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Measuring neutron star radius with second and third generation gravitational wave detector networks

Ananya Bandopadhyay[1,*], **Keisi Kacanja**[1], **Rahul Somasundaram**[1,2], **Alexander H Nitz**[1] **and Duncan A Brown**[1]

[1] Department of Physics, Syracuse University, Syracuse, NY 13244, United States of America
[2] Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, United States of America

E-mail: abandopa@syr.edu, kkacanja@syr.edu, rsomasun@syr.edu, ahnitz@syr.edu and dabrown@syr.edu

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Abstract
The next generation of ground-based interferometric gravitational wave detectors will observe mergers of black holes and neutron stars throughout cosmic time. A large number of the binary neutron star merger events will be observed with extreme high fidelity, and will provide stringent constraints on the equation of state of nuclear matter. In this paper, we investigate the systematic improvement in the measurability of the equation of state with increase in detector sensitivity by combining constraints obtained on the radius of a $1.4\,M_\odot$ neutron star from a simulated source population. Since the measurability of the equation of state depends on its stiffness, we consider a range of realistic equations of state that span the current observational constraints. We show that a single 40 km Cosmic Explorer detector can pin down the neutron star radius for a soft, medium and stiff equation of state with a precision of 10 m within a decade, whereas the current generation of ground-based detectors like the

* Author to whom any correspondence should be addressed.

Advanced LIGO-Virgo network would take $\mathcal{O}(10^5)$ years to do so for a soft equation of state.

Keywords: compact binaries, neutron stars, gravitational waves

1. Introduction

Cosmic Explorer is a groundbreaking next-generation gravitational wave detector, representing a significant leap in our capacity to explore a plethora of compact binary merger events. The development of Cosmic Explorer and other advanced gravitational wave detectors, such as Einstein Telescope [1, 2], will allow us to understand neutron stars and their internal structure. Cosmic Explorer will detect hundreds of signals with high signal-to-noise ratios (SNR), greater than 100 [3, 4]. With the improvement in sensitivity of the detectors, it will be possible to measure the radius of a neutron star with unprecedented accuracy. Inference from simulated populations performed by combining information from a large number of observed events, allows us to obtain improved constraints on the equation of state [5–10]. Previous works have demonstrated the feasibility of combining constraints from multiple observations using statistical tools, to obtain a tighter constraint on the equation of state.

In particular, Finstad *et al* [10] combined $\mathcal{O}(300)$ signals in an Advanced LIGO-Virgo network and compared it to the constraints obtained by stacking observations for one year of operation of a single Cosmic Explorer detector, showing the order of magnitude improvement in measurement accuracy achieved by a singular Cosmic Explorer. However, Cosmic Explorer will operate as a network which calls for the need to systematically investigate different detector configurations in order to fully gauge the improvement in sensitivity that Cosmic Explorer would accomplish. Therefore, in this paper, we extend the work of Finstad *et al* [10] to systematically study the improvement in the measurability of the equation of state with the increase in network sensitivity from the current generation of Advanced LIGO-Virgo detectors at design sensitivity to the planned Cosmic Explorer detectors, which are expected to begin operations in 2030 s. We simulate an astrophysical population of neutron stars consistent with current observational constraints on the rate of binary neutron star mergers in the Universe [11]. We perform a full Bayesian parameter estimation to obtain a forecast on the expected number of years of observation at different detector sensitivities required to measure the radius of a neutron star accurate to 10 m, by simulating different realizations of the Universe from the above population. We present our results for an Advanced LIGO-Virgo network, a post O5 network comprised of three LIGO detectors at $A^\sharp$ [3] sensitivity, a 20 km and a 40 km Cosmic Explorer detector [4], and two other next generation detector networks. The first of these is comprised of a 40 km Cosmic Explorer along with two detectors at $A^\sharp$ sensitivity, and the second consists of a 40 km Cosmic Explorer, a 20 km Cosmic Explorer and one detector at $A^\sharp$ sensitivity. For each of the networks, we combine equation of state constraints (in terms of measurement of radius of a $1.4\,M_\odot$ neutron star) from multiple events, observed across a given observing period for each detector (network). We study the measurability of the neutron star radius for three different equation of state models drawn from [12] that are calibrated with nuclear theory at lower densities and span the current observational constraints at densities relevant for neutron stars. We find that, while the current generation of ground-based gravitational wave detector network would require thousands of years to accurately measure the

[3] https://dcc.ligo.org/LIGO-T2300041/public.

[4] https://dcc.cosmicexplorer.org/CE-T2000017.

neutron star radius even for a stiff equation of state (i.e. relatively more measurable), a single 40 km Cosmic Explorer can do so within a year. Multi-detector networks including at least one Cosmic Explorer detector could constrain the radius to 10 m accuracy within a few years for even the softest equation of state considered here. Our result clearly demonstrates the potential of next generation detector networks to constrain the equation of state of dense nuclear matter to a higher accuracy than is possible for any existing experimental facility.

The rest of this article is organized as follows. In section 2, we describe the properties of the population of neutron stars that we are examining for our study. In section 3, we describe the Bayesian inference framework that is used to obtain combined constraints on the equation of state from the measurement of neutron star radius across an observed population. In section 4, we present the results of our analysis, i.e. the projected constraints on the radius of a $1.4\,\mathrm{M}_\odot$ neutron star for different detector networks, and conclude the study in section 5.

2. Neutron star population

To study the measurability of tidal deformability from the gravitational wave signals of inspiralling binary neutron star systems, we generate a population of binary neutron star systems with component masses drawn from a Gaussian centered at $1.4\,\mathrm{M}_\odot$ and having a standard deviation $\sigma_m = 0.05\,\mathrm{M}_\odot$. The spins are chosen to be aligned with the orbital angular momentum, and are drawn from a Gaussian distribution with zero mean and $\sigma_\chi = 0.02$. The events are chosen to be isotropically distributed across sky locations, and the inclination and orientation of the binary systems are distributed uniformly on the sphere. The luminosity distances of the systems are drawn from a uniform distribution in the range $d_L \in [20\,\mathrm{M}pc, 20\,\mathrm{Gpc}]$. Events at larger distances, i.e. having low values of signal-to-noise ratios, contain roughly the same amount of information about the measured parameters, thus drawing events uniformly in distance reduces the amount of computational resources spent on sampling over the parameters of a large number of 'uninformative' events occurring at higher distances in the simulated astrophysical population. It also gives us a greater selection of 'informative' events (occurring at nearby distances) to choose from, in constructing different realizations of the Universe, as compared to what we would have had if our simulated population of neutron stars was the true astrophysical population. We use the median merger rate for binary neutron stars $\mathcal{R} = 320^{+490}_{-240}\,\mathrm{Gpc}^{-3}yr^{-1}$ from [11] to normalize the astrophysical population, and reweight our neutron star population to a uniform in volume distribution. Our final distribution is uniform in volume. The uniform in volume assumption is reasonable for sources which contribute substantially to the equation of state measurement, since only the close by sources would be detected with signal-to-noise ratios high enough to extract equation of state information from them.

Since the measurability of the tidal deformability depends strongly on the equation of state, we choose a set of three equations of state representing a soft, a medium and a stiff one, for this study. These are then used to assign tidal deformabilities to the individual neutron stars in each simulated signal. They are chosen from a set of 2000 equations of state also used in [10], calibrated with chiral effective field theory calculations up to nuclear saturation density, n_{sat}. Beyond this point, their construction is based on general considerations of thermodynamic stability and causality, and employing a general extension scheme in the speed of sound c_s^2 for densities between $[n_{\mathrm{sat}}, 12n_{\mathrm{sat}}]$ [12]. Figure 1 shows the full set of 2000 equations of state, which we use as the prior for our parameter estimation runs, with the soft, medium and stiff equations of state labelled in blue, green and orange respectively. We note here that many of the equations of state that we use as our prior are already in conflict with current astrophysical

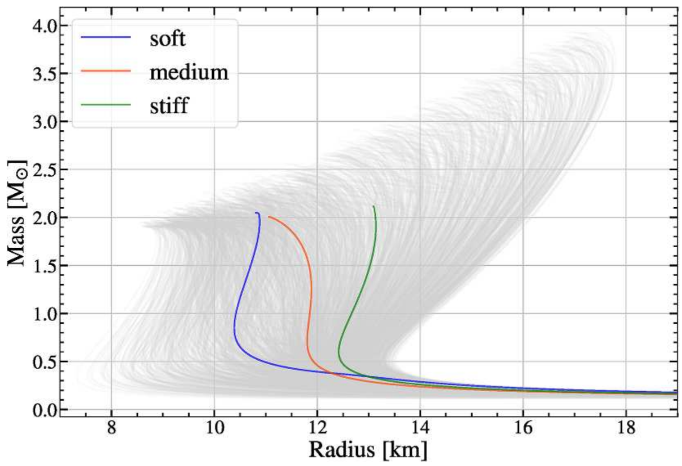

Figure 1. Neutron star mass-radius curves for the soft (blue), medium (orange) and stiff (green) equations of state used in our analysis. The curves shown in grey depict the full set of 2000 equations of state used as the prior for the Bayesian analysis. The equation of state set is constructed in such a way that the distibution of radii for a $1.4\,M_\odot$ neutron star is roughly uniform.

constraints. However, as we go on to show, the posteriors obtained on the neutron star radius from the gravitational wave analysis, particularly for the next generation detectors, are strongly likelihood-informed, and the choice of a broad range of prior does not affect the results of our analysis.

Parameter estimation runs are performed for the simulated population separately for each equation of state. The waveform model used for generating the simulated signals is the `IMRPhenomD_NRTidal` approximant. This is a frequency domain approximant that is based on a phenomenological inspiral-merger-ringdown model `IMRPhenomD`, originally constructed to model binary black hole systems with spins aligned with their angular momentum axis [13, 14]. Tidal effects are superposed on the binary black hole waveforms and calibrated with Numerical Relativity simulations of binary neutron star systems to obtain the binary neutron star waveform approximant [15, 16]. For high SNR signals, which will be detected abundantly by the next generation gravitational wave detectors, waveform systematics can become a dominant source of error in the extraction of parameters [9]. Earlier works have shown that tidal parameters can be recovered reliably using the `IMRPhenomD_NRTidal` waveform for systems with small mass ratios and spins [17]. Stiffer equations of state have been shown to introduce a bias in the recovery of non-tidal parameters, which in turn would affect the measurement of the equation of state [17, 18]. Systems with unequal mass ratios and highly spinning systems also show an increased bias in the estimation of tidal parameters [19], thus indicating the need for improved waveform modelling for binary neutron star systems for the next generation of ground based detectors.

3. Injection study

To quantify the improvement in measurability of the equation of state through neutron star radius and tidal deformability measurements with increase in detector sensitivity, we perform a Bayesian Inference based parameter estimation for the injected signals, which are projected onto simulated detector noise. We consider six different detector networks for the purpose of this study. The LIGO-Virgo network consists of the LIGO Handford, LIGO Livingston [20, 21] and Virgo detectors [22] simulated at design sensitivity [23]. The post-O5 network is also comprised of three detectors, LIGO Handford, LIGO Livingston and LIGO India, at $A^\sharp$ sensitivity [24]. The two Cosmic Explorer detectors are considered as single detector networks, a 40 km Cosmic Explorer detector off the coast of Washington, and a 20 km Cosmic Explorer detector off the coast of Texas. Additionally, we consider two more next generation detector networks. The first of these is comprised of a 40 km Cosmic Explorer detector off the coast of Washington, and two detectors at Livingston and India, operating at $A^\sharp$ sensitivity (henceforth referred to as CE40+2$A^\sharp$). The final network consists of a 40 km Cosmic Explorer, a 20 km Cosmic Explorer and LIGO India operating at $A^\sharp$ sensitivity (henceforth referred to as CE40+CE20+$A^\sharp$). Since the site locations for the Cosmic Explorer detectors are yet to be determined, the aforementioned fiducial locations are chosen as being close enough to a wide range of potential sites for the detectors to be representative from the point of view of the science goals that we address here.

To assess the performance of each detector network, we simulate a population of binary neutron star sources and apply a lower bound of 30 on the signal-to-noise ratio (SNR) to determine the events for which parameter estimation runs are performed to infer source parameters. As noted in [10], the measurement of the neutron star radius $R_{1.4}$ is dominated by the loudest signals, which come from the local Universe. At present, the only gravitational-wave event that has been used to successfully constrain the neutron star radius is GW170817, which had a signal-to-noise ratio of $\sim$33. Considering the SNR threshold of 30 chosen for this work, any single event is either comparable to, or more informative than GW170817, as regards constraining the equation of state. At a population level, the contribution of the distant low-SNR sources to the measurement of equation of state parameters becomes negligible. For signals observed with lower signal-to-noise ratio than the threshold value, the error bars associated with radius measurement are large, implying that these signals contribute negligibly to the combined equation of state inference.

Parameter estimation for the injected signals is based on the Bayes theorem. Here we briefly review the theorem in the context of its application to gravitational wave astronomy. It states that for a set of parameters $\vec{\theta}$ that can be used to construct a model M for the gravitational waveform, and given $\vec{d}(t)$, the gravitational wave strain data from the detectors,

$$p\left(\vec{\theta}|\vec{d}(t),M\right) = \frac{\pi\left(\vec{\theta}(t)|M\right)\mathcal{L}\left(\vec{d}(t)|\vec{\theta},M\right)}{\mathcal{Z}}, \tag{1}$$

where $p(\vec{\theta}|\vec{d}(t),M)$ is the data-informed posterior distribution of the parameter $\vec{\theta}$, $\pi(\vec{\theta}(t)|M)$ is our prior knowledge of the parameter vector $\vec{\theta}$, conditional on the model, and $\mathcal{L}(\vec{d}(t)|\vec{\theta},M)$ is the likelihood of obtaining data $\vec{d}(t)$ given the waveform model M and the parameters $\vec{\theta}$. Assuming that each detector produces stationary, Gaussian noise that is uncorrelated between different detectors in the network, the likelihood function is given by [25]

$$\begin{aligned}\mathcal{L}\left(\vec{d}|\vec{\theta},M\right) &= \exp\left(-\frac{1}{2}\sum_{i=1}^{N}\langle n_i|n_i\rangle\right)\\ &= \exp\left(-\frac{1}{2}\sum_{i=1}^{N}\langle d_i - h_i\left(\vec{\theta}\right)|d_i - h_i\left(\vec{\theta}\right)\rangle\right)\end{aligned} \tag{2}$$

where

$$\langle a|b\rangle = 4\mathbb{R}\int_{f_{\min}}^{f_{\max}} \frac{\tilde{a}^{*}(f)\,\tilde{b}(f)}{S_n(f)} \tag{3}$$

is the noise-weighted inner product of two functions.

$\mathcal{Z} = \int \pi(\vec{\theta}(t)|M)\mathcal{L}(\vec{d}(t)|\vec{\theta},M)d\vec{\theta}$ is the Bayesian evidence, which acts as a normalization constant. We use ensemble samplers, like Markov Chain Monte Carlo samplers, that sample the parameter space of interest, and trace out the posterior probability distribution. For this work, we use the parallel-tempered `emcee` sampler [26], invoking it through the `PyCBC Inference` library [27], to estimate posterior distributions of all of the parameters used to model the binary neutron star signals, as described in section 2. We use the heterodyne likelihood model [28–30] in `PyCBC` for likelihood estimation, which evaluates the likelihood in discrete bins close to the peak value, to speed-up sampler convergence.

In the process of recovering the signal parameters through Markov-Chain Monte Carlo sampling, the sampling parameters are chosen to be the source frame chirp mass and mass ratio, component spins along the direction of the angular momenta, sky location, distance, geocentric time of coalescence, inclination, polarization angle, and the equation of state. We sample over the equations of state in the same procedure as [10]. The equations of state are indexed in increasing order of radii for a $1.4\,\mathrm{M}_{\odot}$ neutron star, and each index corresponds to a unique mass-radius (or equivalently, mass-tidal deformability) curve in the equation of state parameter space. The lower frequency cutoff for the signals analyzed are chosen to be higher than the seismic frequency limit for the detectors. For the LIGO-Virgo network at design sensitivity, we use a lower frequency cutoff of 20 Hz, for the detectors at $\mathrm{A}^{\sharp}$ sensitivity, we use 10 Hz, and for the Cosmic Explorer detectors, the lower frequency cutoff is chosen to be 7 Hz. The high frequency cutoff for all signals is 2048 Hz.

Once the sampler converges, we obtain the full set of posterior samples for the parameters of each of the analyzed events. This represents our knowledge of the source parameters for the gravitational wave event, conditioned on the data. We then combine multiple events hierarchically, to obtain a hyperposterior for the neutron star radius, that represents our integrated knowledge of the equation of state from the observed population of neutron stars. To do this, we first convert the equation of state samples to radius samples for a $1.4\,\mathrm{M}_{\odot}$ neutron star (denoted as $\mathrm{R}_{1.4}$). This is done by using the mass-radius curve corresponding to each equation of state sample to determine the radius value for that sample. We then use a Gaussian Kernel Density Estimation [31] to obtain the posterior distribution on the radius for each event. The combined posterior for N events $\{s_1, s_2, \ldots s_N\}$ is then obtained as

$$p(\mathrm{R}_{1.4}|s_1,\ldots s_N) = p(\mathrm{R}_{1.4})^{(1-N)}\prod_{i=1}^{N} p(\mathrm{R}_{1.4}|s_i) \tag{4}$$

where $p(\mathrm{R}_{1.4})$ is the radius prior, uniform across all events.

4. Results

We simulate a realistic observing scenario for binary neutron star mergers by creating a population of neutron stars distributed uniformly in distance (details in section 2) and re-weighting our events to a population distributed uniformly in volume. We then make random draws from this population to construct different realizations of the Universe. We also randomize

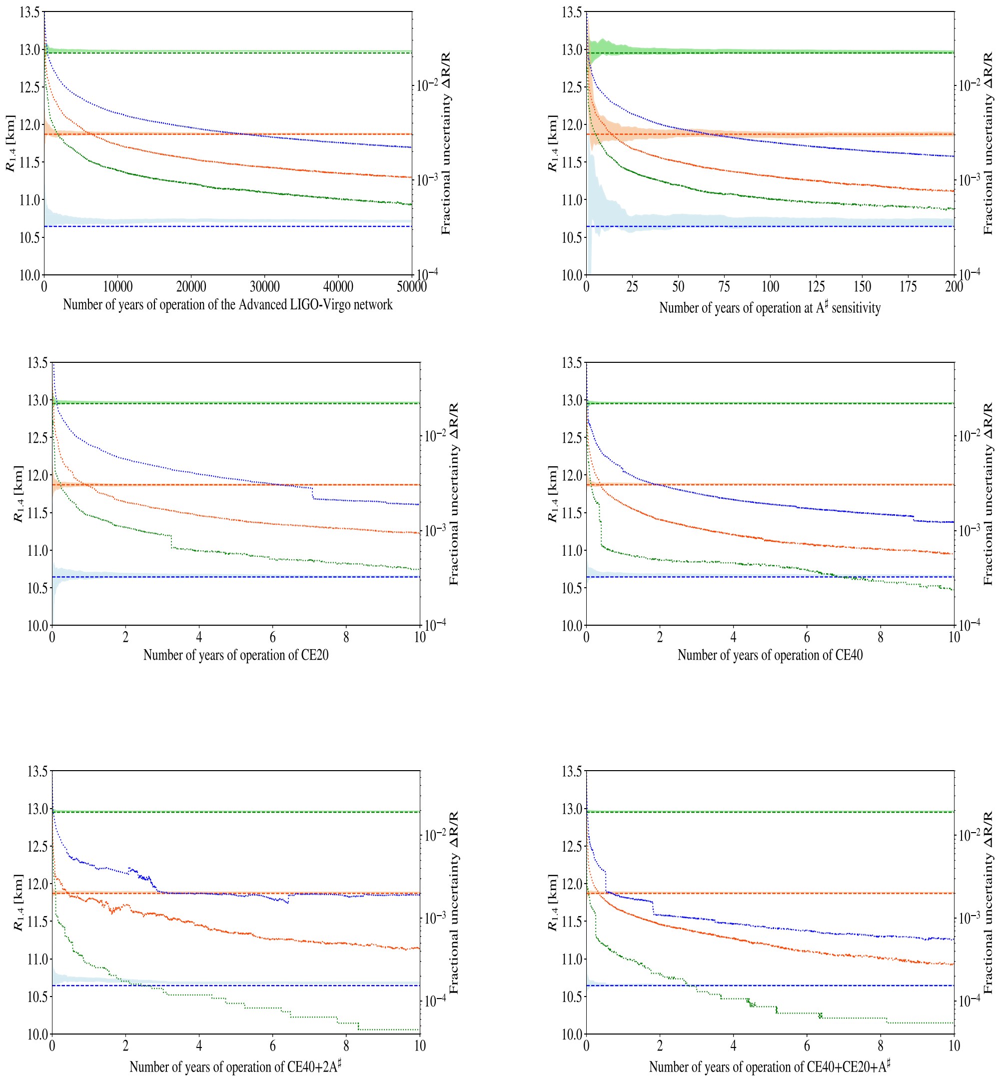

Figure 2. Combined constraints on the measurement of $R_{1.4}$ for the six different detector networks considered in this study. The blue, orange and green colors represent the constraints for the soft, medium and stiff equations of state respectively. The shaded regions in the figures represent the 90% credible intervals for the combined radius posteriors obtained using equation (4). The dotted lines depict the fractional error bars on radius measurement after a given number of years of observation. The dashed lines are the injected radius values.

the sequence of detection of the events and combine the posteriors for $R_{1.4}$ using equation (4) to obtain the projected accuracy in the measurement of neutron star radius for a given observation period for each detector network. In figure 2, we show the projected constraints on the radius of a $1.4M_{\odot}$ neutron star for a given random drawn population for the six detector networks described in section 3. For all of the detector networks, we find that the combined radius posterior converges towards the injected value. For the soft equation of state (depicted in blue in the figure), the shifting of the confidence intervals above the horizontal line representing the injected value suggests a possible systematic bias that causes the radius to be over-estimated for all of the detector networks. This can be attributed, at least partially, to the discrete sampling in the equation of state index, which causes a mismatch between the equation of state prior and the population distribution. We return to a discussion of this bias in section 5. We find that a next generation detector configuration with one 40 km Cosmic Explorer, one 20 km Cosmic Explorer and one $A^\sharp$ detector converges to the injected radius the fastest. Even one Cosmic Explorer detector converges faster to the radius than a three-detector Advanced LIGO-Virgo configuration.

As pointed out in [10], the loudest events have the most significant contribution towards lowering the fractional error bars on radius measurement. Thus, the exact time required to measure the radius of a neutron star to a certain accuracy will be affected by the order of occurrence of the events. To mitigate the effect of the ordering of events used in obtaining the projected constraints in figure 2, we draw different populations of the Universe for each detector network 500 times and plot the distribution of the number of years of operation it would take to constrain the radius of a $1.4\,M_\odot$ neutron star with a precision of $10\,$m. These results are shown in figure 3 and a summary is also shown in table 1. From these, we can see that even for the stiffest equation of state, the current generation of ground based detectors like the Advanced LIGO-Virgo network would take several thousand years to measure the radius of a neutron star with a precision of 10 m, and this improves only slightly with the $A^\sharp$ upgrade. For a very stiff equation of state which gives a radius of $\sim 13\,$km for a $1.4\,M_\odot$ neutron star, the $A^\sharp$ network may be able to constrain the radius with a precision of 10 m within 20 years, but this worsens significantly if the true equation of state is moderately soft. On the other hand, a single 40 km Cosmic Explorer detector will be able to constrain radius with a precision of 10 m within 12 years for the softest equation of state considered here, and within a year for the stiffest. In order to measure the equation of state for a neutron star in a viable time-frame, building Cosmic Explorer is absolutely essential.

5. Conclusion

In this paper, we have performed a comparative study of the potential of different second and third generation gravitational wave detector networks to precisely constrain the nuclear equation of state. We perform a full Bayesian inference on a simulated population of neutron stars to be observed at different network sensitivities. The equation of state models used in the analysis span the plausible range of stiffness allowed by current observational constraints for densities relevant for neutron stars, and are calibrated with chiral effective field theory at the low density regime. We combine constraints from individual events, and show that this procedure can be used to obtain a stringent constraint on the radius of a $1.4\,M_\odot$ neutron star.

Our results demonstrate the systematic improvement in the measurability of the nuclear equation of state with improvement in sensitivity of gravitational detectors, from the current generation of Advanced LIGO-Virgo detectors, to the $A^\sharp$ upgrade, and subsequently the next generation of detectors like Cosmic Explorer. The constraints obtained on the equation of state follow the expected hierarchy of measurability, with the stiffest equations of state giving the most stringent constraints, and the softer models leading to weaker constraints. Based on the current estimate for the merger rate from [11], we find that the Advanced LIGO-Virgo detectors operating at design sensitivity would take $2 \times 10^{5}{}^{+4\times10^4}_{-4\times10^4}$, $5 \times 10^{4}{}^{+1\times10^4}_{-1\times10^4}$ and 7000^{+900}_{-900} years respectively to constrain the radius of a $1.4\,M_\odot$ with a precision of 10 m, for the soft, medium and stiff equations of state respectively. With the detectors operating at $A^\sharp$ sensitivity, the same

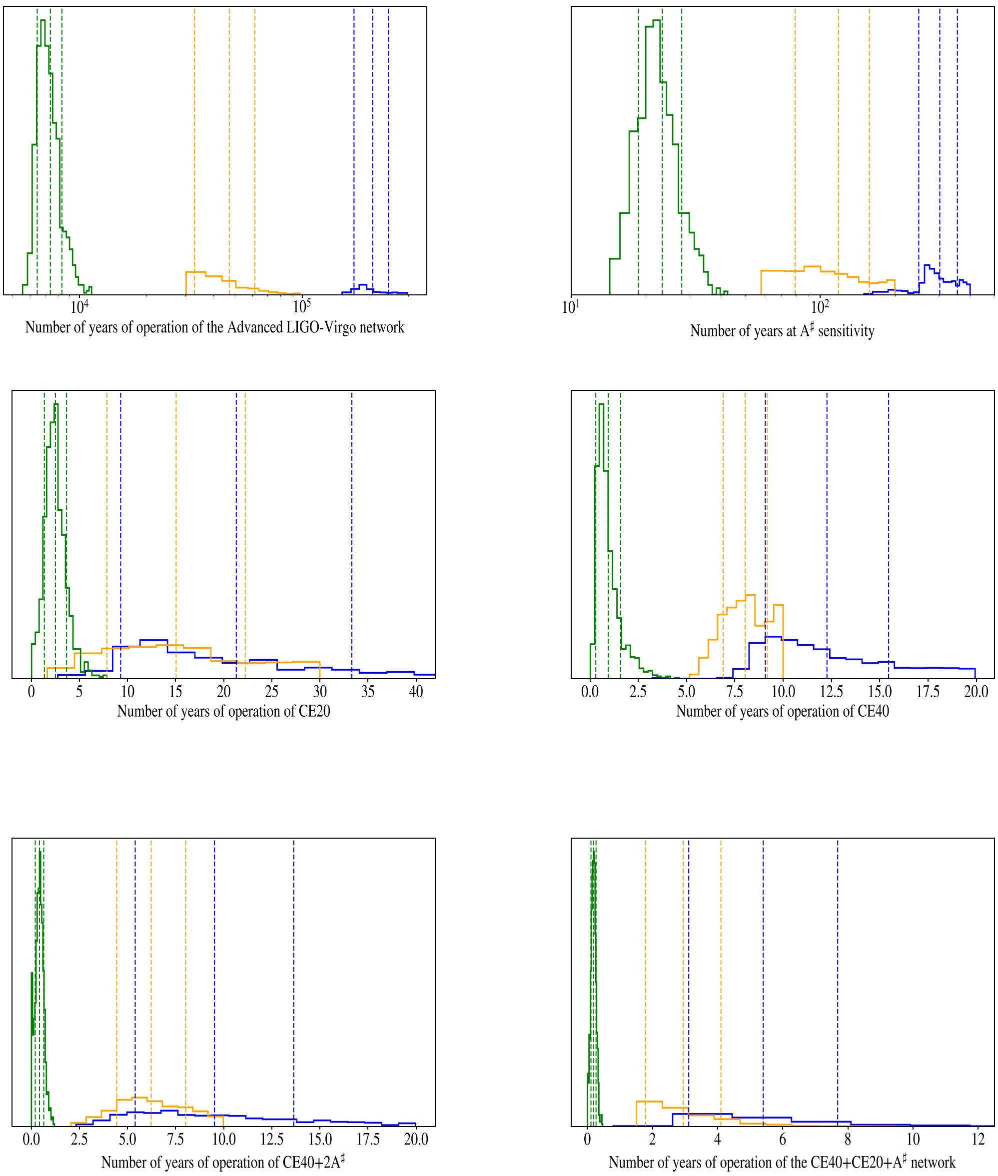

Figure 3. Distribution of the number of years of observation at different network sensitivities required to constrain the radius of a $1.4\,\mathrm{M}_{\odot}$ neutron star down with a precision of 10 m. The blue, orange and green histograms depict the expected distributions for the soft, medium and stiff equations of state respectively. For each equation of state (labeled by a different color) the leftmost and rightmost vertical dashed lines encompass the 90% confidence interval, and the central dashed line represents the median value of the distribution.

accuracy can be reached in 300^{+50}_{-50}, 100^{+40}_{-40} and 20^{+4}_{-4} years. However, with the next generation of ground based detectors like the Cosmic Explorer, the radius of a $1.4\,\mathrm{M}_{\odot}$ neutron star can be measured with a precision of 10 m well within a human lifetime, with a single 20 km detector

Table 1. A summary of the result of drawing 500 random realizations of the Universe from our simulated population for each detector network and each equation of state. For each realization of the Universe, the events are shuffled to randomize order of occurrence, and then used to determine the average number of years required to measure the radius of a $1.4\,\mathrm{M}_\odot$ neutron star with a precision of 10 m.

Detector network	Soft EoS	Medium EoS	Stiff EoS
LIGO-Virgo	$2\times10^{5}{}^{+4\times10^4}_{-4\times10^4}$	$5\times10^{4}{}^{+1\times10^4}_{-1\times10^4}$	7000^{+900}_{-900}
$3\,\mathrm{A}^\sharp$	300^{+50}_{-50}	100^{+40}_{-40}	20^{+4}_{-4}
CE20	21^{+10}_{-10}	15^{+7}_{-7}	3^{+1}_{-1}
CE40	12^{+3}_{-3}	8^{+1}_{-1}	$1^{+0.6}_{-0.6}$
CE40+2$\,\mathrm{A}^\sharp$	9^{+4}_{-4}	6^{+2}_{-2}	$0.4^{+0.2}_{-0.2}$
CE40+CE20+$\mathrm{A}^\sharp$	5^{+2}_{-2}	3^{+1}_{-1}	$0.2^{+0.07}_{-0.07}$

doing it in 21^{+10}_{-10}, 15^{+7}_{-7} and 3^{+1}_{-1} years, and a 40 km detector requiring only 12^{+3}_{-3}, 8^{+1}_{-1} and $1^{+0.6}_{-0.6}$ years respectively for the soft, medium and stiff equations of state. The multi-detector network comprised of a 40 km Cosmic Explorer and 2 detectors at $\mathrm{A}^\sharp$ does it in 9^{+4}_{-4}, 6^{+2}_{-2} and $0.4^{+0.2}_{-0.2}$ years, whereas the network comprised of a 40 km Cosmic Explorer, a 20 km Cosmic Explorer and an $\mathrm{A}^\sharp$ takes 5^{+2}_{-2}, 3^{+1}_{-1} and $0.2^{+0.07}_{-0.07}$ years respectively to measure the radius with a precision of 10 m for the soft, medium and stiff equations of state. To assess the impact that a measurement accuracy of 10 m on $R_{1.4}$ has on constraining the nuclear equation of state, one can overlay the confidence interval on the mass-radius curves constituting the prior (figure 1). This was done, for e.g. in [12], where using the same set of 2000 equations of state as the prior in their analysis for GW170817, the authors ruled out a significant fraction of the mass-radius curves at 90% confidence with a $\sim$2 km accuracy on the measurement of neutron-star radius for a single mass (as shown in figure 2 of [12]). With a 10 m precision, a much larger fraction of the mass-radius curves can be ruled out, thus yielding substantially stronger constraints on the equation of state than obtainable from other astrophysical observations, such as NICER or electromagnetic counterparts of binary neutron star mergers. Also, while the 10 m precision used here to constrain $R_{1.4}$ is an arbitrary choice, and the exact limits on the required precision for neutron star radius will be set by nuclear theorists as their models continue to be upgraded with time, 10 m is a sufficiently high precision, that will allow us to measure subtle features of the equation of state, for e.g. a weak phase transition that is not manifested as a sharp discontinuity in the mass-radius curve. Thus, the next generation of ground based detectors will be instrumental in constraining the nuclear equation of state to a level of accuracy that can not be reached with any existing facilities.

In this paper, we have focussed our results on the measurement of the radius of a $1.4M_\odot$ neutron star. In order to truly constrain the nuclear equation of state, one has to constrain the mass-radius curve (or equivalently, the mass-tidal-deformability curve), thus necessitating the measurement of radii of neutron stars along the full mass spectrum. However, with our approach of sampling directly over the equation of state, rather than in the tidal parameters such as $\Lambda_{1,2}$, we are not restricted to $R_{1.4}$, but can instead map the equation of state posteriors to the neutron star radius for any other mass. As the equation of state priors continue to be updated with constraints from nuclear theory and other astrophysical and terrestrial experiments, this approach will also allow us to place constraints on the equation of state at higher and lower densities, especially as we observe events involving neutron stars along the full mass spectrum. The posteriors obtained from the analysis of gravitational-wave signals detected from binary neutron stars can also be combined with other astrophysical observations, such as pulsar timing and pulse profile modeling [32–34], to improve the combined constraints on the equation of state, as well as mitigate some of the systematic uncertainties involved in the models used to constrain the equation of state [35]. Finally, the advantage of representing the equation of state constraints as single parameter such as $R_{1.4}$ is that it allows us to directly compare the posteriors on $R_{1.4}$ to those on nuclear parameters obtained from terrestrial experiments, such as the neutron skin thickness of ^{208}Pb [36, 37], to set tighter constraints on the equation of state.

Some of the source bias in the process of obtaining combined constraints by this method are pointed out in [10]. These biases include the imperfect knowledge of the mass distribution of neutron stars and the potential absence of an electromagnetic counterpart, posing challenges in distinguishing a binary neutron star system from a neutron star black hole system. In this study, we circumvent the bias caused by the imperfect knowledge of mass distribution by choosing the mass prior used in our analysis to be the same as the one used to simulate the neutron star population. However, in practical applications, as more and more signals from binary neutron star mergers are detected, the mass distribution used in the inference should also be simultaneously updated to mitigate this bias. Further, as seen in figure 2, for some of the equations of state, there is a systematic bias that causes the combined posteriors to deviate from the injected value. This can be interpreted as a finite-sampling effect, which we anticipate would be alleviated by sampling the equation of state in a continuous manner, instead of using a finite set of discrete equation of state samples as the prior. Solving the Tolman–Oppenheimer–Volkoff (TOV) equations that determine the neutron-star structure is a computational bottleneck that limits us from implementing a continuous sampling of the equation of state. Progress has been made in this direction through the use of machine learning algorithms to emulate solutions to the TOV equations, yielding results that are in good agreement with those obtained using the full TOV solver [38]. We leave it to a future work to investigate the improvement brought about through a continuous sampling of the equation of state. Another source of bias in obtaining constraints on the equation of state, particularly in terms of the neutron star radius, is that we do not measure radius directly from gravitational wave signals, but use the measurement of tidal deformability to obtain an estimate of the radius. Since the love number k_2 (as defined in e.g. [39–41]), relating tidal deformability to radius, is also a function of radius, even an extremely precise measurement of the tidal deformability leads to a multimodal distribution for measured radius, which can ultimately tend to bias the median value of the measured radius away from its true value. Future works can extend this work to constrain the radius not only for a $1.4\,\mathrm{M}_\odot$ neutron star but for the whole mass range allowed by plausible equations of state. In conclusion, our work highlights the imperative need to build Cosmic Explorer in order to be able to accurately constrain the radius of a neutron star within a human lifetime.

Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: https://github.com/sugwg/bns-eos-nggw.

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ORCID iDs

Ananya Bandopadhyay https://orcid.org/0000-0002-5116-844X
Keisi Kacanja https://orcid.org/0009-0004-9167-7769
Rahul Somasundaram https://orcid.org/0000-0003-0427-3893
Alexander H Nitz https://orcid.org/0000-0002-1850-4587
Duncan A Brown https://orcid.org/0000-0002-9180-5765

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