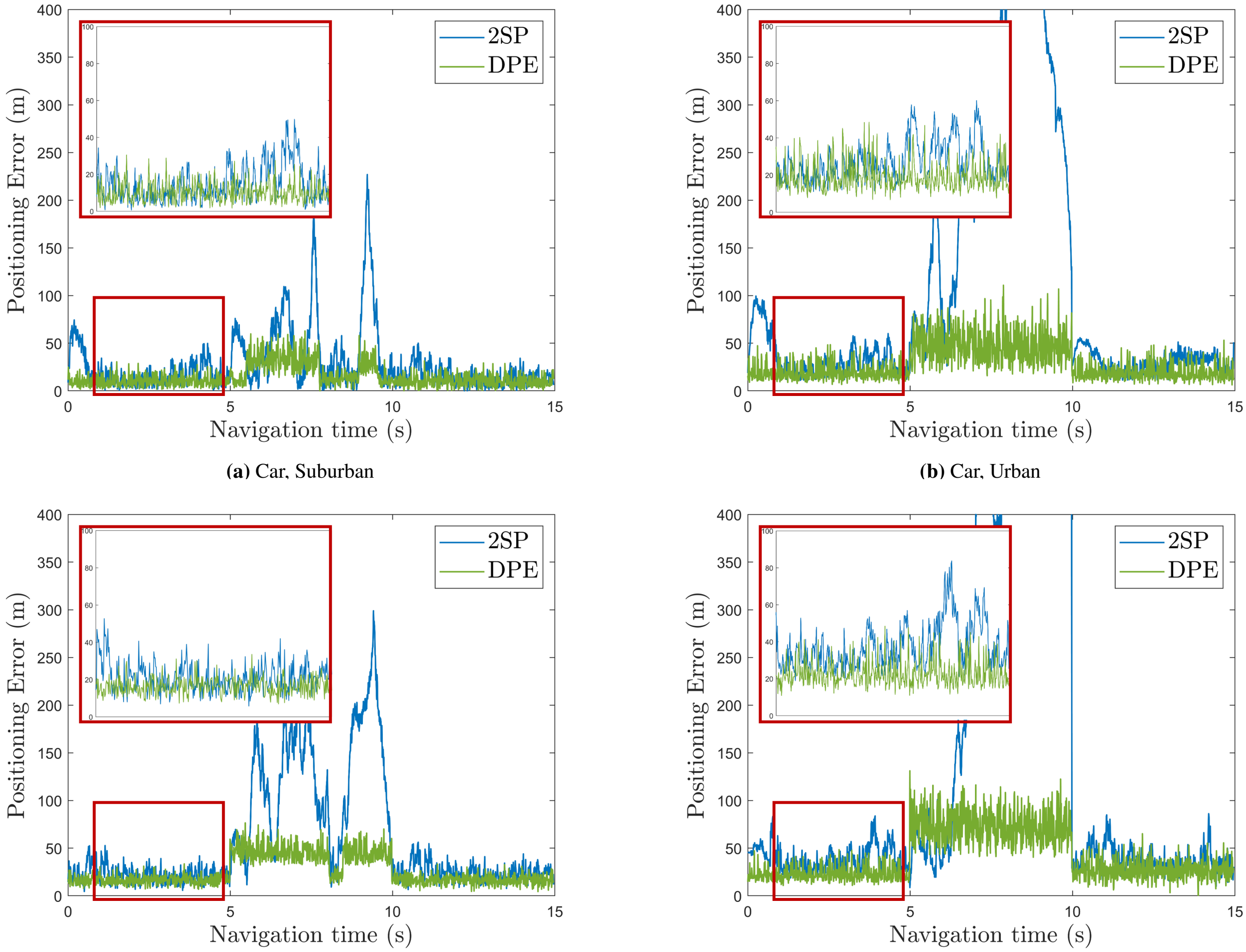
(b) MEEs under the varying number of multipath-suffering satellites. There are 8 satellite in total.

Figure 5: Multipath error envelopes in DPE and 2SP: positioning error vs multipath error.

3. LMSCM Simulation

In this section, we provide the assessment of DPE positioning performance in a more realistic multipath environment created by LMSCM. The so-called LMSCM is developed by the German Aerospace Center (DLR) under the ITU-R P.681 recommendation [ITU-R P.2145-2, 2017] to investigate the land mobile satellite navigation multipath channel. The model takes deterministic effects and statistical distributions from the measurement [Steingass Alexander and Lehner Andreas , 2019]. With the real data sampled around Munich, Germany, it is able to simulate shadowed, blocked and reflected path signal in a practical environment. Besides, the scenery parameters and the statistics can be adjusted to satisfy the particular applications, such as the urban, suburban or rural scenarios for car or pedestrian users. The parameters include the height of the buildings, height of the antenna, distance to the middle of the road, etc. Here, we categorize them into 4 scenarios, which are car-in-urban, car-in-suburban, pedestrian-in-urban and pedestrian-in-suburban scenarios. For each scenarios, 15 sec I&Q samples are recorded. Both 2SP and DPE were implemented with 1ms integration period. It is noted that the maximum velocities of car and pedestrian are set to be 50km/h and 5km/h. Due to these dynamics of the receiver, the LMSCM sampling rate is set to be 100Hz, which means the multipath delay and amplitude were continuously changing for every 10ms. Thus the navigation solutions were also computed at rate 100Hz. To demonstrates the possible NLOS effect on 2SP and DPE, which means only reflected signal without LOS is received, the LOS signal of satellite GPS $\{8, 10, 11\}$ were removed during $5.5 - 9.5$ sec. Fig. 6 shows the positioning error during the navigation tour under 4 scenarios. To improve the readability, the 0.8 and 0.5 quantile position errors of the normal multipath and 3-satellite LOS removal under each scenario are summarized in Tab. 1 and 2.

In all scenarios and cases, the DPE approach outperforms 2SP in terms of the precision. The DPE approach improves the positioning performance more in the urban scenario than it does in the suburban scenario. When the multipath sources become more complex in urban scenario, the delay estimation of traditional 2SP is more likely to deviate from the true value. Similarly, the enhancement of a car from the DPE approach is more than the one for a pedestrian. This may be because the high dynamics of the receiver enables the fast change of the multipath signal and thus make the tracking more difficult. It is interesting that when LOS signals of 3 satellite were removed from the received signal, the 2SP lost the tracking of the satellite very soon, but still used the wrong information of those 3 satellites to solve the position. This gives the bad positioning result, especially in urban scenario when the multipath source takes a nontrivial part in the received signal. However, the DPE approach can still help improve the positioning performance in this case, since the extra NLOS signals will only increase the noise floor of the cost function, but not provide the wrong information.



(a) Car, Suburban

(b) Car, Urban

(c) Pedestrian, Suburban

(d) Pedestrian, Urban

Figure 6: The positioning error of 2SP and DPE under 4 scenarios simulated by LMSCM during 15s navigation recording. The signal is contaminated by multipath all the time and 3 satellites become only NLOS in the middle of the tour.

Scenario	0.8 Quantile Error (Multipath)		0.8 Quantile Error (3 Sat NLOS)	
	2SP	DPE	2SP	DPE
Car, Suburban	27.5476	14.4974	85.9248	37.1772
Car, Urban	49.3010	28.9873	502.9913	61.5329
Pedestrian, Suburban	33.2774	22.2475	187.1261	52.5042
Pedestrian, Urban	52.7748	35.3397	672.2483	88.4740

Table 1: Comparison of 2SP and DPE Methods (0.8 Quantile Error)

Scenario	0.5 Quantile Error (Multipath)		0.5 Quantile Error (3 Sat NLOS)	
	2SP	DPE	2SP	DPE
Car, Suburban	14.6566	9.3712	36.2984	26.5652
Car, Urban	30.4551	17.4800	283.8466	45.0629
Pedestrian, Suburban	22.2874	16.8982	133.3272	42.9657
Pedestrian, Urban	37.0379	23.8987	431.0419	71.3826

Table 2: Comparison of 2SP and DPE Methods (0.5 Quantile Error)

IV. CONCLUSION

The DPE approach naturally has its potential to resist the multipath interference, because it jointly employs the raw signal, other than the delay estimation, from each satellite to solve the position. This work analyzed and assessed the multipath tolerance of the DPE approach. The explanation we provided emphasizes on its naturally earlier combination of the satellite information than the traditional 2SP method. To validate and assess the positioning performance of the DPE approach in multipath environments, the MEEs, which were used here to appeal the relation between the multipath error and the positioning error, were computed and plotted. Besides, a realistic controlled multipath channel model generator, the LMSCM by the ITU-R P.681 recommendation, was used to assess the DPE positioning performance in different environments, such as dense urban or suburban scenarios, moving car or slow-dynamics pedestrian user. The 2SP solutions were also provided by implementing GNSS-SDR for the aforementioned simulations. It turns out that the DPE approach outperforms the 2SP method in terms of positioning precision under multipath environments. Given that the Kalman filter is widely used in conventional 2SP method and it holds a higher estimation accuracy than the WLS approach under the multipath influence with the prior information from the past observations, future work could discuss the multipath toleration ability of Bayesian DPE compared to conventional 2SP method with Kalman filter.

ACKNOWLEDGEMENTS

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