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Lack of Integrated Number Sense Among College Students: Evidence From Rational Number Cross-Notation Comparison

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Growing evidence highlights the predictive power of cross-notation magnitude comparison (e.g., 2/5 vs. 0.25) for math outcomes, but whether these relations persist into adulthood and the underlying mechanisms remain unknown. Across two studies during the 2021–2022 academic year, we investigated undergraduates' cross-notation and within-notation comparison skills given equivalent fractions, decimals, and percentages (Study 1, $N = 220$ and Study 2, $N = 183$). We found participants did not perceive equivalent rational numbers equivalently. Cluster analyses revealed that approximately one-quarter of undergraduates exhibited a bias to select percentages as larger in cross-notation comparisons. Compared with the other cluster of undergraduates who showed little-to-no bias, the percentages-are-larger bias cluster performed worse on fraction number line estimation and fraction arithmetic (exact and approximate), as well as reporting lower Scholastic Aptitude Test/American College Test (SAT/ACT) scores. Hierarchical linear regression analyses demonstrated that cross-notation comparison accuracy accounted for variance in SAT/ACT beyond within-notation accuracy. Mediation analyses were consistent with a potential mechanism: Stronger cross-notation knowledge equips individuals to evaluate the reasonableness of fraction arithmetic solutions. Together, these results suggest the importance of an integrated understanding of rational number notations, which may not be fully assessed by within-notation measures alone.

Public Significance Statement

Our analyses showed that cross-notation understanding (e.g., fraction-to-decimal comparison) is an important predictor of self-reported Scholastic Aptitude Test/American College Test (SAT/ACT) scores beyond within-notation understanding (e.g., fraction-to-fraction comparison). In fact, in cross-notation comparisons, undergraduate students showed that they did not perceive equivalent fractions, decimals, and percentages as being equivalent; many students perceived percentages as larger. Further, the cluster of undergraduate students who exhibited this percentages-are-larger bias performed worse on a variety of math measures, as well as reporting lower SAT/ACT scores, when compared with the high-performing cluster of students.

Keywords: fractions, decimals, percentages, rational numbers, magnitude understanding

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Daily decisions, such as deciding whether a package of 1.05 lbs. of ground beef is sufficient for a recipe that calls for 1½ lbs. of meat, require rational number understanding. Such decision-making engages cross-notation number sense, the ability to compare across different notations such as determining the larger number (e.g., 2/5 vs. 0.25). This kind of number sense differs from within-notation number sense, the ability to compare within notations to determine the larger value (e.g., 2/5 vs. 1/4). Both research and curricula largely focus on within-notation knowledge, leaving open the question of the role that cross-notation knowledge plays in informal daily decisions and more formal school-based mathematics. Recent research has suggested that cross-notation knowledge is essential for success in middle school, even beyond within-notation knowledge (Schiller & Siegler, 2023). In the present research, we examined whether this relation between cross-notation number-sense and mathematical achievement persists to the end of K–12 education, as measured by self-reported SAT/ACT scores. Further, we employ person-centered cluster analyses to uncover distinct profiles of students' cross-notation performance by drawing upon recent work that has uncovered distinct profiles of student misconceptions with fractions (Abreu-Mendoza et al., 2023; Gómez & Dartnell, 2019; Reinhold et al., 2020, 2023). Finally, we test a theory about a possible underlying mechanism for the importance of cross- over within-notation knowledge in mathematical achievement. Specifically, we theorize that cross-notation knowledge helps individuals estimate solutions to fraction arithmetic problems, which helps improve fraction arithmetic calculation accuracy, a necessary skill for math achievement.

Within-Notation Understanding

Rational numbers are a persistent and pervasive challenge for both children and adults. Children frequently make mistakes with fractions, such as indicating that 12/13 + 7/8 is approximately 1, 19, or 21, rather than 2 (Carpenter et al., 1980). These issues extend to decimals and percentages. For example, In Hiebert and Wearne (1985), close to half of fifth-grade students aligned the right most digits and hence added 6 + 0.32 = 0.38. Even understanding of percentages is limited. For example, in Gay and Aichele (1997), less than half of middle school students correctly answered that 87% of 10 is less than 10. Difficulty with rational numbers is problematic given their importance for success in advanced math courses, such as algebra (Booth & Newton, 2012; Siegler et al., 2012), overall math achievement (Siegler et al., 2011), and in everyday life, such as in the workplace (Handel, 2016).

To address these difficulties, researchers have primarily focused on children's within-notation understanding of fractions, decimals, and, to a lesser extent, percentages (e.g., Durkin & Rittle-Johnson, 2012; Gay & Aichele, 1997; Stafylidou & Vosniadou, 2004). Whole number bias, also termed natural number bias, is thought to be at the root of many of the difficulties individuals experience with rational numbers (Alibali & Sidney, 2015; Ni & Zhou, 2005). Whole number bias is the tendency to inappropriately apply principles of whole numbers to fractions, decimals, and percentages.

Within-notation knowledge of fractions is especially difficult (e.g., Behr et al., 1980; Fyfe & Brown, 2018; Iuculano & Butterworth, 2011; Knuth et al., 2005; Matthews et al., 2012) and whole number bias likely contributes to such difficulties (Alibali & Sidney, 2015; Ni & Zhou, 2005). When comparing fractions (e.g., 3/7 vs. 2/3), individuals may be focused on the magnitudes of the numerators (e.g., $3 > 2$) and the denominators (e.g., $7 > 3$) leading them to incorrectly select the smaller fraction as the larger number (Meert et al., 2010; Reinhold et al., 2020). Relatedly, solution times may be affected such that individuals are slower comparing fractions incongruent with whole number thinking (e.g., 3/7 vs. 2/3) than comparisons that are congruent with whole number thinking (e.g., faster at correctly comparing 3/8 vs. 5/9 because $3 > 5$ and $8 > 9$) (Obersteiner et al., 2013; Vamvakoussi et al., 2012; Van Hoof et al., 2013). There is also evidence that individuals deploy other biased responding patterns with fractions (Gómez & Dartnell, 2019) and recent work has suggested that understanding profiles of individuals' misconceptions with fractions could improve math outcomes by targeting the specific misconceptions (Reinhold et al., 2023).

Individuals are also likely to fall prey to whole number bias with understanding of decimals (Durkin & Rittle-Johnson, 2015; Moloney & Stacey, 1997). In whole numbers, a longer-string length indicates a larger magnitude (e.g., $25 > 9$) but this is not always the case for decimals (e.g., 0.25 is not greater than 0.9). Indeed, children exhibit such a whole number misconception, as well as two other related misconceptions: role of zero and fraction misconceptions (Durkin & Rittle-Johnson, 2015; Moloney & Stacey, 1997). In regards to the role of zero misconception, there is evidence that children disregard the role of zero in decimals (e.g., $0.07 = 0.7$ because 07 is the same as 7 in whole numbers) and indicate that affixing a zero to the end of a decimal makes it larger because this is the case with whole numbers (e.g., $320 > 32$ but 0.80 is not greater than 0.8). Finally, in regard to the fraction misconception with decimals, there is evidence that when children start to understand more about fractions and decimals, they inappropriately apply understanding of smaller fractional parts to decimals (e.g., 0.723 is smaller than 0.2 because thousandths are smaller than tenths). Together, these misconceptions could lead to issues in decimal comparison (e.g., incorrectly stating $0.9 > 0.23$) and even underestimation of decimals (Coulanges et al., 2021; Desmet et al., 2010; Durkin & Rittle-Johnson, 2015; Huber et al., 2014; Roell et al., 2017, 2019; Schiller et al., 2023; Varma & Karl, 2013). For example, undergraduate students often estimate 0.80 as larger than 0.8 on the number line (Schiller et al., 2023), likely because the zero in the hundredths place in 0.80 makes it appear more similar to the whole number 80, and 0.8 appears more similar to the whole number 8. That being said, directly comparing notations reveals that relative to each other, within-notation knowledge of decimals is typically better than that of fractions (Binzak & Hubbard, 2020; Mock et al., 2018, 2019).

While a good amount is known about within-notation understanding of fractions and decimals, less is known about understanding of percentages. We would expect performance with percentages to be the highest, given how children estimate them on number lines similarly to whole numbers (Schiller, Abreu-Mendoza, Thompson, & Rosenberg-Lee, 2024). Only one study has compared children's within-notation magnitude representations for the three rational number notations through magnitude comparison (Schiller & Siegler, 2023). They found that middle school students performed best on percentage magnitude comparisons, then decimal magnitude comparisons, and worst on fraction magnitude comparisons. However, it is unclear whether this pattern of results would also hold for undergraduate students, who are far removed from rational number instruction. Thus, the first aim of the current work was to assess undergraduate students' within-notation magnitude comparison accuracy for fractions, decimals, and percentages.

Cross-Notation Understanding

Though whole number bias is thought to be at the root of difficulties for within-notation understanding, less is known about the nature of difficulties that individuals experience with cross-notation understanding. Typically, cross-notation knowledge is measured with tasks that involve more than one notation, such as magnitude comparison (e.g., select the larger value: 0.22 vs. 3/5, Hurst & Cordes, 2016, 2018a, 2018b) or ordering tasks (e.g., rank order from smallest to largest: 0.6, 10/10, 20/100, 0.002; Mazzocco & Devlin, 2008). Similar to within-notation magnitude comparison, both children and adults represent magnitudes along an integrated continuum, such as showing evidence of distance effects when comparing fractions, decimals, and whole numbers (Binzak & Hubbard, 2020; Ganor-Stern, 2013; Hurst & Cordes, 2016, 2018a, 2018b; Mazzocco & Devlin, 2008). Recent work has suggested that cross-notation understanding warrants more careful attention. Specifically, children's cross-notation understanding (i.e., comparing fractions vs. decimals) predicted fraction and decimal arithmetic (Braithwaite et al., 2022) and math achievement (Schiller & Siegler, 2023) beyond within-notation understanding. That being said, it is unclear whether cross-notation knowledge is always weaker than within-notation knowledge or the reverse. For example, adults' fraction–decimal magnitude comparison accuracy was worse than their decimal–decimal comparison but better than their fraction–fraction comparison accuracy (Binzak & Hubbard, 2020; Hurst & Cordes, 2016).

Further, while whole number bias seems to impact within-notation processing of rational numbers, the difficulties involved in adults' cross-notation number processing are not clear. A percentages-are-larger bias was apparent in an investigation with middle school children (Schiller, 2020; Schiller & Siegler, 2023). In other words, middle school students were more accurate on cross-notation comparison trials where the percentage was the larger value (e.g., 40% vs. ¼) than when it was not the larger value (e.g., 2/5 vs. 25%), despite the numerical magnitudes for the compared values being the same (Schiller & Siegler, 2023). It is not clear whether such biases extend into adulthood. Thus, the second goal of this study was to examine undergraduates' cross-notation comparison accuracy to determine whether there are performance differences depending on whether the larger magnitude is expressed as a percentage, decimal, or fraction.

A third goal was to determine what proportion of undergraduates exhibit the percentages-are-larger bias and whether there are differences in math skills among students who do versus do not exhibit the bias. Recent work involving person-centered cluster analyses has revealed distinct profiles of students' fraction misconceptions (Abreu-Mendoza et al., 2023; Gómez & Dartnell, 2019; Reinhold et al., 2020, 2023). Further, these recent studies suggest that characterizing such profiles could allow targeted instruction to address the specific fraction misconception rather than a one-size-fits-all approach. Employing a similar methodology could characterize distinct profiles of misconceptions in cross-notation understanding, which could have implications for instruction. Thus, we employed such person-centered analyses to describe the profiles of undergraduates who exhibit or do not exhibit biases based on notational format. Specifically, we compare Scholastic Aptitude Test/American College Test (SAT/ACT) scores (Study 1 and 2) and other rational number skills (i.e., number line estimation accuracy and confidence, and fraction arithmetic estimation and calculation in Study 2) between biased and high performing profiles.

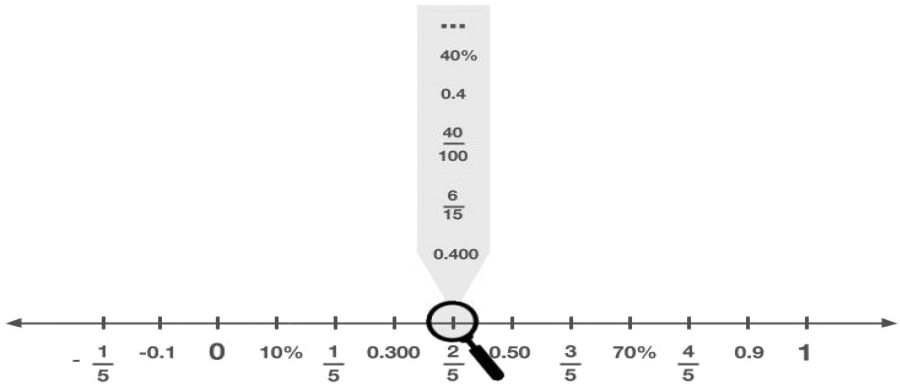
Integrated Numerical Development

Finally, as a fourth goal, we seek to uncover a potential mechanism for previously reported relations between cross-notation knowledge and math achievement, relative to within-notation knowledge. For the purposes of the present work, when discussing cross- and within-notation knowledge, we are referring to magnitude comparison accuracy, which we operationalize as percent accuracy on the cross-notation or within-notation comparison tasks but, as noted earlier, such understandings could be assessed through other measures such as numerical magnitude ordering. Magnitude comparison accuracy is often thought to assess the precision of individuals' magnitude representations, the degree to which individuals represent numbers on a mental number line (Dehaene et al., 1993; Moyer & Landauer, 1967; Siegler et al., 2011). We argue that building a fully integrated number line for fractions, decimals, and percentages is not as straightforward for children to construct as one for whole numbers. Many individuals seem to have difficulty representing the same quantity in multiple ways. If individuals understood that every location on the number line has infinite ways to describe the same location, then they should estimate the magnitudes for equivalent fractions at the same position on the number line. However, even about one-third of eighth grade students estimate the magnitude of fractions with larger numerator and denominator components as being larger than equivalent fractions with smaller components (e.g., placing 8/10 further to the right on the number line than 4/5; Braithwaite & Siegler, 2018). And, as noted earlier, college students underestimate single digit decimals (e.g., 0.8) more than their double-digit equivalents (e.g., 0.80) (Schiller et al., 2023). Beyond these within-notation misconceptions, individuals do not seem to conceive of fractions, decimals, and percentages as being on the same number line. For example, even high school students often claim that only fractions exist between fractions, and only decimals exist between decimals (Vamvakoussi & Vosniadou, 2010). Together, these results suggest that many individuals have not internalized a mental model of numbers that includes fractions and decimals (and likely not percentages) on the same mental number line.

Integrated number sense—understanding of the relations among all rational number notations—has been proposed as a desired end state for numerical development (Schiller & Siegler, 2023; see Figure 1). In the current context, we define integrated number sense as involving a mental number line, which incorporates both within and cross-notation magnitude representation, such that an individual can both identify any location on the number line and abstract that the position can be represented in infinite ways (e.g., $4/5 = 8/10 = 80\% = 0.8 = 0.80$, etc.).

Our definition encompasses the strong view that individuals should be secure in their knowledge of rational numbers as magnitudes on the number line, regardless of notational format. In essence, integrated number sense could be instantiated such that all different notational formats are organized on a mental number line. The number line becomes the organizing mental structure where all notations are represented in the same space. Further, integrated number sense does not have to involve exact translations and therefore, strategies like benchmarking (e.g., determining whether a number is greater or less than 1/2, 1/4, 2/3, etc.) contribute to this organizing structure. Integrated number sense obliviates whole number bias such that individuals should be able to get a ballpark approximation of numerical magnitudes, without being swayed by whole number components of the numbers. For example, in comparing 3/7 and 2/3, individuals

Figure 1
Theorized Integrated Mental Number Line



Note. Individuals who possess an integrated mental number line can abstract that every location on the number line can be represented in infinite ways. Adapted from "Integrated Knowledge of Rational Number Notations Predicts Children's Math Achievement and Understanding of Numerical Magnitudes," by L. K. Schiller and R. S. Siegler, 2023, *Cognitive Development*, 68, Article 101380 (https://doi.org/10.1016/j.cogdev.2023.101380). Copyright 2023 by Elsevier.

might be influenced by the individual whole number components to incorrectly select the larger number (i.e., $3 > 2$ and $7 > 3$, suggesting incorrectly that $3/7 > 2/3$). Integrated number sense enables an individual to flexibly use whatever within- or cross-notation means for approximating these numbers, such as approximately translating to another notation (e.g., $3/7 \sim 40\%$ and $2/3 \sim 60\%$) (Fitzsimmons et al., 2020; Sidney et al., 2021; Siegler et al., 2011; Siegler & Thompson, 2014) and using those approximations to organize numerical magnitudes on their mental number line. In line with Overlapping Waves Theory (Siegler, 1996), people have at their disposal a variety of skills to use depending on the context. In this situation, individuals can approximately translate rational numbers into another notation at any given time and will likely be more accurate and quicker. That being said, even expert mathematicians fall prey to whole number bias (Obersteiner et al., 2013). So, integrated number sense may not totally obliviate whole number bias.

Even still, if integrated number sense is the desired end state of numerical development, then both cross-notation and within-notation number sense should be an important part of mathematical success. However, most educational practices and research focuses on within-notation magnitude representation. This focus on within-notation magnitude representation is not unreasonable, as individual differences for each distinct notation are related to success in mathematics. Specifically, magnitude comparison for children's and adults' non-symbolic quantities, whole numbers, fractions, and decimals are all related to math achievement (Coulanges et al., 2021; Fazio et al., 2014; Schneider et al., 2009, 2017, 2018; Siegler et al., 2011; Torbeyns et al., 2015). Yet, recent research has suggested that cross-notation measures might capture important aspects of individual differences that are not indexed by within-notation measures. Specifically, individual differences comparing across fraction and decimal notations was predictive of children's fraction and decimal arithmetic skill (Braithwaite et al., 2022), algebra ability (Hurst & Cordes, 2018a, 2018b), and was used as a classroom diagnostic tool to identify those at risk for failure (Mazzocco & Devlin, 2008). Further, middle school students' cross-notation magnitude understanding accounted for variance in math achievement and other math outcomes beyond that accounted for by their within-notation understanding (Schiller & Siegler, 2023).

Ideally, students should possess integrated number sense by the end of their formal mathematics education. Thus, those who have achieved this end state of numerical development, by completing K–12 education, should have stronger mathematical skills. We test this hypothesis by investigating whether undergraduates' cross-notation abilities are predictive of achievement on college admissions exams such as SAT/ACT, which are designed to assess overall math readiness for college (see ACT College and Career Readiness Standards, 2022; Allen & Radunzel, 2017; Clough & Montgomery, 2015; Westrick et al., 2020). Though rational numbers may appear in problems on the SAT/ACT, there is likely much less emphasis on understanding of basic concepts (e.g., magnitude comparison) than is apparent on measures of math achievement for middle school students, who more recently encountered this material. Therefore, the fourth goal of the study was to investigate the contribution of undergraduates' within- and cross-notation comparison accuracy on their self-reported college admissions exam performance.

Moreover, we theorize about a potential mechanism underlying why cross-notation number sense better predicts math outcomes than within-notation measures. Specifically, we argue that cross-notation measures might better assess individuals' integrated number sense and that integrated number sense may play a role in helping individuals estimate solutions to fraction arithmetic problems, which in turn helps improve fraction arithmetic calculation accuracy. From a very early age (i.e., preschool/kindergarten), mapping between different number formats (e.g., spoken words, nonsymbolic quantities, symbols, etc.), together with number relations (e.g., magnitude comparison), and number operations (e.g., verbal arithmetic) become intertwined as they support success in mathematics (Jordan et al., 2022). Relatedly, it has been proposed that stronger fraction number sense supports arithmetic accuracy because magnitude knowledge enables one to estimate reasonable solutions and thus select an arithmetic strategy that does not yield implausible results (Braithwaite & Siegler, 2021; Siegler et al., 2011, 2020). Here, we suggest that fraction knowledge alone does not incorporate mapping between notational formats. Cross-notation skill in mapping between rational number formats (e.g., 4/16 = ¼ = 0.25 = 25%), whether exact or approximate mapping, is likely to support skill in number relations (e.g., 2/5 vs. 25%) and also number operations (e.g., is 1/2 + 1/4 closer to 2/6, 2/8, or 8/10?). Harnessing these interwoven skills, individuals can estimate reasonable solutions to arithmetic problems by making approximate translations to organize numerical magnitudes on their mental number lines and use this knowledge to select appropriate calculation strategies.

To see whether this might be the case, consider a fraction addition problem: 2/5 + 1/7. The incorrect strategy of adding numerators and denominators separately would yield the answer 3/12, which might not seem unreasonable to many students. However, approximately translating 2/5 to 0.40, 1/7 to 0.10, and 3/12 to 0.25 would make it more obvious that the estimated sum of the decimal equivalents, 0.50, was greater than the proposed answer, 0.25. That realization could motivate such students to try a different strategy that produced a more plausible answer. If this hypothesis is correct, then the relation between within-notation comparison accuracy and fraction arithmetic skill should be mediated by cross-notation comparison accuracy and arithmetic estimation ability. Indeed, to preview our results, within-notation comparison accuracy was no longer a significant predictor when cross-notation comparison accuracy was added to a model predicting SAT scores in Study 1, suggesting that cross-notation comparison accuracy mediates the relation between within-notation comparison accuracy and mathematical success. Thus, in Study 2, we collected fraction arithmetic estimation and calculation performance accuracy in order to conduct a mediation analysis to test the hypothesis that cross-notation skills (i.e., cross-notation comparison accuracy) support fraction arithmetic estimation, which contributes to fraction arithmetic skill.

The Present Study

The present study tested the premise that if undergraduates possess integrated number sense, there should be no differences in performance whether numbers are expressed as fractions, decimals, or percentages. Therefore, deviations from this result suggest lack of integrated number sense in undergraduates. We examined this premise across four goals. Our first goal was to investigate undergraduates' within-notation magnitude representation for matched fractions, decimals, and percentages. That is, does accuracy differ whether comparisons are presented as fractions, decimals, or percentages? We hypothesized that undergraduates' within-notation comparison accuracy would be lowest for fractions, followed by decimals, and finally percentages.

Our second goal was to investigate undergraduates' cross-notation understanding to determine whether there are any biases based on whether magnitudes are expressed as a fraction, decimal, or percentage. Prior work has shown that middle school students demonstrate a bias toward selecting the percentage as the larger magnitude in percent–fraction and percent–decimal comparisons but no such bias exists in fraction–decimal comparisons (Schiller & Siegler, 2023). Theoretically, if undergraduates have attained integrated number sense, the end state of numerical development, there should be no difference in accuracy for magnitude comparison whether the numbers are expressed as fractions, decimals, or percentages. However, based on the similarity of percentages to whole numbers, we hypothesized that undergraduates would also exhibit this percentages-are-larger bias, but no bias would be present in fraction–decimal comparisons.

The third goal was to determine whether undergraduates could be categorized as students who do or do not exhibit the percentages-are-larger bias. Specifically, first, we performed data-driven, person-oriented analyses (i.e., cluster analyses) to uncover the underlying strategy profiles for comparing percentages and fractions, as it is where children showed the largest bias (Schiller & Siegler, 2023). We hypothesized that similar to children, a proportion of undergraduate students would show a percentages-are-larger bias profile. Then, we examined how this profile, as well as the others found by the cluster analyses, performed in the other types of cross-notation comparisons, as well as the within-notation comparisons. Finally, we investigated whether this profile differed from the undergraduates who do not exhibit the bias, on a range of mathematical skills. Study 1 was our initial foray into this investigation, and as such, we only collected self-reported SAT/ACT scores. Recognizing the limitations of self-reported scores (Cole & Gonyea, 2010; Sticca et al., 2017), in hindsight, it would have been useful to collect scores directly from the relevant institution in our follow-up study. However, in Study 2, we wanted to determine whether we could replicate findings with self-reported SAT/ACT scores and also extend our findings with other directly assessed math outcome measures: number line estimation accuracy and confidence, and fraction arithmetic estimation and calculation.

Our fourth goal was to investigate the relations among within- and cross-notation number sense, together with self-reported college admissions exam scores and fraction arithmetic calculation/estimation accuracy. Specifically, we examined individual differences in cross-notation magnitude comparison and whether these differences explained variance in mathematical success on college admission exams (i.e., SAT/ACT). Further, we conducted a mediation analysis to test the hypothesis that cross-notation skills support fraction arithmetic estimation, which contributes to fraction arithmetic skill. Together, these analyses characterize undergraduate students' cross- and within-notation comparison abilities and their relation to a wide range of mathematical skills, thus specifying the extent of their integrated number sense and its role in mathematical success.

Study 1

Method

Transparency and Openness Statement

In accordance with the journal's Transparency and Openness Promotion, we have made available all analyses in an html version of a R-markdown file (https://osf.io/8z9mt/?view_only=fbe0a894ec444b3fb9064e362bf742a8). This file is a dynamic document based on our R code used for analyses, including the code, results, rendered output, and a brief description of analyses.

Participants

We recruited 220 undergraduate students from Rutgers University-Newark (M_{age} = 20.50 year, SD = 4.14, 73.18% female, 24.45% male, 1.36% other; 65% Latino/Hispanic, 38% Black/African American, 30% Asian, 26% Middle Eastern/North African, 20% White, 19% South Asian/Indian, 6% multiracial, 1% Native Hawaiian/Pacific Islander, 1% American Indian/Alaska Native, 2% another identity, and 4% declined to answer) from a Northeastern university's psychology department subject pool. Specific demographic information about undergraduate major appears in Table 1 in the online supplemental materials. Sixteen additional participants were removed because they did not complete the entire assessment. Prior to data collection, participants provided informed consent and after completing the tasks, they received course credit for their participation in the experiment. All protocols were in accordance with the Rutgers University-Newark Institutional Review Board.

Data were collected online, and we used the end of the academic term as a stopping rule. According to a sensitivity power analysis, the sample size of Study 1 ($N = 220$) allows the detection of small effect sizes for dependent-sample differences (Cohen's $d > 0.19$), with a power of 80% and an alpha level of .05. As the smallest significant cross-notation difference reported in Schiller and Siegler (2023) was 0.42 (percent vs. decimal comparisons), our current sample size was sufficiently powered to detect these effects. Regarding the cluster analyses, our sample size allows us to test up to a 10-cluster solution, assuming participants were distributed equally across clusters, as the recommended minimum number of observations per cluster is between 20 and 30 (Dalmaijer et al., 2022).

Design and Procedure

Participants completed an assessment in a 1-hr online data collection session on the survey platform Qualtrics (mean completion time: 63 min, $SD = 164$) in an unsupervised testing condition.

Measures

Data from the present study were collected as part of a larger project on undergraduate students' understanding of rational numbers. The primary task of interest in this study was magnitude comparison. Participants were also assessed on a set of math measures, which are not considered here. Specifically, after participants completed the magnitude comparison task, they completed tasks in the following order: rational number questions (adapted from Van Hoof et al., 2018), approximate fraction–decimal translation questions strategy reports, exact fraction–decimal translation questions, and decile number line estimation. Problems were randomized within tasks. Participants were told not to use a calculator.

Participants provided demographic information, including their gender, race, math background/major, and ACT/SAT scores. Specifically, participants were asked to share whether they took the ACT or SAT and if they remember their exact or approximate score for each subset (i.e., math, reading, writing). Most participants ($n = 121$ provided SAT scores and the remainder ($n = 99$) did not provide any score).

Within-Notation Magnitude Comparison. On this task, participants selected the larger of two magnitudes presented in the same notation; all magnitudes were between 0 and 1. The same values (i.e., 0.40 vs. 0.25, 0.35 vs. 0.60, and 0.38 vs. 0.08) were presented for all three notations (e.g., 3/5 vs. 7/20, 0.6 vs. 0.35, and 60% vs. 35%). There were three such values, resulting in nine within-notation comparisons; each comparison appeared twice, so that the correct answer appeared once on the left and once on the right resulting in 18 trials (see Table A1 for full set of stimuli). The stimuli were selected to have small, medium, and large numerical distances between the compared values (i.e., 0.15, 0.25, 0.30). We operationalized performance as percent correct across all within-notation trials.

Cross-Notation Magnitude Comparison. Participants selected the larger of two magnitudes that were presented in different notations. The same three pairs of values used for within-notation comparison were used for cross-notation comparison. These values were presented as fraction, decimal, and percentages to be compared where there was an instance in which fraction > decimal (e.g., 3/5 vs. 0.35), decimal > fraction (e.g., 0.6 vs. 7/20), fraction > percent (e.g., 3/5 vs. 35%), percent > fraction (e.g., 60% vs. 7/20), decimal > percent (e.g., 0.6 vs. 35%), and percent > decimal (e.g., 60% vs. 0.35). Each comparison appeared twice: once with the correct value appearing on the left, and once with the correct value appearing on the right. Values were chosen such that whenever possible, the compared values shared digits (e.g., 3/5 vs. 0.35). Since we were controlling for magnitude, it was not always possible to have the same digits in comparisons (e.g., 60% vs. 7/20). Also, the number of decimal digits was varied as much as possible, between one and two digits, given the constraint for controlling for magnitude. There were 36 cross-notation comparison trials (see Table A2 for full set of stimuli). We operationalized performance as percent correct across all cross-notation trials.

Data Analyses

We conducted analyses using R 4.2.0 (R Core Team, 2022). First, we conducted a repeated-measures analysis of variance (ANOVA) and planned pairwise comparisons to determine whether there were differences in magnitude comparison by notation. Then, we conducted cluster analyses to examine the different comparison profiles for fraction–percentage problems. These clusters were based on participants' accuracy on items where the fraction was greater than the percentage and where the percentage was greater than the fraction in cross-notation comparisons. We selected these comparisons because they were most problematic with children and we were particularly interested in whether any of these profiles resembled the percentages-are-larger bias shown by children. The cluster analyses were conducted using the kmeans function from the stats package (R Core Team, 2022). We determined the optimal number of clusters with a consensus method using the n_clusters function from the parameters package (Ludecke et al., 2020). This function performs several methods to determine the optimal number of clusters (e.g., Elbow, Gap, Silhouette, indices included in Nbclust package, among others). Then, it selects as the optimal number of clusters the one suggested by the majority of the methods. As the recommended minimum number of observations per cluster is between 20 and 30 (Dalmaijer et al., 2022), we tested for a maximum of 10 clusters, the default number used by this function. We also reported the percentage of explained variance (R^2) and the Akaike information criterion (AIC) for each of the cluster solutions. We used the fraction–percent magnitude comparison accuracy to generate the clusters, as these comparisons were most problematic for children (Schiller & Siegler, 2023), but we also sought to examine how the clusters differed in performance on other tasks purported to tap magnitude understanding (e.g., percent–decimal magnitude comparison, fraction number line estimation, fraction arithmetic, etc.). Third, we conducted hierarchical linear regressions to determine whether cross-notation comparison accuracy explained variance in SAT scores beyond that explained by within-notation accuracy for the subset of participants who provided SAT math scores. We used average cross- and within-notation magnitude comparison accuracy scores as respective composite scores for analyses involving SAT scores.

All analyses include the full sample of participants ($N = 220$), except for analyses involving SAT scores, which only include those who provided SAT Scores ($n = 125$). Descriptive statistics for all tasks appear in Tables 3, 4, and 6 in the online supplemental materials.

Results

Contrasting Performance on Within and Cross-Notation Comparison

On average, participants were less accurate on cross-notation comparisons ($M = 84.48\%$, $SD = 17.00$) than within-notation comparisons ($M = 87.22\%$, $SD = 17.00$), $t(219) = 4.00$, $p < .001$, Cohen's $d = 0.267$.

Within-Notation Magnitude Comparison. A repeated-measures ANOVAs comparing accuracy for fraction–fraction, decimal–decimal, and percent–percent magnitude comparison showed a main effect of notation, $F(2, 438) = 50.12$, $p < .001$, $\eta^2_g = .084$. Comparisons of percentages were more accurate ($M = 96.44\%$, $SD = 12.18$) than comparisons of decimals ($M = 84.02\%$, $SD = 25.94$), $t(219) = 7.44$, $p < .001$, Cohen's $d = 0.50$, and fractions ($M = 81.21\%$, $SD = 24.89$), $t(219) = 9.122$, $p < .001$, Cohen's $d = 0.61$. Accuracy of comparisons of decimals was slightly higher than that of fraction comparisons, $t(219) = 1.85$, $p = .07$, Cohen's $d = 0.13$.

Cross-Notation Magnitude Comparison. Consistent with prior research with middle-school students (Schiller & Siegler, 2023), undergraduates exhibited a bias to select percentages as larger than both fractions and decimals (Figure 2). Comparison accuracy was higher when the percentage was larger than the fraction ($M = 90.91\%$, $SD = 17.61$) relative to when the fraction was larger than the percentage ($M = 73.39\%$, $SD = 29.85$), $t(219) = 8.40$, $p < .001$, Cohen's $d = 0.57$. Comparison accuracy also was higher when the percentage was larger than the decimal ($M = 93.64\%$, $SD = 15.44$) than when the decimal was larger than the percentage ($M = 81.44\%$, $SD = 25.74$), $t(219) = 6.69$, $p < .001$, Cohen's $d = 0.45$. Accuracy was similar when the decimal was larger than the fraction ($M = 84.17\%$, $SD = 23.58$) and when the fraction was larger than the decimal ($M = 82.35\%$, $SD = 24.78$), $t(219) = 0.98$, $p = .327$, Cohen's $d = 0.07$.

Cluster Analyses

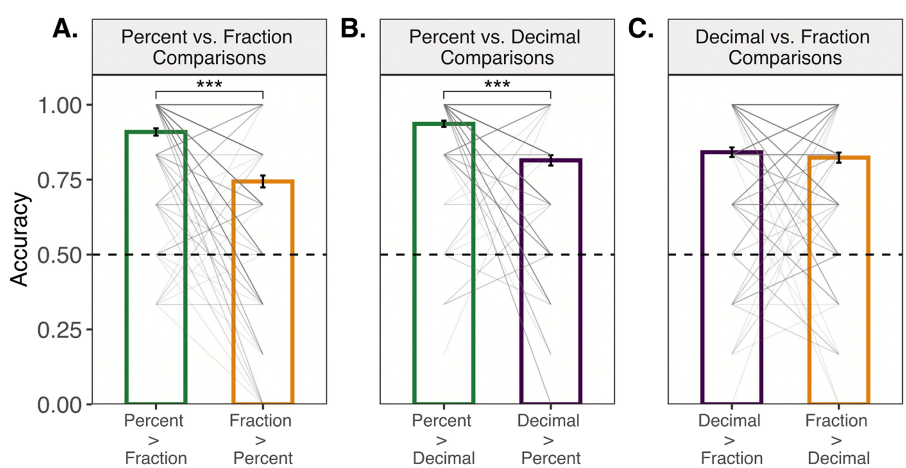
Cross-Notation Comparison Profiles. The bias toward perceiving percentages as larger than fractions and decimals was not exhibited by all students. Some individuals performed similarly on all types of comparisons and others showed the percentages-are-larger bias. To test this interpretation, we used a *k*-means clustering algorithm with a two-dimensional space, where each participant was represented by two data-points: the averaged performance on percent > fraction versus fraction > percent trials. This unsupervised, data-driven algorithm classifies data into *k* groups or clusters. We focused on percent–fraction comparisons (rather than ones involving decimals) as they showed the strongest bias in children (Schiller & Siegler, 2023).

Table 2 in the online supplemental materials shows the percentage of explained variance (R^2), the AIC, and the number of methods that suggested *k* clusters for the 1-to-10 cluster solutions. A four-cluster solution was the number of clusters that was suggested by most methods was seven of 28 (25.00%). This solution explained .809 of the variance (R^2) and had an AIC of 20.95.

Figure 3 shows the accuracy in the different types of cross-notation comparisons for the four clusters. The most common cluster was composed of undergraduates who performed almost at ceiling on all comparisons with no bias or minimal bias based on notation of compared numbers ("high performing" cluster, $n = 126$ [57% of the sample]). The second and third most prevalent clusters were

Figure 2

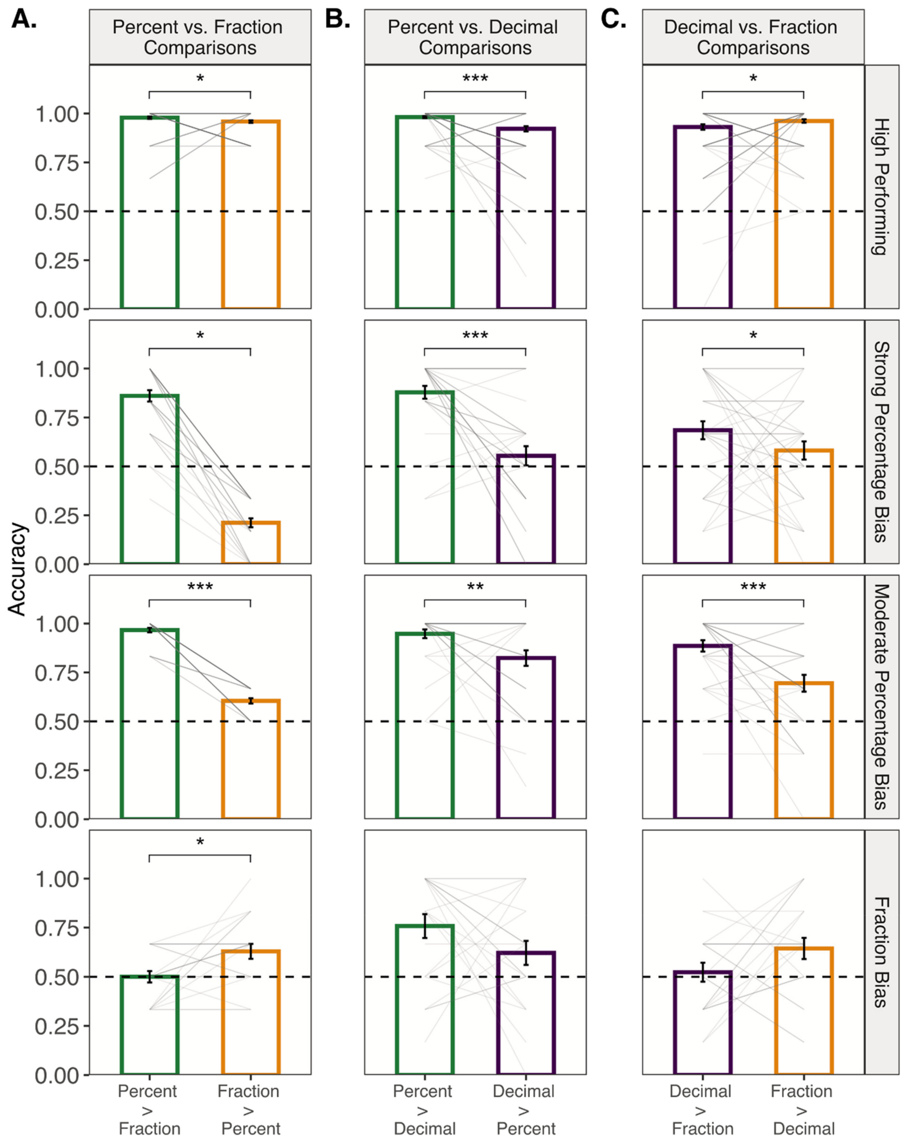
Percent Correct for Cross-Notation Magnitude Comparison



Note. (A) Percent versus fraction comparisons (e.g., 2/5 vs. 25%), (B) percent versus decimal comparisons (e.g., 40% vs. 0.25), and (C) decimal versus fraction comparisons (e.g., 0.40 vs. 1/4). Participants exhibited a bias to select the percentages as larger than fractions and decimals; however, there was no bias among the fraction vs. decimal comparisons. Gray lines represent individual participants' average scores in each of the conditions. Thicker gray lines indicate more participants with the same scores. Error bars represent ± 1 *SE*. See the online article for the color version of this figure.
*** $p < .001$.

Figure 3

Cross-Notation Comparison Accuracy



Note. (A) Percent versus fraction comparisons, (B) percent versus decimal comparisons, and (C) decimal versus fraction comparisons, based on the four-cluster model: high performing profile ($n = 126$), strong percentage bias profile ($n = 37$), moderate percentage bias profile ($n = 35$), and fraction bias profile ($n = 22$). Gray lines represent individual participants' average scores in each of the conditions. Thicker gray lines indicate more participants with the same scores. Error bars represent $\pm$ 1 *SE*. See the online article for the color version of this figure.
* $p < .05$. ** $p < .01$. *** $p < .001$.

composed of undergraduates who performed better on tasks where the percentage was larger but differed in the strength of this bias ("strong percentage bias" cluster, $n = 37$ [17% of the sample] and "moderate percentage bias" cluster, $n = 35$ [16% of the sample]). Finally, a fourth group of students were more accurate when the fraction was larger than the percentage ("fraction bias," $n = 22$ [10% of the sample]). See Figure 1 in the online supplemental materials for accuracy with the different types of within-notation comparisons for the four clusters.

Of note, this cluster analysis process does not merely differentiate high-achieving versus low-achieving participants. Specifically, when we conducted an analysis using a median split, we found that 30 of the 126 participants identified as high-performing (i.e., possessing integrated number sense) would have been identified as low-achieving based on the median split for overall performance. In actuality, these 30 participants are not biased in cross-notation comparison, they just have slightly lower magnitude processing abilities. Further, two of the 35 participants identified as moderate percentage bias (i.e., lacking integrated number sense) would have been identified as high-achieving based on the median split for overall performance. In actuality, these participants are actually more biased in their responding to cross-notation comparisons than other high performing participants. Thus, the cluster analysis examines the degree to which clusters of participants are influenced by a percentage bias, in a more sophisticated process than simply dividing participants based on high- and low-achieving groups.

Finally, to determine whether there were differences within the high performing and biased participants (strong and moderate percentage bias and fraction bias clusters) in their within- and cross-notation comparison skills, we performed a mixed repeated-measures ANOVA with cluster (high performing and biased profiles) as between-subject factor and type of comparison (within and cross) as a within-subject factor and accuracy as the dependent variable. This analysis yielded main effects of cluster, $F(1, 218) = 212.09$, $p < .001$, $\eta_g^2 = .451$, and type of comparison, $F(1, 218) = 17.33$, $p < .001$, $\eta_g^2 = .012$, which were qualified by an interaction between them, $F(1, 218) = 23.61$, $p < .001$, $\eta_g^2 = .016$. Notably, high-performing participants did not show differences between their within ($M = 95.57\%$, $SD = 6.16$) and cross-notation performance ($M = 95.55\%$, $SD = 8.89$), $t(125) = 0.036$, $p = .971$, Cohen's $d = 0.003$, suggesting that these students had integrated number sense. In contrast, biased students had stronger within comparison skills ($M = 76.06\%$, $SD = 18.77$) than cross-comparison ones ($M = 69.62\%$, $SD = 14.69$), $t(93) = 4.92$, $p < .001$, Cohen's $d = 0.507$, indicating that they had yet to develop an integrated number sense.

Math Achievement Differences Between Cross-Notation Profiles. Slightly more than half of participants (125 of 220, 56.81%) reported their SAT scores. The percent who reported their scores varied greatly across profiles (high performing = 84 of 126 [66%], strong percentage bias = 17 of 37 [46%], moderate percentage bias = 20 of 35 [57%], fraction bias = 4 of 22 [18%]). Therefore, we combined the three groups of biased students to analyze whether their SAT scores differed from those of the high performing students. Figure 5A shows that students in the high performing cluster ($M = 575.06$, $SD = 85.39$) had higher SAT scores than students in the biased cluster ($M = 516.92$, $SD = 114.25$), $t(123) = 3.19$, $p = .002$, Cohen's $d = 0.61$.

While these results trend in the direction suggesting best performance for high performing, followed by the percentage bias profiles, and finally the fraction bias group on SAT scores, these results are likely underestimated because participants with lower SAT scores are less likely to self-report their SAT scores (Trice, 1990; Flake & Goldman, 1991).

Individual Differences in Cross- and Within-Notation Accuracy and Their Relation to Math Achievement

The differences in SAT performance among the different profiles suggest that integrated cross-notation ability might be an important aspect of general math achievement. However, it is unclear whether these cross-notation abilities are important beyond within-notation abilities. To address this question, we conducted hierarchical linear regression analyses.

As evidenced by Table 4 in the online supplemental materials, within-notation and cross-notation magnitude comparison accuracy were correlated with SAT and ACT scores. Hierarchical regression analyses were used to determine whether cross-notation accuracy accounted for variance in SAT scores beyond that explained by within-notation accuracy. Within-notation magnitude comparison was added to the model first to account for general rational number magnitude representation; it predicted 5% of the variance in performance (Step 1, $p = .005$). Cross-notation magnitude comparison was entered next; it added 4% additional variance ($p = .015$), and within-notation was no longer a significant predictor. Table 1 displays results from the hierarchical regression.

Table 1

Cross-Notation Comparison Accuracy Explains Variance in Self-Reported SAT Scores Beyond Within-Notation Magnitude Comparison Accuracy

Independent variables	*b* (unstandardized)	*SE b*	β (standardized)
Step 1			
Constant	409.02	52.53	
Within-notation	162.89	57.43	.248**
Step 2			
Constant	372.63	53.55	
Within-notation	−24.47	94.56	−.037
Cross-notation	232.28	94.21	.355*

Note. $R^2 = .054$ for Step 1**; $\Delta R^2 = .0375$ for Step 2*. SAT = Scholastic Aptitude Test.
* $p < .05$. ** $p < .01$.

One possibility for the lack of significance for within-notation comparison accuracy in Model 2 but significance in Model 1 could be multicollinearity. However, tests to see if the data met the assumption of collinearity indicated that multicollinearity was not a concern (within-notation, tolerance = .35, variance inflation factor [VIF] = 2.82; cross-notation, tolerance = .35, VIF = 2.82).

Discussion

The present findings suggested that nearly half of undergraduates do not have well-integrated number sense. On within-notation magnitude comparisons, participants performed better on comparisons involving percentages than on fraction and decimal comparisons. Moreover, undergraduates were less accurate on cross-notation magnitude comparisons than on within-notation comparisons for the same magnitudes. They also perceived percentages as larger than fractions and decimals. Cluster analyses revealed that a third of undergraduates (33%) displayed a percentages-are-larger bias (with 17% displaying a strong percentage bias and 16% displaying a moderate percentage bias), as demonstrated by higher accuracy when the percentage stimulus was the larger number than when it was the smaller number in cross-notation comparisons. A smaller proportion (10%) of undergraduates demonstrated a fractions-are-larger bias, as demonstrated by higher accuracy when the fraction stimulus was the larger number than when it was the smaller number in cross-notation comparison. This fractions-are-larger bias has not been documented previously in the literature. Together, these biases suggest that nearly half of undergraduate students did not fluidly switch between notations to judge values in cross-notation comparisons and instead are responding systematically based on heuristic thinking that either percentages or fractions are larger in cross-notation comparisons. Interestingly, the participants in the fractions-are-larger bias cluster reported lower SAT scores than the percentages-are-larger bias cluster. Third, there was a significant association between SAT scores and the profiles based on rational number magnitude comparisons: Undergraduate students who exhibited a bias also performed worse on the SAT relative to the high performing cluster, who exhibited no bias or minimal bias. Fourth, individual differences in cross-notation accuracy were predictive of self-reported SAT scores beyond within-notation accuracy. In fact, within-notation comparison accuracy was no longer a significant predictor of SAT scores when the model controlled for cross-notation comparison accuracy. These findings suggest undergraduates' cross-notation understandings could play an important role in overall math achievement, though the results are correlational in nature. Together, these results motivate further investigation to determine whether they replicate in another sample and also whether they extend to other directly assessed rational number outcomes. Furthermore, we sought to test a specific hypothesis that cross-notation skills support fraction arithmetic estimation, which contributes to fraction arithmetic skill.

Study 2

Method

In Study 2, we examined whether the percentages-are-larger bias observed in Study 1 replicated with new measures of rational number knowledge and a new sample of undergraduate students. Specifically, we tested whether the clustering results based on biases in magnitude comparison extend to fraction arithmetic estimation, fraction arithmetic, and number line estimation accuracy and confidence. Further, we extended our investigation to examine the mechanism for the relations among cross- and within-notation knowledge and math outcomes. Specifically, we argue that cross-notation measures might better assess individuals' integrated number sense and that integrated number sense may play a role in helping individuals estimate solutions to fraction arithmetic problems, which in turn helps improve fraction arithmetic calculation accuracy. Using both cross- and within-notation magnitude comparison accuracy as measures of magnitude representation, we tested a specific hypothesis that stronger magnitude knowledge may be related to higher arithmetic accuracy because magnitude knowledge enables one to estimate reasonable solutions and thus select an arithmetic strategy that does not yield implausible results (Braithwaite & Siegler, 2021; Siegler et al., 2011, 2020). Thus, we examined whether the relation between within-notation knowledge and fraction arithmetic skill was serially mediated by cross-notation knowledge and the ability to estimate fraction arithmetic solutions. This data set is drawn from a larger study examining the effects of two number line interventions on fraction arithmetic estimation and calculation performance. The larger intervention study was preregistered (https://osf.io/wh8eg/?view_only=ba0f7d589aa34ae890bcb8ac8cd80b35), and one secondary analysis will be examined here using pretest data only: "Using hierarchical linear regression analyses, we will examine whether cross-notation magnitude comparison at pretest predicts ACT/SAT beyond within-notation magnitude comparison."

Participants

There were 185 participants recruited from Kent State University in the United States. Two participants failed to answer two of three attention check questions and thus were excluding, leaving a final sample of 183 participants (74% female, 20% male, 3% other; 3% preferred not to respond; 76% White, 9% Black/African American, 3% American Indian/Alaskan Native, 3% Hispanic/Latino, 7% mixed, <2% other/preferred not to report). Participants received course credit for participating. Two participants were excluded because they missed two of three attention checks, for a final sample of 183 students. For Study 2, we powered for the intervention study as described in the preregistration (https://osf.io/wh8eg/?view_only=ba0f7d589aa34ae890bcb8ac8cd80b35) and conducted the preregistered secondary analysis with this sample. Prior to data collection, participants provided informed consent and after completing the tasks, they received course credit for their participation in the experiment. All protocols were in accordance with the Kent State University Institutional Review Board.

Design and Procedure

Participants completed a pretest-training-posttest design in a 1-hr online data collection session on the survey platform Qualtrics (mean completion time: 62 min, $SD = 120$ min) in an unsupervised testing condition. The program randomly assigned participants to one of two training conditions, which were designed to last about 20 min. The findings reported here consist of pretest data involving planned secondary analyses from the preregistration.

Measures

The measures included magnitude comparison (within- and cross-notation), fraction arithmetic estimation and calculation, number line estimation accuracy and confidence judgments, and a math anxiety questionnaire. For the purposes of the secondary analyses reported here, we exclude the results from the math anxiety questionnaire, because the topic is beyond the scope of this article. Participants completed tasks in the order in which they are listed below, and problems were randomized within tasks. Calculator use was not allowed. The fraction arithmetic calculation task was the only one on which participants were allowed to use paper and pencil. Participants provided demographic information, including gender, race, math background/major, and ACT/SAT information (116 of the 193 participants, 63% provided ACT scores; the remainder did not provide any score).

Cross-Notation and Within-Notation Magnitude Comparison

The magnitude comparison stimuli and procedures were the same as in Study 1.

Fraction Arithmetic Estimation

Participants were presented 12 fraction addition estimation problems, each with three multiple-choice alternatives. None of the choices were the exact sum, but one option was very close. Half of the trials had one "lure," involving independent whole number calculation errors—for example, 5/6 + 2/4 had (a) 7/10, (b) 1/3, and (c) 1 ¼ as answer choices, where 7/10 was considered a "lure" because it involved adding the numerators and denominators separately. The measure was percent correct.

Number Line Estimation and Confidence

Participants completed a number line estimation task that was adapted from Siegler and Pyke (2013), with the same fractions being presented (1/5, 7/8, 11/7, 9/5, 13/6, 7/3, 13/4, 10/3, 9/2, and 19/4).

Each number line had a 0 at the left end, 5 at the right end, and the to-be-estimated fraction above the center of the line. Immediately after providing an estimate for the fraction's location on the number line, participants rated their confidence in their placement from 0% to 100% (i.e., "I am not confident at all" to "I am totally confident"). The participant could not see their previous number line estimate nor change their response when providing confidence judgments.

Fraction Arithmetic Calculation

Participants were asked to solve four fraction arithmetic problems, one each of addition, subtraction, multiplication, and division, consisting of various combinations of 3/5, 1/4, and 2/3. Specifically, the problems participants solved were 3/5 + 1/4, 2/3–3/5, 3/5 ÷ 1/4, 2/3 × 3/5. We operationalized performance as percent correct.

Analytic Plan

As in prior work (e.g., Siegler et al., 2011), estimation accuracy was measured by percent absolute error [PAE]), calculated as PAE = (|participant's answer − correct answer|)/numerical range.

Statistical analyses were performed using R 3.5.3 (R Core Team, 2022) and Statistical Package for the Social Sciences Version 28 (IBM Corp, 2021). Repeated-measures ANOVAs and planned pairwise comparisons were conducted to determine whether magnitude comparison accuracy differed by notation. Hierarchical linear regressions were conducted to determine the variance accounted for by cross-notation and within-notation comparisons to ACT scores. As in Study 1, cross- and within-notation magnitude comparison accuracy scores were used as composite scores for these analyses. All analyses include the full sample of participants ($N = 183$), except for analyses involving ACT scores, which only include the 116 participants who provided ACT Scores. Cluster analyses were conducted according to the methods described in Study 1. Finally, we conducted a serial mediation model in Statistical Package for the Social Sciences using the PROCESS macro (Hayes, 2017). We tested the indirect path from within-notation accuracy to cross-notation accuracy, cross-notation accuracy to fraction arithmetic estimation, and fraction arithmetic estimation to fraction arithmetic calculation. We considered the indirect effect significant if the 95% confidence interval for the indirect effect did not contain zero.

Results

Contrasting Performance on Within and Cross-Notation Comparison

Similar to Study 1, accuracy was lower on cross-notation comparisons ($M = 87\%$, $SD = 14.72$) than on within-notation comparisons ($M = 91\%$, $SD = 13.14$), $t(182) = 5.733$, $p < .001$, Cohen's $d = 0.42$.

Within-Notation Performance

A repeated-measures ANOVA comparing accuracy on fraction–fraction, decimal–decimal, and percent–percent magnitude comparisons yielded a main effect of notation, $F(1, 364) = 35.56$, $p < .001$, $\eta^2_g = .088$. As in Study 1, percent–percent comparison accuracy ($M = 99\%$, $SD = 6.51$) was higher than decimal–decimal accuracy ($M = 88\%$, $SD = 21.77$), $t(182) = 6.37$, $p < .001$, Cohen's $d = 0.47$, and fraction–fraction accuracy ($M = 85\%$, $SD = 22.53$), $t(182) = 7.89$, $p < .001$, Cohen's $d = 0.58$. Accuracy of decimal–decimal comparisons tended to exceed that of fraction–fraction comparisons, $t(182) = 1.85$, $p = .066$, Cohen's $d = 0.14$.

Cross-Notation Performance

As in Study 1, participants exhibited a bias to select percentages as larger than fractions and decimals (Figure 2 in the online supplemental materials).

As in Study 1, magnitude comparison accuracy was higher when the percentage was larger than the fraction than when the fraction was larger than the percentage, 94% ($SD = 13$) versus 75% ($SD = 30$), $t(182) = 9.23$, $p < .001$, Cohen's $d = 0.68$. Similarly, accuracy was higher when the percentage was larger than the decimal than the opposite, 97% ($SD = 9$) versus 85% ($SD = 23$), $t(182) = 7.00$, $p < .001$, Cohen's $d = 0.52$. Unlike in Study 1, magnitude comparison accuracy was also higher when the decimal was larger than the fraction than when the fraction was larger than the decimal, 87% ($SD = 21$) versus 82% ($SD = 24$), $t(182) = 2.90$, $p = .004$, Cohen's $d = 0.21$.

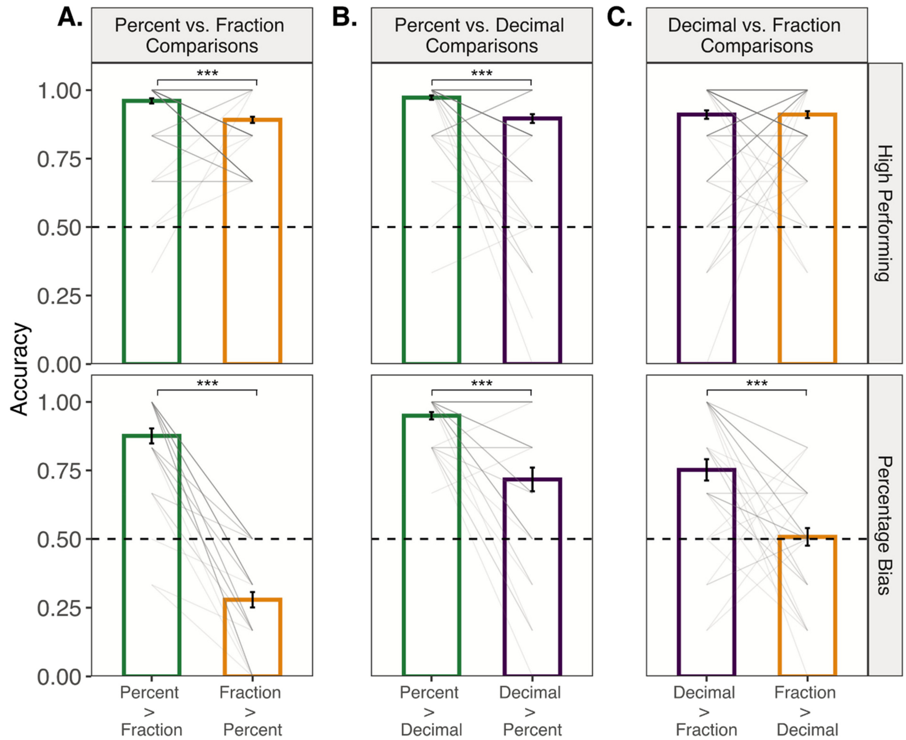
Cluster Analyses

Cross-Notation Comparison Profiles. In Study 1, we identified a subset of students who departed from the overall bias toward percentages being larger than equivalent decimals and fractions. Of note, we determined that this cluster analysis process did not merely differentiate high- versus low-achieving students but rather revealed the degree to which clusters of participants are influenced by a bias. For example, 23% of participants that would have been identified as low-achieving based on median split were identified as high-performing in regards to the bias based on our cluster analysis process. Further, 6% of participants that would have been identified as high-achieving based on median split were identified as moderate percentage bias based on our cluster analysis process. Thus, to investigate whether these profiles of misconceptions are present in this sample as well, we conducted cluster analyses based on accuracy on percent–fraction cross-notation comparisons, as we did in Study 1. Unlike Study 1 where the greatest number of clustering methods suggested a four-cluster model, for Study 2 most clustering methods indicated a two-cluster model, six methods of 30 (20%). This solution explained 64.36% of the variance and had an AIC of 14.91 (see Table 5 in the online supplemental materials).

The most prevalent profile is the "high-performing" cluster ($n = 140$), composed of undergraduates who perform almost at ceiling with no bias or only a minimal percentages-are-larger bias. The other cluster is the "percentages-are-larger bias" cluster ($n = 43$), composed of undergraduates who perform better on tasks where the percentage is larger than the compared number, suggesting a percentages-are-larger bias. Unlike Study 1, there was no "fraction bias" cluster in the Study 2 sample (Figure 4). Further, results consistently point to a percentages-are-larger bias whether we incorporate into the cluster analysis only fraction–percent comparison trials or include all six trial types (see Figures 2 and 5 in the online supplemental materials).

In Study 1, we found a difference in math achievement (i.e., self-reported SAT scores) between high performing and biased profiles. In Study 2, we investigated whether this difference replicated, and if any such differences extended to number line estimation accuracy (PAE), confidence judgments 0%–100%), fraction arithmetic estimation, and fraction arithmetic calculation accuracy (% accuracy). As shown in Figure 5, fraction arithmetic estimation and calculation

Figure 4
Percent Correct



Note. (A) Percent versus fraction comparisons, (B) percent versus decimal comparisons, and (C) decimal versus fraction cross-notation comparison based on a two-cluster model: "percentages bias" cluster ($n = 43$) and "high performing" cluster ($n = 140$). Most participants exhibit a bias to select the percentages as larger than fractions and decimals, with the percentage-bias profile exhibiting a more drastic difference in performance than the high performing profile. The percentage-bias cluster also demonstrated a bias to select the decimal as larger than fractions. However, there was no bias among the fraction–decimal comparisons for the high performing cluster. Gray lines represent individual participants' average scores in each of the conditions. Thicker gray lines indicate more participants with the same scores. Error bars represent ± 1 *SE*. See the online article for the color version of this figure.
*** $p < .001$.

were more accurate among students in the "high performing" profile than among those in the "percentage bias" profile. Moreover, PAE was lower (i.e., more accurate) for the "high performing" profile, and they also rated their confidence higher (i.e., 100 being "totally confident"). Finally, college admission scores (i.e., SAT in Study 1 and ACT in Study 2) were higher in the "high performing" profile than the biased profiles. Figure 3 in the online supplemental materials shows the accuracy in the different types of within-notation comparisons of the two clusters.

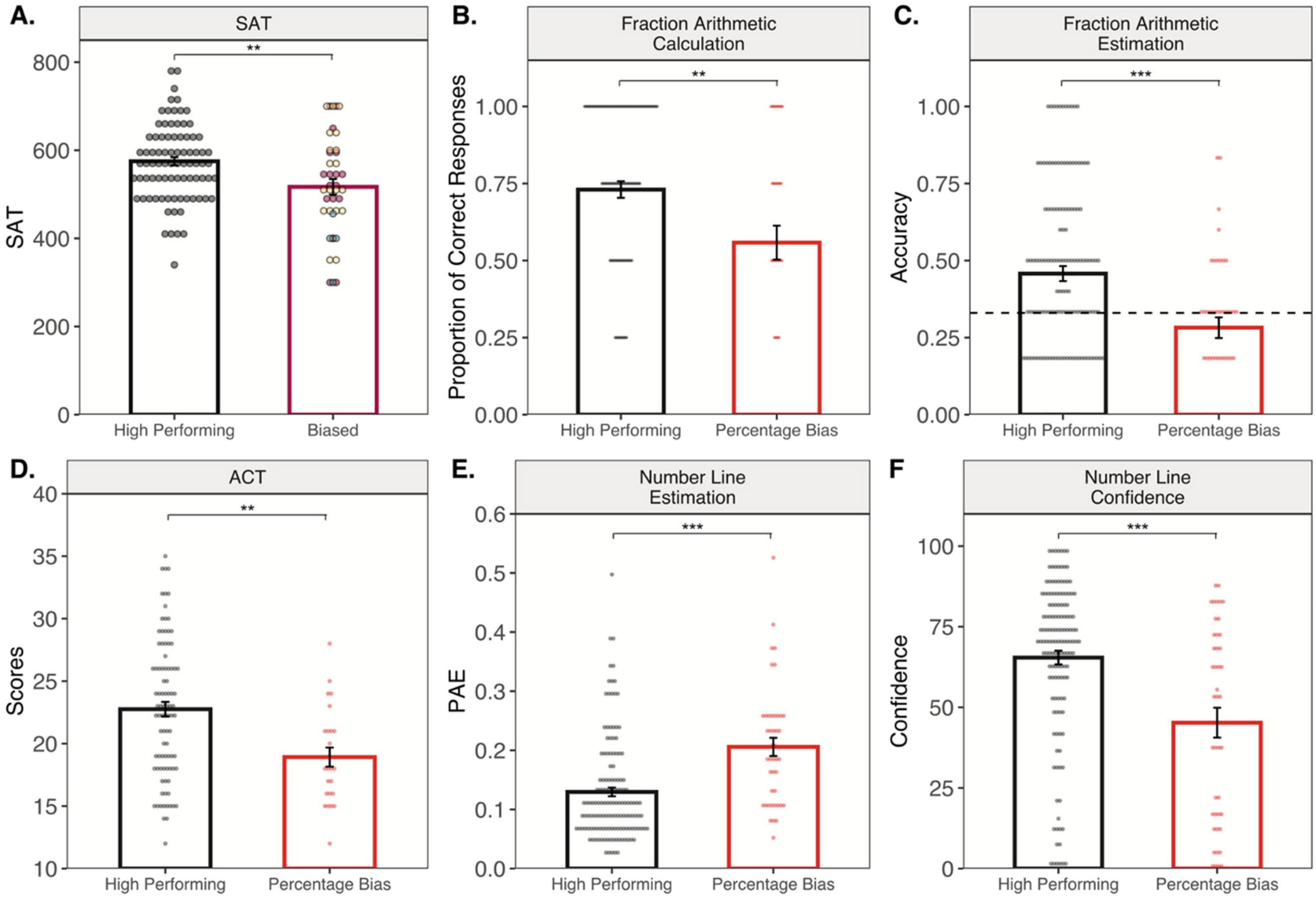
Similar to Study 1, we performed a mixed repeated-measures ANOVA with cluster (high performing and percentage bias profiles) as between-subject factor and type of comparison (within and cross) as within-subject factors and accuracy as the dependent variable. This analysis yielded main effects of Cluster, $F(1, 181) = 127.47$, $p < .001$, $\eta^2_g = .37$, and type of comparison, $F(1, 181) = 38.97$, $p < .001$, $\eta^2_g = .033$, and an interaction between them, $F(1, 181) = 34.81$, $p < .001$, $\eta^2_g = .03$. High performing participants showed small differences in their within ($M = 94.32\%$, $SD = 9.73$) and cross-notation performance ($M = 92.38\%$, $SD = 10.04$), $t(139) = 2.986$, $p = .003$, Cohen's $d = 0.252$. In contrast, percentage biased students were more accurate on within-notation ($M = 79.07\%$, $SD = 15.89$) than cross-notation comparisons ($M = 68.02\%$, $SD = 11.88$), $t(93) = 6.102$, $p < .001$, Cohen's $d = 0.930$.

Math Achievement Differences Between Cross-Notation Profiles. To investigate whether ACT scores differed between students who fit the different profiles, we performed an independent sample t-test with ACT scores from the 116 students who reported their scores (high performing, $n = 91$; percentage bias, $n = 25$). The high performing students reported higher ACT scores ($M = 22.75$, $SD = 5.55$) than peers in the percentage bias cluster ($M = 18.92$, $SD = 3.83$), $t(114) = 3.24$, $p = .002$, Cohen's $d = 0.73$.

Individual Differences in Cross- and Within-Notation Accuracy and Their Relation to Math Achievement and Fraction Arithmetic Calculation/Estimation

Hierarchical linear regression analyses were used to determine whether cross-notation magnitude comparison predicts college admission exams above and beyond within-notation comparison accuracy, as it did in Study 1. However, undergraduate students in

Figure 5
Descriptive Statistics



Note. (A) Self-reported SAT scores collected in Study 1 only and measures (B–F) collected in Study 2 only, by cluster: percentage bias (or biased) and high performing. For the SAT scores from Study 1, the "biased" cluster depicted here includes the three biased clusters from Study 1 with dots in red (strong percentage bias), yellow (moderate percentage bias), and blue (fraction bias). Error bars represent ± 1 *SE*. Dashed line in Part C represents chance performance for that task. Exact test statistics appear in the online supplemental materials. SAT = Scholastic Aptitude Test; ACT = American College Test; PAE = percent absolute error. See the online article for the color version of this figure but for interpretation in grayscale: dots in medium gray (strong percentage bias), light-gray (moderate percentage bias), and dark gray (fraction bias).
* $p < .05$. ** $p < .01$. *** $p < .001$.

Study 2 provided ACT scores, as opposed to SAT scores, which were reported in Study 1. Table 4 in the online supplemental materials displays the mean, standard deviation, and correlations among these variables and other measures in Study 2.

Within-notation magnitude comparison was added to the model first to account for general magnitude representation; it predicted 17% of the variance in self-reported ACT scores (Step 1, $p < .001$). Cross-notation magnitude comparison was entered next; it added 7% additional variance ($p < .001$), and within-notation was no longer a significant predictor (Table 7 in the online supplemental materials). These findings replicated the result of fraction magnitude representation in explaining math achievement (Siegler et al., 2011) and extend the findings to suggest cross-notation also predicts mathematics achievement as measured by college admission exams, consistent with recent findings with middle school students (Schiller & Siegler, 2023).

One possibility for the lack of significance for within-notation comparison accuracy in Model 2 but significance in Model 1 could be multicollinearity. However, multicollinearity proved not to be a concern (within-notation, tolerance = .47, VIF = 2.14; cross-notation, tolerance = .467, VIF = 2.14). Another possibility for this finding is that the within-notation composite score might be inflated because it includes percent–percent comparisons (e.g., 35% vs. 60%). This may relegate the task to that of a whole-number comparison task, since participants could use knowledge of numbers between 0 and 100 for the percentage comparisons. As such, we reran the analyses above but removed the percent versus percent comparisons from the within-notation composite. The pattern of results for the final model was unchanged. Further, the pattern was unchanged even when comparison accuracy for fraction versus fraction or decimal versus decimal are added as the initial predictor. Together, these results indicate that cross-notation understanding accounts for variance in math achievement beyond that which can be explained by within-notation understanding.

Mediation Analysis

In the hierarchical linear regression, within-notation magnitude comparison accuracy accounted for variance in self-reported ACT scores when it was added first to the model, suggesting as expected, that within-notation understanding is important for math achievement. However, within-notation magnitude comparison accuracy was non-significant in the model with cross-notation magnitude comparison accuracy. Why might this be?

Here, we tested the hypothesis that greater cross-notation rather than within-notation knowledge, as measured by magnitude comparison accuracy, indicates stronger magnitude representation organized on their mental number line. In other words, those who flexibly switch between notations for equivalent rational-number magnitudes (e.g., 4/16 = ¼ = 0.25 = 25% = …) may be more likely to evaluate the reasonableness of solutions on fraction arithmetic problems because they can approximate numerical magnitudes in whatever notational format that helps organize their mental number line. We tested this contention by examining whether the relation between within-notation knowledge and fraction arithmetic is serially mediated by cross-notation knowledge and fraction arithmetic estimation ability (Figure 6).

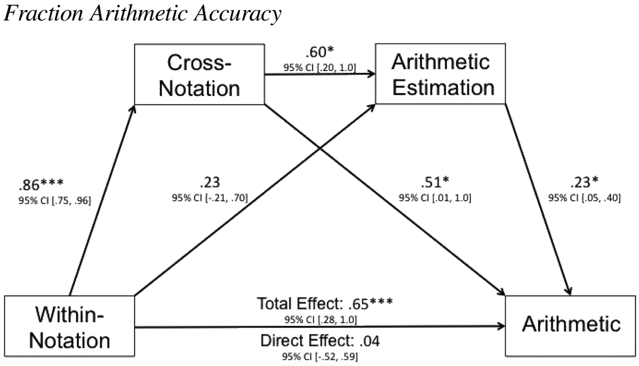
Indeed, the indirect effects path from within-notation to cross-notation to fraction arithmetic estimation to fraction arithmetic was $b = 0.12$, 95% confidence interval [.025, .259]. Further, controlling for cross-notation accuracy and fraction arithmetic estimation ability, the direct effect of within-notation accuracy was not a significant predictor of fraction arithmetic accuracy, $b = 0.04$, $t(183) = 0.1276$, $p = .899$. Hence, cross-notation accuracy and fraction arithmetic estimation ability are mediators of within-notation accuracy on fraction arithmetic (Figure 6).

Discussion

The present findings suggested that many undergraduates do not have strongly integrated number sense. Replicating Study 1, for the within-notation comparisons, undergraduate students performed better on percentage comparisons relative to fraction and decimal comparisons, and there was also a marginally significant difference between fraction and decimal within-notation comparisons. Second, replicating Study 1, undergraduates performed worse on cross-notation magnitude comparison than within-notation comparison for the same magnitudes and demonstrated biases based on notational form. Specifically, there was a bias for perceiving percentages as larger than fractions or decimals. Unlike in Study 1, there was also a bias for perceiving decimals as larger than fractions in fraction–decimal comparisons. Third, as in Study 1, cluster analyses revealed that a sizable portion of undergraduates (23% in Study 2) demonstrated a "percentages-are-larger bias," with the remaining students in the high performing group demonstrating "no bias" or only a slight bias. These cluster analyses also revealed striking differences between those who did or did not have a percentages-are-larger bias. Specifically, biased students performed worse on a measure of math achievement (self-reported ACT scores) and on a wide range of rational number tasks requiring magnitude understanding: number line estimation accuracy and confidence, fraction arithmetic estimation and calculation. That being said, unlike Study 1, there was no cluster that exhibited a fractions-are-larger bias in Study 2. In Study 1, participants with the fractions-are-larger bias reported substantially lower SAT scores than the percentages-are-larger bias. It is possible that the reason for a lack of replication of the fractions-are-larger bias had to do with the overall math achievement of the sample in Study 2 being somewhat higher than that of Study 1. Future research should examine whether the fractions-are-larger bias exists in lower achieving samples. Fourth, hierarchical linear regression analyses revealed that individual differences in cross-notation accuracy were predictive of self-reported ACT scores above and beyond within-notation accuracy, mirroring the SAT results in Study 1. In fact, within-notation comparison accuracy was no longer a significant predictor of these math outcomes when the model controlled for cross-notation comparison accuracy. These findings converge with conclusions from Study 1 that suggest undergraduates' who have better cross-notation understanding tend to have higher math achievement. Finally, mediation analyses allowed us to uncover a possible mechanism for the result of lesser importance of within-notation understanding than cross-notation understanding in predicting mathematical outcomes. Specifically, the relation between individual differences in within-notation magnitude representation and in fraction arithmetic skill was mediated by cross-notation magnitude representation and fraction arithmetic estimation ability, lending support to the hypothesis that greater magnitude knowledge enables individuals to reject implausible solutions and select correct calculation strategies (e.g., Siegler et al., 2011). Ultimately, flexibly thinking about numbers across notations may better equip individuals to evaluate the reasonableness of solutions, supporting their arithmetic performance.

Figure 6

Unstandardized Estimated Coefficients for the Hypothesized Model Pathway From Within-Notion to Cross-Notation Magnitude Comparison Accuracy to Fraction Arithmetic Estimation Ability to Fraction Arithmetic Accuracy



Note. Total effect refers to the effect of within-notation to fraction arithmetic through all possible paths (direct and all mediational effects). It does not control for the other variables in the model. Direct effect refers to the effect of within-notation to fraction arithmetic controlling for the mediational variables. CI = confidence interval.
* $p < .05$. ** $p < .01$. *** $p < .001$.

General Discussion

We examined undergraduates' integrated number sense across multiple tasks with multiple analysis techniques. Undergraduates' within-notation magnitude representation for equivalent rational numbers differed with whether comparisons were presented as fractions, decimals, or percentages. Specifically, percent–percent comparison accuracy was greater than either fraction–fraction or decimal–decimal comparisons. Moreover, undergraduate students were more accurate at within-notation than cross-notation comparison, with their cross-notation magnitude comparison being biased by notational format. In particular, a subset of undergraduates exhibited a percentages-are-larger bias, such that they were more likely to select the percentage as the larger number even when it was not. Through cluster analysis methods, we determined that a sizable proportion of undergraduate students demonstrated a lack of integrated number sense; approximately 23%–33% of undergraduate students in our samples exhibited the percentages-are-larger bias, and an additional 10% exhibited a fractions-are-larger bias (Study 1 only). This outcome suggests that a rather large portion of undergraduate students persist in displaying biases based on notational format, consistent with the percentages-are-larger bias that was found in middle-school students (Schiller & Siegler, 2023). Importantly, there were striking differences in a wide range of outcome measures between the cluster of undergraduates exhibiting a bias and the high performing cluster, who displayed a slight bias. Specifically, across the two studies, those exhibiting a bias reported lower college admissions test scores (SAT or ACT) and, in Study 2, they performed worse on number line estimation, with lower confidence in their estimates, and lower accuracy estimating and calculating answers for fraction arithmetic problems. These cluster analyses underscored the importance of cross-notation understanding but did not empirically test whether cross-notation understanding accounts for variance in math achievement beyond within-notation understanding.

Indeed, integrated number sense, what we have operationalized as cross-notation magnitude comparison accuracy, accounted for variance in self-reported college admission scores (i.e., SAT/ACT scores) above and beyond within-notation magnitude comparison accuracy. The integrated theory of numerical development (Siegler et al., 2011) defines successful numerical development as a gradual broadening of understanding of number to incorporate all types of rational numbers. Here, we proposed and found evidence in support of a new dimension: It is not simply enough to understand different types of numbers in isolation, successful numerical development involves understanding the notations in relation to one another. Mediation analysis was consistent with a potential mechanism for this result: Within-notation understanding supports cross-notation understanding, which in turn enables one to estimate reasonable solutions for arithmetic problems, leading to greater overall arithmetic accuracy. In fact, we found no relation between within-notation understanding and arithmetic accuracy when controlling for the mediators of cross-notation understanding and arithmetic estimation ability. These results support the conclusion that many undergraduates have not achieved the desired end state for numerical development, which consists of having a mental number line that integrates within- and cross-notation understanding.

Within-Notation Comparison Accuracy Differs by Notation

Although our primary focus was on cross-notation understanding, within notation performance bears on the concept of integrated number sense: If one has maximally achieved this state, there should be no differences between accuracy with different notational formats. Instead, we found that percentage–percentage comparisons were more accurate than fraction–fraction and decimal–decimal comparisons. Decimal–decimal comparisons were also marginally better than fraction–fraction comparisons in both samples, consistent with prior work (DeWolf et al., 2014; Ganor-Stern, 2013; Hurst & Cordes, 2016). However, not all studies find this decimals advantage (see for a review Tian & Siegler, 2018), suggesting this may not be a robust effect. Rather than speculate on potential reasons for the marginal difference observed here between fraction and decimal comparisons, we focus on the novelty revealed by including percentage–percentage comparisons in our measures of within-notation understanding.

This is the first study to directly compare undergraduate students' within-notation comparison accuracy with matched fractions, decimals, and percentages. Most studies exclude percentage–percentage comparisons, presenting only matched fraction–fraction and decimal–decimal comparisons, perhaps reasoning that within-notation percentage comparisons are too easy. Indeed, accuracy in percent–percent comparisons (e.g., 40% vs. 25%) was near-perfect (99% accuracy). However, had we not included percentage–percentage comparisons, we would not have evidence that even the lowest performing profile (fraction bias) had relatively high accuracy in percentage–percentage comparisons (84.09% accuracy). Likely, whole-number knowledge played a role in the overall high accuracy for percentage–percentage comparisons, as it should be fairly easy for adults to compare whole numbers (e.g., 40 vs. 25). These results suggest that among the rational number notations, percentages may be easiest to represent and perhaps the reason for the success of interventions that introduce rational concepts first through percentages, connecting them to fractions and decimals (Kalchman et al., 2001; Moss, 2005; Moss & Case, 1999). A potential explanation for the better performance with percentages could have to do with some decimal comparison trials including differing number of digits (i.e., it is difficult to compare 0.6 vs. 0.35 because people often activate whole number components $6 < 35$ (Ren & Gunderson, 2019) but $0.6 > 0.35$, whereas percentages have a fixed implicit denominator of 100 and, thus, can be treated similarly to whole numbers without issue (e.g., $60\% > 35\%$ because $60 > 35$). However, percentages are not immune to the difficulties that individuals experience with rational numbers (Tian, 2018; Siegler & Tian, 2022). For example, more than half of middle school students judged 87% of 10 to be more than 10 (Gay & Aichele, 1997). In this example, individuals may not be treating percentages as proportions but as whole numbers (i.e., $87 > 10$). This strategy serves individuals well when comparing percentages within-notation (e.g., comparing battery charge levels). However, this type of reasoning is problematic when percentages are represented in relation to other types of numbers, as evident in the cross-notation results, which we discuss next.

Percentages-Are-Larger Bias in Cross-Notation Comparison

Across two studies, we found that undergraduates exhibited a percentages-are-larger bias in cross-notation comparisons. For example, comparing 25% and 2/5, undergraduate students were more likely to select 25% as the larger number, resulting in lower accuracy than when the equivalent numbers were expressed as 1/4 and 40%. As noted, comparing 25% and 40% was not a difficult task, even for the lowest performing students. However, even the highest performing students were slightly biased to select the percentage as the larger number in these cross-notation comparisons.

The percentages-are-larger bias was shown to differentiate undergraduates on a host of rational number skills and more general measures of math achievement. With the integrated theory of numerical development, Siegler et al. (2011) theorize about the importance of a gradual broadening of knowledge of numbers to include fractions, decimals, and percentages in numerical development. They provide evidence for within-notation knowledge of fractions predicting math achievement and suggest that cross-notation knowledge of decimals and percentages could potentially be important. We verify this suggestion by demonstrating that cross-notation understanding of fractions, decimals, and percentages has important implications for rational number skill and math achievement, in general.

From a cognitive development standpoint, this design is analogous to tasks measuring conservation of quantity (Piaget, 1952). Children must conserve quantity if a row of objects is elongated or liquid is poured into a taller glass. Similarly, children must come to understand that a number's magnitude (e.g., 1/4) is conserved when it is translated (e.g., 0.25, 25%, 2/8, 0.250, etc.). Here, we showed that many undergraduates, like middle school children (Schiller & Siegler, 2023), do not conserve magnitude across notations.

Cluster Analyses Revealed Differences in Math Outcomes Based on Biased or High Performing Profiles

Cluster analysis has been gaining popularity in rational number cognition (Abreu-Mendoza et al., 2023; Gómez & Dartnell, 2019; Reinhold et al., 2020) based on its utility for identifying distinct performance/strategy profiles. While we find a percentages-are-larger bias exists in two large samples of undergraduate students, not all students followed this general trend (see gray lines in Figure 2). Person-centered cluster analyses thus allowed us to uncover distinct profiles of students' cross-notation performance. Across two studies, we identified high performing and percentage bias groups (as well as a less common fraction bias profile, in Study 1). We found that participants' profiles were associated with performance on a host of math outcomes, including math achievement, as measured by self-reported SAT/ACT scores, fraction number line estimation and confidence, and fraction arithmetic estimation and calculation accuracy. In essence, those who were more biased in cross-notation comparisons performed worse than the high-performing group. Interestingly, even among this high-performing profile, there was a slight bias overall, suggesting the intractability of this bias, even among students with stronger math skills.

The cluster analysis helped us identify distinct profiles of student cross-notation misconceptions, which might make it possible to tailor instruction to target students' specific needs. For example, students with a stronger percentage bias may need instruction that links all three rational number notations on the number line. Students with a fraction bias (which was found in Study 1 only) may require further clarification about the specifics of the fraction notation. For example, secondary math educators were trained to diagnose specific misconceptions in algebra and tailor their instruction explicitly to address those misconceptions (Holmes et al., 2013). Such targeted instruction based on misconceptions could have beneficial effects on students' learning (Booth et al., 2014). The current study also converges with other work (Mazzocco & Devlin, 2008), which suggests the potential benefit of including cross-notation measures in the classroom as a diagnostic tool.

Integrated Number Sense

Our work demonstrated that at least a quarter of undergraduate students lacked a strong integrated number sense. In theory, individuals who possess "perfect" integrated number sense should perform equally well on within-notation and cross-notation comparisons, and there should be no bias based on notational format of the numbers being presented. However, as demonstrated here, cross-notation performance was worse than within-notation performance, even among the high performing students in Study 2 only, and there were differences based on notation for both within- and cross-notation performance across studies.

This relatively weak integrated number sense found in undergraduates is problematic, given the role that integrated number sense potentially plays in many math outcomes. We demonstrated for the first time that cross-notation magnitude comparison accuracy accounts for variance in self-reported SAT/ACT scores, above and beyond within-notation magnitude comparison accuracy. Additionally, this lack of integrated number sense in at least approximately a quarter of undergraduate students is problematic given the role of integrated number sense in math achievement and other math outcomes. In other words, those with percentages-are-larger biased responding exhibited lower performance on fraction number line estimation, fraction arithmetic estimation/calculation accuracy, and lower confidence in estimating fraction magnitudes. These results are parallel to results found with children in one study examining within- and cross-notation comparison accuracy and other math outcomes in children (Schiller & Siegler, 2023). Future investigations across multiple age groups with the same outcome measures are needed to establish whether this pattern holds across development.

Our findings with fractions, decimals, and percentages are consistent with prior work that has suggested that cross-notation understanding (i.e., fraction–decimal) is important for rational number arithmetic (Braithwaite et al., 2022) but add to the literature by demonstrating the importance of including percentages in a measure of cross-notation knowledge for undergraduate students and extend this relation to math achievement. Prior work with children (Schiller & Siegler, 2023) called for including percentages in a measure of cross-notation knowledge, but it was not known whether differences in percentage cross-notation understanding would be relevant for undergraduate students. Indeed, the current study revealed differences based on the degree to which participants exhibited a percentages-are-larger bias on a number of measures (i.e., SAT/ACT, number line estimation accuracy and confidence, fraction arithmetic estimation and calculation).

In addition to replicating results in an undergraduate population, these results build upon prior work with children (Schiller & Siegler, 2023) by offering a potential mechanism for the importance of cross-notation understanding in math outcomes. Specifically, we argue that cross-notation measures might better assess individuals' integrated number sense and that integrated number sense may play a role in helping individuals estimate solutions to fraction arithmetic problems, which in turn helps improve fraction arithmetic calculation accuracy. We reasoned that those who flexibly switch between notations for equivalent rational-number magnitudes (e.g., 4/16 = ¼ = 0.25 = 25% = …) may be more likely to evaluate the plausibility of solutions on fraction arithmetic problems, because they can translate to easier-to-process numbers. Indeed, we found that within-notation understanding supported cross-notation understanding, which in turn supported estimation accuracy for answers to fraction arithmetic problems, contributing to better fraction arithmetic calculation skill. These results support the hypothesis that greater magnitude knowledge enables individuals to reject implausible solutions and select calculation strategies that yield plausible solutions (e.g., Siegler et al., 2011). An open question is whether training in cross-notation understanding could have an effect on fraction arithmetic estimation and calculation accuracy.

Instructional Implications, Limitations, and Future Directions

The current work highlights the importance of integrated number sense, incorporating both within- and cross-notation magnitude representation into one's mental number line. Moss and Case (1999) provided evidence toward this idea with their successful intervention highlighting connections among notations. Though that experiment has been replicated with similar results (Kalchman et al., 2001), it was not clear whether highlighting relations among notations was the key to the intervention's success. Success could stem from the constructivist nature of instruction with beakers of water or from introducing percentages first, especially since a more recent study (Malone et al., 2019) did not show benefits with fraction/decimal cross-notation training over fractions-only training when they were not related to percentages. Here, we provide initial evidence of cross-notation's importance for math outcomes. However, the findings reported here are correlational. Future research examining the effects of cross-notation interventions are needed to establish the mechanistic relationships between these constructs and provide insights for classroom instruction. For example, there may be value in helping students notice the relationships between notations by vertically aligning the magnitudes on the number line (Thompson & Opfer, 2010; Yu et al., 2022) or directly practicing comparison of magnitudes expressed in different notations.

Furthermore, we found that fraction arithmetic estimation accuracy predicted fraction arithmetic calculation accuracy. However, we did not directly test whether it would be possible to improve one's fraction arithmetic estimation ability, and if so, whether that would result in higher fraction arithmetic calculation accuracy. Indeed, Braithwaite and Siegler (2021) demonstrated that number line training could improve fraction arithmetic estimation accuracy when estimation accuracy for sums was assessed using number lines. Braithwaite and Siegler (2021) did not test whether such improvements in fraction arithmetic estimation might transfer to symbolic sum estimation (e.g., which is the best estimate for 12/13 + 7/8: 1, 2, 19, or 21), nor did they test whether improved fraction sum estimation accuracy resulted in improved fraction calculation accuracy. Recent work with a "stop and think" intervention (Wilkinson et al., 2020) has shown that training students on exerting inhibitory control in the math and science domain is beneficial for learning outcomes. Perhaps, a similar "stop and think" intervention aimed at evaluating the plausibility of fraction arithmetic sums could have a positive impact on improving fraction arithmetic calculation accuracy. Thus, future studies might try to improve fraction arithmetic estimation accuracy by providing number line and inhibitory control type training (i.e., a focus on executive function combined with math) and determine whether it results in improvements in symbolic sum estimation accuracy and also calculation accuracy.

Although it is important to note a limitation of the present work is that it includes self-reported SAT/ACT scores as a measure of math achievement and previous studies have highlighted the unreliability of self-reported scores/grades (Cole & Gonyea, 2010; Sticca et al., 2017). That being said, the present findings with undergraduate students' self-reported achievement data is consistent with similar findings with middle school children, using achievement data collected directly from the school district (Schiller & Siegler, 2023). Future work should consider directly collecting ACT/SAT scores from the relevant institution and/or triangulating with other directly assessed measures of math achievement, rather than self-report. Relatedly, another limitation of the current work is that it involves an online rather than laboratory-based research experiment, which may explain the relatively lengthy and varied task completion duration. Future work should also consider a supervised laboratory-based environment for task completion.

Furthermore, an open question is whether the percentages-are-larger bias observed here differs from the well-establish phenomenon of whole number bias. Whole number bias involves the misapplication of whole number principles to tasks involving rational numbers, such as magnitude comparison, number line estimation, and arithmetic (Christou, 2015; Fitzsimmons et al., 2020; McMullen & Van Hoof, 2020; Ni & Zhou, 2005; Obersteiner et al., 2013; Vamvakoussi & Vosniadou, 2010; Van Hoof et al., 2015). Perhaps, undergraduates are selecting the percentage as larger in cross-notation comparisons because it appears more similar to whole numbers. If this is the case, perhaps what we are detecting is individuals who are heavily swayed by whole number bias and viewing fractions as an entity smaller than one (Kallai & Tzelgov, 2009). Our limited stimulus set does not enable us to check for typical manifestations of whole number bias, such as selecting the fraction with the larger numerals (e.g., 3/8 vs. 2/3, where the fraction with smaller numerals has the larger magnitude) or the decimal with more digits (e.g., 0.8 vs. 0.27, where the number with the shorter digit train has the larger magnitude). The percentages-are-larger bias might also be a reflection of participants' preferences: Adults report that they like percentages more than fractions (Mielicki et al., 2022; Sidney et al., 2021). If adults show a preference toward percentages, they might be inclined to select percentages more frequently in cross-notation comparisons. Thus, future research might seek to determine if the percentages-are-larger bias observed in this study is a distinct phenomenon, or a newly identified instantiation of whole number bias, and asses the role notational preference plays in these findings.

Future investigations might longitudinally follow children as they develop understanding of fractions, decimals, and percentages to determine what other factors might influence integration of these concepts. Furthermore, we only collected data at one time point. Addressing reliability of these cross-notation measures could be an avenue for future research. Alternatively, future work might seek to experimentally manipulate integrated number sense by improving cross-notation knowledge (Schiller, Abreu-Mendoza, Siegler, et al., 2024) to determine whether improvements might transfer to other areas of mathematics. Consistent with Mazzocco and Devlin (2008) who advocate for measures of cross-notation knowledge to be used as a diagnostic tool, we suggest that measures of cross-notation knowledge such as these percent–fraction/decimal comparisons could provide useful information to classroom teachers about students who may or may not struggle in rational number concepts and math, in general. Future research might examine the practicality of using such an assessment in the classroom.

Conclusion

Current instructional practices focus sequentially on fractions, decimals, and percentages with little emphasis on their relations (Common Core State Standards Initiative, 2010; Moss, 2005; Siegler & Oppenzato, 2021). The present findings suggest that cross-notation understanding is a key aspect of numerical development, beyond within-notation understanding. Critically, we found that approximately a quarter of undergraduate students at two selective universities have only weakly integrated number sense. This is concerning, given that undergraduate students have recently completed K–12 formal schooling in mathematics; thus, it seems K–12 mathematics education has not helped a sizeable population of students to establish an understanding of the relations among fractions, decimals, and percentages. Together, these results provide a comprehensive assessment of college students' cross-notation understanding and indicate that fully integrated number sense still remains a goal for many students.

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Appendix

Experimental Stimuli

Table A1
Within-Notation Comparison Problems

F vs. F	D vs. D	P vs. P
2/5 vs. 1/4	0.25 vs. 0.4	40% vs. 25%
7/20 vs. 3/5	0.6 vs. 0.35	35% vs. 60%
3/8 vs. 8/100	0.08 vs. 0.38	38% vs. 8%
1/4 vs. 2/5	0.4 vs. 0.25	25% vs. 40%
3/5 vs. 7/20	0.35 vs. 0.6	60% vs. 35%
8/100 vs. 3/8	0.38 vs. 0.08	8% vs. 38%

Note. F = fraction; D = decimal; P = percentage.

Table A2
Cross-Notation Comparison Problems

F > D	D > F	F > P	P > F	D > P	P > D
2/5 vs. 0.25	¼ vs. 0.40	2/5 vs. 25%	40% vs. ¼	0.40 vs. 25%	40% vs. 0.25
0.35 vs. 3/5	0.6 vs. 7/20	35% vs. 3/5	7/20 vs. 60%	35% vs. 0.6	0.35 vs. 60%
3/8 vs. 0.08	8/100 vs. 0.38	3/8 vs. 8%	38% vs. 8/100	0.38 vs. 8%	0.08 vs. 38%
0.25 vs. 2/5	0.40 vs. 1/4	25% vs. 2/5	1/4 vs. 40%	25% vs. 0.40	0.25 vs. 40%
3/5 vs. 0.35	7/20 vs. 0.6	3/5 vs. 35%	60% vs. 7/20	0.6 vs. 35%	60% vs. 0.35
0.08 vs. 3/8	0.38 vs. 8/100	8% vs. 3/8	8/100 vs. 38%	8% vs. 0.38	38% vs. 0.08

Note. F = fraction; D = decimal; P = percentage.

(*Appendix continues*)

Experiment 2 Only

Fraction Arithmetic Estimation

Directions: select the best estimate for the following problems

Problem	Answer choices
Lure: fraction across answer choices	
$\frac{5}{6}+\frac{2}{4}$	$\frac{7}{10}$ $\frac{1}{3}$ $1\frac{1}{4}$
$\frac{3}{4}+\frac{1}{10}$	$\frac{4}{14}$ $1\frac{1}{4}$ $\frac{9}{10}$
$\frac{1}{5}+\frac{1}{2}$	$\frac{2}{7}$ $\frac{1}{3}$ $\frac{3}{4}$
Lure: fraction hybrid across answer choices	
$\frac{3}{5}+\frac{8}{9}$	$\frac{11}{45}$ 2 $1\frac{1}{2}$
$\frac{2}{9}+\frac{3}{5}$	$\frac{5}{45}$ $\frac{4}{18}$ $\frac{4}{5}$
$\frac{3}{4}+\frac{2}{10}$	$\frac{5}{40}$ $1\frac{1}{2}$ 1
No lure: fraction answer choices	
$\frac{3}{5}+\frac{2}{3}$	$\frac{1}{4}$ $\frac{7}{10}$ $1\frac{1}{4}$
$\frac{5}{9}+\frac{1}{3}$	$\frac{1}{2}$ $1\frac{1}{2}$ 1
$\frac{3}{8}+\frac{1}{3}$	$\frac{1}{10}$ $\frac{2}{5}$ $\frac{3}{4}$
$\frac{4}{5}+\frac{2}{3}$	$\frac{1}{2}$ 2 $1\frac{1}{2}$
$\frac{2}{10}+\frac{2}{4}$	$\frac{1}{5}$ $\frac{1}{3}$ $\frac{2}{3}$
$\frac{3}{7}+\frac{5}{9}$	$\frac{3}{4}$ 1 $1\frac{1}{2}$

0–5 Number Line Estimation

Trials: 7/8, 13/4, 11/7, 19/4, 10/3, 9/5, 9/2, 7/3, 1/5, 13/6,

Number line confidence: After each number line estimate, participants were shown where they placed their estimate, and then, participants were asked to estimate their confidence (without being able to change their original answer):

How sure are you that you estimated X at close to the correct spot on the number line from 0% = I am not confident at all to 100% I am totally confident? (Participants then entered their confidence level as a number in an open response text box.)

Fraction Arithmetic Calculation

3/5 + 1/4
2/3 − 3/5
3/5 ÷ 1/4
2/3 × 3/5

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