Price Coordination for Electric Vehicle Fleet Using Mean Field Game Theory

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***Abstract*—This paper proposes an advanced approach to control and coordinate a large number of electric vehicles to optimize their charging and discharging strategies using mean field game theory. Due to high-dimensional complexity, studying a system with a swarm of agents is computationally expensive. Therefore, the system can be structured as a game using mean field game theory to handle this complexity. Mean field game facilitates the interactions between players by considering the collective behavior of all agents. The finite difference method integrated with Bayesian optimization is utilized to solve the mean field game system, which consists of coupled Hamilton-Jacobi-Bellman and Kolmogorov forward equations. Those formulas guide electric vehicle owners' decisions to avoid penalties. This paper aims to determine the optimal parameters that enhance the numerical stability and accuracy of the finite difference method. Then, these parameters are utilized to solve the system of mean field game to control the actions of electric vehicle owners and analyze the impact of the estimated mass function of the entire population on their decision-making process. In addition, the reliability is evaluated to assess the effectiveness of price coordination in enhancing energy management. Comprehensive economic analyses for a fleet of electric vehicles are also conducted through a numerical example to validate the efficiency of the proposed method.**

***Index Terms*—Economic analysis, electric vehicle, finite difference, mean field game, reliability.**

I. Introduction

The combustion engines used in various means of transportation are considered one of the main sources of increasing levels of greenhouse gases and air pollution [1]. As a result, the use of electric vehicles (EVs) has increased worldwide because of their vital role in reducing emissions and the dependency on fossil fuel sources [2], [3]. However, the electricity required to charge an EV battery entirely is equivalent to that needed for a household during peak demand, which can strain the local grid [4]. On the other hand, EVs can enhance the grid's resilience and reliability by setting up rational policies and strategies. This can be achieved by encouraging the owners of EVs to charge during times of low-cost electricity and supply the grid in high-demand periods [5], [6]. Thus, controlling a large scale of EVs in a grid becomes critical.

One of the strategies that can be utilized to achieve this goal is the mean-field game (MFG). This is an advanced method used to guide the owners of EVs in their decisions. While it is difficult for players to gather comprehensive information about the behaviors and strategies of others, MFG depends on the global distribution of other agents to guide their decisions [7]. In the context of information flow, there are two types of MFG: decentralized and centralized. In the former, players make decisions individually and according to feedback from the local environment; in the latter, players' decisions are guided by a central authority [8].

In comparison to other multi-agent optimization techniques, such as asynchronous distributed optimization, distributed stochastic algorithm, and the maximum gain message, MFG shows a lot of advantages. Plug-in EVs are coordinated using an asynchronous distributed optimization method in [9]. However, this method is limited by communication capabilities. The maximum gain message is used to manage the charging process for a group of EVs [10]. However, the technique showed limitations in scalability. Traditional optimization methods show limitations and a lack of coordination when dealing with a swarm of players. In contrast, MFG needs less communication and is more suitable for highly dynamic environments [11].

Researchers have shown interest in optimal control, especially the applications of MFG for controlling and coordinating EVs and energy storage systems (ESS) within power grids. The authors of [12] applied the generalized Nash game method to model the charging process of EVs. However, the study does not consider the local constraints for the state of charge and charging rates. Moreover, the convergence analysis is neglected. MFG is used to study the effect of electricity prices on EV owners in [13]. The results showed how the decisions of EV owners influenced the electricity market. In [14], an optimal charging strategy is suggested using MFG to control the charging rate for a fleet of EVs. The study showed how encouraging EV owners to charge simultaneously at certain intervals at a clustered charging station can minimize costs. However, the paper does not consider potential difficulties, such as charging all EVs at an aggregated station.

The work in [15] used a decentralized game to optimize the interactions between EV owners and decentralized EV supply, such as solar energy. Despite the advantages of decentralized game theory, the level of satisfaction decreases with the number of EVs. Moreover, while the paper considered switching to the grid in case of insufficient supply from solar energy, it does not study the effect of this switching on the grid. MFG is also used to optimize the charging patterns for plug-in EVs and minimize battery degradation using a sequential quadratic programming (SQP) algorithm [16]. Traveling time is considered a constraint to controlling the decision of EV owners in switching between electricity and gasoline mode.

However, the paper did not consider the pricing technique for each agent and the impact of it on the power grid. Linear quadratic (LQ) MFG is also utilized to manage the interactions between three parties: the power grid, parking area, and EV owners to optimize the charging cost [17]. Despite the benefit of the LQ MFG approach, it is hindered by the linear dynamic state assumption.

The work in [18], [19] uses reinforcement learning to learn the optimal charging protocol for EVs. Deep reinforcement learning (DRL) controls the charging plan for a fleet of EVs in [20]. The results showed that increasing the number of agents will not change the computational complexity. However, the effect of the residential load is neglected. Furthermore, using reinforcement learning requires a significant amount of data for training. MFG offers an effective solution due to its unique ability to reduce dimensions by mapping a high-dimensional game into a low-dimensional one. This feature differs from traditional multiagent systems, which require extensive communication tools to capture the interactions between agents and experience high computation costs due to high dimensionality.

The contribution of this paper is as follows: 1) Developing a model to study the interactions between a fleet of EVs in a smart grid using the finite difference method. 2) Addressing numerical instabilities of finite difference method using sensitivity analysis and convergence criteria such as the Bayesian optimization method. 3) Evaluating the impact of managing the charging protocols of EV owners on enhancing reliability.

The remainder of the paper is structured in the following manner. Section II presents the formulation of the game. Section III represents the finite difference method based on MFG. Section IV includes a simulation and the main results, followed by the conclusion in section V.

II. Mean field game formulation

For classical games, the interactions and decision-making process are studied for a limited set of players. As the number of players rises, coordinating the process that governs agent interactions becomes computationally expensive. Furthermore, the availability of reliable means of communication is limited. Thus, MFG approximates the swarm of agents and transforms the game from a multiplayer to a two-player game. In the context of EVs, EV owners are handled as the players in the game. The game's rules that govern the dynamic environment of the players consist of the electricity price, battery state of charge, consumption rate, and the reward function.

A. Game Formulation

Suppose a group of battery electric vehicles (BEVs) denoted as N, where the evolution of the battery level X_t can be expressed as the difference between the consumption rate D_t^i and the control rate β_t^i, with i representing the individual EVs, as follows [21].

$$dX_t^{(i)} = \beta_t^{(i)}\,dt - D_t^{(i)}\,dt \tag{1}$$

Due to the stochastic nature of the battery level, the consumption rate is described using the Brownian motion W_t [21].

$$dX_t = (\beta_t - D_t)\,dt - D_t\gamma_t\,dW_t \tag{2}$$

Without suitable coordination to govern the process of charging and discharging, EVs can cause grid congestion, especially if many of them charge simultaneously during peak demand. Consequently, a game formulation is necessary. In other words, EV owners will play a game to optimize their battery level at all times. Thus, their decision to charge or discharge is directed by the reward function consisting of: 1) The cost of charging or discharging the battery $\beta_t p_t(m)$. 2) The cost of using the battery at a particular moment in the day $h(t,\alpha)$. 3) The cost associated with the lack of satisfaction when the battery capacity is low $f(t,X)$. 4) Safety costs $S(t,X)$. Therefore, for N EVs, the value function is calculated by (3)

$$V(t,X);\beta) = \mathbb{E}\left(\int_0^T C(\beta,X,m)\,dt + F(X_R)\right) \tag{3}$$

where $F(X_R)$ represents the final penalty at time T, which prevents the unwanted effect of charging in the last moments. It is important to mention that addressing the individual differences between players such as the capacity of batteries and charging speed allows more realistic simulation. However, assuming homogeneous EVs simplifies the computational complexity and lowers the problem's dimensionality.

As the number of EVs, N, increases, the classical game shows many system control limitations. For instance, managing a system with a significant number of agents is computationally expensive. In addition, intensive knowledge about agents makes them prone to the dimensionality problem [22]. In contrast, MFG can overcome classical game limitations. The merits of MFG are evident through its ability to simplify the dimensions of control problems. The formation of MFG is based on the assumption that the players are considered almost identical and continuum, which gives it the flexibility to optimize the agent actions based on the collective statistical behavior of the group. Furthermore, MFG is robust since it can handle uncertainty by capturing the average dynamic of the system. Therefore, it mitigates the effects of individual agents' random actions [23].

In a dynamic environment, MFG often consists of the Hamilton-Jacobi-Bellman (HJB) and Fokker-Planck-Kolmogorov (FPK) equations. HJB describes the objective function $u(x,t)$ over space and time, while FPK illustrates the global distribution of the overall population $m(x,t)$. Equations (4)-(7) show the general formulation for the MFG system. Equations (4) and (5) show HJB and FPK formulas, respectively [24], [25].

$$-\frac{\partial u}{\partial t} - \nu\Delta u + H(x,\nabla u) = R(x,m) \quad \text{in } \mathbb{R}^d \times (0,T) \tag{4}$$

$$\frac{\partial m}{\partial t} - \nu\Delta m - \nabla\cdot(D_p H(x,\nabla u)m) = 0 \quad \text{in } \mathbb{R}^d \times (0,T) \tag{5}$$

$$m(0,x) = m_0(x) \tag{6}$$

$$u(x,T) = L(x) \tag{7}$$

It is clear from equations (4) and (5) that the reward function $u(x,t)$ of each player is affected by the density $m(x,t)$ of all agents. The function $H(x,p)$ is the Hamiltonian, and the function $R(x,m)$ describes the connection between $u(x,t)$ and $m(x,t)$. Equations (6) and (7) show the initial condition of system density and the terminal condition for reward function, respectively. It is important to mention that HJB is solved backward in time while FPK is solved forward in time.

The interaction between HJB and FPK equations means that HJB updates its solution using the probability distribution from FPK. In contrast, FPK updates the global distribution according to the solution of HJB. This process occurs iteratively until the solution has converged. Thus, finding the solution for the MFG model analytically can be a challenging task. As a result, numerical methods are needed to learn PDE solutions. In this context, we will utilize the finite difference method to determine the MFG system.

III. FINITE DIFFERENCE METHOD BASED ON MFG

A. MFG Equations

To implement MFG using finite difference, the MFG system should be discretized after setting up the boundary conditions. In general, the domain is divided into a consistent grid. Accordingly, sets of discrete numerical approximations to the derivative are produced. PDEs can be discretized using (8) [26].

$$\begin{aligned} \text{Forward Difference:} \quad & \frac{\partial V}{\partial X} \approx \frac{V_{i+1,j} - V_{i,j}}{\Delta X} \\ \text{Backward Difference:} \quad & \frac{\partial V}{\partial X} \approx \frac{V_{i,j} - V_{i-1,j}}{\Delta X} \\ \text{Central Difference:} \quad & \frac{\partial V}{\partial X} \approx \frac{V_{i+1,j} - V_{i-1,j}}{2\Delta X} \end{aligned} \tag{8}$$

For $V(t,X;\beta)$ given in (3), the corresponding HJB and FPK are presented in (9) and (10):

$$\begin{aligned} \frac{\partial V(t,X)}{\partial t} = \frac{1}{2\zeta_t}\left(\frac{\partial V(t,X)}{\partial X} + p_t(m_t^{FPK})\right)^2 + D_t \frac{\partial V(t,X)}{\partial X} \\ - f(t,X) - S(t,X) - \frac{1}{2}\Gamma_t^2 D_t^2 \frac{\partial^2 V(t,X)}{\partial X^2} \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{\partial m(t,X)}{\partial t} &= \left(\frac{1}{\zeta_t}\left[\frac{\partial V^*(t,X)}{\partial X} + p(m)\right] + D_t\right)\frac{\partial m(t,X)}{\partial X} \\ &+ \frac{1}{\zeta_t}\frac{\partial^2 V^*(t,X)}{\partial X^2} m(t,X) \\ &+ \frac{1}{2} D_t^2 \gamma_t^2 \frac{\partial^2 m(t,X)}{\partial X^2}. \end{aligned} \tag{10}$$

The optimal policy that maximizes or minimizes the objective function given in (11) can be derived using the HJB equation.

$$\beta_t^* = -\frac{1}{\zeta_t}\left[\partial_X V(t,X) + p_t(m_t^{FPK})\right] \tag{11}$$

It is worth mentioning that the HJB and FPK systems need to be solved sequentially. Thus, the optimal value V^* from the HJB equation will be used to find the global distribution m^{FPK} using (10), and vice versa. The probabilistic feedback from FPK assists HJB in finding the optimal policy that maintains the state of charge constraints. Notably, if the coupled equations HJB and FPK are solved with stability and accuracy, the optimal policy exists. Furthermore, the iterative process between HJB and FPK enables individual decisions to be adjusted to reach a stable solution that optimizes the charging protocols of the swarm of EVs. Thus, for an optimal policy to exist, it is important that numerical methods can solve the MFG system with sufficient accuracy and under well-defined circumstances and parameters.

As previously mentioned, finding solutions for such systems requires numerical methods. While these methods provide approximate solutions, knowing the source of errors is crucial. In regards to finite difference, one of the main drawbacks is numerical instability. For instance, the grid quality used to discretize a function affects the precision and stability of the solution. This is known as a truncation error, a disparity between the exact quantity assuming perfect arithmetic and the precise solution of the original differential equation. Furthermore, there is a loss of precision due to the computer adjusting decimal numbers. To avoid these problems, the Bayesian optimization technique can be used to find the optimal hyperparameters to enhance stability. The mechanism used to solve the MFG system is shown in Fig. 1. It portrays the primary goal of MFG in reducing the state space to only two players: the individual EV and the total mass. The mass of agents will represent the local environment that provides agents with the necessary information to follow the optimal path. To illustrate the process, HJB will find the value and policy that controls EV decisions at each time step. Then, the output of HJB will be used to determine the density function through the FPK equation, which describes how the control strategies evolve; the process continues until the optimal values are found, ensuring reliable performance.

B. Reliability evaluation

The measure of reliability comes from the system's ability to purchase electricity at high and low prices efficiently. By evaluating the purchased electricity against the average levels of these periods, the simulation estimates the overall reliability as follows :

$$\text{HPR} = \frac{\sum_{t\in\mathcal{T}_{\text{high}}}(\mathcal{E}_t < \overline{\mathcal{E}})}{|\mathcal{T}_{\text{high}}|} \tag{12}$$

$$\text{LPR} = \frac{\sum_{t\in\mathcal{T}_{\text{low}}}(\mathcal{E}_t > \overline{\mathcal{E}})}{|\mathcal{T}_{\text{low}}|} \tag{13}$$

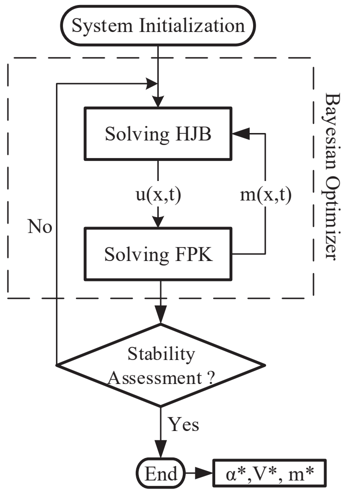

Fig. 1: The process for solving MFG system.

$$\text{Total Reliability} = \frac{\text{HPR} + \text{LPR}}{2} \tag{14}$$

where HPR, LPR are high and low price reliability, respectively. $\mathcal{T}_{\text{high}}$, $\mathcal{T}_{\text{low}}$ represent the set of high price and low price intervals. $\mathcal{E}_t$ presents the electricity purchased at time t.

IV. Simulation and the Key Findings

This proposed approach will be applied to manage the charging strategies for the EV fleet. EVs are treated as a continuum; hence, the number of EVs approaches infinity. To begin with, the regulations for the EVs game have been established. The energy consumption of EVs is considered to be three days, Saturday to Monday. We assumed that the EVs would consume more energy on Monday than on Saturday or Sunday (weekend days). Fig. 2 depicts the average energy consumption pattern D_t for EVs. The data is hypothetical and estimated based on the expected usage patterns of EVs in real life [27].

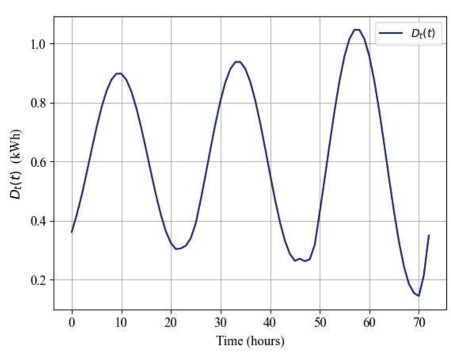

Fig. 2: Energy consumption D_t by EVs (kWh) .

Due to the assumption of indistinguishability, EVs are considered identical. The boundary conditions for both m and V are $\frac{\partial X_m}{\partial t}(0,\theta) = \frac{\partial X_m}{\partial t}(1,\theta) = \frac{\partial X_v}{\partial t}(0,\theta) = \frac{\partial X_v}{\partial t}(1,\theta) = 0$ to force the battery level X to be within $[0,1]$. As previously discussed, m^{FPK} represents the global distribution of battery levels X_t across EVs at time t. The battery level is constrained not to reach extreme values, fully charged or completely discharged, to model realistic and practical scenarios. Considering the changing rate of energy storage in EV batteries, the interaction between EVs and the grid can be tracked. In other words, selling or purchasing electricity for or from the grid can be calculated using (15):

$$ET = \left(D_t + \partial_t \left(\int X \, m^{FPK}(t, dX) \right) \right) \tag{15}$$

The price of electricity is defined as follows:

$$p_t = \left((ET)^+ + d_t \right)^2 \tag{16}$$

where d_t reflects the demand of other facilities.

Bayesian optimization is used to mitigate the numerical instability resulting from the finite difference method. The objective function for Bayesian optimization is maximizing energy efficiency. The best parameters according to this optimization are: $\triangle t$ = 0.005, $\triangle X$ = 0.1, and $\zeta_t = 13.68$. It is important to mention that those values meet the Courant-Friedrichs-Lewy condition [28].

Fig. 3 reflects the solution of (9). The value function $V(t, X)$ illustrates the cost associated with different battery levels X over time. As stated previously, HJB is critical in finding the policy that minimizes the cost function. This illustrates the descending pattern for the value function, which reflects the cost associated with EV behaviors. The peaks and valleys correspond to higher and lower costs, respectively. At high-price times, the value function should be less expensive with a high battery level. Moreover, it shows high costs at higher battery levels when the price is low. At the final condition $V(t = T, X) = (1 - X)^2$, the high battery level is desirable, as shown in Fig. 4. This condition will prevent EV owners from charging at the last moment. Thus avoiding grid overload.

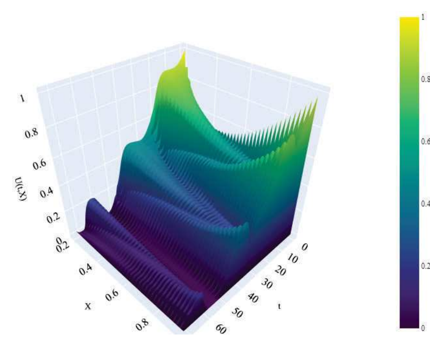

Fig. 3: Value function evolution concerning time and battery level.

The evolution of EV global mass m^{FPK} is shown in Fig. 5, which reflects the optimal solution of (10). EVs tend to charge at night time and discharge during the day. For instance, the battery level increases from midnight until about 6 AM, which

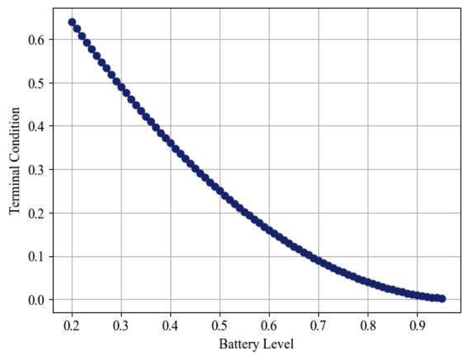

Fig. 4: Value function at boundary $t = T$.

indicates that energy is bought at night and utilized during the day. It is important to mention that the initial distribution for battery levels follows a triangular distribution.

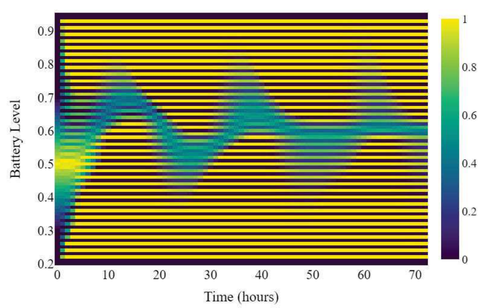

Fig. 5: The global distribution m^{FPK} .

Electricity trading and price are calculated using (15) and (16), as shown in Fig. 6, which reflects the electricity purchasing process over time. EV owners are incentivized to charge their batteries during the low-price period, while they will tend to discharge in the peak demand periods. Notably, some players are still purchasing electricity in high price periods. This pattern reflects the real-life scenario and shows one limitation of MFG: neglecting individual diversity and the assumption that agents are homogeneous. However, the use of MFG offers numerous advantages for EV owners. It enhances battery health by controlling the charging process and reducing battery degradation. Moreover, scheduling the charging protocols reduces energy consumption and minimizes power outages. In addition, MFG facilitates the exchange of information between EV drivers using the average distribution of EVs.

It is important to mention that different optimization metrics will lead to different results. To illustrate that, we choose the optimization metric to ensure system reliability. The reliability objective measures the ability of a system to manage EV consumption under different prices and state of charge levels. For that performance metric, the overall reliability evaluation is 94.73%, and the best parameters are $\triangle X$ = 0.447, $\triangle t$ = 0.0999 , and ζ_t = 30. Electricity trading according to the reliability metric is shown in Fig. 7. It is noticeable in Fig. 7 that the surge in the final moments reflects a drastic increase in demand due to relaxing the terminal cost condition. In addition, tuning the performance metric leads to adjustments in the electricity purchasing process, whereas the best parameters are changed to maximize the performance objective.

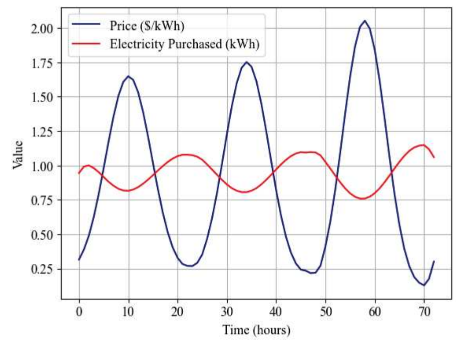

Fig. 6: Electricity price and trading.

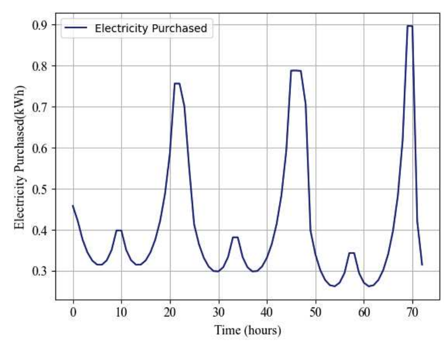

Fig. 7: Electricity trading after terminal condition softening.

Fig. 8 shows the benefit of the charging policy in reducing the total cost. The cost is mainly affected by the price of electricity and the control rate. The drop in cost is a result of the charging policy's ability to identify periods of low electricity prices. When electricity prices are high, drivers tend to avoid purchasing electricity and prefer to charge their vehicles when prices are low. To demonstrate the effectiveness of the

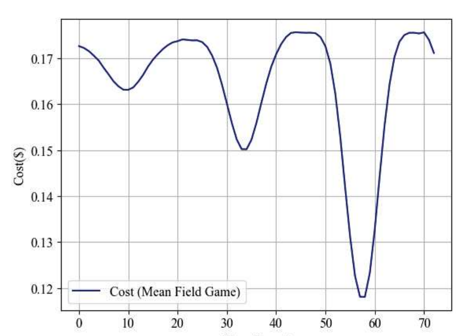

Fig. 8: Mean field cost.

proposed method, a comparison with other references shows the proposed method's cost reduction and energy utilization in Table I. Our approach shows about a 27% increase in off-peak electricity use and a 20% cost reduction. The electricity purchases in this study are managed based on electricity prices to optimize the reward function. Reference [15] showed that the peak-to-average ratio is reduced from 4.73 to 1.21, indicating significant cost reduction. The exact percentage of cost reduction is not provided. In [19], results showed that the percentage of both energy utilization and cost reduction are almost 15%.

TABLE I: Comparison of different methods

Method	**Finite difference**	**SQP [15]**	**DRL [19]**
Convergence	Achieved	Achieved	Achieved
Energy Utilization	27.7%	PAR= 1.21	15%
Cost Reduction	20%	not specified	15%
Electricity Purchase	Varies by price periods	Minimize cost and degradation	Multiagent coordination

V. Conclusion

This paper uses MFG to schedule the charging protocol for a swarm of EVs. The finite difference method is used to find the solution of the MFG system. To tackle numerical instability, Bayesian optimization is employed to obtain the best parameters to solve the MFG system. Moreover, the reward function and the collective mass are presented regarding electricity prices to coordinate electricity trading. Numerical simulations are carried out to show the efficacy of our approach. The reliability of the proposed algorithm is evaluated to ensure its efficiency in obtaining the optimal charging behavior of an EV fleet. The results showed that optimizing for system reliability yields a high level of reliability. Furthermore, MFG reduces the overall EV operating costs and charging times in the peak load periods, indicating its role in energy management and cost reduction for both EV owners and grid operators. The proposed method can be applied not only to control homogeneous EVs but also to heterogeneous agents and other components of power systems, such as renewable energy resources and energy storage systems.

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