# On the Existence of Envy-Free Allocations Beyond Additive Valuations

GERDUS BENADE, Boston University, USA
DANIEL HALPERN, Harvard University, USA
ALEXANDROS PSOMAS, Purdue University, USA
PARITOSH VERMA, Purdue University, USA

We study the problem of fairly allocating $m$ indivisible items among $n$ agents. Envy-free allocations, in which each agent prefers her bundle to the bundle of every other agent, need not exist in the worst case. However, when agents have additive preferences and the value $v_{i,j}$ of agent $i$ for item $j$ is drawn independently from a distribution $D_i$, envy-free allocations exist with high probability when $m \in \Omega(n \log n / \log \log n)$.

In this paper, we study the existence of envy-free allocations under stochastic valuations far beyond the additive setting. We introduce a new stochastic model in which each agent's valuation is sampled by first fixing a worst-case function, and then drawing a uniformly random renaming of the items, independently for each agent. This strictly generalizes known settings; for example, $v_{i,j} \sim D_i$ may be seen as picking a random (instead of a worst-case) additive function before renaming. We prove that random renaming is sufficient to ensure that envy-free allocations exist with high probability in very general settings. When valuations are non-negative and "order-consistent," a valuation class that generalizes additive, budget-additive, unit-demand, and single-minded agents, SD-envy-free allocations (a stronger notion of fairness than envy-freeness) exist for $m \in \omega(n^2)$ when $n$ divides $m$, and SD-EFX allocations exist for all $m \in \omega(n^2)$. The dependence on $n$ is tight, that is, for $m \in O(n^2)$ envy-free allocations do not exist with constant probability. For the case of arbitrary valuations (allowing non-monotone, negative, or mixed-manna valuations) and $n = 2$ agents, we prove envy-free allocations exist with probability $1 - \Theta(1/m)$ (and this is tight).

A full version of this paper can be found at: https://arxiv.org/abs/2307.09648

Additional Key Words and Phrases: stochastic fair division, envy-freeness with high probability, arbitrary valuation functions

**ACM Reference Format:**
Gerdus Benade, Daniel Halpern, Alexandros Psomas, and Paritosh Verma. 2024. On the Existence of Envy-Free Allocations Beyond Additive Valuations. In *Conference on Economics and Computation (EC '24), July 8–11, 2024, New Haven, CT, USA.* ACM, New York, NY, USA, 1 page. https://doi.org/10.1145/3670865.3673466



## Acknowledgments

Daniel Halpern is supported in part by the National Science Foundation Graduate Research Fellowship Program under Grant No. DGE1745303. Alexandros Psomas and Paritosh Verma are supported in part by an NSF CAREER award CCF-2144208, a Google AI for Social Good award, and research awards from Google and Supra.

Authors' Contact Information: Gerdus Benade, benade@bu.edu, Boston University, Boston, MA, USA; Daniel Halpern, dhalpern@g.harvard.edu, Harvard University, Boston, MA, USA; Alexandros Psomas, apsomas@cs.purdue.edu, Purdue University, West Lafayette, IN, USA; Paritosh Verma, verma136@purdue.edu, Purdue University, West Lafayette, IN, USA.



*EC '24, July 8–11, 2024, New Haven, CT, USA*



ACM ISBN 979-8-4007-0704-9/24/07
https://doi.org/10.1145/3670865.3673466