On the Fairness of Normalized p-Means for Allocating Goods and Chores

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Allocating items in a fair and economically efficient manner is a central problem in fair division. We study this problem for agents with additive preferences, when items are all goods or all chores, divisible or indivisible. The celebrated notion of Nash welfare is known to produce fair and efficient allocations for both divisible and indivisible goods; there is no known analogue for dividing chores. The Nash welfare objective belongs to a large, parameterized family of objectives called the p-mean welfare functions, which includes other notable members, like social welfare and egalitarian welfare. However, among the members of this family, only the Nash welfare produces fair allocations for goods. Incidentally, Nash welfare is also the only member that satisfies the axiom of scale invariance, which is crucially associated with its fairness properties.

We define the class of "normalized p-mean" objectives, which imparts the missing key axiom of scale invariance to the p-mean family. Our results show that optimizing the normalized p-mean objectives produces fair and efficient allocations when the items are goods or chores, divisible or indivisible. For instance, the normalized p-means gives us an infinite class of objectives that produce (i) proportional and Pareto efficient allocations for divisible goods for all $p \leq 0$, (ii) approximately proportional and Pareto efficient allocations for divisible chores for all $p \geq 1$, (iii) EF1 and Pareto efficient allocations for indivisible goods for two agents for all $p \leq 0$, and (iv) EF1 and Pareto efficient allocations for indivisible chores for two agents for all $p \geq 2$.

A full version of this paper can be found at: https://arxiv.org/abs/2402.14996

CCS Concepts: • **Theory of computation → Algorithmic game theory and mechanism design**.

Additional Key Words and Phrases: fair division, p-mean welfare

ACM Reference Format:
Owen Eckart, Alexandros Psomas, and Paritosh Verma. 2024. On the Fairness of Normalized p-Means for Allocating Goods and Chores. In *Conference on Economics and Computation (EC '24), July 8–11, 2024, New Haven, CT, USA.* ACM, New York, NY, USA, 1 page. https://doi.org/10.1145/3670865.3673445

Acknowledgments

Alexandros Psomas and Paritosh Verma are supported in part by an NSF CAREER award CCF-2144208, a Google AI for Social Good award, and research awards from Google and Supra. Owen Eckart is supported in part by an NSF CAREER award CCF-2144208.

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EC '24, July 8–11, 2024, New Haven, CT, USA

ACM ISBN 979-8-4007-0704-9/24/07
https://doi.org/10.1145/3670865.3673445