Strategies for Rate Optimization in Joint Communication and Sensing over Channels with Memory

Siva Aditya Gooty
Department of Electrical and Computer Engineering
Northeastern University
360 Huntington Ave, Boston, MA 02115, United States
Email: gooty.s@northeastern.edu

Hessam Mahdavifar
Department of Electrical and Computer Engineering
Northeastern University
360 Huntington Ave, Boston, MA 02115, United States
Email: h.mahdavifar@northeastern.edu

***Abstract*—Joint communication and sensing (JCAS) is one of the promising technologies envisioned for future generations of wireless systems. Several attempts have been made to formulate the problem from an information-theoretic perspective, assuming underlying discrete memoryless channels and leveraging classical rate-distortion trade-offs. However, communication channels often have memory and strategies to improve data rates with the help of sensing in such scenarios is not yet well understood. In this paper, we consider a generic system model for JCAS, and present a novel and greedy-type strategy to achieve optimal average rate, assuming a channel model with memory. In particular, in the considered channel model, the channel state follows a finite state Markov chain (FSMC). We also characterize how the solutions to communication and sensing problems complement each other in such scenarios. To the best of authors' knowledge, this is the first attempt towards understanding rate-optimized strategies in JCAS systems involving channels with memory from a communication-theoretic perspective.**

I. Introduction

Joint communication and sensing systems are widely considered as one of the main enabling technologies for the next generation, 6G and beyond, wireless networks [1], [2]. The problem of designing JCAS systems and understanding their fundamental limits has received significant attention recently and has been investigated from various perspectives. Several research directions are being actively pursued for complex design of JCAS systems while addressing challenges from the perspectives of distributed sensing, channel modeling, and characterizing radio frequency (RF) parameters [1]. Furthermore, due to a major trend towards utilizing higher frequency bands in wireless systems, the authors of [3] proposed a framework for JCAS systems in THz frequency bands. The potential applicability of multiple-input multiple-output antenna systems in JCAS systems is analyzed in [4], with extensions to the usage of re-configurable intelligent surfaces.

To fundamentally characterize the system performance, it is essential to define a mathematical model that can capture the essence of operations of the channel in JCAS systems. To this end, various efforts have been made in the literature. For instance, a simplified channel model for JCAS systems was provided and analyzed in [5], discussing the receiver design from a radar communication perspective. In another work, the capacity of additive white Gaussian noise (AWGN) channels in the presence of radar interference was discussed in [6]. Furthermore, once the channel model is established, understanding what is fundamentally achievable, in terms of the achievable rate of communication and some measure of success for the sensing operation, can play a vital role in the selection of various design parameters in the real system. Next, we review some of the prior work aiming to tackle this problem from an information-theoretic perspective.

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A. Related prior work

In an earlier work, Zhang et al. [7] considered the problem of joint communication and channel state estimation, and presented the fundamental trade-off between the transmission rate and state estimation distortion for various memoryless channels. In [8], an iterative algorithm was presented to optimize the input distribution at the transmitter to achieve optimal trade-offs in joint state sensing and communication in memoryless channels. Very recently, an information-theoretic capacity-distortion trade-off in point-to-point and single transmitter two receiver JCAS systems was presented in [9]. Furthermore, in [10], optimal trade-offs were characterized when the transmitter does not utilize the state feedback. In [11], the capacity-distortion trade-off were captured with subspaces of channels of sensing and communication and various degrees of freedom. In another work, similar trade-offs were explored in the finite block length regime [12].

It is worth noting that in most of these prior works the underlying channel is assumed to be memoryless. And the trade-offs are characterized for a *one-shot* scenario, e.g., by characterizing the optimal input distribution. In this paper, we make the first attempt, to the best of our knowledge, to design rate-optimizing strategies for a certain family of channels with memory.

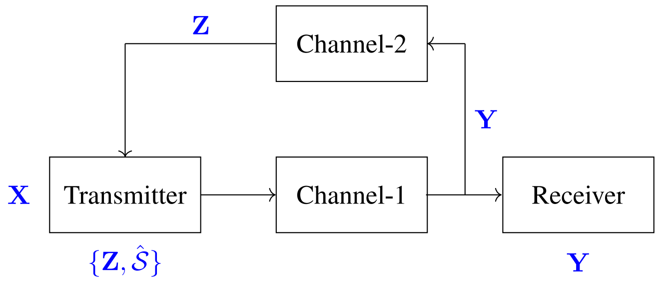

Fig. 1. JCAS system model

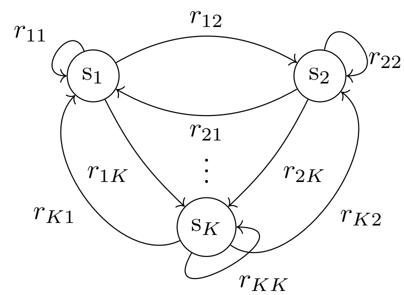

Fig. 2. Channel state $\mathcal{S}$ representation as multi-state Markov chain with transition probabilities $\mathbf{T}$

B. Our contributions

In our work, we consider a generic system model for JCAS systems with an emphasis on maximizing the communication data rate in such systems when the channel has memory. In particular, we study a scenario where the channel state follows a FSMC. We develop a greedy strategy to maximize the rate of communication at every round of communication given the past observations of the channel output available at the transmitter. We show that the greedy strategy also leads to the maximum throughput over several rounds of communication. It is demonstrated that maximizing the rate does not affect the Maximum a posteriori (MAP) estimation of the channel state at the transmitter, an observation that is primarily due to the symmetry of the underlying channels. We also compare and demonstrate the advantages of our optimized greedy-type strategy with strategies that are solely based on utilization of the MAP estimate of the channel state.

The rest of the paper is organized as follows. In Section II we discuss the considered system model for the JCAS problem and the FSMC modeling of the channel. In Section III we propose a novel strategy to maximize the data rate and the throughput of the communication system in the considered JCAS scenario. In Section IV, we demonstrate a numerical evaluation of the systems to test the performance compared with other MAP-based strategies. The paper is concluded in Section V.

II. System model

A. Notation

In our notation, bold letters represent vectors, capital letters of the form X represent random variables, and lower-case letters of the form x denote realizations of the random variable. The quantity $\mathrm{x}_j^{(i)}$ denotes realization x_j observed at a time instant i. And calligraphic letters, e.g., $\mathcal{X}$, represent sets.

The system model in Fig. 1. represents co-existence of traditional communication between a transmitter and a receiver along with the sensing information available at the transmitter. The input $\mathbf{X}$ is a random variable where each realization $\mathbf{x}$ represents binary data bits of length N from the transmitter. It is assumed that the bits in $\mathbf{X}$ are uniform i.i.d., i.e.,

$$\mathbf{X} \sim \mathrm{U}[0, 2^N]. \tag{1}$$

When the transmitter uses a specific codebook of dimension k, one can note that $\mathbf{X} \sim \mathrm{U}[0, 2^k]$. The output $\mathbf{Y}$ at the receiver is the result of input $\mathbf{X}$ undergoing transmission through Channel-1. The quantity $\mathbf{Z}$ can be regarded as the reflected signal available at the transmitter. Thus, $\mathbf{Z}$ is assumed to be the output of Channel-2 with input $\mathbf{Y}$. A pivotal assumption to consider at this point is regarding the channel type offered by Channel-1 and Channel-2.

B. Channel model

The channels are assumed to be binary-input symmetric channels with varying states that follow a FSMC, see, e.g., Fig. 2, where s_j's represent channel states and r_{ji}'s represent transition probabilities. Since the receiver is not aware of individual states of the channel realization, the FSMC channel model is not memoryless. In fact, it was shown in [13] that if a genie were to disclose channel states to receiver, then the channel decomposition would then be Markovian. This model has practical relevance in fading scenarios, e.g., [14] utilized Kullback–Leibler (KL) divergence criterion to fully characterize the Rayleigh and log-normal fading channel profiles as a variable length Markov chain.

Another assumption in our work is that the channels are reciprocal, see, e.g., [15], [16] for practical considerations of such an assumption. Note that this assumption only helps us to simplify the formulations, and the setup and the analysis can be extended to the cases where the channels are not reciprocal as well. More specifically, Channel-1 and Channel-2 in our JCAS model are assumed to follow the same Markov state realization at any time instant. For illustration purposes, in this work, we use the FSMC model with the underlying channels to be from the classes of binary symmetric channel (BSC), AWGN, and fading channels.

In addition, the state realization remains unchanged for the total duration of N bits of the input $\mathbf{x}$ making it a quasi-static channel. Suppose the channel has finite number of K states in total. The set $\mathcal{S}$ represents state space of possible K states of a channel, i.e.,

$$\mathcal{S} = \begin{bmatrix} \mathrm{s}_1 & \mathrm{s}_2 & \dots & \mathrm{s}_K \end{bmatrix}.$$

The finite states have a certain transition probabilities summarized in the transition matrix $\mathbf{T}$ as

$$\mathbf{T} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1K} \\ \vdots & \vdots & \vdots & \vdots \\ r_{K1} & r_{K2} & \dots & r_{KK} \end{bmatrix}.$$

C. Data rate

The transmitter and the receiver are agreed upon a specific codebook design before the rounds of transmission of $\mathbf{X}$. In this work, the strategies are oblivious to the codebook design. Instead, we simply assume that there is a certain data rate associated with each channel state at which the transmitter can *reliably* communicate to the receiver in N uses of that channel state. The communication rounds, each round consisting of communication at a fixed state N times, are indexed by positive integers from 1 to n. The transmitter attempts to transmit N bits of the input $\mathbf{X} = \mathbf{x}$ at a rate $R^{(i)}$ in the i-th round. Suppose $\mathcal{R}$ denote the ordered set of data rates, in a decreasing order, i.e.,

$$\mathcal{R} = \{R_1, R_2, \dots, R_K\},$$

$$\mathcal{S} \cong \mathcal{R}.$$

In other words, R_j is the rate associate with state s_j, and we have $R_1 \geq R_2 \geq \cdots \geq R_K$. Furthermore, we naturally assume that if, at a given round, the rate of communication is $R \leq R_j$, where s_j is the true state of the channel, then the communication is successful. Otherwise, if $R > R_j$, then the communication at that round fails.

III. The Rate-Maximization Strategy

In this section, we present a greedy strategy to achieve the maximum rate in each round of communication and show that it also leads to the maximum average rate/throughput across several rounds of communication.

Let us first define various events of interest. At the beginning of the communication, in the first round, all channel states $\mathrm{s}_j \in \mathcal{S}$ are assumed to be equally likely. Let $\mathrm{S}^{(i)}$ denote the prior event of observing any state $\mathrm{s}_j \in \mathcal{S}$ at round i. Then we have

$$P(\mathrm{S}^{(1)}) \sim \mathrm{U}[1, K]. \tag{2}$$

Note that once the i-th round is concluded, the transmitter is now equipped with $\mathbf{Z}$ and the receiver with $\mathbf{Y}$. In terms of the communication strategy, the goal of the JCAS framework is to make an informed decision at the transmitter with available $\mathbf{Z}$ in order to pick the *optimal* rate for communication at round $i+1$. Hence, it is critical that the transmitter computes the posterior probabilities of the channel state in the i-th round once it is concluded. This is discussed in the next subsection.

A. Maximum a posterior probaility (MAP) rule

The posterior probabilities of the channel state at the transmitter are computed by comparing the reflected signal $\mathbf{Z}$ with the already known input $\mathbf{X}$. For instance, in the case of the underlying channels being BSCs, this can be done as follows. Let $\mathrm{E}^{(i)}$ denote the random variable representing the number of bit errors at the i-th round. Note that the number of bit errors observed at the transmitter is simply the Hamming distance between $\mathbf{Z}$ and $\mathbf{X}$, i.e.,

$$\mathrm{e}^{(i)} = d_H(\mathbf{x}, \mathbf{z}), \tag{3}$$

where d_H denotes the Hamming distance between two binary vectors. Let ζ_j represents the conditional distribution of observing $\mathrm{E}^{(i)} = \mathrm{e}^{(i)}$ errors given $\mathrm{S}^{(i)} = \mathrm{s}_j$. Then the MAP estimate of the state $\mathrm{s}^{(i)}$, denoted by $\hat{\mathrm{s}}^{(i)}$, can be found as follows:

$$\zeta_j = P\{\mathrm{E}^{(i)} = \mathrm{e}^{(i)} | \mathrm{S}^{(i)} = \mathrm{s}_j\}, \tag{4}$$

$$\hat{\mathrm{s}}^{(i)} = \arg\max_{\mathrm{s}_j} \Big(\zeta_j * P\{\mathrm{S}^{(i)} = \mathrm{s}_j\}\Big). \tag{5}$$

Let $\hat{\mathrm{s}}^{(i+1)} = \mathrm{s}_j$, where the MAP estimate is made at the end of the i-th round. Then the *MAP-rule* strategy chooses R_j as the rate of communication in the next round. This MAP-rule strategy is straightforward, and yet intuitive, to make a decision of picking the rate for the next round. However, it fails at maximizing the overall rate, as we discuss next.

B. Transmitter preprocessing

Let $\mathrm{q}_j^{(i)}$ denote the posterior probability of state s_j available at the transmitter at the end of the i-th round. The quantities $\mathrm{q}_j^{(i)}$ are critical in the design of the greedy strategy and showing its optimality.

Before presenting the strategy, let us define and characterize the throughput of the JCAS system under consideration. As in the convention, the throughput is defined as the average rate of successful communication across a given number of rounds of communication.

Lemma 1. *At the end of the i-th round, the expected value of throughput in the next round is given by*

$$\textit{Expected throughput} = \left[\sum_{j=1}^{d} P(\mathrm{S}^{(i+1)} = \mathrm{s}_j)\right] R_d, \tag{6}$$

where R_d is the designated rate of communication for the next round.

Proof. Note that, as mentioned at the end of Section II, the rates R_j's are in decreasing order. This implies that if the designated rate is R_d, then the communication is successful if and only if the true state at the next round is s_j for some $j \leq d$. Otherwise, the communication fails. Hence, taking the expected value with respect to the random variable $\mathrm{S}^{(i+1)}$ representing the events for the random state in the next round would yield (6). □

Next, we describe how the transmitter computes the prior distribution of $\mathrm{S}^{(i+1)}$ using the available $\mathrm{q}_j^{(i)}$'s at the end of the i-th round. This is done as follows:

$$P(\mathrm{S}^{(i+1)}) = \mathbf{T}'\mathbf{Q}^{(i)} \tag{7}$$

where $\mathbf{T}$ is the transition matrix of the FSMC, and $\mathbf{Q}^{(i)} = \begin{bmatrix}\mathrm{q}_1^{(i)} & \mathrm{q}_2^{(i)} & \dots \mathrm{q}_K^{(i)}\end{bmatrix}$ is the vector of the posterior probabilities available from the previous round with

$$\mathrm{q_j}^{(i)} = \frac{\zeta_j * P\{\mathrm{S^{(i)} = s_j}\}}{\sum_{g=1}^{K} \zeta_g * P\{\mathrm{S^{(i)} = s_g}\}}, \tag{8}$$

where ζ_j's are defined in (4).

The following lemma sums up the discussion in this subsection.

Lemma 2. *The posterior probabilities of states $\mathrm{q}_j^{(i)}$ at the end of the i-th round together with the designated rate for the next round uniquely determine the expected value of throughput in the next round.*

Proof. The proof is by noting (8) and (7) together with Lemma 1. □

What remains is to specify how to choose the designated rate for the next round in order to maximize the expected throughput. This is discussed in the next subsection.

C. The greedy strategy

Suppose that the communication is at the end of the i-th round. Then the greedy strategy is essentially picking the designated rate for the next round, i.e., round $i+1$, by picking the index d, i.e., the rate R_d, that maximizes the average throughput for the next round, as given by (6). More specifically, let

$$d_{\max} = \underset{d\in\{1,2,\dots,K\}}{\arg\max} \left[\sum_{j=1}^{d} P(\mathrm{S}^{(i+1)} = \mathrm{s}_j)\right] R_d. \tag{9}$$

Then the greedy strategy picks $R_{d_{\max}}$ for the communication rate at round $i+1$.

Theorem 3. *The greedy strategy described in this section achieves the maximum average throughput.*

Proof. Note that by the definition of throughput and the expression for the average throughput, given in Lemma 1, the greedy strategy provides the maximum expected throughput for the next immediate round of communication. The key point to conclude the proof is that, the rate of communication at round i does not affect the distribution of the states in future rounds, i.e., $P(\mathrm{S}^{(i')})$, for $i' > i$. The reason behind this important observation is that the underlying communication channels in our framework are symmetric and hence maximizing rate, and, more specifically, the realization of the binary input sequence $\mathbf{X}$ to the channel does not affect the posterior distribution of channel states. Thus, the greedy strategy for maximizing rate at every round results in maximizing the overall average throughput and data rate for any number of rounds of communication. □

In fact, a more general result can be shown for the greedy strategy. In particular, even if one considers a randomized strategy with either deterministic or probabilistic distribution of picking the rates for subsequent rounds for communication, we claim that it can not perform better than the greedy strategy. The performance of the strategy is guaranteed in terms of both the individual rates of communication across rounds as well as the average rate over all rounds. This is discussed in the next Corollary.

Corollary 3.1. *The average rate achievable through greedy strategy is the maximum across what is achievable by any strategy including both deterministic and probabilistic strategies.*

Proof. The proof follows from the fact that the rate achievable by any randomized strategy can be expressed as a convex combination of rates achievable by a certain set of deterministic strategies. This together with Theorem 3 completes the proof. □

Besides maximizing the average rate, the other objective in JCAS systems is to minimize the cost of errors in sensing, i.e., the estimation of the channel states in our framework. This is discussed in the next subsection.

D. Estimation cost

The state estimation process has a certain cost of error which is a function of the state estimate and the true state. For instance, a general cost function is defined and considered in [9]. However, since the setup is different in our case and involves a finite number of states in an FSMC, we consider a Hamming-type estimation cost function defined as follows:

$$\mathrm{c}^{(i)}[\hat{\mathrm{s}}^{(i)}, \mathrm{s}^{(i)}] = \begin{cases} 0 & \text{if } \hat{\mathrm{s}}^{(i)} = \mathrm{s}^{(i)} \\ 1 & \text{if } \hat{\mathrm{s}}^{(i)} \neq \mathrm{s}^{(i)} \end{cases},$$

where i refers to the index of the communication round. Thus, the average cost can now be characterised as a function of estimation process and the total number of transmission rounds as,

$$\mathrm{C}(n, \mathcal{S}, \hat{\mathcal{S}}) = \frac{\sum_{i=1}^{n} \mathrm{c}^{(i)}[\hat{\mathrm{s}}^{(i)}, \mathrm{s}^{(i)}]}{n}. \tag{10}$$

IV. Examples and numerical results

In this section, we present the conceptual idea of applying the strategies for three different channels. Namely, a two state BSC channel (2-BSC), AWGN and real fading channels.

1) BSC Channel: The 2-BSC channel of interest is assumed to have two error probabilities representing good and bad channels. The Fig. 3. represent Markov states and the transition probabilities between the two states. Later in this section, we provide a numerical evaluation for this example.

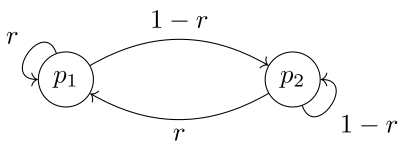

Fig. 3. 2-BSC model representing good and bad channel state as seen by transmitter

2) AWGN Channel: In this example, the wireless channel is a noisy channel whose realization is random. The random realization is a traditional white Gaussian noise which is additive in nature. Consider the simplest observation model:

$$\mathbf{Z} = \underbrace{\left[\mathbf{X} + \mathbf{W_1}\right]}_{\mathbf{Y}} + \mathbf{W_2} \tag{11}$$

$$\mathbf{Z} = \mathbf{X} + \mathbf{W}^* \tag{12}$$

where $\mathbf{W_1} \sim \mathcal{N}\left[0, \sigma_1^2\right]$ & $\mathbf{W_2} \sim \mathcal{N}\left[0, \sigma_2^2\right]$ are real standard Gaussian random variables. The two variables have independent distributions. Thus for the transmitter working with $\mathbf{Z}$, $\mathbf{W}^* \sim \mathcal{N}\left[0, \sigma_1^2 + \sigma_2^2\right]$. The greedy strategy for maximizing average rate can be used noting (9) and the corresponding cost with (10). In a more general case, the channel state can be seen as finite state realization of variances of AWGN, i.e.,

$$\mathcal{S} = \{\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2\}$$

3) Fading Channel: The fading wireless channel assumed for illustration has real co-coefficients along with AWGN in both Channel-1 and Channel-2. At the transmitter, the received information signal $\mathbf{Z}$ can be modeled as,

$$\mathbf{Z} = h_2 \underbrace{\left[h_1\mathbf{X} + \mathbf{W_1}\right]}_{\mathbf{Y}} + \mathbf{W_2} \tag{13}$$

$$\mathbf{Z} = h_2 h_1 \mathbf{X} + \mathbf{W}^* \tag{14}$$

$$\mathbf{Z} = h^2 \mathbf{X} + \mathbf{W}^* \tag{15}$$

where the channel reciprocity assumption is used for $h_1 = h_2 = h$ and $\mathbf{W}^* \sim \mathcal{N}\left[0, \sigma^2(h^2+1)\right]$. The state space $\mathcal{S}$, now consists of real channel coefficients as FSMC states assuming the noise realization $\mathbf{W}^*$ being constant. The greedy strategy for maximizing average rate can be used noting (9) and the corresponding cost with (10).

A. Numerical results for 2-BSC FSMC

In this section, we demonstrate the performance of different strategies for the 2-BSC example in JCAS system. Note that, such an optimization of the strategy is parameterized with values in TABLE I and remains the same with MAP rule for comparison with a baseline. To better analyze the performance of the greedy strategy, the MAP rule is also simulated with updated priors of the greedy strategy, in a strategy referred to as MAP_Q.

TABLE I
THE SIMULATION PARAMETERS FOR THE JCAS SYSTEM INVOLVING TWO STATES FOR THE BSC CHANNEL MODEL. THE PARAMETERS ARE THE SAME FOR ALL THE THREE DIFFERENT STRATEGIES.

Parameter	Value
N	5
n	50
p_1	0.09
p_2	0.25
r	0.85
R_1, R_2	0.3, 0.1

The transmitter maintains the set $\mathcal{R} = \{R_1, R_2\}$ where the two rates are respectively capacity achieving with respect to channel-1 BSC. With the greedy strategy, transmitter adapts R_1 as it's rate when the value of the updated priors are suitable for the optimization as per (9).

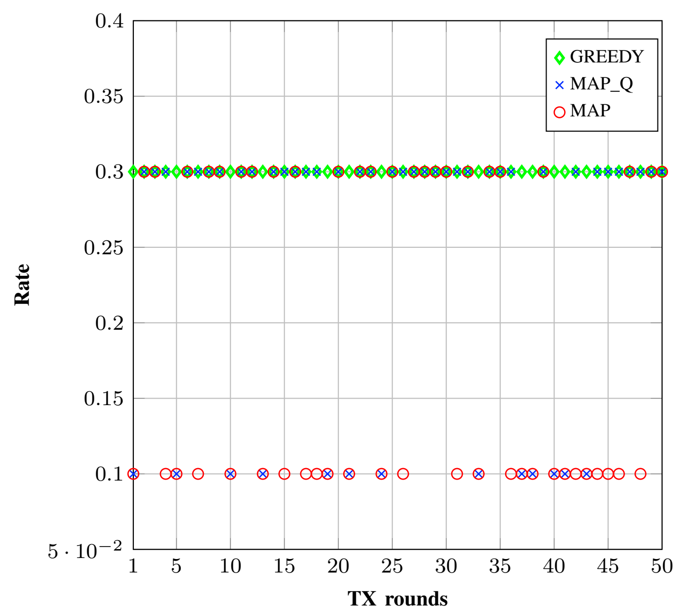

Fig. 4. Rate chosen by TX in each round by different strategies. The greedy strategy performs better than MAP with updated priors and MAP with uniform priors in several rounds.

From Fig. 4. it is evident that the greedy strategy outperforms MAP and MAP_Q where MAP_Q denotes the MAP rule with updated priors of the channel states. The average rates reported are 0.3, 0.268 and 0.18 for the greedy, MAP_Q and MAP strategies. The greedy strategy resulted in using R_1 better than MAP strategy in 60% of total rounds of communication. Similarly, the greedy strategy resulted in using R_1 better than MAP_Q strategy in 16% of total rounds of communication. Whereas, the MAP_Q alone resulted in using R_1, 44% better than MAP. The simulation results guarantee the theoretical analysis and hence proves that our novel strategy outperforms the simple and updated MAP baselines. Note that the estimation cost (10) for MAP and MAP_Q is observed at

0.64 and 0.38 respectively, where cost for the greedy strategy remaining same as MAP_Q.

V. Conclusion and future work

In this paper, we proposed a novel greedy strategy that can maximize the average communication data rate and throughput of the communication in JCAS system with the help of sensing information from the environment. The proposed greedy strategy can guarantee the optimal average data rate among the space of any deterministic and probabilistic strategies. This approach is a stepping stone to establish guaranteed maximization of rates and the throughput, even when the communication system utilizes modern coding schemes.

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