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Children's estimates of equivalent rational number magnitudes are not equal: Evidence from fractions, decimals, percentages, and whole numbers

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A R T I C L E I N F O

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A B S T R A C T

Integration of rational number knowledge with prior whole number knowledge has been theorized as critical for mathematical success. Fractions, decimals, and percentages are generally assumed to differ in difficulty based on the degree to which their structure is perceptually similar to whole numbers. Specifically, percentages are viewed as most similar to whole numbers with their fixed unstated denominator of 100. Decimals are often assumed to be easier than fractions because their place-value structure is an extension of the base-ten system for whole numbers, unlike fractions, which have a bipartite structure (i.e., *a*/*b*). However, there has been no comprehensive investigation of how fraction, decimal, and percentage knowledge compares with whole number knowledge. To assess understanding of the four notations, we measured within-participants number line estimation of equivalent fractions and decimals with shorter string lengths (e.g., 8/10 and 0.8) and longer string lengths (e.g., 80/100 and 0.80), percentages (e.g., 80%), and proportionally equivalent whole numbers on a 0–100 scale (e.g., 80.0). Middle school students (N = 65; 33 female) generally underestimated all formats relative to their actual values (whole numbers: 3% below; percentages: 2%; decimals: 17%; fractions: 5%). Shorter string-length decimals and fractions were estimated as smaller than equivalent longer string-length

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equivalents. Overall, percentages were estimated similarly to corresponding whole numbers, fractions had modest string-length effects, and decimals were the most underestimated, especially for single-digit decimals. These results highlight the strengths and weaknesses of children's understanding of each notation's magnitudes and challenge the assumption that decimals are easier than fractions.

Introduction

Rational number difficulties are thought to stem from *whole number bias,* which is difficulty in integrating prior whole number knowledge with contradictory knowledge about other notations (Ni & Zhou, 2005). In the integrated theory of numerical development, Siegler and colleagues (2011) proposed that children gradually incorporate new knowledge about rational numbers into existing whole number knowledge. Yet, Siegler and colleagues (2011) focused solely on estimation of fractions and did not investigate how decimal and percentage magnitude knowledge are incorporated and whether these notations are similarly affected by interference from whole number knowledge.

Fractions, decimals, and percentages are generally assumed to differ in difficulty based on the degree to which their structure is perceptually similar to whole numbers. For example, decimals are often assumed to be easier than fractions because their place-value structure is an extension of the base-ten system for whole numbers, unlike fractions, which have a bipartite structure (i.e., a/b) (for a review, see Tian & Siegler, 2018). However, unlike whole numbers, a longer string length for decimals (and fractions) does not always signify a larger magnitude (e.g., 0.23 is not greater than 0.9). Indeed, decrements in performance resulting from string-length effects have been observed in rational number magnitude comparison (Coulanges et al., 2021; Desmet et al., 2010; Durkin & Rittle-Johnson, 2015; Huber et al., 2014; Roell et al., 2017, 2019; Varma & Karl, 2013), ordering (Van Hoof et al., 2018), and number line estimation (Braithwaite & Siegler, 2018; Schiller et al., 2024). Moreover, percentages are viewed as most similar to whole numbers with their fixed unstated denominator of 100 (Moss & Case, 1999), but this assumption has not been systematically investigated. Thus, the degree to which fraction, decimal, and percentage magnitude estimation is similar to that of whole number estimation has not been empirically tested in children.

This lack of investigation has practical implications. Students' understanding of rational numbers is often weak (Lortie-Forgues et al., 2015), which is especially concerning given their importance in later math achievement (Siegler et al., 2012) and frequent use in the workplace (Handel, 2016). Moreover, difficulties in understanding rational numbers have downstream consequences (Rosenberg-Lee, 2021), for example, in understanding information relevant to health (Cavanaugh et al., 2008; Fitzsimmons et al., 2023, 2024; Thompson et al., 2023) and finances (Gerardi et al., 2013). One controversial proposal to remediate these challenges is to alter the order in which the notations are introduced from the typical fractions, decimals, and then percentages (Common Core State Standards Initiative) to introducing notations from most to least similar to whole numbers—that is, percentages, decimals, and then fractions (for a review, see Tian & Siegler, 2017). Indeed, an experimental curriculum that followed such a sequence demonstrated greater gains in learning over the typical sequence (Kalchman et al., 2001; Moss & Case, 1999). Yet, there have been no studies of children's magnitude estimation that directly contrast each notation for equivalent fractions, decimals, percentages, and proportionally equivalent whole numbers. If percentages are estimated most similarly to whole numbers, perhaps percentages should be the notation taught immediately after whole numbers.

Therefore, the current investigation aimed to elucidate the degree to which magnitude estimation is affected by notation in children. We used a number line task to investigate how magnitude estimation varies by notation (whole number vs. fraction vs. decimal vs. percentage); we also examined how estimates of fractions and decimals are affected by string length and magnitude. Thus, this design allowed us to examine to what extent the superficial similarity of each notation to whole number notation is predictive of estimation performance.

Number line estimation is often used to measure an individual's understanding of the magnitude of numbers (Booth & Siegler, 2006; Laski & Siegler, 2007; Opfer & Siegler, 2007; Schneider et al., 2018; Siegler & Booth, 2004; Siegler & Opfer, 2003; Thompson & Opfer, 2008). Although theoretical debate surrounds the nature of the capacities indexed by number line estimation (Barth & Paladino, 2011; Barth et al., 2011; Cantlon et al., 2009; Cohen & Blanc-Goldhammer, 2011; Thompson et al., 2022), the practical relevance of the task is clear. Higher precision on number line estimation tasks for whole numbers, fractions, and decimals is correlated with whole number and fraction calculation accuracy and overall math achievement (Booth & Siegler, 2006, 2008; Schneider et al., 2009, 2018; Siegler & Booth, 2004; Siegler & Pyke, 2013; Siegler et al., 2011). However, prior rational number estimation studies have focused on a subset of notations in a given experiment and have not examined performance for equivalent numbers expressed in different notations. The current study expands on prior work on children's number line estimation (e.g., Iuculano & Butterworth, 2011) that examined fractions, decimals, whole numbers, and money but excluded percentages. Furthermore, this earlier work did not explicitly manipulate string length of the notations being compared, which recent work has highlighted as a crucial feature in decimal comparison and estimation (Rosenberg-Lee et al., 2023; Schiller et al., 2024).

Decimal underestimation relative to whole numbers

Given what is known about whole number knowledge interfering with new knowledge of rational numbers (Ni & Zhou, 2005), it would seem likely that decimal estimation performance might be worse than whole number estimation. Here, we predict a specific direction of effects for children following prior research with young adults. Notably, Schiller et al. (2024) found that undergraduates underestimated decimals relative to whole numbers by about 4%. With regard to children, prior research has shown that they typically have similar or worse estimation performance than adults (e.g., Iuculano & Butterworth, 2011), although there are instances where children perform better than adults, especially at estimating unit fractions (e.g., 1/5) (Opfer & DeVries, 2008; Thompson & Opfer, 2008). Thus, the first aim of this study was to investigate whether children underestimate decimal stimuli relative to their whole number equivalents. Specifically, in this context, we hypothesized that children, like adults, would underestimate decimals relative to proportionally equivalent whole numbers, especially given a tendency to view fractions and decimals as small entities (Kallai & Tzelgov, 2009, 2014).

String length and magnitude interference effects

Decimal string length

Many researchers have suggested that decimals might be easier for children to learn than fractions because of their similarity to whole numbers (for a review, see Tian & Siegler, 2017). Indeed, decimals share the same place value as whole numbers. As with processing whole numbers, individuals show distance effects for decimals. That is, individuals have worse accuracy and slower response times for near comparisons (e.g., 8 vs. 9, 0.80 vs. 0.90) than for far comparisons (e.g., 2 vs. 9; 0.20 vs. 0.90) (DeWolf et al., 2014; Ganor-Stern, 2013; Hurst & Cordes, 2016, 2018; Kallai & Tzelgov, 2014; Wang & Siegler, 2013). However, an inherent difference between whole numbers and decimals is the role of string length. For whole numbers, a longer string length always signifies a larger magnitude (e.g., 23 > 9), but this is not the case for decimals (e.g., 0.23 < 0.9). The number of digits can interfere with decimal comparison, such that participants are slower and less accurate when the larger decimal has a shorter string length (e.g., 0.23 vs. 0.9) (Coulanges et al., 2021; Desmet et al., 2010; Durkin & Rittle-Johnson, 2015; Huber et al., 2014; Roell et al., 2017, 2019; Varma & Karl, 2013; Wang & Siegler, 2013).

One explanation for these decrements in accuracy and speed based on decimal string length is the *string length congruity effect* (Huber et al., 2014). Based on experiments with human participants and computational modeling, Huber and colleagues (2014) proposed that decimal comparison involves a left-to-right serial comparison for each decimal digit as well as a comparison of the physical length of the entire decimal string, leading to string length interference (Nuerk, Kaufmann, et al., 2004; Nuerk et al., 2001; Nuerk, Weger, et al., 2004; Nuerk & Willmes, 2005).

A second explanation is the *semantic interference effect* (Varma & Karl, 2013), which suggests that decimal numbers automatically activate whole number referents, causing interference in decimal comparison. Specifically, participants ignore the decimal point and treat the decimals as whole numbers, (e.g., 0.9 and 0.23 would be treated as 9 and 23). Indeed, participants are faster and more accurate when comparing 0.90 and 0.23, suggesting that the semantic interference dissipates when the string lengths are congruent (Coulanges et al., 2021; Varma & Karl, 2013). Thus, both theories predict that single-digit decimals (e.g., 0.8) should be estimated as smaller than equivalent double-digit decimals (e.g., 0.80). This prediction was borne out in adults (Schiller et al., 2024), and the second aim of this study was to test this prediction in children and determine the extent of underestimation relative to the 4% in adults.

A further prediction, unique to the semantic interference effect, proposes that the magnitude of the decimal digits will also affect number line estimates, such that single-digit decimals with a larger magnitude (e.g., 0.8) will be estimated as proportionally smaller than single-digit decimals with a smaller magnitude (e.g., 0.2). This result is exactly what was found in adults (Schiller et al., 2024), and the authors reasoned that if adults were estimating 0.8 as proportionally equivalent to 8 (i.e., 0.08), this results in a difference of 72 from its actual value, causing far more underestimation than estimating 0.2 as proportionally equivalent to 2 (i.e., 0.02), a difference of 18. Thus, as an exploratory analysis, we sought to determine whether there was greater underestimation for decimals with larger magnitudes (Exploratory Question 1).

Fraction string length

The bipartite (i.e., a/b) structure of fractions is also thought to pose problems for children because many students consider the parts of fractions as independent whole numbers (Alibali & Sidney, 2015; Ni & Zhou, 2005; Thompson et al., 2022). Consistently, double-digit fractions (e.g., 16/20) were estimated as larger than single-digit fractions (e.g., 4/5), solely based on surface-level fraction components of the number being presented (Braithwaite & Siegler, 2018; Fitzsimmons et al., 2020). In addition, there was a similar effect within double-digit fractions (e.g., 16/20 was perceived as larger than 12/15), suggesting that the numerals themselves (rather than the number of digits) influence magnitude perception. This result is in line with what would be predicted by the semantic interference account. Although children have been shown to exhibit estimation patterns similar to adults (Iuculano & Butterworth, 2011), the effect of fraction string length has not been explicitly manipulated. Thus, as part of our second research question, we tested whether fraction string length, as well as decimal string length, affected magnitude processing in children (e.g., is 8/10 perceived as smaller than 80/100?). To preview, we found string length effects for decimals and fractions, so we directly compared the extent of effects on estimation for each notation in Exploratory Question 2.

Percentages estimation relative to whole numbers

In comparison with the depth of research on other rational number notations, understanding of percentages is relatively understudied. A review found only 10 studies examining children's understanding of percentages as compared with hundreds of studies on fractions (Tian & Siegler, 2017) despite the greater prevalence of percentages than fractions or decimals in written and oral language (Yang & Wang, 2022). Moreover, textbooks devote far less coverage to percentages than fractions or decimals (Siegler & Tian, 2022).

A potential reason for the lack of research and educational emphasis is that percentages appear easy to understand and use. Indeed, many students reported translating fractions to approximate percentages to help them estimate fractions on number lines (Siegler et al., 2011). Furthermore, adults show a preference for percentages over other notations in certain situations (Mielicki et al., 2022; Tian et al., 2020) and for rate percentages more positively (and more similarly to whole numbers) than fractions (Sidney et al., 2021).

However, results from the National Assessment of Educational Progress (NAEP) paint a more complicated picture; only 38% of eighth graders correctly estimated a 15% tip (NAEP, 1996), and only 37% of eighth graders correctly identified the number of employees that a 10% increase in 90 employees would yield (NAEP, 2005). These results are not limited to standardized test outcomes. When asked in an experimental setting to decide whether 87% of 10 is greater or less than 10, more than half of middle school students incorrectly claimed that it is greater than 10 (Gay & Aichele, 1997). Children's attention was perhaps captured by the whole number component 87, and they judged 87 > 10.

This interpretation dovetails with findings demonstrating a bias to select percentages as larger than fractions and decimals in cross-notation comparison with both undergraduates (Schiller et al., 2023) and middle school students (Schiller, 2020; Schiller & Siegler, 2023). For example, when asked whether 3/5 or 35% was larger, most middle school children incorrectly chose 35%. Individual differences in this bias are related to a variety of other outcomes. In particular, stronger percentages-are-larger biases correlate with lower performance on other rational number tasks and lower SAT/ACT scores (Schiller, 2020; Schiller et al., 2023; Schiller & Siegler, 2023). The authors suggested that individuals with this bias may be inappropriately treating percentages as whole numbers.

Applying whole number principles to understanding of percentages might not always be detrimental. As noted, percentages are thought to be a good starting place to introduce proportional thinking because they are assumed to be estimated most similarly to whole numbers (Moss & Case, 1999). If this were the case, then there should be no difference in estimating percentages on a 0%–100% number line relative to estimating whole numbers on a 0–100 number line (e.g., 80% should be estimated in the same position relative to 80). Thus, the third goal of the study was to investigate whether participants estimate percentages similarly to whole numbers.

The current study

This study, which was preregistered (https://osf.io/m9c53/?view_only=a50def93ca21403db6ecb22367c811fe), had three main goals. The first was to investigate whether children underestimate decimals relative to proportionally equivalent whole numbers, as was found with adults (Schiller et al., 2024). The second goal of the study was to determine whether digit length affects decimal and fraction estimation. We predicted that children would underestimate single-digit decimals more relative to their double-digit equivalents; similarly, we predicted parallel fraction underestimation based on string length. The third goal was to examine whether whole numbers and percentages were estimated similarly. Percentage estimation on the number line could yield high levels of performance, comparable with whole number estimation and better than the other rational number notations, if participants are thinking about percentages as whole numbers. Together, this research presents the first comprehensive analysis of children's magnitude estimation for all rational number notations—fractions, decimals, percentages, and whole numbers.

Method

Participants

Participants were sixth-grade students from five classrooms, containing 77 students, of a northeastern United States public school who were participating in a larger intervention study. Because of our opt-out procedure, which included all students in the classroom, we did not collect exact ages or birthdates. However, to provide age information for those not familiar with the grade/age equivalents in the United States, these students were approximately 11 to 12 years old. We excluded 2 participants who answered less than 70% of problems in each of the number line conditions (decimals, fractions, percentages, and whole numbers), as per our preregistration criteria (https://osf.io/m9c53/?view_only=a50def93ca21403db6ecb22367c811fe). Due to absences, only 69 data points were collected for each measure. We also excluded 2 participants because we had an instance of a duplicate participant ID number and could not determine whether it was two participants who completed the study twice or there was an error in assigning ID numbers (e.g., 2 students were assigned the same student ID number by the classroom teacher). Our final sample size was 65 sixth-grade students (32 male and 33 female). School demographics include 20.3% Black/African American, 40.4% Hispanic/Latino, 15.2% Asian, 0.4% Native American, 3% multi-race non-Hispanic, and 20.6% non-Hispanic White children, with 75% of the school population receiving free or reduced-price lunch.

Using G*Power (Faul et al., 2009), we performed an a priori power analysis based on the effect of notation (i.e., decimal or whole number) with partial η^2 = .17 (reported as generalized η^2 = .04 in Schiller et al., 2024), with 80% power and alpha = .05. For a repeated-measures analysis of variance (ANOVA) within factors, G*Power indicated that 22 participants would be needed based on the effect size of Schiller et al. (2024). The current sample is larger than what G*Power indicated would be required because the sample size was determined by the classes participating in the larger intervention study, as per consent/assent procedures approved by the institutional review board.

General procedure

Data were collected in-person during regular classroom hours. Participants were assessed on a set of math measures, most of which are not reported here. Specifically, participants completed tasks in the following order: fraction arithmetic estimation, magnitude comparison, a number line estimation task with stimuli of different notations (our main task of interest described in detail below), and rational number arithmetic. Except for the rational number arithmetic task, which was completed on paper and then entered into Qualtrics, all the other tasks were completed on school-issued Chromebook laptop computers without access to paper and pencil. The whole pretest session took approximately 20 min, with the number line estimation task that was the focus of this study lasting less than 5 min.

Number line estimation

The number line estimation task was implemented with a slider question in Qualtrics, which has been shown to yield similar results to point-and-click versions of the task (Oppenzato et al., 2022). Number line estimation notation type was counterbalanced by participant, with each notation presented individually in one of four counterbalanced orders: (a) whole percentage decimal fraction (WPDF), (b) WPFD, (c) PWDF, or (d) PWFD. We had hoped to randomly assign participants to all combinations of orders (e.g., DFWP, DFPW) but knew we would not have enough participants and opted to present orders with whole numbers/percentages first because these two notations are generally assumed to be most similar. Furthermore, we anticipated that equal numbers of students would be assigned to each of the four order presentations, but due to a technical error regarding how counterbalancing was set up in Qualtrics, the numbers of students in the conditions were unequal: WPDF (n = 25), WPFD (n = 16), PWDF (n = 10), and PWFD (n = 14). However, a chi-square test revealed that the numbers of participants who estimated fractions or decimals after whole/percentages (WPDF/PWDF = 35 vs. WPFD/PWFD = 30) were equally distributed, $\chi^2(1, N = 65) = 2.27$, p = .13).

Each block of the task began with a practice trial consisting of 1/2, 0.50, 50%, or 50.0, depending on the notation type. Participants did not receive feedback; this practice trial was just to orient them to the activity in case they needed to ask questions. Participants were given these instructions: "Please slide to estimate the [decimal, fraction, percent, or whole number] on the number line." To minimize visual dissimilarity between whole numbers and decimals, the whole numbers were presented with a decimal point and a zero in the tenths place (e.g., 80.0) and the decimals were presented with a zero in the ones place (e.g., 0.80). For fraction and decimal trials, a 0–1 number line was presented and participants used the computer's track pad to select the position they thought best estimated the magnitude of the number. For whole number and percentage trials, the only aspect that differed from the fraction/decimal trials was that participants were presented with a 0–100 and 0%–100% number line, respectively (Fig. 1). This range was chosen because it is a relatively easy task where middle school children should be extremely accurate in their estimates (Siegler et al., 2009) and enabled us to measure their estimates for the proportionally equivalent fraction, decimal, and percentage values on the 0–1 number line.

Across the notations, we had three number types: decade, tenths, and within-decades. We illustrate these first based on their decimal versions. Specifically, *decade* decimals include all two-digit decimal

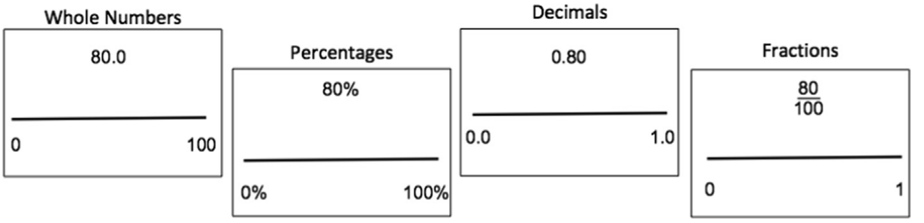

Fig. 1. Schematic of the number line task for whole numbers (W), percentages (P), decimals (D), and fractions (F). Each notation was presented in a block in a counterbalanced order of WPDF, PWDF, WPFD, or PWFD.

numbers between 0 and 1 consisting of a 0 in the hundredths place (e.g., 0.10, 0.20, 0.30). The decade fractions included all equivalent fractions (e.g., 10/100, 20/100, 30/100), with corresponding decade percentages (e.g., 10%, 20%, 30%) and decade wholes (e.g., 10, 20, 30). We also wanted to examine string-length differences for decimals and fractions, recognizing that our decade stimuli (e.g., 0.80 and 80/100) could also be expressed as a shorter string length (e.g., 0.8 and 8/10). Thus, the *tenths* decimal and fraction stimuli involved shorter string-length equivalents to the decade stimuli (e.g., 0.1, 0.2, 0.3 and 1/10, 2/10, 3/10). Note that there are no corresponding tenths stimuli for percentages or wholes. Finally, we wanted to include numbers that were *within-decades,* which are numbers that fell between the decades/tenths (e.g., 0.88 is a within-decade stimulus because it falls between 0.80/0.8 and 0.90/0.9). To select the within-decade stimuli, we used the same fractions (and equivalent decimals) as those presented in Siegler and Pyke (2013) with two exceptions, namely that (a) 2/9 replaced the 1/5 (because 1/5 corresponds to a decade for decimals [i.e., 1/5 = 0.20] rather than a within-decade decimal [i.e., 2/9 = 0.22]) and (b) 1/19 was excluded (because 1/19 is smaller than the first decade [i.e., 0.10] in our stimulus set). The previous within-decades decimals of Schiller et al. (2024) did not map cleanly onto simple fractions without 100 in the denominator (e.g., 0.27 = 27/100), so we opted instead to use the fraction stimuli from Siegler and Pyke (2013): 2/13, 2/9, 1/3, 3/7, 7/12, 5/8, 3/4, 7/8, and 13/14. We also included the corresponding within-decade decimal (e.g., 0.15, 0.22, 0.33), percentage (e.g., 15%, 22%, 33%), and whole numbers (e.g., 15, 22, 33).

Participants completed a total of 40 trials. There were 8 stimuli within each condition given that we removed a half (e.g., 0.50, 0.5, 5/10) from all notations (see Table 1 for examples by types, and see Table 4 in the online supplementary material the full stimuli set). Given the length of the larger intervention study, we opted to reduce the stimuli set for individual participants. Table 4 in the supplementary material includes the full set of stimuli, with half of the participants estimating stimuli corresponding to 0.10, 0.30, 0.60, and 0.80 (Set A) and the other half of the participants estimating stimuli corresponding to 0.20, 0.40, 0.70, and 0.90 (Set B). An ANOVA with type (e.g., fraction decade or decimal tenth) and stimulus set (Set A or Set B) revealed that there was only an effect of type whether students completed one stimulus set or the other, $F(1, 53) = 1.18$, $p = .28$. Thus, we did not include this factor in further analysis.

Statistical analyses

Statistical analyses were conducted in R Version 1.2.1578 (R Core Team, 2019), and an html version of an R-markdown file reporting all analyses, along with de-identified data, is posted to Open Science

Table 1
Examples from each category of stimuli.

	Notation			
Type	Whole	Percentage	Decimal	Fraction
Within-decades	88.0	88%	0.88	7/8
Decades	80.0	80%	0.80	80/100
Tenths	–	–	0.8	8/10

Framework (https://osf.io/m9c53/?view_only=a50def93ca21403db6ecb22367c811fe). Typically, number line estimation analyses employs percentage absolute error (PAE) (e.g., Siegler & Pyke, 2013; Siegler et al., 2011), which measures the absolute value of the difference between the actual and estimated values (e.g., Booth & Siegler, 2008; Hamdan & Gunderson, 2017; Rivers et al., 2020). However, following Schiller et al. (2024), we opted to focus on directional error because PAE, by design, masks the effects of direction. Thus, we measured performance on the number line estimation task using directional error (i.e., Estimate − Actual Magnitude) (Schiller et al., 2024). This measure allows for distinguishing whether numbers are over- or under-estimated. For purposes of analysis, the whole number magnitudes and corresponding errors were put in the same scale as the rational numbers (i.e., the whole number magnitudes were divided by 100). ANOVAs were used to determine whether notation (fraction, decimal, percentage, or whole number), type (within-decades, decades, or tenths), and/or their interaction had an effect on directional error of estimation.

To determine whether number line estimation depended on the magnitude for each notation, we used linear mixed effects models. All mixed model analyses used the *lmer* function from the "lme4" package in R (Bates et al., 2015). Satterthwaite's method for estimating degrees of freedom was used. For post hoc analyses, we used functions from the "emmeans" package (*emmeans* and *emtrends*) in R (Lenth et al., 2018). This package allows post hoc analyses in models involving interactions between categorical factors and continuous predictors as well as simple slope analyses. Results were considered marginal if.05 < p < .10 and were considered significant if p < .05, as is commonplace in this field.

We preregistered three analyses:

Research Question 1 (RQ1): Do children underestimate decimals relative to whole numbers? We wanted to determine whether children, like adults (Schiller et al., 2024), also underestimated decimals (e.g., both 0.8 and 0.80) relative to proportionally equivalent whole numbers (e.g., 80).

Research Question 2 (RQ2): Do children underestimate decimals and fractions with fewer digits relative to the equivalent numbers with more digits? This analysis was conducted to determine whether string length plays a role in decimal and fraction number line estimation given that these findings for fractions are somewhat mixed in the literature, with consistent effects for decimals (Schiller et al., 2024) but not for fractions (e.g., Braithwaite & Siegler, 2018, found string-length effects for fractions, whereas Tian & Siegler, 2017, did not).

Research Question 3 (RQ3): Do children estimate percentages differently than whole numbers? This analysis sheds light on whether percentages are indeed estimated most similarly to whole numbers.

We also conducted three exploratory analyses (see supplementary material) to follow up on the finding that both decimal and fraction estimation were influenced by string length. First, to determine whether number line estimation was dependent on magnitude for each notation, given that decimal estimation for adults was affected by magnitude (Schiller et al., 2024), we examined children's number line estimation based on the presented magnitude for each notation (e.g., whether larger numbers were more underestimated). Second, to determine whether decimals or fractions were more affected by string length than the other, we investigated how directional error differed between those notations. Finally, based on the striking underestimation of decimals, we conducted a third exploratory analysis to investigate order effects. In adults, prior work found that estimating decimals immediately after whole numbers exacerbated the underestimation (Schiller et al., 2024) relative to completing them before the whole numbers. Therefore, we investigated whether estimating decimals immediately after whole numbers/percentages versus not immediately after whole numbers/percentages exacerbated the underestimation (i.e., after estimating fractions). We anticipated that estimating whole numbers/percentages immediately before decimals would lead to more decimal underestimation for children as well.

Results

In general, across all formats and number types, children underestimated the magnitude of the numbers (Fig. 2; see Table 1 in supplementary material for full results). The exceptions to this general pattern were fraction decades (e.g., 80/100), which were overestimated, and percentage decades (e.g., 80%), which were not significantly different from 0. Now, we turn to our specific research questions.

RQ1: Do children underestimate decimals relative to whole numbers?

Fig. 2 indicates that children underestimate decimals relative to whole numbers. To quantify this observation and determine whether the notation (decimal or whole), type (decades or within-decades), and/or their interaction had an effect on directional error of estimation, we conducted a 2 × 2 repeated-measures ANOVA (see Fig. 2 in supplementary material). This analysis revealed the expected main effect of ntation, $F(1, 64) = 10.54$, $p = .002$, $\eta_g^2 = .07$, with more underestimation in decimals (10%, SD = 18%) than in whole numbers (3%, *SD* = 4%) (supplementary material presents full results of effect for type/interaction). These analyses indicate that children, just like adults (Schiller et al., 2024), systematically underestimated even double-digit decimals relative to their corresponding whole number equivalents.

RQ2: Do children underestimate decimals and fractions with fewer digits relative to equivalent numbers with more digits?

RQ2a: Shorter versus longer string-length decimals

To examine whether there was a difference in directional error on stimuli with comparable proportional decimal magnitudes (decimal tenths, decimal decades, and whole decades) but different string lengths, we conducted a one-way ANOVA with three levels. Specifically, we wanted to determine whether expressing a decimal as a tenth (e.g., 0.8) relative to a decade (e.g., 0.80) exacerbated the directional error relative to its proportionally equivalent whole number (e.g., 80). For this analysis, there was a main effect of type, $F(2, 128) = 74.59$, $p < .001$, $\eta_g^2 = .39$. Post hoc *t* tests confirmed that decimal tenths (e.g., 0.8) were underestimated more ($M = 31\%$, $SD = 20\%$) than their comparable decimal

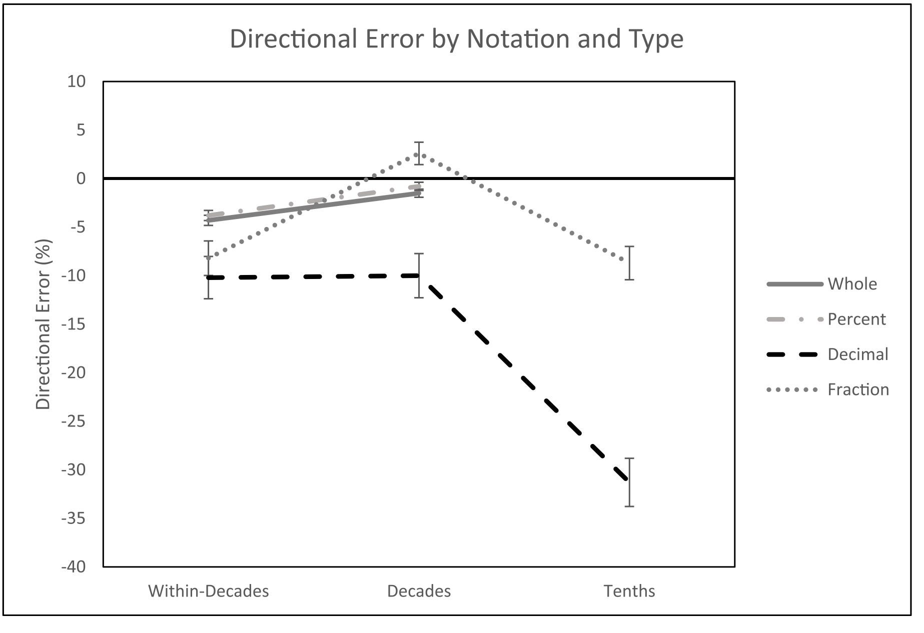

Fig. 2. Line graph displaying average directional error by notation and type. Error bars represent standard errors. As depicted here, most notations/types are underestimated. Strikingly, decimal tenths (e.g., 0.8) were underestimated by quite a bit more than decimal decades (e.g., 0.80) and all other notations/types.

decade trials (e.g., 0.80) (M = 10%, SD = 18%), $t(64) = 7.98$, $p < .001$, and whole decade trials (e.g., 80) (M = 1%, SD = 3%), $t(64) = 11.86$, $p < .001$. Consistent with RQ1, for decade trials, decimals were underestimated relative to whole numbers, $t(64) = 3.61$, $p = .001$. To better understand the sources of these effects, we plotted individuals' directional errors across these conditions along with rain clouds to display the distribution of performance (Fig. 3). Inspection of the figure revealed bimodal distributions in estimation performance in the decimal conditions, with subsets of participants showing more than 10% underestimation. Using this cut point to categorize participants reveals that 72% of the sample underestimated decimal tenths by more than 10%, whereas only 25% of the sample underestimated decimal decades by more than 10% and no one underestimated whole decades by the same amount. Thus, the decimal underestimation relative to whole numbers may be largely driven by subsets of participants with considerable underestimation.

In sum, decimals were underestimated relative to whole numbers (RQ1), and single-digit decimals exacerbated the underestimation (RQ2a). In particular, this underestimation was exacerbated by string length, such that tenths (e.g., 0.8) were underestimated by 31% and decade stimuli (e.g., 0.80) were underestimated by 10% (see Fig. 2 and Table 1 in supplementary material).

RQ2b: Shorter versus longer string-length fractions

Like decimals, fraction string length exacerbated underestimation, such that fraction tenths (e.g., 8/10) were perceived as smaller than their comparable fraction decade (e.g., 80/100) (the supplementary material presents full analyses on fraction estimation). Unlike decimals, fraction decades (e.g., 80/100) were perceived as larger than their proportionally equivalent whole number trials (e.g., 80). Perhaps the whole number component in the denominator (i.e., 100) biased participants to estimate 80/100 on a 0–1 number line as slightly larger than 80 on the 0–100 number line.

RQ3: Do children estimate percentages differently than whole numbers?

To determine whether the notation (percentages or whole), type (decades or within-decades), and/or their interaction had an effect on directional error of estimation, we conducted a 2 × 2 repeated-

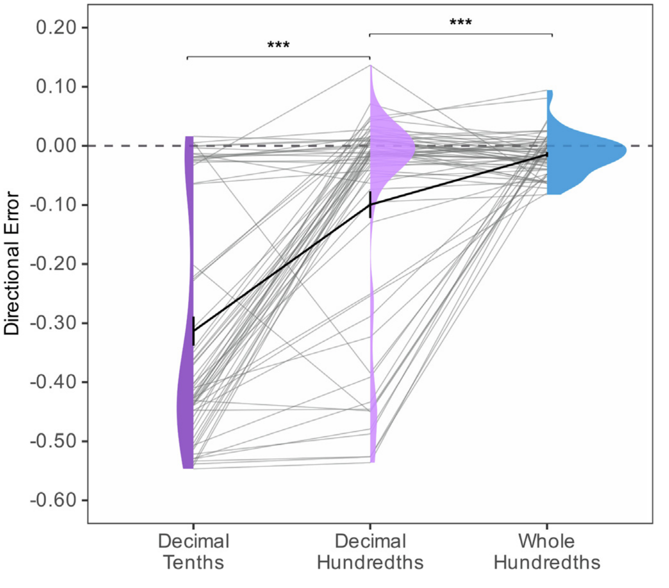

Fig. 3. Directional error for each type of proportionally equivalent number (decimal tenths, decimal decades, and whole decades). Decimals are underestimated compared with their proportionally equivalent whole numbers (e.g., 0.8 and 0.80 underestimated relative to 80). Shorter decimal string length (i.e., single-digit vs. double-digit decimals) exacerbates the underestimation. ***$p < .001$.

measures ANOVA. This analysis revealed a main effect of type (decades or within-decades), $F(1, 64) = 72.80$, $p < .001$, $\eta_g^2 = .13$, but did not reveal a main effect of notation (percentage or whole), $F(1, 64) = 2.85$, $p = .10$, $\eta_g^2 = .01$, or an interaction between notation and type, $F(1, 64) = 0.03$, $p = .87$. Percentages and whole numbers were equally underestimated relative to their actual values (percentages: $M = 2\%$, $SD = 4\%$; whole numbers: $M = 3\%$, $SD = 4\%$). These results suggest that children estimated percentages and whole numbers from 0 to 100 equivalently.

Exploratory analyses

We also investigated three exploratory questions, which we report on in the supplementary materials and summarize here:

1. Does under- or overestimation for each notation depend on magnitude? There was an effect of magnitude—that is, more underestimation for larger numbers (e.g., 0.8) than for smaller numbers (e.g., 0.2)—as proposed by the semantic interference account (Varma & Karl, 2013). This magnitude-based underestimation was most pronounced for decimals, particularly decimal tenths relative to percentages and fractions (Fig. 4; see full analyses in supplementary material).
2. How do decimals and fractions compare in terms of directional error? Children showed less underestimation when estimating fractions as compared with decimals for fractions and decimals equivalent to a decade (e.g., 8/10, 80/100, 0.8, 0.80). In contrast, there was no difference in estimation for fractions and decimals for within-decades (e.g., 7/8, 0.88).
3. Does estimating decimals immediately after whole numbers/percentages versus after fractions affect estimation performance? Estimating decimals immediately after whole numbers/percentages resulted in strikingly more underestimation for decimals (24% vs. 10%). Notably, Fractions did not show this same dependence on task order (5% vs. 3%, with no significantly more underestimation when fractions were estimated third immediately after whole numbers/percentages).

Discussion

The current work investigated children's number line estimation for equivalent single- and double-digit fractions and decimals, the corresponding percentages, and the proportionally equivalent whole numbers (e.g., 8/10, 80/100, 0.8, and 0.80 on a 0–1 number line, 80% on a 0%–100% number line, and 80 on 0–100 number line). We found that numbers were generally underestimated. Replicating prior work with young adults (Schiller et al., 2024), children underestimated decimals relative to whole numbers but to a greater extent (adults underestimated decimals relative to whole numbers by 4% as compared with 10% relative underestimation by children). Shorter string-length fractions and

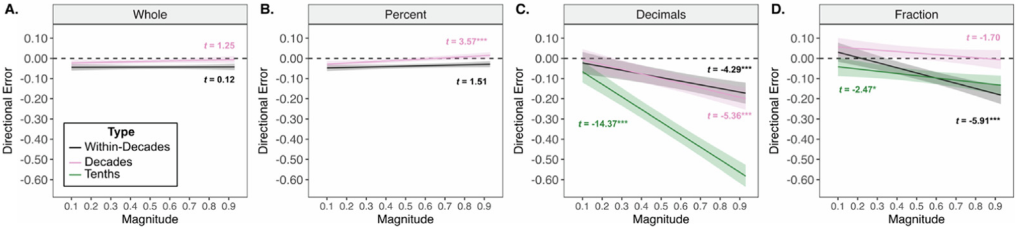

Fig. 4. Directional error by type and magnitude, with separate panels for each notation: (A) whole number; (B) percentage; (C) decimal; and (D) fraction. For whole numbers (0–100 range), there is no effect of magnitude, suggesting that estimation does not depend on the magnitude of the presented stimuli for either type of whole number. For percentages, there is an effect of magnitude, suggesting slightly larger estimates for larger percentage decades. For decimals, there was general effect of magnitude, such that larger decimals were more underestimated than smaller decimals for all the decimal types. The effect was strongest for decimal tenths and did not differ for decimal decades and within-decades. For fractions, larger presented fractions were significantly underestimated for within-decades and tenths. For fraction decades, there was only a marginal effect of magnitude and in fact smaller fractions were slightly overestimated.

decimals were more underestimated than their equivalents with longer string lengths. Specifically, the underestimation was more dramatic for decimal tenths (e.g., 0.8) than for fraction tenths (e.g., 8/10), with decimal tenths being underestimated by 31% and fraction tenths being underestimated by 8%. These results support the interpretation that whole number knowledge about shorter string length indicating smaller magnitude interferes with the estimation of fractions and decimals. Whole numbers and percentages were estimated similarly, supporting the previously untested assumption that children estimate percentages similarly to whole numbers.

Decimals and fractions are underestimated relative to whole numbers and percentages

Our findings with children replicate the systematic underestimation of decimals found with adults (Schiller et al., 2024), albeit to a greater extent for children. The current work extends those findings to show that decimals are also underestimated relative to percentages.

Similarly, we extend the findings of Schiller et al. (2024) to show that fractions are also largely underestimated relative to whole numbers and percentages, but this effect depended on the denominator of the fractions. Among this fraction stimulus set, most were underestimated on average except for fractions that had 100 as the denominator (i.e., fraction decades). For fraction decades (e.g., 80/100), the stimuli were slightly overestimated, perhaps because the magnitude of the numerals may have counteracted the bias of thinking that fractions are small (Kallai & Tzelgov, 2009). More research is needed to replicate this effect and perhaps examine whether problems like 800/1000 show even more overestimation, especially in relation to 1 in *X* statistics which are commonplace in health statistical information (Thompson et al., 2023).

Underestimation in mathematics is not a new phenomenon. Underestimation has been observed with non-symbolic quantities, including dot arrays (Izard & Dehaene, 2008) and bisection of physical lines (Longo & Lourenco, 2007). With symbolic quantities, individuals tend to show a left bias in generating random numbers (Loetscher & Brugger, 2007), suggesting a tendency to underestimate. Fractions, even improper fractions (e.g., 3/2), are perceived as entities smaller than 1 (Kallai & Tzelgov, 2009). Here, we found a pattern where fractions and decimals are generally underestimated relative to percentages and whole numbers. That being said, children showed less underestimation of fractions than decimals for decades (i.e., 8/10, 80/100, 0.8, 0.80; see ExpQ2 in supplementary material), which could have important implications for education.

The finding that fractions and decimals are estimated on the number line as smaller than equivalent percentages, which did not differ from whole numbers, parallels cross-notation magnitude comparison research (Schiller, 2020; Schiller et al., 2024; Schiller & Siegler, 2023). In particular, prior research has revealed that many children and adults exhibit a percentages-are-larger bias in cross-notation comparisons involving fractions, decimals, and percentages, and this bias has ramifications for math outcomes. Specifically, given a percentage versus fraction or percentage versus decimal comparison, performance was better when the larger value was presented as a percentage than as a fraction or decimal. In other words, accuracy was higher for items such as 40% versus 1/4 and 40% versus 0.25 than when the larger magnitude was equivalently expressed as a fraction or decimal (e.g., 2/5 vs. 25%, 0.40 vs. 25%).

The authors suggested that participants may have been treating percentages as whole numbers in the cross-notation comparisons, reasoning that fractions and decimals are smaller than whole numbers. Indeed, the current work shows that children estimate fractions and decimals as smaller than equivalent percentages and proportionally equivalent whole numbers, thereby providing a potential explanation for prior cross-notation findings where fractions/decimals are perceived as smaller than percentages (Schiller, 2020; Schiller & Siegler, 2023). Whether this result reflects the idea that fractions and decimals are viewed as entities smaller than 1 (Kallai & Tzelgov, 2009) or just as "small" is a fruitful area for future work.

Decimal and fraction number line estimation is affected by string length and digit magnitude

The findings here converge with other research demonstrating that rational number understanding is affected by string length and digit magnitude (Avgerinou & Tolmie, 2020; Coulanges et al., 2021;

Huber et al., 2014; Ren & Gunderson, 2019; Varma & Karl, 2013). Specifically, numerous studies of decimal comparison have found lower accuracy and slower response times for comparisons involving unequal digits, specifically when the larger number has the shorter string length (e.g., 0.9 vs. 0.23) relative to when the larger number has longer (e.g., 0.93 vs. 0.2) or equivalent lengths (e.g., 0.90 vs. 0.23) (Avgerinou & Tolmie, 2020; Coulanges et al., 2021; Huber et al., 2014; Ren & Gunderson, 2019; Varma & Karl, 2013). Similarly, string length has been shown to affect fraction estimation (Braithwaite & Siegler, 2018), especially for those who have lower fraction equivalence knowledge (Fitzsimmons et al., 2020).

Two theories have been offered to explain these effects in the case of decimals: the string-length congruity effect (Huber et al., 2014) and the semantic interference effect (Varma & Karl, 2013). The *string length congruity effect* suggests that magnitude processing involves a left-to-right serial comparison for each decimal digit as well as comparing the physical length of the entire decimal string (Huber et al., 2014; Nuerk, Kaufmann, et al., 2004; Nuerk et al., 2001; Nuerk, Weger, et al., 2004; Nuerk & Willmes, 2005). The *semantic interference effect* (Varma & Karl, 2013) suggests that decimal numbers automatically activate whole number referents (e.g., 0.9 and 0.23 would be treated as 9 and 23), causing interference in decimal comparison. Such interference dissipates when the string lengths are congruent (e.g., 0.90 and 0.23) (Coulanges et al., 2021; Varma & Karl, 2013).

Our results provide evidence in support of both a string length and semantic interference effect for decimals; we found that shorter string length resulted in greater underestimation than longer strings, but we also found a magnitude effect, such that larger decimals (e.g., 0.8) display more underestimation than smaller ones (e.g., 0.2 in ExpQ1 in the supplementary material; comparable but smaller effects were found for fractions in ExpQ1 and ExpQ2). This finding is consistent with evidence that approximately 40% of fifth- and sixth-grade students underestimated one-digit numbers (marking 0.7 at 0.07) and overestimated three-digit numbers (e.g., 0.289) by approximately 10% larger than their actual value (Rittle-Johnson et al., 2001). Interference from the magnitude of whole number referents has also been demonstrated in decimal comparison. Specifically, for problems of equal rational distance (e.g., 0.9 vs. 0.81 and 0.3 vs. 0.21), problems involving whole number referents that are further apart (i.e., 9 vs. 81) are more difficult than problems involving whole number referents that are closer together (e.g., 3 and 21) (Rosenberg-Lee et al., 2023). In other words, when comparing decimals, corresponding whole number magnitudes are activated leading to interference in decimal processing. The current results suggest that this effect is robust across measures of magnitude understanding given that they are found in both comparison and number line estimation.

Percentages and whole numbers are estimated similarly

Our findings suggest that percentages are not estimated differently from whole numbers and that, among rational number notations, percentages were the most accurate. This finding is consistent with the lay intuition that percentages and whole numbers are interchangeable. Even very young children know when their tablet device is about to run out of battery power (i.e., a percentage close to 0), likely because they understand that whole numbers closer to 0 signify a smaller amount. In this case, harnessing knowledge of whole numbers in accessing the magnitudes of percentages is advantageous and helpful. However, depending on whole number knowledge can lead them astray in the case of deciding whether 87% of 10 is smaller or larger than 10 (Gay & Aichele, 1997) or whether 25% is larger than 2/5 (Schiller et al., 2024; Schiller & Siegler, 2023).

The field has debated about which rational number notation is best to learn first in school curriculum (for a review, see Tian & Siegler, 2017). Some argue that decimals might be the most natural segue from whole numbers into other rational numbers. However, the current study found that decimals are represented least similarly to whole numbers. In fact, single-digit decimals (e.g., 0.8) were underestimated the most of any notational type. Given that our results demonstrate that percentages are estimated most similarly to whole numbers, perhaps instruction that progresses from teaching about percentages after whole numbers and before fractions and decimals could potentially circumvent the many problems that children encounter with rational numbers. Indeed, an experimental curriculum that progressed from whole numbers to percentages and then decimals and fractions demonstrated greater gains in rational number concepts for treatment relative to business-as-usual control students (Moss & Case, 1999). Taken together, results from Moss and Case (1999) and the current study suggest that appropriately relating whole numbers and percentages could serve as a springboard to learning about other rational numbers. Future research should directly measure order effects for learning about fractions, decimals, and percentages and whether a particular order might have increased benefits in learning.

Limitations and future directions

The current study has several limitations. The presentation order (presenting whole and percentages first) may have exacerbated underestimation (given the priming results in Schiller et al. (2024). Indeed, estimating percentages/whole numbers immediately before decimals worsened decimal underestimation (i.e., 24% vs. 10%; see ExpQ3 in supplementary material). It would have been advantageous to compare these results with children who estimated decimals or fractions before whole numbers/percentages or before a non-numerical control. For example, the non-numerical Flanker Task, which was interleaved with other numeric tasks in Ren and Gunderson (2019), did not activate whole number knowledge. However, we knew we would not have enough participants to investigate all combinations of order presentations, and our main focus here was not on presentation order. Future research should carefully consider and investigate further the effects of presentation order in studies of children's magnitude estimation of rational numbers.

The presentation format of the stimuli and number line end points may have affected the number line estimation for the different notations. In regard to the number line end points, all rational number lines could have been 0–1 number lines used for fractions rather than 0%–100% for percentages and 0.0–1.0 for decimals. Concerning the format of the whole number stimuli, we had decided to present whole numbers with ".0" affixed to the end of the number (e.g., 80.0) in order to minimize the visual dissimilarity between decimals (e.g., 0.80) and their proportionally equivalent whole numbers (e.g., 80). Future work might examine whether there are any differences if whole numbers are presented in a "pure" whole number format (e.g., 80). Similarly, the percentages should have been presented as 80.0% rather than 80% to capitalize on perceptual similarity between percentages and whole numbers in terms of string length (80% vs. 80.0). Future research should carefully consider the presentation format of the stimuli and end points of the number line.

Despite these limitations, the current study is the first to investigate children's number line estimation within-participants for equivalent fractions, decimals, and percentages and proportionally equivalent whole numbers. This investigation allowed us to examine the degree to which perceptual similarity between whole numbers and fraction, decimal, and percentage notations predicts magnitude estimation performance. Percentages were most closely estimated to whole numbers. The current study also demonstrated that fraction and decimal notations present further challenges for magnitude estimation, with decimals presenting the most challenging perceptual novelty when it comes to string length. Specifically, single-digit decimals were underestimated more than double-digit ones, with greater underestimation for larger single-digit decimals (e.g., 0.8 was more underestimated than 0.2). Future research might also want to consider investigating, within-participants, how estimation for these notations differ from 1 in *X* statistics (such as 1 in 4 pregnancies result in miscarriage), especially given adults' overestimation of such number formats (Thompson et al., 2022). Future investigations could also include intermixed formats (e.g., 80.45%) to see whether percentages are still estimated similarly to whole numbers when there is a decimal component. This investigation also has practical relevance given that mortgage rates are often presented in intermixed formats.

Future work might also consider including as stimuli numbers greater than 1 and longer string-length decimals as well as the role of education in magnitude estimation for the distinct notations. The current study only included fractions, decimals, and percentages less than 1; it would be interesting to see whether findings are similar for numbers greater than 1 given that children often perceive fractions as entities smaller than 1 (Kallai & Tzelgov, 2009). Moreover, the stimuli included only one- and two-digit decimals; perhaps we might see a reverse effect where decimals are overestimated relative to their magnitude if children think about them as three- and four-digit whole numbers. Finally, here we only examined number line estimation and did not consider direct educational implications.

Longitudinal or training studies should examine how notational effects may increase or diminish over time.

Conclusion

The current study is the first to attempt to capture a comprehensive within-participant picture of children's magnitude estimation for equivalent fractions, decimals, percentages, and proportionally equivalent whole numbers. Of the rational number notations, percentages were estimated most similarly to whole numbers. Both fractions and decimal number line estimation differed substantially from whole number estimation. Rational numbers are often thought to differ in difficulty, based on the degree to which their format is perceptually similar to that of whole numbers, suggesting that decimals may be more similar to whole numbers than fractions. The results here are contrary to this assertion; decimals were underestimated far more than fractions (17% vs. 5% underestimation, respectively), highlighting the unique challenges in understanding the decimal format. Curricula may benefit from a different presentation order of the notations or focus on building integrated number sense (Schiller & Siegler, 2023; Schiller et al., 2024) among whole numbers, fractions, decimals, and percentages. However, future research is needed in this regard. At the very least, educators need to be made aware of the unique challenges of decimals.

CRediT authorship contribution statement

Lauren K. Schiller: Writing – review & editing, Writing – original draft, Visualization, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Roberto A. Abreu-Mendoza:** Writing – review & editing, Formal analysis, Data curation. **Clarissa A. Thompson:** Writing – review & editing, Methodology, Investigation, Data curation, Conceptualization. **Miriam Rosenberg-Lee:** Writing – review & editing, Supervision, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Data availability

In accordance with the journal's Transparency and Openness Promotion, we have made available all analyses in an html version of an R-markdown file. This file is a dynamic document based on our R code used for analyses, including the code, results, rendered output, and a brief description of analyses. We have also included the de-identified data and the preregistration information for this study. All materials appear on the Open Science Framework (https://osf.io/m9c53/?view_only=a50def93ca21403db6ecb22367c811fe).

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jecp.2024.106030.

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