# Synthetic Diversity for Interference Mitigation in Widely Tunable Receiver Frontends

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Abstract—Dynamic spectrum access relies fundamentally on the ability to tune radio transceivers to frequencies that are deemed to be available. Consequently, radio hardware must support tuning over a wide range of frequencies. For the receiver, this precludes the use of fixed frontend filters to reject out-ofband interfering signals. Instead, widely tunable receivers rely on filtering after down-conversion either at IF or baseband. This approach relies on linearity and an ideal mixer to keep the desired signal and interfering signals separated. However, practical receivers exhibit non-linearity, phase noise, and oscillator spurs that cause mixing of the signal of interest and interfering signals. As a result, portions of the interfering signals may appear in the band of the desired signal; this causes interference that cannot be mitigated by filtering. Synthetic diversity mitigates this problem by combining analog and digital processing techniques. In the analog domain, the wide-band RF signal is passed through a passive, lossless multi-port diversity network. Each output from this network is then down-converted and digitized so that multiple versions of the signal are available at digital baseband. As the desired signal and the interfering signals experience different frequency response as they pass through the diversity network, it is possible to employ beam forming methods in digital baseband processing to mitigate the interfering signals while preserving the desired signal. The performance of the proposed synthetic diversity receiver is analyzed and it is shown that excellent interference rejection can be achieved. Rejection performance can be increased even further when the circuit elements in the diversity network can be adapted.

Index Terms—Radio Receiver, Interference Rejection

#### I. Introduction

Dynamic spectrum access relies fundamentally on the ability to identify and exploit spectrum opportunities. The exploitation of such opportunities relies in turn on the ability of the transmitter and receiver to tune to the frequency where the opportunity exists. It is desirable to tune over a wide frequency range to be able to exploit any opportunities that exist within the tuning range.

A wide tuning range precludes the use of fixed filters between the antenna and the receiver as such filters limit the tuning range to the passband of the filters. Commercial receivers rely on banks of front-end filters to allow operation in a small set of frequency bands.

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The purpose of these filters is to suppress strong out-of-band interference. If the analog parts of the receiver were ideal, the front-end filters would not be required since the interfering signal could be rejected by the anti-aliasing (or IF) filter after down-conversion and just prior to analog-to-digital conversion. In practical receivers, however, non-ideal behavior, including non-linearities, oscillator spurs, or phase noise can cause parts of the interfering signal to be mixed into the band of the signal of interest. Once that occurs, a simple filter cannot separate the desired signal from the interference that is now in-band.

A receiver with selective front-end filters can reject interference before it becomes a problems — as long as the interferer is outside the passband of the filters. However, the filters severely restrict the tuning range. For dynamic spectrum access, both a wide tuning range and resilience to the effects of strong interference are required,

A novel method for rejecting out-of-band interference, after it has mixed into the band of interest, was proposed originally in [1] and refined in [2]. The fundamental idea of this approach is to pass the received signal through a passive, lossless, multiport circuit network and to process each of the outputs from the network by separate analog receiver frontends. The network causes the signal of interest and any interfering signals to experience different frequency responses as they are located at different frequencies. Since the effect induced by the multiport network is akin to the diversity known from multi-antenna systems, this techniques is named *synthetic diversity* and the multi-port network is referred to as a diversity network. The synthetic diversity can be used in the digital back-end, to combine the signal if interest coherently and to reject in-band components due to interference.

Goals: This paper aims to augment the experimental results in [1], [2]. Specifically, we address the optimal digital combing of signals from the diversity network and quantify the performance of the complete receiver. Additionally, we demonstrate that the overall performance can be greatly enhanced by modest tuning of the circuit elements of the diversity network. As a result, the synthetic diversity concept is established as a viable approach for the design of widely tunable receivers that are resilient in the presence of strong interference.

## II. SYNTHETIC DIVERSITY

The synthetic diversity concept is illustrated in Figure 1. The received signal from the antenna is passed through a passive, lossless [3] circuit with M outputs in addition to the single input. The frequency response between the input and each of the M outputs of this diversity network is frequency dependent. Consequently, signals at different frequencies  $f_k$  "see" experience different complex gain vectors  $\mathbf{H}(f_k)$  before they enter the active receiver array.

In each of the M analog receiver front-ends, the signal is down-converted, filtered, and finally A/D-converted, As the analog receivers contain active element, the signals along each path are subject to imperfections, including non-linear distortion, oscillator spurs, and phase noise. It is these imperfections that can cause interfering signals to be mixed into the band of the desired signal; once this occurs, the interference cannot be suppressed by filtering.

However, we will show in Section III that the synthetic diversity enables simultaneous coherent combining of the desired signal and rejection of the in-band interference due to receiver imperfections.

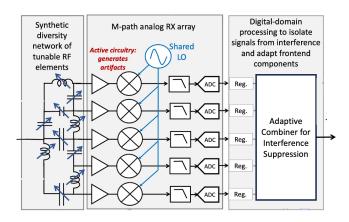


Fig. 1. The Synthetic Diversity Concept.

# A. Causes of In-band Interference

There are multiple potential causes for mixing an interfering signal into the band of the desired signal. In all cases, the in-band interference terms are proportional to the frequency response vector  $\mathbf{H}(f_I)$  at the frequency of the interfering signal or a deterministic function of these vectors.

The most obvious case occurs when the oscillator has spurs. Assume that the oscillator is tuned to the frequency  $f_0$  of the desired signal and that there is a spur at frequency  $f_I$ . Then, any interfering signal located near  $f_1$  will be down-converted to baseband where it will interfere with the down-converted signal of interest. A similar argument holds for phase noise that extends to the frequency  $f_I$  of the interferer. It is important that all receiver paths are driven by the same oscillator as indicated in Figure 1. Then, the interfering components will be proportional to the elements of the frequency response vector  $\mathbf{H}(f_I)$  at the frequency of the interfering signal.

Assume now that the receiver is non-linear and that the non-linearity can be model by a third order polynomial. Let there be two interfering signals at frequencies  $f_1$  and  $f_2$ , respectively. Both are outside the desired band near  $f_0$ . The non-linearity causes the two interfering signals to mix and create spectral components at frequencies  $\pm mf_1 \pm nf_2$  for certain m and n and one of these components may coincide with the desired frequency  $f_0$ . For example,  $f_1$  and  $f_2$  may be such that  $2f_2 - f_1 = f_0$ . Then, a signal with "effective" frequency response vector  $\mathbf{H}^2(f_2) \cdot \mathbf{H}^*(f_1)$  falls into the band of the desired signal.

In either case, the interfering signal can be rejected in digital baseband processing based on the differences between the frequency response vectors at the interfering frequencies and the frequency of the desired signal.

# B. Parallel Two-Ports

The original work on synthetic diversity relied on a fully connected mesh as the diversity network. This network is very difficult to analyze in closed form. Instead, the diversity network in this work is constructed systematically to allow analysis in closed form — both in terms of frequency and in terms of the values of circuit elements.

As shown in Figure 2, our diversity network consists of M parallel two-port circuits, each terminated by a load  $Z_L$  that represents the receiver inputs. Moreover, we limit the two-ports to T-circuits with impedances  $Z_A$  (series),  $Z_B$  (shunt), and  $Z_C$  (series). These impedances may be open or short circuits, single inductors or capacitors, or parallel or series resonant circuits.

With these choices, it is straightforward to determine the input impedance  $Z_{\mathrm{in},m}(s)=\frac{V_{\mathrm{in},m}(s)}{I_{\mathrm{in},m}(s)}$  and the voltage gain  $G_m(s)=\frac{V_{\mathrm{out},m}(s)}{V_{\mathrm{in},m}(s)}$  for each terminated two-ports.

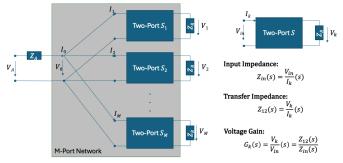


Fig. 2. Synthetic Diversity Network of Parallel Two-Ports.

With these two quantities, the overall system can be analyzed. To begin, the input impedance  $Z_{\rm in}$  of the parallel two-ports  $(V_0/I_0$  in Figure 2) is given by

$$Z_{\rm in}(s) = \frac{1}{\sum_{m=1}^{M} \frac{1}{Z_{\rm in,m}(s)}}.$$
 (1)

This input impedance is important as it determines how much power is transmitted from the antenna into the diversity network. For maximum power transfer, the input impedance must be matched to the antenna impedance  $Z_{\rm ant}$ , i.e.,  $Z_{\rm in}=Z_{\rm ant}^*$ . The *insertion loss* S(s) is the fraction of the available power (i.e., the power with perfect matching) that enters the diversity network. It is given by

$$S(s) = \frac{4\Re\{Z_{\rm in}(s)\}\Re\{Z_{\rm ant}(s)\}}{|Z_{\rm in}(s) + Z_{\rm ant}(s)|^2}.$$
 (2)

The insertion loss S(s) provides the fraction of the power that enters the diversity network. Since the diversity network is lossless, all the power that enters the network is delivered to the M connected receivers. The specific power delivered to the m-th receiver is proportional to the voltage gain of the m-th two-port,  $G_m(s)$ . The lossless condition implies that for each frequency f, the power delivered to the m-th receiver is

$$P_m = P_{\text{in}} \cdot \frac{|G_m(j2\pi f)|^2}{\sum_{n=1}^M |G_n(j2\pi f)|^2} = P_{\text{in}} \cdot \frac{|G_m(j2\pi f)|^2}{\|\mathbf{G}(j2\pi f)\|^2}, \quad (3)$$

where  $P_{\rm in}$  denotes the power that enters the diversity network. For our further development, it will be useful to scale the voltage gains to unit norm, i.e., we define

$$\tilde{G}_m(s) = \frac{G_m(s)}{\|\mathbf{G}(s)\|}.$$
(4)

With these definitions and results in place, we can now turn our attention to the digital backend.

## III. DIGITAL COMBINING

The purpose of the digital backend is to coherently combine the desired signal across the M receiver paths while simultaneously suppressing the interfering signal that appears in-band.

For this purpose, we can rely on well-known results from beamforming [4] for optimally combining the signals from the M receiver paths. The problem is to find a length-M vector  $\mathbf{w}$  of weights to optimally combine the baseband outputs  $\mathbf{v}_{\text{out}}$  from the M receiver paths,

$$v_{\text{comb}} = \sum_{m=1}^{M} w_m^* v_{\text{out},m} = \langle \mathbf{v}_{\text{out}}, \mathbf{w} \rangle.$$
 (5)

The minimum mean-squared-error (MMSE) combiner, also called the Wiener filter, provides the wight vector that minimizes the MSE of the difference between the filter output  $v_{\rm comb}$  and the desired signal. The MMSE weights are given by

$$\mathbf{w} \sim \mathbf{R}_n^{-1} \tilde{\mathbf{G}}(s),$$
 (6)

where  $\mathbf{R}_n^{-1}$  is the correlation matrix of noise and interference. For example, for the important case of a single interferer falling into the band of the desired signal, the noise and interference correlation matrix  $\mathbf{R}$  is equal to

$$\mathbf{R} = \sigma^2 + \alpha P_I S(f_I) \tilde{\mathbf{G}}(j2\pi f_I) \tilde{\mathbf{G}}^H(j2\pi f_I),$$

where  $P_I$  is the power of the interferer (at the antenna) and  $\alpha$  is a factor that depends on the mechanism that causes the interference to be mixed into the desired band.

With this choice of weights, the ratio L of signal-to-interference-plus-noise-ratio (SINR) and SNR with perfect

(power) matching and without interference can be computed as

$$L = S(f_0) \cdot \left( 1 - \langle \tilde{\mathbf{G}}(j2\pi f_0), \tilde{\mathbf{G}}(j2\pi f_I) \rangle \cdot \frac{\alpha P_I S(f_I)}{\sigma^2 + \alpha P_I S(f_I)} \right). \tag{7}$$

In this expression are three discernible factors. The factor  $S(f_0)$  reflects the insertion loss due to impedance mismatch. The inner product between (normalized) voltage gains at frequencies  $f_0$  and  $f_i$  reflects the similarity of the frequency responses; ideally this inner product is zero so that all interference is rejected. The fraction involving the noise variance  $\sigma^2$  and the interference. In our experiments, we have focused on the case of strong interference, so that this ratio is equal to one.

With these metrics in place, we can now perform numerical experiments to assess the viability of the diversity concept.

#### IV. RESULTS

For the numerical experiments in this section, we focus on a single structure for the diversity network. Our diversity network consists of four receiver branches, i.e., the diversity network has M=4 output ports. Each of the four two-ports in our network is a T-circuit. The first element  $Z_A$  is a single inductor or capacitor. The shunt in the T-circuit,  $Z_B$  is a series resonator that can short the load at resonance. Finally,  $Z_C$  is a parallel resonator that can decouple the load at resonance. We are investigating tuning of the circuit between 1 and 2 GHz.

To begin, we dimensioned the elements of the circuit to produce 8 evenly spaced resonance between 0.8 and 2.2GHz. The resulting four frequency responses are shown in Figure 3. In this figure, the resonance frequencies are clearly visible both in magnitude and in phase.

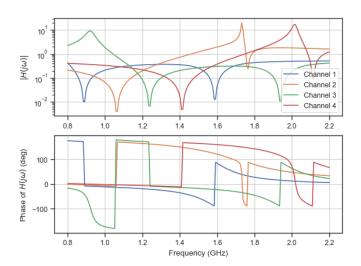


Fig. 3. Frequency Responses. Top: magnitudes, bottom: phases.

For the same configuration, Figure 4 shows the input impedances of the individual two-ports together with the resultant input impedance for the entire diversity network. From the input impedance and the antenna impedance, the

insertion loss can be computed according to (2). Notice that the insertion loss is large when the input impedance is either very large or very small.

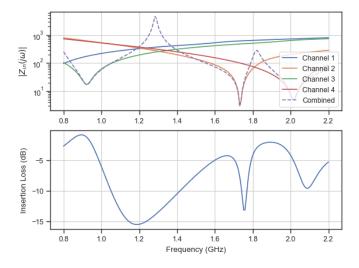


Fig. 4. Top: Input impedance of the individual two-ports and input impedance of the entire diversity network. Bottom: insertion loss for antenna impedance of  $50\Omega$ .

Still for the same configuration, Figure 5 shows the loss that arises because the voltage gains at the desired frequency  $f_0$  and  $f_I$  are not perfectly orthogonal. Specifically, the figure shows  $1 - \langle \tilde{\mathbf{G}}(j2\pi f_0), \tilde{\mathbf{G}}(j2\pi f_I) \rangle$ . Generally, the correlation loss is small when  $f_0$  and  $f_I$  are well separated. As should be expected, large losses occur along the diagonal where  $f_0 \approx f_I$ .

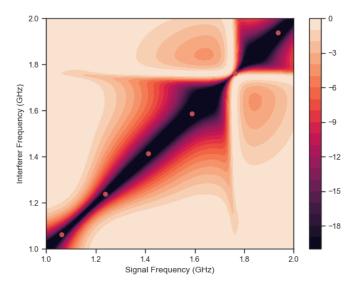


Fig. 5. Correlation loss for signal of interest at frequency on x-axis and interferer at frequency on y-axis. The dots along the diagonal mark the locations of the eight resonance frequencies.

So far, we have considered a static diversity network. An explicit premise of our collaborative research effort is to adapt the circuit elements of the diversity network. To demonstrate the utility of tuning the diversity network, a simple random

search in the vicinity of the initial configuration was conducted. We generated 100 perturbations of the circuit elements such that the resonant frequencies varied by at most 20% from their initial values. For each pair  $(f_0, f_I)$  we kept the best loss among the 100 measurements.

Figure 6 shows that with the relative small perturbations, very good correlation losses are achievable. Even signal in close spectral proximity are rejected without sacrificing SNR. When the insertion loss is included, results are slightly worse.

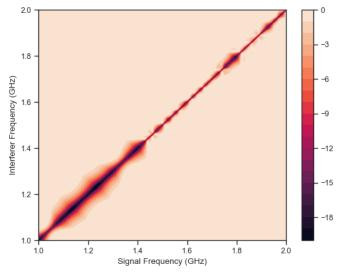


Fig. 6. Best correlation loss over 100 perturbations for signal of interest at frequency on x-axis and interferer at frequency on y-axis.

## V. CONCLUSIONS AND FUTURE WORK

We have shown that synthetic diversity is a promising approach to maintaining wide tunability while protecting the receiver effectively from the detrimental effects of interference. In particular, we demonstrated that even small adaptations to the circuit elements can greatly improve the performance of the receiver.

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