

Article

Studio as a Catalyst for Incremental and Ambitious Teacher Learning

Rebekah Elliott ^{1,*}  and Sarah A. Roberts ² ¹ Department of Education, Practice and Research, Oregon State University, Corvallis, OR 97331, USA² Department of Education, University of California, Santa Barbara, CA 93106, USA;
sarahann_roberts@ucsb.edu* Correspondence: elliotttr@oregonstate.edu

Abstract: This article explores how the practice-focused Studio professional learning (PL) model can provide incremental and ambitious teacher learning opportunities. We argue that when the model's structures and practices are grounded in ambitious and equitable teaching, they catalyze incremental teacher learning. Studio, like lesson study, supports teachers in considering the entailments of lessons, focusing on the live shared enactment to strengthen teaching and learning through collaborative analysis and reflection. To build our argument, we drew from two Studio projects that had shared structures of cycles of learning and routines, as well as shared practices of using rich representations and collective interpretations of teaching. While both projects' structures and practices take up ambitious and equitable teaching, they use different routines and attend to different features of equitable teaching. Building on a history of PL models, such as lesson study, which use similar structures and practices as powerful catalysts of teacher learning, we argue that Studio's structures and practices can catalyze teachers' incremental learning of ambitious and equitable teaching. We discuss the implications for future research based on this argument and for those leading PL.

Keywords: mathematics education; professional learning; studio; multilingual learners; data literacy; pedagogical reasoning

**Citation:** Elliott, R.; Roberts, S.A.

Studio as a Catalyst for Incremental and Ambitious Teacher Learning.

Educ. Sci. **2024**, *14*, 1160.<https://doi.org/10.3390/educsci14111160>

Academic Editor: Amber Simpson

Received: 1 September 2024

Revised: 20 September 2024

Accepted: 29 September 2024

Published: 25 October 2024



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1. Introduction

The editors of this Special Issue identified professional learning (PL) focused on ambitious teaching as different from PL that attended to incremental instructional improvement [1]. They suggested that the aims of these two types of PL emphasize different learning units, but both seek to improve practice and thus engage those involved in teacher learning. In this conceptual paper, we build on a history of research on the PL models that are ongoing, grounded in teaching, and engaged in collective interpretations of teaching to support incremental teacher learning [2]. We draw on two such PL projects, Beachside and Mesa, which used an adaptive practice-focused model of Studio PL (herein called Studio) to provide opportunities for both incremental and ambitious teacher learning [2,3].

Studio is situated in teaching involving cycles of activities used across a school year to improve instruction collectively [4,5]. Like lesson study [6], Studio cycles support teachers in considering the entailments of lessons, focusing on learning from a shared enactment and strengthening the teaching and learning process through collaborative analysis and reflection [4]. Studio differs from lesson study in that it does not focus on just a research lesson. Instead, it focuses on the lesson and the live lesson enactment with students, where a Studio teacher enacts a lesson, and colleagues closely observe students' participation. Analysis and reflection follow, where teachers make sense of the mathematics, lesson design, and pedagogical choices and how these are linked to the documented observations of students' participation. Through a debrief process, all teachers consider their future teaching.

In this conceptual paper, we offer examples from two Studio projects to argue that Studio structures and practices catalyze incremental learning like other similar PL models. Our paper extends the previous arguments and provides insights into how these structures and practices can support ambitious and equitable teaching [7]. Catalysts, by definition, are the people or events that cause change [8]. We aim to explore how Studio structures and practices cause incremental change. Lave called on us to pay attention to the mechanisms or the “ways by which learning comes about” [9] (p. 156). Over a decade ago, Jackson and Cobb [2] challenged the field to understand how districts and school structures support ongoing teacher learning of ambitious and equitable teaching. Given this Special Issue’s focus, we see this call as still relevant. We draw on the work of Borko et al. [10] and Borko et al. [11] to compare these two Studio enactments. We pose what Borko et al. [11] called “an important question” (p. 260), contributing a conceptual argument for the field to consider how Studio structures and practices catalyze incremental teacher learning of ambitious and equitable teaching. As an existence proof, rather than an empirically-based argument, we explore intentionally chosen rich images of Studio structures and practices and how they foster teacher learning. Our argument contributes analytic insights on framing Studio’s teacher learning opportunities. We provide a chain of evidence based on our intentionally-selected illustrations of two instances of Studio to address our important question. We next provide an overview of our important question. We then discuss the perspectives that guide our thinking on our important question for Studio structures. We share the contexts of the two Studio structures before delving into comparing our two Studio enactments. We finish with discussions of our conceptual argument, including the implications for the field.

1.1. Important Question for the Field

In this paper, we build an argument for the following question: How do Studio’s structures and practices act as catalysts for the incremental learning of ambitious and equitable teaching? We address this question by discussing intentionally-selected examples from two Studio settings with similar structures and practices. Beachside and Mesa used a repeated activity cycle across Studios for the teachers’ observation, analysis, and reflection. They also used routines, the sequences of teacher instructional moves guided by student mathematics and participation goals. Routines were the focal “lessons” in Beachside and Mesa Studios [12,13]. Beachside participants used Mathematics Language Routines (MLRs) to meet the multilingual learners’ language and mathematics learning needs [14]. At Mesa, a facilitator-designed data literacy routine was used to serve the school’s detracked mathematics courses, integrating new and innovative content. Although they used different routines, they both involved ambitious and equitable pedagogies. Both routines provided students with opportunities to reason, explain their thinking, and solve mathematics tasks in ways that supported all students to use authentic mathematical tools [14]. Both routines attended to language and mathematics but were designed to support different aims. MLRs specifically supported the language and content needs of multilingual learners in the Beachside community [14]. The data literacy routine supported the language of data literacy for heterogeneous groups of students in the Mesa community. Beachside and Mesa also utilized a similar set of practices, rich public representations of teaching that offered an object for collective interpretations of teaching [15,16]. Studio’s public representations of teaching were rehearsals and live enactments of routines. Teaching representations became objects for analysis and reflection through facilitators’ and teachers’ collective interpretation of teaching. In this paper, we build evidence to argue that the repeated Studio structures and practices enabled facilitators to focus on incrementally supporting teachers’ ambitious and equitable teaching [4,17].

We draw upon pedagogical reasoning to consider teachers’ incremental learning of ambitious and equitable teaching. Greeno and Engeström framed learning as participation in activity systems and, from this perspective, we considered teachers’ understanding of ambitious and equitable teaching as participation in pedagogical reasoning and how

changes in that reasoning were catalyzed by Studio structures and practices [18–20]. Pedagogical reasoning, which we expand on below, entails the Studio facilitators’ and faculty’s discourse on ambitious and equitable instruction. Our views of ambitious and equitable teaching, which we expand on below, follow the current calls for centering students’ discourse, focusing on reasoning versus attending to answer-seeking and (re)positioning the students often marginalized within educational systems [2,21,22]. In the next section, we provide details on the Studio contexts drawn upon for this paper.

1.2. Contexts

While the two Studio studies drawn upon to illustrate our conceptual argument involved similar structures and practices, they had different features, as shown in Table 1. The first key difference was Studio facilitation. In the Beachside Studio, the research team facilitated. In the Mesa Studio, two teacher leaders (herein called facilitators) facilitated.

Table 1. Key differences between Beachside and Mesa Studio.

	Beachside	Mesa
Routine	Mathematics Language Routines (MLRs)	Data Literacy
Grade Level	Junior High School	High School
Facilitator	Researcher Team	Two Teacher Leaders
Cycle	Pre-Studio Day, Studio Day, Post-Studio Day	Pre Studio, Studio Day

A second key difference between the projects was related to the routines they used. While each project centered Studio on routine use, Beachside utilized MLRs [14], and Mesa took up a facilitator-designed routine for data literacy. While these routines each supported teacher and student interactions toward a focus on mathematics and participation goals, including students’ mathematical reasoning, they contrasted in unique ways that we will discuss in Section 2.3.2.

The final key difference between these projects was adapting the Studio cycle to the local needs of teacher availability. Beachside’s Studio cycle took place across three meetings, with only the Studio Day occurring during the school day; the Pre-Studio Days and Post-Studio Days took place after school. In Mesa, the facilitators planned during the Pre-Studio Day, with the Studio teacher involved for a portion of that time. Mesa’s Studio took place on a single day and included activities similar to those across Beachside’s cycle of three meetings.

2. Perspectives

In this paper, we make a conceptual argument that Beachside’s and Mesa’s Studio structures, their cycles and routines, and their practices, the rich public representations of teaching involving collective interpretations of teaching, served as catalysts for the incremental learning of ambitious and equitable teaching. Our paper unfolds in the following manner. Because Studio seeks to change teachers’ practices and knowledge, we conceptualize teacher learning as an act of pedagogical reasoning. Pedagogical reasoning is a lens through which to consider the Beachside and Mesa Studio structures and practices as catalysts for the incremental learning of ambitious and equitable teaching. Within our Perspectives section, we review research on pedagogical reasoning and a definition of ambitious and equitable teaching. We then turn to the slim but compelling body of Studio research for fostering pedagogical reasoning. We conclude this section by describing Beachside’s and Mesa’s Studios’ structures and practices.

2.1. Pedagogical Reasoning

At the center of the Studio structures and practices are teaching and teacher conversations directed toward teacher learning about teaching and the knowledge of teaching. Researchers of teacher conversations as learning opportunities frame this as pedagogical reasoning [23–26]. Loughran and colleagues [26], in their comprehensive review of the evolution of pedagogical reasoning, offered nuanced details on this construct beyond this paper’s scope. However, their work and that of others have highlighted the key features

of pedagogical reasoning that we find compelling. Our conceptualization of pedagogical reasoning involves teachers collaboratively focused on generative teaching dilemmas. These dilemmas are rich in detail, allowing for the elaboration of evidence, explanation, and rationale to interpret classroom events. Further, as teachers engage in pedagogical reasoning, they offer differing analyses and justifications linked to principles of teaching and learning. These conversations lead to action and future teaching [16,26]. Lefstein et al. [25] offered additional insights into our conceptualization of pedagogical reasoning with what they called pedagogical productive talk. In their case-study research, teacher conversations are productive when they fulfil the following: (1) focus on problems of classroom practice; (2) use evidence, explanations, and reasons to interpret events and analyze and justify the courses of action; (3) are anchored in rich representations of practice; (4) involve different perspectives; (5) support a generative stance on the instructional system (content, students, teaching/learning and contexts) and problems of practice; and (6) foster trust, collegiality, and critical inquiry. Loughran [20] and Horn [26] remind us that teachers see their knowledge of teaching through a lens of practice and therefore do not always make explicit their rationale, or what Loughran calls the ‘why’ of their teaching [20]. Moreover, this type of reasoning is often not built into the typical teaching workday.

Horn et al.’s study [16], which built a taxonomy of low- and high-depth teacher conversations, empirically affirmed Loughran’s [20] and Horn’s [26] claim that pedagogically productive, or what Horn and colleagues [16] called high-depth, conversations were rare. Horn et al.’s [16] taxonomy detailed the complexity of features of pedagogically productive talk and our working conception of pedagogical reasoning. Here, they documented how rich representations of teaching, a feature from Lefstein et al. [24] and our working conception, are necessary but insufficient to ensure high-depth conversations. For example, when rich representations were univocally discussed, they were low depth because a teaching event would unfold with little questioning or press for explanation and rationale. In contrast, high-depth conversations involved what Horn and colleagues called a collective interpretation of teaching linked to future teaching [16]. These conversations included a rich representation of teaching. They dialogically elaborated justifications involving mathematics, students, and pedagogy, with multiple participants connecting representations of teaching to teaching and learning principles and future teaching. These conversations focused on the social and analytic scaffolding imperative for ambitious teaching [15,27]. Collective interpretation of teaching is the discursive practice entailed in constructing pedagogical reasoning.

We draw upon the construct of pedagogical reasoning, synthesized from our literature review, as a lens to examine Studio structures and practices as catalysts for the incremental learning of ambitious and equitable teaching. Pedagogical reasoning helps us analyze how Studio cycles and routines engage facilitators and teachers in predictable and repeated opportunities to use rich representations of teaching and the collective interpretation of teaching. This paper illustrates how Studio cycles afforded teachers opportunities to make teaching public. Routines were the shared rich representation of teaching that was both doable and embodied ambitious and equitable teaching principles. Moreover, these structures afforded space for the collective interpretation of teaching, such as taking up dilemmas, pressing for elaboration, examining alternatives, and considering future teaching. Next, we review the research on ambitious and equitable teaching to share our working definition.

2.2. Ambitious and Equitable Instruction

Ambitious teaching aims to ensure that all learners develop conceptual understanding and procedural fluency, engage in mathematical argumentation and reasoning, and communicate that reasoning using different representations [2]. Students’ equitable mathematics learning means that teaching must mitigate learning barriers that shape lesson structures and the discipline of mathematics while also attending to students’ multidimensional identities, status, and teachers’ biases [28]. We additionally argue for attention to the structures of tracking and so-called ability grouping policies and practices that often shape the instruction of students of color and multilingual learners [29–31].

Because multilingual learners, students of lower socioeconomic status, and students of color have often been provided with a less than ambitious pedagogy, it is essential to provide teachers with opportunities to learn about equity-based pedagogies [21,32,33]. Teachers must trace linkages between practice and students' participation and self-reflect on bias [34]. When teachers teach in detracked systems and work with multilingual learners, PL must offer opportunities to engage in pedagogical reasoning to reframe the students' strengths with asset-based language [35], to discuss ways for the students to experience grade-level mathematics [36], and to provide multiple opportunities to share and refine ideas; e.g., rough drafts, [37], before formalizing mathematics [38–40].

Studio structures and practices provided teachers space for these pedagogical conversations that explicitly attend to what it means to have ambitious goals and work toward equitable practice. In the Beachside and Mesa Studios, teachers enacted and reasoned about the ambitious goals of MLRs and a data literacy routine (what we call a Studio structure). Because each Studio cycle included a common teaching experience, time for collective analysis, and consideration of future teaching, Beachside Studio members explicitly considered how to equitably support all the multilingual learners' mathematics and language production [14]. Mesa Studio members considered how heterogeneous student groups in their detracked classes worked with the new and often perceived "advanced" mathematics of data literacy. As illustrated in the selection of the Studio data shared, teachers' practices and pedagogical reasoning changed across cycles, thus advancing our argument that the structures and practices catalyzed the incremental learning of ambitious and equitable teaching.

2.3. Studio Research

A slim but compelling body of mathematics Studio research has emerged documenting mathematics teachers' learning [4,17,41]. We compare the Studio research to similar PL models published in approximately the past decade, which all build on lesson study in Table 2. All these PL models leverage the importance of planning, viewing teaching, and reflecting on teaching in cycles. The models differ in how teaching representations are accessed, either through a live enactment [4,42–47] or through viewing video-recorded enactments and artifacts from enactments [6,48–51]. Further, differences include who teaches lessons and who facilitates the learning. Finally, another difference is the lesson content in these models. Lesson study models are aimed at building a "research lesson" based on children's thinking [6,43–45]. Studio studies vary their content focus across science and mathematics, with most making reference to ambitious teaching. We note that across all these models, teachers work with rich representations of practice that anchor their conversations on practice [19]. Many of the PL models provide opportunities for teachers' incremental learning of ambitious teaching, with some specifically aimed at disrupting classroom inequities [6,10,43,44]. These models build on the lesson study's structure and create opportunities for teachers' sensemaking through the observations of teaching and engagement in a community of other practitioners.

We now delve into the science [45] and mathematics Studio research [4,42,46,47]. Thompson et al.'s [45] study found that the science Studio structures and tools that teachers adapted across Studios were critical to the development of teachers' ambitious science teaching. The researchers focused on teachers' sense-making conversations based on problems of practice they faced and their improvement of ambitious teaching practices and tools as they engaged in Studio cycles. Kim et al. [46] examined elementary mathematics teachers' pedagogical reasoning and the PL opportunities made available by Studio's structure. They found that teachers' pedagogical reasoning about children's thinking shifted and deepened across Studio cycles with the support of the Studio facilitator. Lai et al. [47] in a study of first grade mathematics Studio focused on teachers' pedagogical reasoning found that Studio structures allowed teachers to examine the reasons and purpose of decisions leading to building knowledge of and for teaching. In a unique mathematics leader Studio study, Carlson and colleagues [42] found that they needed to revise their initial goals and prompts to shift the participants' reasoning to support coaches and principals.

The researcher-facilitators leveraged the Studio’s iterative cycle to realize their goals. By formatively tracking participants’ discourse, the authors found that participants focused on students’ content learning even though they were responsible for teacher development. Additionally, changes in facilitation across the cycles shifted the participants’ focus to mathematics for teachers and support of K–12 students. While these authors did not use the construct of pedagogical reasoning, we see the nature of the changes they found aligned with this construct. In another Studio study, Lesseig [4] focused on the teachers’ reasoning with the content and pedagogical knowledge of mathematical conjecturing, generalizing, and justifying. Lesseig’s middle-level mathematics study examined the leader and participant discourse practices, skilled facilitation, and norms of interaction. Studio practices cultivated participants’ inquiry stance, their use of evidence to support claims, their press for others to do so, and their elicitation of alternative interpretations. Again, this author did not use the specific construct of pedagogical reasoning, but the teachers’ ways of reasoning built in Studio align well with our conception of pedagogical reasoning described above [16,26].

Table 2. A Comparison of Studio, Lesson Study, and Lesson Lab.

Professional Learning Model	General Structure	Who Teaches the Lesson	Who Facilitates the Model	Grade Level and Content Focus	Examples of Scholars Who Have Used This Model
Studio	Collaboratively Plan, Gather Data During Live Enactment, Reflect on Observation, Implement Revisions	Teacher with Coaches	Teacher Educator	Elementary, Math	Carlson, et al. [42]
		Teacher (with researchers and other teachers monitoring student progress)	Research Team	Middle School, Science	Thompson et al. [45]
		Teacher	Teacher Educator	Kindergarten, Math	Kim et al. [46]
		Teacher	Teacher Educator	First Grade, Math	Lai et al. [47]
		Teacher	Math Coach	High School Math	Lesseig [4]
Lesson Study	Curriculum Study, Lesson Planning, Observation of Lesson, Debrief	Teacher	Outside Expert	Variety	Morris and Hiebert [48]
		Teachers, Teacher Leaders	Teacher Leader and University-Based Mentor	K-8, Math	Ebby et al. [49]
		Teachers	Teachers	K-8, Math	Fernandez [6]
		Teachers	University-Based Outside Experts	Middle and High School, Math	Lewis [50]
		Facilitator	Facilitator in Participant Classroom	Middle School, Math	Amador et al. [51]
Teaching Lab	Examine Standards and Research, Co-Plan Lesson, Co-Enact Lesson, Debrief	Teacher Educator with Inservice Teachers	Teacher Educator	Elementary, Math	Gibbons et al. [43] Kazemi et al. [44]

These studies suggest Studio cycles enhanced the participants’ pedagogical reasoning and allowed them to move beyond quick solutions to focus on instruction and student learning “problems”. Next, we describe the Studio structures of cycles and routines.

2.3.1. Studio Cycles as a Structure

We illuminate the critical aspects of Studio structure and practices to situate our work as catalysts for incremental learning, beginning with the vital elements of the Studio cycle. Each Studio involved opportunities for an initial collective investigation for a Studio lesson (e.g., the MLRs at Beachside and data literacy routine at Mesa), Studio Lesson enactment with students, where the resident teachers (herein called residents) observed and took notes. The enactment debriefs involved collective descriptive conversations amongst all the residents to take up new instructional actions based on the discussions [4]. Each Studio cycle included these elements, and Studio is generally conducted for multiple cycles in a year. The Studio cycle provided a myriad of opportunities to engage in the collective interpretation of teaching [16] within a cycle, starting with the initial segment in what Beachside called

the “pre-brief” before the Studio lesson and what Mesa called the “rehearsal” and the “rehearsal debrief.” The Studio enactment phase then asked the residents to listen carefully to the students’ participation and take notes (a form of interpretation) to support the third opportunity for collective interpretation of teaching, the enactment “debrief” of the Studio lesson. Studio cycles offered structural coherence from cycle to cycle, using the same activity sequence [52]. There were also opportunities across cycles for the collective interpretations of teaching supported by a coherent set of mathematical and instructional goals that connected one cycle to the next. In the Studio studies highlighted here, conceptual coherence was established and maintained using the same routine within each site [52].

2.3.2. Studio Routine as a Structure

We highlight routines as a structure within Studio that catalyzed incremental teacher learning and their pedagogical reasoning. As other authors have noted, routines are “patterned sequences of interactions that teachers and students jointly enact to organize opportunities for student learning in classrooms” [12] (pp. 1–2). A routine’s sequence offers stable predictability via the actions guiding teachers and students. However, the routine does not dictate lockstep decisions; it provides opportunities for responsiveness to teachers’ localized needs and students’ contributions [53]. Examples of routines abound, such as creating a “five practices” structure for holding a mathematical discussion to confer with students on their reasoning [54–57]. As a bounded representation of practice, with a definite start and a closing set of actions, routines can be “sized” to fit within the classroom curricular materials. Indeed, routines have become more prominent in curricula, such as those developed to support the Common Core State Standards [58] mathematical practices [13] and those embedded in curricula to support the needs of emergent multilingual students [59].

Researchers studying routines suggest that they offer ambitious opportunities for teachers to (re)position learners as agentic and accountable to shared mathematical reasoning, drawing upon the epistemically diverse ways of knowing to solve complex and authentic problems relevant to learners [21,55]. For example, MLRs are instructional routines that expand the diversity, inclusion, and equity of the students’ language and mathematics development [14]. The data literacy routine (described below) designed by Mesa facilitators borrowed structures from Kelemanik and colleagues [13], with embedded participation structures [60] and reflection prompts, but shifted to a goal of fostering students’ data literacy.

The two routines explicitly intended to disrupt inequities that have long shaped school mathematics. The MLRs are specifically designed to challenge the “persistent assumptions about how to support and develop students’ disciplinary language. . . to meet the needs of linguistically and culturally diverse students who are simultaneously learning mathematics while acquiring English” [14] (p. 3). These MLRs provide a pathway through the systemic and systematic barriers that multilingual learners face in mathematics classrooms to tasks, materials, and educators’ expectations [14]. The data literacy routine pressed heterogeneous groups of students to hone their awareness of data and use descriptive language to make sense of data representations widely used in “real life” but not standard to U.S. high school textbooks and curricula, directly disrupting the practices that only advanced students or “math people” work with complex data representations [61]. Routines were the repeated representations of teaching within Studio, which centered on collective investigation, enactment, and debriefing within each cycle. Teachers learned about these routines through the repeated Studio cycle elements and across cycles as the routine was enacted multiple times. Routines were units of analysis for investigating collective learning. Given the similar ambitious goals of MLRs and data literacy routine and their specific aims to disrupt classroom inequities, they were structures for fostering Studio faculty’s incremental learning of ambitious and equitable instruction [2].

2.3.3. Using Rich Representations of Teaching as a Practice within Studio

We examined the Studio practice of using rich representations of teaching to catalyze incremental learning and pedagogical reasoning. Facilitators, Studio teachers, and residents were encouraged to foster norms by making teaching actions and rationales public and available

for discussion as a part of the Studio [23,27]. The Studio teachers, residents, and facilitators constructed a rich representation of teaching by dwelling on classroom events. Studio participants replayed these events by quoting students' and teacher's contributions, viewing lesson video clips, examining student work, or previewing a lesson plan [25]. At Beachside, rich representations of practice included a "pre-brief" discussion of the targeted MLR, observation of the MLR with students in the Studio enactment, a debrief discussion, and the viewing of video clips from the Studio lessons during the Post-Studio Day. Mesa's Studio included multiple opportunities to build insights from rich representations of teaching, starting with the routine rehearsal and debrief and ending with the routine enactment and debrief. Both projects grounded debrief conversations in residents' observational notes.

Through the rich representations of teaching, Studio teachers and residents deprivatized their practice, pressed each other for instructional specificity, unpacked the entailments of the routine, and allowed it to be viewed from a lens of benefits and challenges [24]. Beachside provided residents with an observation sheet to complete during the Studio lessons, prompting them to record what they saw students and Studio teachers doing in various stages of the MLR (i.e., Compare and connect stages of the Compare and Connect MLR), as well as how the teacher attended to other critical goals related to the MLR, such as how the MLR was integrated into the lesson. The observation sheet in Beachside also provided overall reflection questions to start the initial discussions of the lesson debrief, asking Studio teachers and residents to reflect on their strengths, struggles, and how students engaged with the MLR. At Mesa, facilitators framed each observation of teaching (rehearsals and Studio enactment) with a set of directions. The teachers were directed to record their observations linked to the goal and collect direct quotes from the students and the teacher. Each time the facilitators opened the rehearsal and enactment debrief conversations, they reminded the residents to use their notes, saying something similar each time, such as Georgia's quote from Studio 1: "When you start, we want you to say, 'I noticed. . .' then add the student thinking quotes, or 'Oliver said this, and I think.'" Notably, both projects asked the residents to "replay" or cite verbatim the students' or the Studio teacher's quotes in debrief conversations [62]. Observation tools and the consistent reminders for observing and debriefing shaped the norms for participating in Studio, namely the members' collective interpretations of how they made sense of instructions [4,63,64].

2.3.4. Using Collective Interpretations of Teaching as a Practice Within Studio

The second practice within Studio that we examined was using collective interpretations of teaching, which were the discursive practices of pedagogical reasoning. We highlight here what was required to support collective interpretation teaching as a Studio practice. Studio's cycle elements reviewed above offered Studio teachers and residents opportunities for focused discussions of the representations of teaching. Shaped by social norms of mutual trust, the debrief conversations following the enactments provided the Studio teacher and residents opportunities to share their perspectives on the routine, where the facilitator could press everyone to use evidence, explain their reasoning, and build on each other's ideas. For example, in the Beachside Studio, facilitators supported the Studio teacher in identifying a focal lesson goal for the cycle and paying attention to the MLR-centered discussion of the Studio enactment.

Further, the facilitators provided time for the residents to complete their observation handouts and overall reflection questions before beginning the debrief. By doing so, the facilitators prompted the Studio teacher and residents to ground the discussions in the routine and focal goal of the lesson [65]. As a result, Studio members could use evidence, unpack critical features of the MLR to enhance students' mathematical and linguistic learning, and consider their future teaching [14]. In Mesa's Studio, the collective interpretation of teaching was supported in several ways. The facilitators moved to lift and press on the routine's goals in debrief conversations [66]. The facilitators, Studio teachers, and residents upheld normative ways of interacting, including the goal-directed observation of students' and Studio teachers' contributions and cultivating trust while engaging in

critical inquiry [67]. Studio members and facilitators acknowledged the group's distributed expertise, providing insights that beget further inquiry for collective learning about content, pedagogy, and students [24]. We note that these structures and practices within Studio can co-occur and reinforce each other to support Studio members' pedagogical reasoning about ambitious and equitable teaching. We next discuss our materials and methods.

3. Materials and Methods

3.1. Beachside Studio and Participants

Beachside, situated in the Beachside School District (pseudonyms are used for all proper nouns), is a high-needs K–12 school district in California with approximately 15,000 students. At the time of our study, approximately 25% of Beachside's students were classified as multilingual learners, and 48% qualified for free or reduced-price lunch. The district demographics were reported as 60% Latino/a, 31% white, 3% Asian/Pacific Islander, and the remainder of minoritized students.

Beachside occurred as part of a funded research project, which supported Beachside's Studio cycles for three years. Teachers engaged in Studio Day Cycles, as shown in Figure 1. In the data shared in this paper, the Studio teachers (those teaching during Studio lessons) and residents (those who did not teach during Studio lessons) were in the second of three cycles in the first of these three years during Fall 2023. They used the Compare and Connect MLR [14]. The project was focused on working with Studio members to simultaneously attend to language and mathematics through MLRs [14] and multilingual learner mathematics core practices [65]. In these cycles, the primary facilitator of the Studio Day Cycles was the PI of the grant (Roberts), who collaborated with a district mathematics instructional support specialist to implement these cycles. Six teachers who taught at the district's four junior high schools participated in the project during the year, as shown in Table 3. Beachside had a history of Studio (with Roberts) and previously participated in a two-year high school project with Math I teachers in 2019–2021. The two secondary Studio projects focused on using MLRs, which supported the district's interest in providing mathematics-specific PL that attended to multilingual learners. The MLRs were selected to meet the district's goals of accessing text and communicating reasoning. All teachers who participated in Studio consented to participate in the study. Two teachers were identified as bilingual, one bilingual in Russian, and the second teacher was semi-fluent in Spanish; all held a secondary teaching license in mathematics. During the Studio we examine in this paper, Ms. Taylor did not participate, as she left the project after Fall 2023 for personal reasons, and Mr. Valle had not joined the project yet, as he joined the project in Spring 2024.

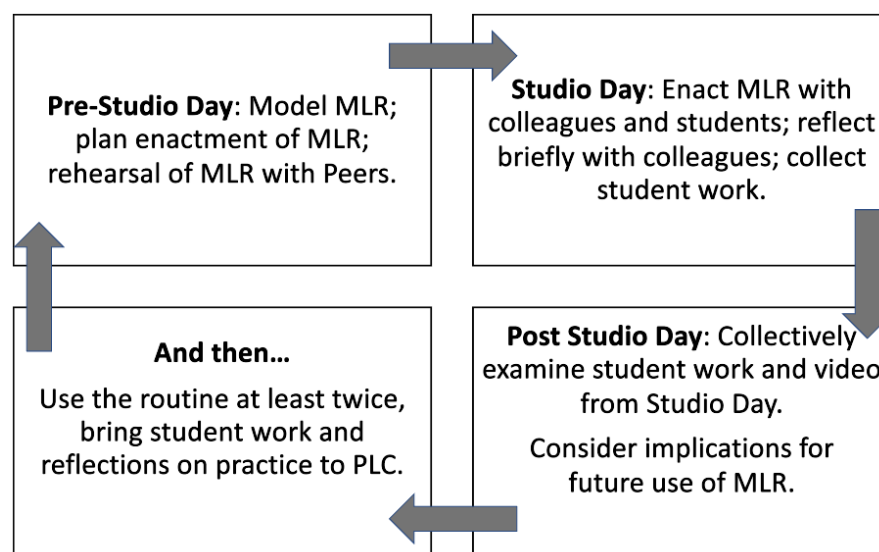


Figure 1. Beachside's Studio Cycle.

Table 3. Pseudonyms, Roles, and History of Beachside Teachers in Studio.

Name (Pseudonym)	Grade Level	Race/Ethnicity (Self-Described)	Years Teaching Mathematics	Studio Role
Ms. Ruth	7th Grade	White	26	Resident
Ms. Severn	7th Grade	Caucasian	4	Studio Teacher/Resident
Ms. Taylor	7th Grade	White	34	Resident
Ms. Foster	8th Grade	White	3	Resident/Studio Teacher
Ms. Penny	8th Grade	White	2	Resident
Mr. Valle	8th Grade	White/Caucasian	5	Resident
Ms. Hope	Instructional Support Specialist	Caucasian	25	Instructional Support Specialist

3.2. Mesa Studio and Participants

Mountainside School District, the site of Mesa Studio, was a K–12 district on the West Coast of the United States in a mid-size city serving approximately 17,400 students. At the time of the study, approximately 7% of students were designated as ever English language learners, and 36% qualified for free/reduced-priced lunch. The district was 80% white, and 20% were minoritized youth, with the schools segregated by socioeconomic background. Mesa HS students were majority white (87% across the district; 1 October 2019 data) and dominant English speakers (93%), with no school-level data available on free or reduced-price lunch because of COVID-19.

Elliott’s funded project supported Mesa’s two Studio cycles during 2020–2021, the facilitator planning and the associated Studio costs. In Fall 2020, Studio 1 was online due to the COVID-19 remote teaching restrictions, with nine teachers and two facilitators. In Spring 2021, Studio 2 was conducted on Mesa’s campus, with seven teachers and the same two facilitators (see Table 2). Two teachers from Studio 1 opted out of Studio 2 because of upcoming job changes. Both Studio cycles focused on enacting the data literacy routine. After being introduced in the Fall 2020 Studio cycle, facilitators asked all teachers if they would commit to regularly using the data literacy routine across all courses. In the past, Mesa teachers had developed routines supporting mathematical practices, and they used their well-established Studio model to create a shared vision to improve instruction. Mesa’s mathematics department had engaged in Studio for ten years, and the Mountainside district sponsored Studio in elementary and middle schools. For the past five years, including the study year, Mesa Studio was facilitated by two teacher-leaders. All the teachers and facilitators in Studio consented to participate in the study.

Eleven educators (Table 4; five presenting female and six presenting male) participated in Studio 1, and nine educators participated in Studio 2. The educators reported having taught 12 years on average, and all held a mathematics teaching license. Studio facilitators Georgia and Jasper taught high school mathematics for over a decade, teaching nearly all their careers at Mesa. They had participated in Studio since they were novice teachers. Six of the 11 Mesa teachers in Studio 1 had participated in Studio for nearly ten years, and only one teacher, Monte, new to the district, was new to Studio.

Table 4. Pseudonyms, Roles, and History of Mesa HS Teachers in Studio 1 and 2.

Participant Pseudonym	Studio Role	Years Teaching Math
Benson	Studio 2 Teacher	8
Brooke	Resident	15
Kay	Resident	26
Georgia	Studio Facilitators	14
Jasper	Studio Facilitators	13
Justine	Resident	13
Oliver	Studio 1 Teacher	1
Tyson	Resident	7
Zandra	Resident	10
Monte	Resident (Studio 1 Only)	Not Reported
Henry	Resident (Studio 1 Only)	Not Reported

3.3. Studio Models

3.3.1. Beachside Studio Cycle

Beachside held three Studio cycles during 2023–2024. The data shared in this paper came from the second cycle, hereafter called Studio 2. Each Studio cycle includes the following four parts: three professional development meetings and considerations for future implementation (see Figure 1). In the Pre-Studio Day meeting, the research team introduced Studio members to the focal MLR through a sample lesson enactment. For example, in Pre-Studio 2, the group participated in a lesson on comparing the average change in sea ice thickness using the Compare and Connect MLR. Compare and Connect engages students in comparing and contrasting the different mathematical approaches by examining different mathematical representations, approaches, examples, or language [14]. The group discussed considerations for their lessons, including one teacher asking about the possibility of using a gallery walk to enact the Compare and Connect, an idea raised during the sample lesson. Residents collaborated during the Pre-Studio Day with residents from their school sites and others across the district. Two residents volunteered to be the Studio teachers and teach the MLR during the Studio Day in two different lessons. We sought consent from parents/guardians and assent from students to study the Studio enactment.

On Studio Day, the two Studio teachers enacted an MLR-focused lesson in their classrooms, and the residents observed it. Teacher observations included engaging with students, asking questions, asking students to share, and sometimes sharing students' responses during small group and whole class discussions when appropriate. The teachers noted their observations and reflections during the lesson, specifically on the MLR. The research team then facilitated a debrief after each implemented lesson. The Studio Teachers and residents identified how they might modify their instruction related to the MLR and their work with their multilingual learners going forward.

During the Post-Studio Day, approximately a week after the Studio Day, teachers came together to reflect on their and their students' work completed during their Studio Day lesson. The Studio members discussed the MLR enactments, analyzed the student's work, viewed video clips from the studio clip enactments, shared challenges and successes, and considered the additional implications for their future practice. Selected video clips from Studio Day were a means to elaborate on the teaching event. As the final component of the Studio Day Cycle, all Studio members were encouraged to implement the MLR at least two more times during the school year. They were also encouraged to bring student work and reflections from their routine implementation to other PL opportunities, such as their professional learning communities (PLCs).

We focus on illustrative examples from Studio 2, which used the Compare and Connect routine. This MLR had a math and language goal of comparing and contrasting different mathematical approaches by examining the different mathematical representations, approaches, examples, or language [14]. While two teachers, Ms. Severn and Ms. Foster, taught Studio lessons in cycle 2, we focus on Ms. Severn's lesson in this manuscript. Ms. Severn noted that they had not performed this MLR previously. She explained that she would use the gallery walk.

Further, part of her goal in enacting the Studio lesson was as follows: How could she get the students to talk to each other about mathematics because the class was "really hesitant to talk at all?" While the gallery walk was an idea that a facilitator had shared during the Pre-Studio Day, Ms. Severn planned her lesson independently. Ms. Severn shared during the pre-brief of the Studio lesson that students would complete the following problem from Desmos Lesson 7.2, Lesson 11: "The two quantities are: Five friends shared three pizzas. The total came to \$36" [59] using a single representation (e.g., ratio, table of values, graph, or equation), which included additional prompts for their given representations (e.g., "Write the equation here."). Students were to post their solutions on poster boards to share them with the class. Ms. Severn planned to have the students rotate around the room and examine their peers' posters. She provided students with a graphic organizer (Figure 2), which asked them to record a ratio, table of values, graph, and equation [59]

from their peers’ solutions during the gallery walk. Students had four minutes to look at how their peers solved the problem and to record this work in the corresponding box on the graphic organizer.

desmos

Unit 7.2, Lesson 11: Four Representations

Name(s) _____

Lesson Synthesis

For each of the representations, describe how the group solved the problem.

Ratio	Table of Values
Graph	Equation

Figure 2. Ms. Severn’s Graphic Organizer for Compare and Connect. Note: Used with permission. Image taken from <https://teacher.desmos.com> (accessed on 15 May 2024).

3.3.2. Mesa Studio Cycle

In 2020–2021, the Mesa teachers used the state’s draft standards to infuse statistics and data reasoning into all their courses. In their work, the Mesa teachers drew on the expertise of the following colleagues who had strong statistics backgrounds: Tyson, the AP statistics teacher, who supported all the Math 10 teachers in developing units, and Benson, who volunteered to teach a data science course for third-year HS students in 2021–2022, the year following the study. Moreover, the Mesa Studio had transitioned over the years to embed innovative pedagogies, such as the *Five Practices for Orchestrating Productive Mathematical Discussions* [57], *Routines for Reasoning* [13], and routines that Mesa teachers designed to leverage mathematical modeling [68,69].

Similar to the Beachside’s Studio cycle described above, Mesa’s Studio cycle (See Figure 3) provided a structure that was a predictable set of elements centered on preparing for the live enactment by the Studio teacher with students and debriefing that event such that everyone had opportunities to take away insights. Before the Studio Day, Jasper and Georgia planned the Studio sequence of activities. They selected a Studio teacher based on the teacher’s and students’ willingness (parent/guardian consent and student assent) and the mathematical topic. The Studio teacher shifted based on who had teaching responsibilities during a time that allowed for Steps 1 and 2 in the cycle to be completed before the live enactment and whose students would profit from the content goals that could be leveraged in the Studio cycle.

In our results, our Mesa illustrative examples will focus on the two Studio cycles from 2020–2021. Three Studio cycles were initially scheduled for the year. However, two cycles were completed because of COVID-19 and the complexity of shifting from distance learning to opening schools for in-person teaching. Jasper and Georgia engaged in extensive Studio planning that was observed and recorded (Studio 1, three hours; Studio 2, four hours) before each Studio Day. Here, the facilitators designed the data literacy routine (described in the next paragraph) used in the two Studio cycles. Planning for Studio 2 included

interactions with the Studio teacher to co-plan. Studio 1 created opportunities for the Studio teacher to plan during the Studio Day.

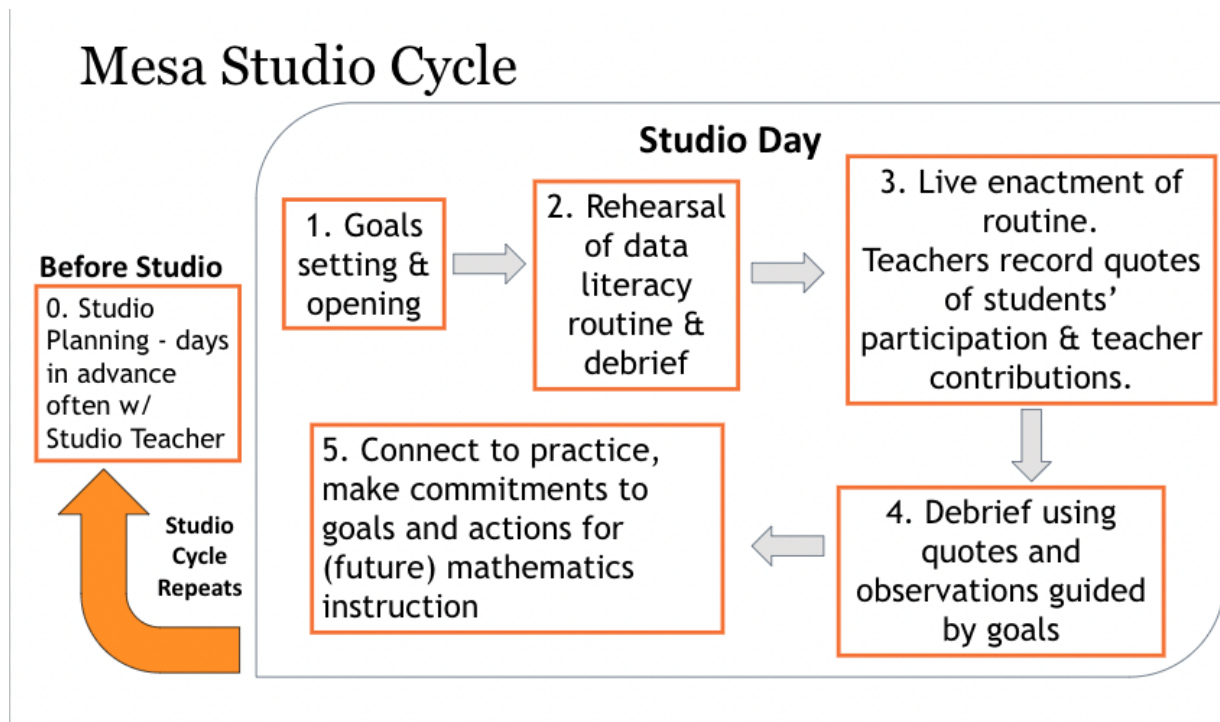


Figure 3. Mesa’s Studio Cycle.

The Studio opened with a presentation of goals and an agenda for the day, followed by some introductory discussion or activity leading to a routine rehearsal (two rehearsals in Studio 1 and one in Studio 2). The rehearsal allowed all the residents to experience the routine as participants and to debrief it to engage in the collective interpretation of teaching. These debriefing discussions were goal-focused and unpacked instructional decisions, considered rationales of decisions, anticipated students’ thinking, contemplated alternatives, and supported the Studio teacher in refining how to anticipate and lead the routine’s important mathematics moments. The Studio teacher then enacted the routine with high school students and residents closely observed, noting students’ participation, and guided by norms for recording specific contributions of students’ mathematical participation and teacher contributions.

Facilitators guided the notetaking and the debriefing, assuring that the goal of recording quotes was not to critique the Studio teacher. For example, Georgia noted, “We are not trying to critique, specifically not trying to make an exemplar.” These verbal reminders, which happened at the start of every debrief, framed the conversations as a space for critical inquiry and a way to build trust [67]. The opening of Studio 2’s enactment debrief also started with a statement from Georgia, “Teacher moves create the student moves”, and she provided reminders not to critique. This framing led us to look for patterns in our data and to uncover what we called a principle that, while unspoken, guided Studio members’ actions and responses. Their principle was that students’ ideas are sensible, and it was the teachers’ responsibility for all students to show their brilliance, exemplified by how they interpreted students’ contributions and critiqued lesson designs and prompts [15,16]. We offer more detail on this idea in the Results section and focus on it as part of Mesa’s collective interpretation of teaching.

The Mesa Studio enactment debriefs focused on the Studio members’ collective interpretation of teaching, which included attention to the students’ mathematical contributions and pedagogical moves concerning the routine’s learning goal. Facilitators were observed pressing the Studio members to consider how the routine supported students’ data literacy.

The enactment debrief was a second opportunity for the collective interpretation of teaching, with the routine typically happening in two phases. The first was an opportunity to build a shared understanding of the enactment and to provide feedback to the Studio teacher. The second phase shifted to connecting the routine to everyone's practice by discussing what "tweaks" the Studio teacher or residents would make to the routine. This pressed everyone to consider their teaching and collective commitments to enact the routine in the future. The data literacy routine was used in Studio 1 (November 2020, online, 4 h), where facilitator Georgia and Studio teacher Oliver each rehearsed, and Oliver enacted the routine online. The Studio teacher, Benson, used the data literacy routine in Studio 2 for in-person teaching (June 2021, in person, 5 h). As stated by the facilitators, the goal of the routine was to "support students' data literacy and support teachers in routine development that focuses on data literacy." At the time of this study, data literacy was a new K–12 content area rapidly emerging in education [61]. The Mesa facilitators were invested in data literacy and data science because the state was moving to rewrite standards for data reasoning (citation blinded for review). Between Studio 1 and 2, the Mesa faculty attended an online data science workshop from Youcubed [70]. There was an additional goal of considering the sociopolitical climate, which Jasper noted as he opened Studio 1's first rehearsal debrief, explaining the goal and their intention to the Studio members.

"We're trying to look at something twice and change your thinking. And, we felt like there's an urgency for that right now, especially if we talk about Coronavirus. If you show people a graph, are people comfortable being calm? So, it seems like there's an urgency for that, which is part of the reason for the routine. And, the second part was about having students be able to recognize the variables that they're paying attention to . . . are they able to make a connection between those things?"

Jasper referenced the United States' sociopolitical climate; when COVID-19 rates were skyrocketing, there was a contentious presidential election, and there was no COVID-19 vaccine. The need to understand data was a shared sentiment across the Studio members, evidenced in their buy-in to plan a routine and commitment to use it with students.

The data literacy routine involved using the Desmos platform [71] to support Studio members with teacher notes and a template that everyone could use for any data representation. The routine created three opportunities to examine a data representation (The routine was similar to slow reveal graphs: see [72]; however, the facilitators were not aware of this site at the time.) by slowly revealing the full representation to see and make sense of it. Prompts organized the teacher and students' interactions with the representation, asking students to "Notice and Wonder" and consider how their thinking had changed as the activity revealed the complete representation. The routine concluded with a set of sentence stems for reflection. In Studio 2, facilitators added a goal of "using the mathematical practices to pass the baton," meaning teachers would pay attention to when students were "meta-cognitive" that they were using mathematical reasoning. For example, when students changed their interpretation of a representation based on their data literacy, the residents' goal was to make them aware of this. Across the two Studio cycles, the Studio members participated in five routine enactments, three via rehearsals and two in classrooms with students.

4. Examples from Beachside and Mesa Studio

In this section of the manuscript, we build our conceptual argument by using a chain of evidence to answer our important question. We share examples of how Beachside's and Mesa's Studio cycles and routines catalyzed incremental learning. We use the Beachside example to bring to the forefront how the first practice, using rich representations of teaching, catalyzed incremental learning. We use the Mesa example to put the second practice, collective interpretations of teaching, at the forefront. In both examples, we examined the Studio structure of routines to catalyze incremental change; however, we focused on a single cycle in Beachside and multiple cycles at Mesa, exploring different types of routines at the sites. We share these examples to argue that Studio structures

and practices catalyzed incremental learning of ambitious and equitable teaching [7] and advanced pedagogical reasoning.

4.1. Beachside Studio Within a Cycle Highlighted a Rich Representation of Teaching

To illustrate Beachside's incremental learning, we share episodes from Studio 2—two from the Studio Day debrief and one from the Post-Studio Day. These episodes provide illustrations of the Studio teacher and the residents engaged with MLRs, with an explicit focus on attending to ambitious and equitable instruction. In this instance, the teachers considered the Compare and Connect MLR such that students could examine and discuss multiple representations. As such, the MLR allowed Studio members to examine ambitious and equitable instruction by focusing on instruction that could meet the needs of linguistically diverse students by providing avenues for rich language opportunities and cognitively demanding mathematics content [12]. We focused on different aspects of the gallery walk related to Ms. Severn's implementation of the MLR, with the Method section providing an initial preview of Ms. Severn's Studio lesson and this gallery walk. Teachers engaged with rich representations of teaching and collective interpretations of teaching during the Studio cycle, and the cycle and Compare and Connect routine were catalysts to foster the incremental learning of ambitious and equitable teaching. We begin with Ms. Severn's debrief of the Studio as an illustrative example of the structures and routines of the studio and the practice of using rich representations of practice.

Once the residents completed their observation form with questions and reflections (introduced in the method section), the debrief of the Studio lesson was further structured with questions from the research team posted on a PowerPoint. Ms. Severn began her debrief first. Some of the questions from the PowerPoint for Ms. Severn were as follows: How do you feel about the lesson? How, if at all, would you consider multilingual learners during the lesson? How did you adapt the MLR in real-time? And how might you modify your Compare and Connect going forward? Ms. Severn explained the following:

"I feel like I spent more time on the actual process [of the gallery walk] than I did on the conversation at the end. And so, I was trying to think of either, 'How can I cut down on some of the process of getting the posters made?' Or, 'How can I incentivize kids to speak a little, to be willing to share a little faster,' because that conversation took a long time to get voices going. And a lot of it was just me being like, 'Are you sure you don't want to share?' So yeah, I'm trying to think of how I can get them talking more? Or, how can I leave more room for that at the end, so that if it does take a long time, we still get to the rich conversation?"

(Debrief, 105–112)

Ms. Severn shared a teaching dilemma with her peers about how she might move her process of using the routine of Compare and Connect more quickly to support students to get to the mathematical conversations that she wanted students to have and how she might make more space for rich conversations as part of using the MLR with her students. She also returned to her original question posed in the pre-brief, which we noted in the Method section, of wanting to get the students to talk to each other about mathematics because the class was "really hesitant to talk at all".

In this Studio debrief, we began by noting some of the roles of structures to catalyze discussion. First, the Studio structure made space for debriefing the lesson and returning to ideas from earlier in the day, such as the goal Ms. Severn had stated in her pre-brief. Second, the routine structure became a focal component of Ms. Severn's reflection, related to how she might move forward with the routine more quickly and create rich mathematical conversations. Further, the structure of the routine grounded the conversation, with Ms. Severn considering the affordances and constraints of the MLR and how she might use the MLR to get students in the class talking while also using it more efficiently. Finally, regarding using rich representations of practice, Ms. Severn started the conversation with the rest of the participants by drawing on her representation of practice. She referred to her

Studio lesson, noting how she felt she spent more time on the process of the routine than on engaging students with the mathematical content of the routine. This initial debrief created a space for incremental learning through pedagogical reasoning for Ms. Severn because she felt comfortable with her peers; Ms. Severn was in a space of mutual trust to attend to this teaching dilemma and could honestly share her thoughts on her teaching from the Studio lesson. The Studio norms created space for Ms. Severn to share her thoughts about the lesson before others did so, honoring the expertise of the teacher who taught the Studio lesson. This norm positively positioned the Studio teacher and their public display of teaching, lifting the teacher's voice.

Following Ms. Severn's sharing, the facilitators opened the reflection to the residents, allowing the group to unpack the lesson. The group tackled Ms. Severn's question of how she could get students to talk more by discussing the value of having students examine multiple mathematical representations and using a mathematics task that allowed students to use multiple modes of communication [73] within their work with their peers. The group then discussed their role during Studio in visiting small groups, including the types of questioning they used with students to support reasoning and increase student dialogue about mathematics, such as by asking students to identify specific points on the graph. The group next attended to the time spent on the MLR routine. They discussed that Compare and Connect could be a short routine or used for an entire class period, as Ms. Severn had enacted. Ms. Hope (the district math specialist) initiated the following conversation.

Ms. Hope: "What I saw is that you took this MLR that some people might think of as small, but it was a lesson structure. And so, you went to the selection of the task, to the way that you work with teams, to the gallery walk, to the debrief. If that lesson structure became a routine, then some of this would speed up for you."

Ms. Severn: "That's true, getting them used to it."

Ms. Hope: "Right, so, that's where then the routine becomes routine. But this evidence of providing students opportunities to use multiple means of communication. ... And so, you created a lesson where you set up this time to, yes, time to listen and share, write, listen, and speak about what they had seen and written, right, and the written part of the representations. There's that lesson [that] captured a bunch of modes of communication, from reading, writing, speaking, listening, whatever. And so, that was the design of the whole lesson. And it was like, I didn't even realize that until I was writing this down. I'm like, oh, just look at how you hit all of those modalities. And if you did that again, and again, students will get better at it."

Ms. Severn: "Yeah, that's a good point. If we keep repeating this structure at least once per unit, they'll get used to it for sure."

Ms. Ruth: "The trick is finding the problem that allows for that structure. But it lent itself nicely because you did have these multiple—no, not entry points so much, but multiple representations or multiple ways of solving it."

Ms. Hope explained that the MLR could be a fast routine or a whole lesson structure, including working with groups, completing a gallery walk, and debriefing the work in the Compare and Connect. The group strategized how teachers could draw upon students' language and mathematical ideas in the MLR. They also considered ways to use the MLR as a complete lesson or a brief routine. Ms. Hope highlighted that in teaching a lesson with Compare and Connect, students would have the opportunity to engage with their multiple modes of communication to address Ms. Severn's concern about students communicating in class. Using this repeated structure within a lesson over time, students could develop familiarity with the structure and routine, supporting their work with the routine, the communication, and the mathematics. Ms. Ruth also raised the challenge of finding suitable tasks to complete this work [74].

The Studio structures of cycles and routines catalyzed learning during the debrief. The Studio structure of a cycle created a space for residents to revisit Ms. Severn's teaching

dilemma from earlier in the Studio cycle. Additionally, the gallery walk initially posed on Pre-Studio Day was a source of continued reflection and a rich representation of practice. During the lesson debrief, Ms. Severn and the other teachers considered it and the MLR. The debrief created opportunities for discussion around practice and supported unpacking the structure and role of Compare and Connect as an MLR, including the gallery walk. The routine was a catalyst for learning in multiple ways. In particular, the group highlighted that Compare and Connect helped illuminate multiple modes of communication [73], and they considered whether the routine might be helpful as a shorter or a whole-period routine, as Ms. Severn had enacted. Drawing on Ms. Severn's lesson as a rich representation of practice, for example, when Ms. Hope noted, "What I saw is that you took this MLR," the residents were taken back to Ms. Severn's classroom enactment during the debrief. In their conversations, there continued to be a mutual trust to unpack ideas around teaching practice. The residents worked together to consider a course of action for moving forward with the MLR—to make it a routine. The debrief asked residents to draw on evidence, including using rich representations of teaching in the Studio lesson and their observation protocols, to consider their observations, to interpret these events, to unpack teaching dilemmas, to develop collective understandings, and to consider a future course of action.

The final episode of the Beachside Compare and Connect Studio came from the Post-Studio Day, a few weeks following Ms. Severn's Studio lesson. At the beginning of the Post-Studio Day, the facilitator asked what the Studio teachers and residents had been thinking about related to the Studio cycle and Compare and Connect since Studio Day. Ms. Severn began by noting how much she enjoyed the routine. However, she was still wondering whether she should "just stick to doing routines that are shorter" (Post Studio, 88–89) because she did not "feel like I have time to do [Compare and Connect as a full day] every unit and throughout a whole day" (Post Studio, 86–87). Ms. Ruth followed by sharing that she had used Ms. Severn's enactment in her class before Thanksgiving.

"I did Ms. Severn's lesson on Thursday and Friday. It was, uh, supposed to be an easy day before Thanksgiving, right? We were just gonna do something that was super easy and didn't require a lot of prep. And, I, you know, I was like, I can do this. And, all I did is create a ton of work [before Thanksgiving]. . . And they were engaged. . . And, it was just really cool. So, thank you Ms. Severn for showing me that I can do it. And, I just took her lesson and implemented it in two days. And it was great. . . And, and well, it was thanks to this whole process. It was, like, 'I can do this!' And, it was really good. Nobody else [in my grade level] did it. I was the one who did it, because I was willing to put in the work."

(Post Studio, 113–143)

Ms. Ruth drew on Ms. Severn's lesson. She highlighted being able to see the lesson in the Studio lesson and how that implementation and seeing it during the Studio allowed her to confidently use Ms. Severn's lesson and the Studio Compare and Connect MLR in her classroom with her students. However, she also highlighted that this work was challenging, particularly before the holiday. Ms. Ruth's sentiments, perhaps, illuminate that the incremental work of ambitious, equitable pedagogy can be demanding. Even so, Ms. Ruth also reflected that "it was really good" and "thank you, Ms. Severn, for showing me that I can do it", illustrating that, sometimes, it can be helpful to have a model or a rich representation of practice to create an image of a new practice. The lesson engaged her students, which was a positive experience for Ms. Ruth. While she noted that it was some work to create the lesson, particularly before Thanksgiving and without any support at her site, she successfully tried something new through the Studio process. The Studio group provided a place of encouragement and support beyond Ms. Ruth's site-based team, where she could try out and debrief ambitious, equitable instruction. Ms. Ruth saw Ms. Severn develop a lesson that included the MLR Compare and Connect, try out the MLR, and publicly debrief the MLR. In this process, Ms. Ruth, with the confidence she gained through the Studio process, developed her version of a lesson that used Compare and

Connect, making it her own, and shared her practice with her peers on Post-Studio Day. In this example, Ms. Ruth exemplified incremental learning during Studio.

On the Post-Studio Day, the structure of the Studio allowed the teachers to return to the routine structure, Compare and Connect. The structure of Studio allowed the group to return to Ms. Severn's use of the gallery walk once more, this time, with Ms. Severn continuing to think about how she could make the routine a routine. They also heard about Ms. Ruth's enactment of Compare and Connect, using Ms. Severn's model. The structure of the routine and the practice of using rich representations of practice provided a shared focus across the conversations. In their work together, the Studio team used prompts from the facilitators to begin their discussions to focus on the Studio lesson, specifically around the Compare and Connect MLR. The group used the structure of the Studio and the rich representations of Ms. Severn's and Ms. Ruth's teaching to make sense of the Compare and Connect MLR. Ms. Ruth, drawing on Ms. Severn's prompt of having time for the Compare and Connect within a unit, shared her experience enacting Ms. Severn's lesson with her students and having such a positive experience, even if it was challenging at times. Ms. Ruth built on Ms. Severn's ideas and those developed during the Studio Day to plan her lesson and enact it with her students, interpreting the classroom enactment she witnessed and debriefed with her peers and developing a course of action. Ms. Ruth then shared her initial impressions of completing her Compare and Connect lesson, describing her representation of practice with her peers. Following Ms. Ruth's sharing, Ms. Severn, Ms. Ruth, and Ms. Hope further discussed how the gallery walk might be enacted aside from using the trifold boards that both Ms. Ruth and Ms. Severn had used, such as using portable whiteboards. Through his process of debriefing in the Post-Studio Day, we saw multiple opportunities for incremental learning, with Ms. Severn considering how to continue to develop her use of the MLR in her teaching with the support of her Studio team and with Ms. Ruth developing, enacting, and reflecting on her Compare and Connect lesson.

The space of the Studio allowed the teacher participants to unpack the seen and unseen aspects of practice in both Ms. Ruth's and Ms. Severn's practice, thereby informing the incremental learning of the group through their pedagogical reasoning [18]. Engaging in discussions during the debrief, guided by discussion prompts, provided a norm during the MLR Studio to guide the pedagogical reasoning, where the teacher participants were able to provide supportive feedback and to push on future practices, such as how to make Compare and Connect a routine and whether to make a gallery walk part of this practice. Teachers drew on their experiences in the Studio, using representations of practice to ground their conversations. The Compare and Connect routine, as a structure, helped to guide conversations. The routine was the lesson's focus in the Studio classroom, and the residents brought their discussions back to what had occurred in the classroom during both the Studio debriefs and the Post-Studio Day by attending to the rich representations of teaching. Teachers noted how they might make incremental changes, such as trying out the routine, as Ms. Ruth had, or using a different method for implementing the gallery walk, such as portable whiteboards.

4.2. Mesa Studio Across Cycles Highlighted the Collective Interpretation of Teaching

The Mesa Studio site highlighted how incremental learning happened across cycles via facilitators' press on Studio members to attend to the mathematics of data literacy, students' reasoning, and teachers' pedagogical moves. They did so by leveraging Studio's practices, building multiple rich representations of teaching using the same data literacy routine and collective interpretation of teaching to analyze the use of the routine. The Mesa site forefronts the Studio members' collective interpretation of teaching driven by their commitment to the principle that students' ideas are sensible, and it is the teachers' responsibility for all students to be able to use their brilliance. The data literacy routine explicitly took up this asset-based principle for student learning by asking teachers to recognize how students used descriptive data literacy language and narrate it for the whole group in ways that would make it accessible to others. We organized the examples in the

following manner. Episode one is Studio 1's discussions from three debriefs of the two rehearsals and the Studio lesson. Episode two is Studio 2's discussions from two debriefs of the rehearsal and the Studio lesson.

4.2.1. Episode 1

In Studio 1, we saw a sequence of facilitator moves and resident contributions aimed at the Studio members learning to lift students' contributions and narrate their data literacy. In this cycle, facilitator Georgia's rehearsal was the Studio members' first opportunity to experience the routine described in the methods. During the rehearsal, Jasper interjected, "I'm noticing that people are thinking about a couple different variables ... [and] the relationship between those ... I just wanted to appreciate the attempt to connect some of the variables that are happening in this [graph]." Jasper's comment lifted the goal for the residents to hear and link to ideas they were generating. The routine debrief opened with Jasper telling residents, "I popped in a couple of times to, [say] we're making connections between the variables that are happening there, and I want to make that transparent".

Recall Studio 1 had two rehearsals as described in the methods. The Studio teacher's rehearsal debrief also called Jasper into action driven by the Studio goals, "I'm going to pause this, [let's] talk more about what [Studio teacher] Oliver was saying to help kids focus on data literacy." Jasper, here, pressed the Studio members to consider the pedagogical moves that lifted the students' contributions and narrated their data literacy. However, we saw a limited uptake of Jasper's press, albeit the debrief was only 12 min long. Instead, the residents focused on the students' engagement, followed by the facilitators summarizing the mathematical and pedagogical ideas based on Oliver's strengths.

The routine rehearsal allowed Studio members to learn more about the routine's ambitious and equitable goals for student learning. Facilitators could formatively attend to the Studio teacher's and residents' learning as they heard and experienced the routine's goals in action. The routine's structure was introduced and unpacked during the rehearsal and debrief, which, similar to Beachside, ground the conversations in the students' brilliance by lifting the students' contributions. Finally, the collective interpretation of teaching in the rehearsal debriefs provided a space for incremental learning through building on teachers' ideas, such as Jasper's role in the debrief of lifting the instructional goal and lifting teachers' contributions, focusing on the collective understanding of the group.

In Studio 1's lesson debrief, residents replayed Oliver's responses such as, "Could you expand on ...", and "Wow, I love your analysis." Residents focused mostly on how Oliver socially scaffolded students' contributions [27]. Numerous Studio members commented on the students' brilliance, demonstrating their principled commitment to the students' ideas are sensible and that it is the teachers' responsibility for all students to show their brilliance. For example, when one student in the enactment conjectured about data beyond the representation, Tyson noted, "I thought that was awesome." Georgia commented on that student's contribution and the brilliance of other's contributions with exuberance, "The students are f**ing rad. How blown away were you guys by how well they interpreted that graph"?

Studio members replayed the students' contributions as assets; however, they did not specifically address how the ideas related to data literacy and routine goals were related. Zandra's comment linked the students' brilliance to the routine design, "They did amazing things that probably wouldn't have happened had they only seen that [one] graph," remarking on the benefits of the routine to reveal the representation slowly. Jasper moved to connect to the routine goal at one point and used a resident's question that pressed Oliver for a rationale to narrate how Oliver emphasized changes in the students' data interpretations, one feature of the routine's goal stressed at the start of Studio 1.

At this point in the Studio cycle, noticeably absent in the conversations were linkages between the pedagogical moves centering students' voices and the teachers' capacities to lift and narrate students' data literacy. Facilitators and Studio members grappled with their data literacy understanding catalyzed by the Studio's routine and practices. This was

confirmed by Georgia's remark reflecting on Studio 1, "I was hoping for more about what it means to be data literate. I'm not sure of that myself or if others really know what it means to be data literate. It's difficult to teach kids how to be data literate when we ourselves don't necessarily know." Jasper lamented, "I wanted to talk more about how to promote teachers to build on talk about [variables] with their kids, but we didn't quite get there". The collective interpretation of teaching involved generative attention to core features of pedagogical reasoning but fell short of advancing the routine goals.

4.2.2. Episode 2

In Studio 2, facilitators and residents focused on the same routine to build a rich representation of teaching and to collectively interpret it. The goal was to support students in data literacy and to support teachers in routine development on data literacy. Jasper and Georgia capitalized on the resident Justine's feedback from Studio 1 to add a goal of using the mathematical practices to pass the baton. This was a teacher learning goal asking the Studio members to make evident in their teaching and their reflections on teaching when the students were using mathematical practices. This included attention to data literacy. Studio 2's rehearsal debrief opened with Jasper saying, "I think we're still missing the moments when we are telling kids when they're getting better at data literacy." Studio teacher Benson agreed and presented a dilemma focused on students' access to the data representation. As a resident started to talk about her data literacy insights from the routine, Jasper quickly remarked, "I'm hearing good data scientists right now . . . but I think we got to be more meta with the kids," as though to counter Benson's dilemma that there was an accessibility issue.

With an invitation to all the Studio members to generate ways to lift students' contribution and to narrate their data literacy in the rehearsal debrief, a multivocal discussion unfolded in which everyone but the Studio teacher narrated when their data literacy ideas emerged and built off these examples to offer alternative "tweaks" to the routine. The collective debated various revisions, clarified student goals aligned to those revisions, anticipated students' responses to revised prompting, and rehearsed ways everyone might lift and narrate students' viable arguments for data literacy (i.e., a mathematical practice linked to data literacy). This discussion differed notably from Studio 1 because Studio members offered generative pedagogical alternatives linked to students' mathematical and contextual knowledge, pressed for specific goals, and analyzed the routine design connected to data literacy opportunities—their collective interpretations of teaching advanced teachers' pedagogical reasoning [15].

After nine minutes of Studio members' collective interpretations of teaching, Benson stepped back into the conversation. "I agree, there wasn't much depth. . . with the first image, we weren't getting very far. What I'm concerned about right now is this thing gonna stall out dead, and there's not gonna be a lot happening?" Benson was concerned that students would not have access to the ideas, and the conversations would fall flat. As the debrief was winding down, the following transcript provides an example of the group's collective interpretation of teaching regarding their future teaching. The principle, students' ideas are sensible, and it is the teachers' responsibility for all students to show their brilliance, underpinned how the Studio members grappled with the routine's design and goals.

1. Benson: *The time's coming up [for class]. I'm just trying to think about how to tweak this.*
2. Georgia: *What would we say is the purpose of the routine?*
3. Zandra: *For me, [it's] that your first moment of looking isn't enough. Keep looking.*
4. Benson: *Yeah, [the routine] draws your attention to what's missing.*
5. Jasper: *Which is making sense and persevering.*
6. Zandra: *And iteration.*
7. Georgia: *We want them to become data literate. So, if that's what the baton is, can we see the baton in action?*
8. Tyson: *If data literacy is the baton, and not the practice, then I feel like, you want to give them everything and see if they will make that connection. If the baton is the practice, critique and*

- debate the same and different. Some kids are going to look at that and still not be able to make sense of it, especially if they are not [aware of] sports [the context of the second use of the routine]. Other kids are gonna look at it in two seconds and know what's going on.*
9. *Benson: Why are we so concerned about passing three batons [math practice 1, 3, and data literacy] in 20 min? When we don't even do data science in the first place!*
 10. *Brooke: We just need to know what the goal is though!*
 11. *Jasper: All I'm saying is, I naturally tend to, and I think we all tend to, leave the data literacy baton off.*
 12. *Benson: Which I am fine with.*
 13. *Georgia: Why don't we try?*
 14. *Jasper: Right, that is why we are here, to try it, because it doesn't have to be an exemplar.*
 15. *Benson: Just tell me before the bell! [the group erupts in laughter]*

In this excerpt, Studio members were engaged in the collective interpretation of teaching with future teaching imminent (lines 1 and 15). Based on his previous remarks, Benson framed his dilemma as a design “tweak” to ensure students’ access (line 1). The facilitators reframed the dilemma to be about clarifying goals, enacting the foundational principle, students’ ideas are sensible, and it is the teachers’ responsibility for all students to show their brilliance [14]. The residents offered ideas (line 6) and unpacked alternative goals (line 8), and the facilitators pressed (lines 2, 7, 11, 13, 14) Benson on a data literacy goal. They reminded Benson and the residents that the Studio was a place to try new ideas. Benson specified his concern (line 9) that the routine had too many goals and, in a cheeky way that elicited laughter, he asked for direction (line 15). As the debrief wrapped up, residents and Benson reviewed the pedagogical moves to support the students’ mathematical ideas. In response, Benson proposed a strategy to attend to data literacy, which he thought he could accomplish.

Studio 2’s lesson debrief demonstrated the teachers’ incremental learning as multiple residents evaluated where “the baton” was passed and the students’ data literacy brilliance shined. This was a shift from Studio 1, when only the facilitators offered this evaluation. Justine said, “The moment that it happened was when the reflection prompt changed into doing. All of a sudden, they all started talking about the things we would want them to do every time they look at graphs.” Residents and facilitators effusively noted the students’ brilliance across Benson’s teaching. After discussing revising the routine, Benson commented, “I agree with Brooke; I would try [next time] to talk about the variables in each graph. I think that is the inroad on it, and that didn’t come out.” Benson showed a change in his understanding of the goal of the routine that echoed many of his colleagues’ collective interpretations of teaching. Benson acknowledged the value of lifting the students’ data literacy contributions and narrating them. Studio 2 debrief documented Benson’s and the residents’ shifting pedagogical reasoning associated with the routine [26]. In Studio 1, residents focused on student engagement and pedagogy without explicitly finding ways to leverage the students’ data literacy. In Studio 2, Studio members concentrated on strengthening the routine for future teaching so that the students’ brilliance was evident every time.

We noted a difference in the collective interpretation of teaching in Studio 1 and 2 debriefing. We hypothesized that this may be due to several factors. Studio teacher Oliver was a first-year teacher, and Benson was at the end of his ninth year. There were also differences in the level of the facilitators’ and residents’ press for specificity and rationale [16]. In Studio 1, the facilitators primarily took on the press, interpreting the routine goals and learning opportunities. In Studio 2, the press interpreting the routine goals was shared by facilitators and residents. Studio cycles that used the same routine catalyzed the members’ incremental learning. Through Studio members’ collective interpretation of teaching, we saw their principled commitment to the students’ ideas are sensible, and it is the teachers’ responsibility for all students to show their brilliance underpin the ways debriefs evolved across the Studio cycles. In Studio 1, the Studio members noted the students’ brilliance, and the facilitators invited, pivoted, and reminded residents of the routine’s goal to lift

students' contributions and leverage their data literacy [28,38]. Residents took seriously their responsibility to center students' brilliance by elevating their voices and appreciating their contributions [35]. In Studio 2, we saw incremental shifts in how residents took up their teacher responsibilities, and brilliance was linked to explicit pedagogical work they needed to do in the routine to deepen students' data literacy [15,21,61].

5. Discussion and Conclusions

We aimed to offer a conceptual argument on how Studio structures and practices catalyze the incremental learning of ambitious and equitable teaching. We offer these ideas building on the research documenting how practice-focused PL models, such as lesson study and teaching cycles reviewed above, are opportunities for the incremental learning of ambitious teaching. In particular, these PL models employ cycles of teacher learning focused on rich representations of teaching and collective interpretation of teaching, structures, and practices also employed by Studio. We also suggest an additional element is necessary, a routine aimed explicitly to disrupt inequities that we posit is essential for incremental learning of ambitious and equitable teaching. The following sections discuss how the illustrative Studio structures and practices catalyzed incremental learning in the Beachside and Mesa Studios. We share insights for research and practice connected to the themes of the Special Issue and the limitations of our ideas.

5.1. Studio Structures for Incremental Learning

Beachside Studio, examined through a single cycle, and Mesa Studio, examined across multiple cycles, were critical structures for the Studio teachers and residents to collaborate on a shared bounded instructional routine. Cycle elements were repeated structures that the residents could depend on to support them in trying new ideas within a trustworthy yet generative space for learning. Consistent with previous research, trying new ideas was not an isolated task but, like at Beachside, meant that residents would support the Studio teacher's experiment, and the innovation could spread to other classrooms, where there was an opportunity to then learn from each other because of the collective experience [24]. Further, the dilemmas of teaching, which drove the MLR's innovative uses, were opportunities for collective support and sensemaking.

Similarly, at Mesa, the Studio teacher's public display of practice meant that residents could take up dilemmas that arose to design more robust opportunities for student learning in the Studio enactment. The Studio teacher was not on display, and teaching was not to be critiqued; their teaching and the classroom was a working space for learning about students' reasoning, mathematics teaching, pedagogical moves, and creating opportunities to "try something new." While the examples we highlighted were similar to previous research on Studio teacher conversations linked to future practice [4,41], we argue that Studio structures and practices catalyzed incremental learning, as evidenced in the spread of ambitious pedagogies and changes in teachers' pedagogical reasoning [9,15,26,44,45].

Although the Beachside and Mesa routines differ, there were commonalities—the MLRs and data literacy routine shaped teacher and student interactions during the studio lessons, and they both attended to students' mathematical language production. At Beachside, integrating the gallery walk into the Compare and Connect routine was a strategy for the students to share their solutions and record solutions to support student learning. Ms. Severn aimed for the gallery walk to facilitate whole group discussion and support students' willingness to speak. While challenges arose, the opportunity to use the MLR and gallery walk uncovered for the Studio members the complexity of supporting students' mathematical and linguistic learning and the interactions among the students [35,73]. Because multiple Studio members enacted the same MLR and used the gallery walk, there were opportunities to unpack that complexity. With the lack of Studio research in the field, Beachside's illustrative examples offer existence proof to show how Studio structures and practices are opportunities for incremental learning that advances ambitious and equitable

mathematics teaching for multilingual learners [34]. Our argument offers insights into how MLRs were used by practicing teachers [5].

At Mesa, Studio members experienced five representations of the data literacy routine to interpret and unpack these repeated opportunities across the routine rehearsals and enactments. These were structural catalysts for the Studio members' shifting interpretations of teaching and their incremental learning [55,56]. The data literacy routine's ambitious goals catalyzed Studio members to build capacity incrementally; they learned how to lift students' data literacy contributions and narrate them in detracked classrooms, thereby taking up ambitious and equitable teaching. Beachside's and Mesa's cycles and routines catalyzed incremental learning, aligning with the limited body of routine research documenting teacher learning [53–56]. Our illustrative Beachside and Mesa examples suggest that not only are cycles critical for the incremental learning of ambitious teaching but routines explicitly focused on disrupting inequities are essential elements of classroom-embedded PL models to catalyze teacher learning of ambitious and equitable teaching [38]. While this may be logically obvious, our illustrative examples provide a proof of concept.

5.2. Studio Practices for Incremental Learning

Studio practices were featured in both Studio sites. Beachside's examples of incremental learning within the cycle with the MLRs demonstrated how the rich representations of teaching catalyzed the Studio teacher's and residents' ambitious and equitable teaching. Ms. Severn's lesson with the MLR offered a rich representation of teaching as a foundation to which others could reason and add based on their experiences and expertise [62]. The Mesa Studio examples of incremental learning across cycles demonstrated how their collective interpretation of teaching was grounded in their principle of students' brilliance and catalyzed the Studio members' ambitious and equitable teaching. Across cycles, Mesa Studio members shifted their focus from social scaffolds supporting students' contributions [27] to deeply engaging in how heterogeneous groups of students might show their data literacy, and teachers might make that evident [38,39]. Our argument that Studio practices catalyze incremental teacher learning of ambitious and equitable teaching aligns with research that asks teachers to disrupt inequities for the most minoritized students and to disrupt systems that create barriers, such as tracking, to access robust mathematics [19,20,28–32]. We framed the Studio practice of collective interpretation of teaching as a form of pedagogical reasoning that involved the Studio members collaboratively focusing on generative teaching dilemmas [20]. These dilemmas were rich in detail, linked to the shared representations of teaching, and fostered the elaboration of evidence, explanation, and rationale to interpret classroom events. Similar to the previous research on teacher conversations, our examples document how the Studio practices led to action and future teaching [16,20,26]. The MLR guided the Beachside Studio's teaching dilemma and, as they worked on it, it also honed the MLR's development. Across elements of one cycle, Studio members engaged in pedagogical reasoning by interpreting classroom events, building on each other's ideas, unpacking dilemmas, and considering future courses of action [16,25,26] to create opportunities to support multilingual learner students. Studio members fostered collegiality, leveraged students' strengths, and attended to language and mathematical goals [14,35,73]. These incremental changes allowed teachers to enact more ambitious and equitable teaching. Mesa Studio's illustrative examples from across cycles on the same routine shifted the nature of Studio members' pedagogical reasoning, honing how they made sense of students' data literacy contributions and how they could lift and narrate students' data literacy. Their pedagogical reasoning led them to consider routine revisions to ensure the students could show their data literacy brilliance using explanations and evidence, building on each other's ideas to develop a collective understanding that would affect future practice [4,16,26,44,74].

We offered two illustrations of our data and examined the underpinnings of Studio members' pedagogical reasoning. Both examples showed how the Studio cycles and the use of a routine supported the Studio members learning from one another through

the shared rich representations of teaching, collective interpretations of teaching, and subsequent opportunities to plan and teach using the routine. Further, by arguing that ambitious and equitable teaching happens through incremental teacher learning catalyzed by Studio structures and practices, we offer an existence proof that ambitious teaching and incremental learning are not mutually exclusive constructs. Instead, incremental learning is a means for taking up ambitious and equitable teaching.

5.3. *Connections to the Special Issue and Implications for Research and Practice*

This paper contributes to the following three areas highlighted in the Special Issue of Educational Sciences: professional development for secondary mathematics instruction, innovative models of professional development, and incremental professional development models. Studio is an innovative PL model supporting secondary mathematics instruction using structures that have been successful for incremental learning [4,23,42]. The Studio model, like lesson study and other practice-focused models that center on the live enactment of teaching [75,76], is widely used in the U.S. The slim yet compelling body of small studies embodies the dimensions of high-quality PL identified in larger-scale studies [77]. As Studio studies are still emerging, it is understandable that small studies are the norm. Drawing from the two Studio study sites, our conceptual argument contributes to unpacking Studio mechanisms that enhance teacher learning [9].

We built a chain of evidence to conceptually argue that Studio structures and practices can catalyze incremental teacher learning of ambitious and equitable teaching. We highlight two research implications from the ideas we raised. First, research that investigates Studio and other practice-focused PL models would need complex and nuanced methodologies to trace the residue of teachers' incremental learning of ambitious and equitable teaching, which often takes the form of changes in pedagogical reasoning and incremental change in instruction across different models [78]. Tracing pedagogical reasoning into practice, including attention to how instruction mitigates inequitable practices (e.g., minoritized students' mathematical learning and participation), is an emerging research area. There are many ways to frame disrupting inequities and define equity [22,39,41]. Such a design would, by necessity, be longitudinal and highly contextualized because disrupting classroom inequities is tied to the specific social characteristics of students [34]. A second research implication from our conceptual argument links to future Studio research. Future research should examine cross-case analyses of Studio projects that could empirically support or refute how structures and practices catalyze the incremental learning of ambitious and equitable teaching. Further, Studio studies across sites could build an empirical case for how Studio models differ and what infrastructures are necessary to sustain Studio.

Practically, we have provided the implications for the facilitators of practice-focused models of PL, like Studio. Our paper highlighted the pivotal role of repeated and predictable structures, such as the Studio cycle and routines, in supporting teacher learning. It also provided images of the practices needed to attend to teachers' pedagogical reasoning, including creating space for rich representations of teaching and collective interpretations of teaching. Selecting a routine with goals to disrupt inequities intentionally is critical to supporting incremental teacher learning of ambitious and equitable teaching. For school-based coaches who may lead professional learning communities of teachers, adaptations of these structures and practices could include identifying a shared routine to disrupt a pressing inequity that all teachers might use across a year. Observations and recording rich representations of teaching could then serve as a foundation to facilitate the collective interpretation of teaching. We argue that the Studio's routines and practices we put forward are necessary elements to support the incremental learning of ambitious and equitable teaching. However, we recognize that they may still be insufficient without the critical role of the skilled facilitator in guiding PL goal-directed activities and facilitating the teachers' pedagogical reasoning while being responsive to the teachers' incremental learning [75,76,79–81].

The limitation of this conceptual manuscript is that our examples are based on limited images from the two Studio sites. As a result, while our examples are rich, we do not

claim that every Studio PL community will support incremental learning. Nor do we suggest that the structures and practices we examined, common to other practice-focused PL models, such as lesson study [6,43–45] or Math Labs [46,47,51], would necessarily catalyze incremental learning. We also did not explore all of Studio’s structures and practices to determine how they might catalyze incremental learning. For example, we did not foreground practices such as norm-setting that would guide the teachers’ generative and collective interpretations of teaching rather than the critique of teaching [64,75]. As noted in our practical implications, we did not highlight the critical role skilled facilitation plays in fostering incremental learning of ambitious and equitable teaching nor track how teachers’ pedagogical reasoning translated into long-term use of classroom practices [25,68,76,79–81]. Future research provides opportunities for exploring teachers’ incremental learning of ambitious and equitable teaching within the context of Studio in the ways we have suggested above.

In this paper, we argue that Studio structures and practices catalyze learning. Like other practice-focused PL models, Beachside’s and Mesa’s teachers had the opportunity to improve their pedagogical reasoning, including pedagogical innovations and ambitious and equitable mathematical goals in the routines they were taking up as part of their practice. PL communities that deprivatize practice through rich representations of teaching and the collective interpretation of learning focused on content, student learning, pedagogy, and future practice are more likely to improve teaching knowledge and practice and enhance student learning in both large-scale studies [77,79,82] and case studies [76], similar to the Beachside and Mesa studies drawn upon for this manuscript.

Lave [9] called us to explore the mechanisms for learning, and the field has built compelling frameworks and investigated PL models, taking up her call [6,7,15,44–46,50,51]. Our paper builds on what Borko et al. [10,11] have called important questions for the field, that uncover constructs for further investigation. Studio’s structures and practices are mechanisms for incremental teacher learning and ambitious and equitable teaching. They offer another PL model for further investigation.

Author Contributions: Rebekah Elliott and Sarah A. Roberts contributed equally to all the manuscript components including conceptualization, writing—original draft preparation, and writing—review and editing. All authors have read and agreed to the published version of the manuscript.

Funding: Rebekah Elliott’s project was funded by CPM Educational Program, Elliott Award #20-0302. Sarah A. Roberts’ project was funded by the National Science Foundation, Award #2144027. The authors would like to thank the funder; the results or conclusions are of the authors, not the funding agent.

Institutional Review Board Statement: All subjects gave their informed consent for inclusion before they participated in the study. The study was conducted in accordance with the Declaration of Helsinki, and the protocol was approved by the Oregon State University, protocol number IRB 2020-0534, and by the University of California, Santa Barbara, protocol number 15-24-0614.

Informed Consent Statement: Informed consent was obtained from all subjects in the study.

Data Availability Statement: The datasets presented in this article are not readily available because of human subject agreements with participants and institutional review boards.

Acknowledgments: The Mesa author thanks the support of the Mesa teachers, facilitators, Mesa principal, and former graduates and undergraduate assistants working on the project. The Beachside author thanks the support of the Beachside teachers and former and current graduate and undergraduate assistants working on the project.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Otten, S.; de Araujo, Z.; Candela, A.G.; Vahle, C.; Stewart, M.E.; Wonsavage, F.P. Incremental Change as an Alternative to Ambitious Professional Development. In *Proceedings of the Forty-Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*; Middle Tennessee State University: Murfreesboro, TN, USA, 2022.
2. Jackson, K.; Cobb, P. Refining a Vision of Ambitious Mathematics Instruction to Address Issues of Equity. In *Proceedings of the Annual Meeting of the American Educational Research Association*, Denver, CO, USA, 30 April–4 May 2010.

3. Koellner, K.; Jacobs, J. Distinguishing Models of Professional Development: The Case of an Adaptive Model's Impact on Teachers' Knowledge, Instruction, and Student Achievement. *J. Teach. Educ.* **2015**, *66*, 51–67. [CrossRef]
4. Lesseig, K. Investigating Mathematical Knowledge for Teaching Proof in Professional Development. *Int. J. Res. Educ. Sci.* **2016**, *2*, 253–270. Available online: <https://eric.ed.gov/?id=EJ1105101> (accessed on 15 August 2024). [CrossRef]
5. Von Esch, K.S.; Kavanagh, S.S. Preparing Mainstream Classroom Teachers of English Learner Students: Grounding Practice-Based Designs for Teacher Learning in Theories of Adaptive Expertise Development. *J. Teach. Educ.* **2018**, *69*, 239–251. [CrossRef]
6. Fernandez, C. Learning from Japanese Approaches to Professional Development: The Case of Lesson Study. *J. Teach. Educ.* **2002**, *53*, 393–405. [CrossRef]
7. Brehmer, D.; Ryve, A. Facilities for Mathematics Teachers' Learning from Professional Development Programmes: A Qualitative Systematic Review. *Math. Teach. Educ. Dev.* **2024**, *26*, 2. Available online: <https://mtd.merga.net.au/index.php/mtd/article/view/870> (accessed on 15 August 2024).
8. Merriam-Webster. Available online: <https://www.merriam-webster.com/> (accessed on 1 August 2024).
9. Lave, J. Teaching, as Learning, in Practice. *Mind Cult. Act.* **1996**, *3*, 149–164. [CrossRef]
10. Borko, H.; Koellner, K.; Jacobs, J.; Seago, N. Using video representations of teaching in practice-based professional development programs. *ZDM* **2011**, *43*, 175–187. [CrossRef]
11. Borko, H.; Jacobs, J.; Seago, N.; Mangram, C. Facilitating video-based professional development: Planning and orchestrating productive discussions. In *Transforming Mathematics Instruction*; Li, Y., Silver, E.A., Li, S., Eds.; Springer International: Berlin, Germany, 2014; pp. 259–281.
12. DeBarger, A.H.; Penuel, W.R.; Harris, C.J.; Schank, P. Teaching Routines to Enhance Collaboration Using Classroom Network Technology. In *Techniques for Fostering Collaboration in Online Learning Communities: Theoretical and Practical Perspectives*; IGI Global: Hershey, PA, USA, 2011; pp. 224–244.
13. Kelemanik, G.; Lucenta, A.; Creighton, S.J.; Lampert, M. *Routines for Reasoning: Fostering the Mathematical Practices in All Students*; Heinemann: Portsmouth, NH, USA, 2016.
14. Zwiers, J.; Dieckmann, J.; Rutherford-Quach, S.; Daro, V.; Skarin, R.; Weiss, S.; Malamut, J. *Principles for the Design of Mathematics Curricula: Promoting Language and Content Development*; UL/SCALE; Stanford University: Stanford, CA, USA, 2017; Available online: https://ul.stanford.edu/sites/default/files/resource/2021-11/Principles%20for%20the%20Design%20of%20Mathematics%20Curricula_1.pdf (accessed on 30 August 2024).
15. Horn, I.; Garner, B. *Teacher Learning of Ambitious and Equitable Mathematics Instruction: A Sociocultural Approach*; Routledge: England, UK, 2022.
16. Horn, I.S.; Garner, B.; Kane, B.D.; Brasel, J. A Taxonomy of Instructional Learning Opportunities in Teachers' Workgroup Conversations. *J. Teach. Educ.* **2017**, *68*, 41–54. [CrossRef]
17. Dexter Torti, C.; Roberts, S.A.; Bianchini, J.A. Mathematics teachers' perceptions of their instruction for multilingual learners through professional development experiences. In *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*; Sacristán, A.I., Cortés-Zavala, J.C., Ruiz-Arias, P.M., Eds.; Mazatlan: Sinaloa, Mexico, 2020; pp. 1853–1857. [CrossRef]
18. Greeno, J.G.; Engeström, Y. Learning in activity. In *The Cambridge Handbook of the Learning Sciences*, 2nd ed.; Sawyer, R.K., Ed.; Cambridge University Press: Cambridge, UK, 2006; pp. 79–96.
19. Horn, I.; Garner, B.; Chen, G.A. How Concepts Change as Teachers Learn Ambitious and Equitable Instruction. In *Teacher Learning of Ambitious and Equitable Mathematics Instructions: A Sociocultural Approach*; Horn, I., Garner, B., Eds.; Routledge: England, UK, 2022; pp. 25–54. [CrossRef]
20. Loughran, J. Pedagogical Reasoning: The Foundation of the Professional Knowledge of Teaching. *Teach. Teach.* **2019**, *25*, 523–535. [CrossRef]
21. Boaler, J.; Staples, M. Creating Mathematical Futures Through an Equitable Teaching Approach: The Case of Railside School. *Teach. Coll. Rec.* **2008**, *110*, 608–645. [CrossRef]
22. Joseph, N.M.; Hailu, M.F.; Matthews, J.S. Normalizing Black Girls' Humanity in Mathematics Classrooms. *Harv. Educ. Rev.* **2019**, *89*, 132–155. [CrossRef]
23. Horn, I.S.; Kane, B.D. Opportunities for Professional Learning in Mathematics Teacher Workgroup Conversations: Relationships to Instructional Expertise. *J. Learn. Sci.* **2015**, *24*, 373–418. [CrossRef]
24. Lefstein, A.; Vedder-Weiss, D.; Segal, A. Relocating Research on Teacher Learning: Toward Pedagogically Productive Talk. *Educ. Res.* **2020**, *49*, 360–368. [CrossRef]
25. Loughran, J.; Keast, S.; Cooper, R. Pedagogical Reasoning in Teacher Education. In *International Handbook of Teacher Education*; Loughran, J., Hamilton, M.L., Eds.; Springer: Berlin/Heidelberg, Germany, 2016; pp. 387–422. [CrossRef]
26. Horn, I. Teachers Learning Together: Pedagogical Reasoning in Mathematics Teachers' Collaborative Conversations. *Proc. 12th Int. Congr. Math. Educ.* **2015**, *52*, 33342. Available online: http://link.springer.com/chapter/10.1007/978-3-319-17187-6_19 (accessed on 30 August 2024).
27. Baxter, J.A.; Williams, S. Social and analytic scaffolding in middle school mathematics: Managing the dilemma of telling. *J. Math. Teach. Educ.* **2010**, *13*, 7–26. [CrossRef]
28. Gutiérrez, R. Embracing the Inherent Tensions in Teaching Mathematics from an Equity Stance. *Democr. Educ.* **2009**, *18*, 9–15.

29. Gamoran, A. In High School Math, More Instructional Time Helps, but the Tracking Dilemma Remains. *Proc. Natl. Acad. Sci. USA* **2021**, *118*, e2109648118. [CrossRef] [PubMed]
30. Mosqueda, E. Compounding Inequalities: English Proficiency and Tracking and Their Relation to Mathematics Performance among Latina/o Secondary School Youth. *J. Urban. Math. Educ.* **2010**, *3*, 57–81. Available online: <https://journals.tdl.org/jume/index.php/JUME/article/view/47> (accessed on 30 August 2024). [CrossRef]
31. Thompson, K.D. What Blocks the Gate? Exploring Current and Former English Learners' Math Course-Taking in Secondary School. *Am. Educ. Res. J.* **2017**, *54*, 757–798. [CrossRef]
32. Iddings, A.C.D. Linguistic Access and Participation: English Language Learners in an English-Dominant Community of Practice. *Biling. Res. J.* **2005**, *29*, 165–183. [CrossRef]
33. Planas, N.; Gorgorió, N. Are Different Students Expected to Learn Norms Differently in the Mathematics Classroom? *Math. Educ. Res. J.* **2004**, *16*, 19–40. [CrossRef]
34. Herbal-Eisenmann, B.; Shah, N. Detecting and Reducing Bias in Questioning Patterns. *Math. Teach. Middle Sch.* **2019**, *24*, 282–289. [CrossRef]
35. Celedón-Pattichis, S.; Borden, L.B.; Pape, S.J.; Clements, D.H.; Peters, S.A.; Males, J.R.; Chapman, O.; Leonard, J. Asset-Based Approaches to Equitable Mathematics Education Research and Practice. *J. Res. Math. Educ.* **2018**, *49*, 373–389. [CrossRef]
36. National Council of Teachers of Mathematics. Available online: https://www.nctm-org.webpkgcache.com/doc/-/s/www.nctm.org/uploadedFiles/Standards_and_Positions/Position_Statements/Opportunity%20Gap.pdf (accessed on 30 August 2024).
37. Jansen, A. *Rough Draft Math: Revising to Learn*; Routledge: New York, NY, USA, 2023.
38. Hunter, J.; Hunter, R.; Anthony, G. Shifting Towards Equity: Challenging Teacher Views about Student Capability in Mathematics. *Math. Educ. Res. J.* **2020**, *32*, 37–55. [CrossRef]
39. Liljedahl, P. *Building Thinking Classrooms in Mathematics, Grades K-12: 14 Teaching Practices for Enhancing Learning*; Corwin Press: Thousand Oaks, CA, USA, 2021.
40. Louie, N.L. The Culture of Exclusion in Mathematics Education and Its Persistence Inequity-Oriented Teaching. *J. Res. Math. Educ.* **2017**, *28*, 488–519. [CrossRef]
41. Roberts, S.A.; Bianchini, J.A.; Pillsbury-Fischler, J. Supporting the development of adaptive expertise through distributed leadership: An examination of a mathematics specialist's work with math 1 teachers. (under review).
42. Carlson, M.A.; Heaton, R.; Williams, M. Translating Professional Development for Teachers into Professional Development for Instructional Leaders. *Mathematics Teacher Educator*. **2017**, *6*, 27–39. [CrossRef]
43. Gibbons, L.K.; Lewis, R.M.; Nieman, H.; Resnick, A.F. Conceptualizing the work of facilitating practice-embedded teacher learning. *Teach. Teach. Educ.* **2021**, *101*, 103304. [CrossRef]
44. Kazemi, E.; Ghouseini, H.; Cordero-Siy, E.; Prough, S.; McVicar, E.; Resnick, A.F. Supporting teacher learning about argumentation through adaptive, school-based professional development. *ZDM* **2021**, *53*, 435–448. [CrossRef]
45. Thompson, J.J.; Hagenah, S.; McDonald, S.; Barchenger, C. Toward a practice-based theory for how professional learning communities engage in the improvement of tools and practices for scientific modeling. *Sci. Educ.* **2019**, *103*, 1423–1455. [CrossRef]
46. Kim, H.J.; Metzger, M.; Heaton, R.M. Teacher planning sessions as professional opportunities to learn: An elementary mathematics teacher's re-conceptualization of instructional triangles. *Int. J. Sci. Math. Educ.* **2020**, *18*, 1207–1227. [CrossRef]
47. Lai, Y.; Carlson, M.A.; Heaton, R.M. Giving reason and giving purpose. In *Mathematics Matters in Education: Essays in Honor of Roger Howe*; Howe, E., Li, Y., Lewis, W.J., Madden, J.J., Eds.; Springer International Publishing: Cham, Switzerland, 2018; pp. 149–171.
48. Morris, A.K.; Hiebert, J. Effects of teacher preparation courses: Do graduates use what they learned to plan mathematics lessons? *Am. Educ. Res. J.* **2017**, *54*, 524–567. [CrossRef]
49. Ebby, C.B.; Hess, B.; Pecora, L.; Valerio, J. Facilitating collaborative inquiry into practice around artifacts of mathematics teaching. *J. Math. Teach. Educ.* **2024**, 1–24. [CrossRef]
50. Lewis, J.M. Learning to lead, leading to learn: How facilitators learn to lead lesson study. *ZDM* **2016**, *48*, 527–540. [CrossRef]
51. Amador, J.M.; Gillespie, R.; Carson, C.; Kruger, J. Online teaching labs: Changes in design and facilitation for teacher learning in synchronous professional development. *Prof. Dev. Educ.* **2021**, 1–17. [CrossRef]
52. Lindvall, J.; Ryve, A. Coherence and the Positioning of Teachers in Professional Development Programs: A Systematic Review. *Educ. Res. Rev.* **2019**, *27*, 140–154. [CrossRef]
53. Pentland, B.T.; Feldman, M.S. Organizational Routines as a Unit of Analysis. *Ind. Corp. Change* **2005**, *14*, 793–815. [CrossRef]
54. Leinhardt, G.; Steele, M.D. Seeing the Complexity of Standing to the Side: Instructional Dialogues. *Cogn. Instr.* **2005**, *23*, 87–163. [CrossRef]
55. Thanheiser, E.; Melhuish, K. Teaching Routines and Student-Centered Mathematics Instruction: The Essential Role of Conferring to Understand Student Thinking and Reasoning. *J. Math. Behav.* **2023**, *70*, 101032. [CrossRef]
56. Shaughnessy, M.; Ghouseini, H.; Kazemi, E.; Franke, M.; Kelley-Petersen, M.; Hartmann, E.S. An Investigation of Supporting Teacher Learning in the Context of a Common Decomposition for Leading Mathematics Discussions. *Teach. Teach. Educ.* **2019**, *80*, 167–179. [CrossRef]
57. Smith, M.S.; Stein, M.K. *Five Practices for Orchestrating Productive Mathematics Discussions*; National Council of Teachers of Mathematics: Reston, VA, USA, 2011.
58. National Governors Association (NGA). *Center for Best Practices and the Council of Chief State School Officers*; Common Core State Standards for Mathematics; NGA: Washington, DC, USA, 2010.

59. Amplify. Available online: <https://teacher.desmos.com> (accessed on 30 August 2024).
60. Philips, S.U. *The Invisible Culture: Communication in Classroom and Community on the Warm Springs Indian Reservation*; Waveland Press: Long Grove, IL, USA, 1992.
61. National Academies of Sciences, Engineering, and Medicine. *Foundations of Data Science for Students in Grades K-12: Proceedings of a Workshop*; The National Academies Press: Washington, DC, USA, 2023. [CrossRef]
62. Horn, I.S.; Little, J.W. Attending to Problems of Practice: Routines and Resources for Professional Learning in Teachers' Workplace Interactions. *Am. Educ. Res. J.* **2010**, *47*, 181–217. [CrossRef]
63. Elliott, R.; Kazemi, E.; Lesseig, K.; Mumme, J.; Carroll, C.; Kelley-Petersen, M. Conceptualizing the work of leading mathematical tasks in professional development. *Journ of Math Teach. Educ.* **2009**, *60*, 364–379. [CrossRef]
64. Dick, L.K.; Sztajn, P.; White, T.F.; Heck, D.J. Investigating Sociopedagogical Norms: Teachers' Discussions about Own and Others' Instruction. *Teach. Teach. Educ.* **2018**, *71*, 297–307. [CrossRef]
65. Roberts, S.A.; Olarte, T.R. Enacting multilingual learner core practices: A PST's approximations of practice of mathematics language routines. *J. Math. Teach. Educ.* **2023**. [CrossRef]
66. Sleep, L. The work of steering instruction toward the mathematical point: A decomposition of teaching practice. *AERJ* **2012**, *49*, 935–970. [CrossRef]
67. Grossman, P.; Wineburg, S.; Woolworth, S. Toward a theory of teacher community. *TCR* **2001**, *103*, 942–1012.
68. Elliott, R.; Lesseig, K. Productive disciplinary engagement as a framework to support mathematics teacher leaders. *Investigations in Mathematics Learning.* **2023**, *15*, 29–49. [CrossRef]
69. Garfunkel, S.; Montgomery, M. (Eds.) *GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education*, 2nd ed.; COMAP and SIAM: Philadelphia, PA, USA, 2019; Available online: <https://www.siam.org/publications/reports/guidelines-for-assessment-and-instruction-in-mathematical-modeling-education-gaimme/> (accessed on 30 August 2024).
70. youcubed. Explorations in Data Science. Available online: <https://hsdatascience.youcubed.org/> (accessed on 30 August 2024).
71. Desmos. Available online: <https://www.desmos.com/> (accessed on 30 August 2024).
72. Laib, J. Slow Reveal Graphs. Available online: <https://slowrevealgraphs.com/> (accessed on 19 September 2024).
73. Moschkovich, J. Examining mathematical discourse practices. *FLM* **2007**, *27*, 24–30.
74. Stein, M.K.; Grover, B.W.; Henningsen, M. Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms. *AERJ* **1996**, *33*, 455–488. [CrossRef]
75. Coles, A. Facilitating the use of video with teachers of mathematics: Learning from staying with the detail. *J. STEM Educ.* **2019**, *6*, 1–13. [CrossRef]
76. Gibbons, L.K.; Nieman, H. Examining the facilitation of generative teacher workgroup conversations. *Teach. Teach. Educ.* **2024**, *142*, 104542. [CrossRef]
77. Desimone, L.M. Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Ed. Res.* **2009**, *38*, 181–199. [CrossRef]
78. Hall, E.; Wall, K. *Research Methods for Understanding Professional Learning*; Bloomsbury Publishing: New York, NY, USA, 2019.
79. Garet, M.I.S.; Andrew, C.P.; Desimone, L.; Birman, B.F.; Yoon, K.S. What makes professional development effective? Results from a national sample of teachers. *Am. Educ. Res. J.* **2001**, *38*, 915–945. [CrossRef]
80. Sebring, P.B.; Allensworth, E.; Bryk, A.S.; Easton, J.Q.; Luppescu, S.; The Essential Supports for School Improvement. Research Report. Consortium on Chicago School Research. 2006. Available online: <https://files.eric.ed.gov/fulltext/ED498342.pdf> (accessed on 30 August 2024).
81. Andrews-Larson, C.; Wilson, J.; Larbi-Cherif, A. Instructional improvement and teachers' collaborative conversations: The role of focus and facilitation. *Teach. Coll. Record.* **2017**, *119*, 1–37. [CrossRef]
82. Desimone, L.M.; Garet, M.S. Best practices in teachers' professional development in the United States. *Psychol. Soc. Educ.* **2015**, *7*, 252–263. [CrossRef]

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