Cell-free Joint Sensing and Communication MIMO: A Max-Min Fair Beamforming Approach

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***Abstract*—This paper considers a cell-free integrated sensing and communication (ISAC) MIMO system, where distributed MIMO access points are jointly serving the communication users and sensing the targets. For this setup, we first develop two baseline approaches that separately design the sensing and communication beamforming vectors, namely communication-prioritized sensing beamforming and sensing-prioritized communication beamforming. Then, we consider the joint sensing and communication (JSC) beamforming design and derive the optimal structure of these JSC beamforming vectors based on a max-min fairness formulation. The results show that the developed JSC beamforming is capable of achieving nearly the same communication signal-to-interference-plus-noise ratio (SINR) that of the communication-prioritized sensing beamforming solutions with almost the same sensing SNR of the sensing-prioritized communication beamforming approaches, yielding a promising strategy for cell-free ISAC MIMO systems.**

I. Introduction

The integration of sensing functions into the communication systems is envisioned to be an integral part of the 6G and future communication systems [1], [2]. If the hardware and wireless resources are efficiently shared, this will enable the communication infrastructure to have sensing capabilities at minimal cost and open the sensing frequency bands for wireless communication operation. Achieving that, however, requires the careful design of the various aspects of the integrated sensing and communication (ISAC) systems, including the transmission waveform, the post-processing of the received signals, and the MIMO beamforming. While these problems have recently attracted increasing research interest, the prior work has mainly focused on the single ISAC basestation case. In practice, however, multiple ISAC basestations will operate in the same geographical region, frequency band, and time, causing interference on each other for both the sensing and communication functions. This motivates the coordination between these distributed nodes to improve both communication and sensing performance. This ultimately leads to cell-free ISAC MIMO systems, where distributed ISAC basestations jointly serve the same set of communication users and sense the same targets. With this motivation, this paper investigates the joint sensing and communication beamforming design of these cell-free ISAC MIMO systems.

Prior work has mainly focused on the single-node case and investigated the design of the joint-sensing and communication (JSC) waveform [3], and beamforming [4]. For beamforming, the work in [4] investigated the JSC beamforming design of co-located MIMO system with monostatic radar that serves multiple users. For distributed nodes, but assuming that each basestation serves only one user, i.e., not in a cell-free MIMO setup, [5]–[7] studied the power allocation and beamforming design problems. More relevantly, the optimization of the JSC power allocation has been investigated for distributed multi-antenna systems that consider a single user served by basestations or in a cell-free setup [8]. The authors in this work, however, adopted fixed beam designs, i.e., regularized zero beamforming for the communication with the sensing beamforming in the nullspace of the the communication channels without further optimization, and focused on optimizing the power allocated to these beams. Since these cell-free ISAC MIMO systems rely mainly on beamforming in their dual-function operation, it is very important to optimize the design of these JSC beams, which to the best of our knowledge, has not been previously investigated. With this motivation, we propose and compare various beamforming strategies for the cell-free ISAC MIMO systems.

To investigate the JSC transmit beamforming in cell-free massive MIMO systems, in this paper, we consider a system model with many APs and UEs, where the APs jointly serve the UEs and sense the targets in the environment. For beamforming, we first consider two baseline strategies that we call *communication-prioritized sensing* and *sensing-prioritized communication beamforming*. In these strategies, either the sensing or the communication beamforming is given the priority to be design first without accounting for the other function, and then the beamforming of the other function is designed in a way that does not affect the performance of the higher-priority function. After that, we consider the case when the sensing and communication beamforming is jointly designed. For this, we formulate a JSC beamforming problem, that aims to maximize the sensing SNR while satisfying the communication SINR constraints. We then re-formulate this problem as a non-convex semidefinite problem (SDP) and apply semidefinite relaxation (SDR) to find the optimal beamforming structure for a large set of classes. Using numerical simulations, We then evaluate the proposed approaches and show that the JSC design provides near-optimal performance for both sensing and communication thanks to the co-design for the two functions.

II. System Model

We consider a cell-free massive MIMO ISAC system with M access points (APs) and U communication users, as illustrated in Fig. 1. In the downlink, and without loss of generality, we assume that a subset $\mathcal{M}_t$ (out of the M APs) are transmitting communication and sensing waveforms to

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jointly serve the U users, where $|\mathcal{M}_t| = M_t$. Simultaneously, a subset $\mathcal{M}_r$ (out of the M APs) is receiving the possible reflections/scattering of the transmitted waveforms on the various targets/objects in the environment, with $|\mathcal{M}_r| = M_r$. It is important to note here that the subsets $\mathcal{M}_t$ and $\mathcal{M}_r$ may generally have no, partial, or full overlap, which means that none, some, or all the APs could be part of $\mathcal{M}_t$ and $\mathcal{M}_r$ and are simultaneously transmitting and receiving signals. The transmitting and receiving APs are equipped with N_t and N_r antennas. Further, for simplicity, all the APs are assumed to have digital beamforming capabilities, i.e., each antenna element has a dedicated radio frequency (RF) chain. The UEs are equipped with single antennas. The APs are connected to a central processing unit that allows joint design and processing, and they are assumed to be fully synchronized for both sensing and communication purposes.

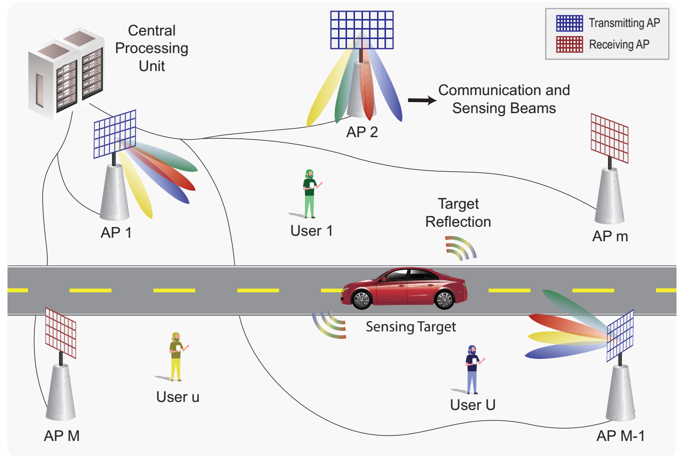

Fig. 1. The system model with the joint sensing and communication transmissions is illustrated. The APs serve multiple users while aiming to sense the target.

A. Signal Model

In this subsection, we define the joint sensing and communication signal model for the downlink transmissions. The APs jointly transmit U communication streams, $\{x_u[\ell]\}_{u\in\mathcal{U}}$, and Q sensing streams, $\{x_q[\ell]\}_{q\in\mathcal{Q}}$, where $\mathcal{Q} = \{U+1,\ldots,U+Q\}$ and with ℓ denoting the ℓ's symbol in these communication/sensing streams. For ease of exposition, we also define the overall set of streams as $\mathcal{S} = \mathcal{U}\cup\mathcal{Q} = \{1,\ldots,S\}$ with $S = U+Q$. If $\mathbf{x}_m[\ell] \in \mathbb{C}^{N_t}$ denotes the transmit signal at the transmitting AP m due to the ℓ-th symbol, we can then write

$$\mathbf{x}_m[\ell] = \underbrace{\sum_{u\in\mathcal{U}} \mathbf{f}_{mu} x_u[\ell]}_{\text{Communication}} + \underbrace{\sum_{q\in\mathcal{Q}} \mathbf{f}_{mq} x_q[\ell]}_{\text{Sensing}} = \sum_{s\in\mathcal{S}} \mathbf{f}_{ms} x_s[\ell], \quad (1)$$

where $x_s[\ell] \in \mathbb{C}$ is the ℓ-th symbol of the s-th stream, $\mathbf{f}_{ms} \in \mathbb{C}^{N_t}$ is the beamforming vector for this stream applied by AP m. The symbols are assumed to be of unit average energy, $\mathbf{E}[|x_s|^2] = 1$. The beamforming vectors are subject to the total power constraint, P_m, given as $\mathbf{E}[\|\mathbf{x}_m[\ell]\|^2] = \sum_{s\in\mathcal{S}} \|\mathbf{f}_{ms}\|^2 \leq P_m$. Further, by stacking the beamforming vectors of stream s of all the APs, we define the beamforming vector $\mathbf{f}_s = \left[\mathbf{f}_{1s}^T \quad \ldots \quad \mathbf{f}_{M_t s}^T\right]^T \in \mathbb{C}^{M_t N_t}$.

For each stream s, we denote the sequence of L transmit symbols as $\mathbf{x}_s = \left[x_s[1],\ldots,x_s[L]\right]^T$. Given this notation, we make the following assumption, which is commonly adopted in the literature [4]: The messages of the radar and communication signals are statistically independent, i.e., $\mathbf{E}[\mathbf{x}_s\mathbf{x}_s^H] = \mathbf{I}$ and $\mathbf{E}[\mathbf{x}_s\mathbf{x}_{s'}^H] = \mathbf{0}$ for $s, s' \in \mathcal{S}$ with $s \neq s'$. Note that the radar signal generation with these properties may be achieved through pseudo-random coding [4].

B. Communication Model

We denote the communication channel between UE u and AP m as $\mathbf{h}_{mu} \in \mathbb{C}^{N_t}$. Further, by stacking the channels between user u and all the APs, we get $\mathbf{h}_u \in \mathbb{C}^{M_t N_t}$. Next, considering a block fading channel model, where the channel remains constant over the transmission of the L symbols, we can write the received signal at UE u as

$$\begin{aligned} y_u^{(c)}[\ell] &= \sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{x}_m[\ell] + n_u \\ &= \underbrace{\sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu} x_u[\ell]}_{\text{Desired Signal (DS)}} + \underbrace{\sum_{u'\in\mathcal{U}\setminus\{u\}} \sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu'} x_{u'}[\ell]}_{\text{Multi-user Interference (MUI)}} \\ &\quad + \underbrace{\sum_{q\in\mathcal{Q}} \sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mq} x_q[\ell]}_{\text{Sensing Interference (SI)}} + \underbrace{n_u[\ell]}_{\text{Noise}}, \end{aligned} \quad (2)$$

where $n_u[\ell] \sim \mathcal{CN}(0, \sigma_u^2)$ is the receiver noise of UE u. Then, the communication SINR of UE u can be obtained as given in (3).

C. Sensing Model

For the sensing channel model, we consider a single-point reflector, as commonly adopted in the literature [8]. Specifically, the transmit signal is scattered from the single-point reflector and received by the receiving APs in $\mathcal{M}_r$. With a single path, the channel between the transmitting AP m_t and the receiving AP m_r through the reflector is defined as

$$\mathbf{G}_{m_t m_r} = \alpha_{m_t m_r} \mathbf{a}(\theta_{m_r}) \mathbf{a}^H(\theta_{m_t}), \quad (4)$$

where $\alpha_{m_t m_r} \sim \mathcal{CN}(0, \zeta_{m_t m_r}^2)$ is the combined sensing channel gain, which includes the effects due to the path-loss and radar cross section (RCS) of the target, and $\mathbf{a}(\theta)$ is the array response vector. The angles of departure/arrival of the transmitting AP m_t and receiving AP m_r from the point reflector are respectively denoted by θ_{m_t} and θ_{m_r}. We consider the Swerling-I model for the sensing channel [9], which assumes that the fluctuations of RCS are slow and the sensing channel does not change within the transmission of the

$$\mathrm{SINR}_u^{(\mathrm{c})}=\frac{\left|\sum_{m\in\mathcal{M}_t}\mathbf{h}_{mu}^H\mathbf{f}_{mu}\right|^2}{\sum_{u'\in\mathcal{U}\setminus\{u\}}\left|\sum_{m\in\mathcal{M}_t}\mathbf{h}_{mu}^H\mathbf{f}_{mu'}\right|^2+\sum_{q\in\mathcal{Q}}\left|\sum_{m\in\mathcal{M}_t}\mathbf{h}_{mu}^H\mathbf{f}_{mq}\right|^2+\sigma_u^2}=\frac{\left|\mathbf{h}_u^H\mathbf{f}_u\right|^2}{\sum_{u'\in\mathcal{U}\setminus\{u\}}\left|\mathbf{h}_u^H\mathbf{f}_{u'}\right|^2+\sum_{q\in\mathcal{Q}}\left|\mathbf{h}_u^H\mathbf{f}_q\right|^2+\sigma_u^2}. \quad (3)$$

L sensing and communication symbols in $\mathbf{x}_s$. With this model, the signal received at AP m_r at instant ℓ can be written as

$$\begin{aligned}\mathbf{y}_{m_r}^{(\mathrm{s})}[\ell]&=\sum_{m_t\in\mathcal{M}_t}\mathbf{G}_{m_tm_r}\mathbf{x}_{m_t}[\ell]+\mathbf{n}_m[\ell]\\&=\sum_{m_t\in\mathcal{M}_t}\alpha_{m_tm_r}\mathbf{a}(\theta_{m_r})\mathbf{a}^H(\theta_{m_t})\mathbf{x}_{m_t}[\ell]+\mathbf{n}_{m_r}[\ell],\end{aligned} \quad (5)$$

where $\mathbf{n}_{m_r}[\ell]\in\mathbb{C}^{N_r}$ is the receiver noise at AP m_r and has the distribution $\mathcal{CN}(0,\varsigma_{m_r}^2\mathbf{I})$. To write the received radar signal due to the L symbols in a compact form, we introduce

$$\overline{\mathbf{F}}_m=[\mathbf{f}_{m1},\ldots,\mathbf{f}_{mS}]\in\mathbb{C}^{N_t\times S}, \quad (6)$$

$$\overline{\mathbf{X}}=[\mathbf{x}_1,\ldots,\mathbf{x}_S]^T\in\mathbb{C}^{S\times L}. \quad (7)$$

Then, as an equivalent of (1), we can write the transmit signal of the L symbols from each AP m_t as

$$\mathbf{X}_{m_t}=\overline{\mathbf{F}}_{m_t}\overline{\mathbf{X}}\in\mathbb{C}^{N_t\times L}. \quad (8)$$

With that, we can re-write the sensing signal in (5) at each receiving AP m_r, due to the L symbols, in a compact form as

$$\mathbf{Y}_{m_r}^{(\mathrm{s})}=\underbrace{\sum_{m_t\in\mathcal{M}}\underbrace{\alpha_{m_tm_r}\mathbf{a}(\theta_{m_r})\mathbf{a}^H(\theta_{m_t})\overline{\mathbf{F}}_{m_t}}_{\triangleq\overline{\mathbf{G}}_{m_tm_r}}}_{\triangleq\overline{\mathbf{G}}_{m_r}}\overline{\mathbf{X}}+\mathbf{N}_{m_r}, \quad (9)$$

with $\overline{\mathbf{G}}_{m_r}$ denoting the beam-space sensing channel of the receiving AP m_r and $\mathbf{N}_{m_r}=[\mathbf{n}_{m_r}[1],\ldots,\mathbf{n}_{m_r}[L]]$.

With the purpose of having a general sensing objective that is correlated with the performance of various sensing tasks (e.g., detection [7], range/Doppler/angle estimation and tracking), we adopt the joint SNR of the received signals as the sensing objective. Note that utilization of the joint SNR requires a joint processing of the radar signal at the $\mathcal{M}_r$ sensing receivers. The sensing SNR can be written as

$$\begin{aligned}\mathrm{SNR}^{(\mathrm{s})}&=\frac{\mathbb{E}\left[\sum_{m_r\in\mathcal{M}_r}\left\|\overline{\mathbf{G}}_{m_r}\overline{\mathbf{X}}\right\|_{\mathrm{F}}^2\right]}{\mathbb{E}\left[\sum_{m_r\in\mathcal{M}_r}\left\|\mathbf{N}_{m_r}\right\|_{\mathrm{F}}^2\right]}\\&=\frac{\sum_{m_r\in\mathcal{M}_r}\sum_{m_t\in\mathcal{M}_t}\zeta_{m_tm_r}^2\left\|\mathbf{a}^H(\theta_{m_t})\overline{\mathbf{F}}_{m_t}\right\|^2}{\sum_{m_r\in\mathcal{M}_r}\varsigma_{m_r}^2}.\end{aligned} \quad (10)$$

in equation (10). Here we note that the sensing SNR is scaled with the contribution of all the (communication and sensing) streams.

Our objective is then to design the cell-free communication beamforming $\{\mathbf{f}_u\}_{u\in\mathcal{U}}$ and the sensing beamforming $\{\mathbf{f}_q\}_{q\in\mathcal{Q}}$ to optimize the communication SINR and the sensing SNR defined in (3) and (10). It is important to note here that, in this paper, we focus on the beamforming design problem assuming that the communication channel and the sensing target angles are known to the transmit APs. In the next three sections, we present the proposed beamforming strategies.

III. Communication-Prioritized Sensing Beamforming Design

In this section, we investigate the scenario where the communication has a higher priority, and where the communication beams are already designed a priori. In this case, the objective is to design the sensing beams to optimize the sensing performance while not affecting the communication performance (i.e., not causing any interference to the U communication users). Note that in this section and the next section, Section IV, we assume that $Q=1$ since we have one sensing target and that the total power is divided with a fixed ratio ρ, leading to $P_m^{\mathrm{c}}=\rho P_m$ for the communication power and $P_m^{\mathrm{s}}=P_m-P_m^{\mathrm{c}}$ for the sensing power. This makes it interesting for the future work to explore the joint optimization of the beamforming and power allocation in cell-free ISAC MIMO systems. Next, we present two sensing beamforming design solutions for the cases (i) when the communication users are not present and when (ii) they are present.

Conjugate Sensing Beamforming: When the communication users are not present (i.e., $U=0$), for example during downtimes, the system can completely focus on the sensing function. In this case, and given the single target sensing model, the conjugate sensing beamforming solution becomes optimal, as it directly maximizes the sensing SNR. With this solution, the sensing beamforming vectors can be written as

$$\mathbf{f}_{mq}^{\mathrm{CB}}=\sqrt{\frac{p_{mq}}{N_t}}\,\mathbf{a}(\theta_m), \quad (11)$$

where $p_{mq}=P_m^s$ is the power allocated for the sensing beam.

Communication-Prioritized Optimal Sensing Solution: When the communication users exist (i.e., $U\geq 1$), and since the communication has a higher priority, a straightforward optimal sensing beamforming approach is to project the optimal sensing beams (constructed through conjugate beamforming) to the null-space of the communication channels. This way, the interference contribution of the sensing beam to the communication channels is eliminated while the sensing SNR is maximized within the communication null space. Let $\mathbf{H}_m=[\mathbf{h}_{m1},\ldots,\mathbf{h}_{mU}]\in\mathbb{C}^{N_t\times U}$ denote the full channel matrix from the transmit AP m to all the UEs, then the NS sensing beamforming can be constructed as

$$\mathbf{f}_{mq}^{\mathrm{NS}}=\sqrt{p_{mq}}\frac{\left(\mathbf{I}-\mathbf{H}_m\left(\mathbf{H}_m^H\mathbf{H}_m\right)^{\dagger}\mathbf{H}_m^H\right)\mathbf{a}(\theta_m)}{\left\|\left(\mathbf{I}-\mathbf{H}_m\left(\mathbf{H}_m^H\mathbf{H}_m\right)^{\dagger}\mathbf{H}_m^H\right)\mathbf{a}(\theta_m)\right\|}, \quad (12)$$

where we again set the allocated power $p_{mq}=P_m^s$ as we consider a single sensing beam.

IV. Sensing-Prioritized Communication Beamforming Design

In this section, we consider the scenario where the sensing has a higher priority, and where the sensing beams are already designed a priori. In this case, the objective is to design the communication beams to optimize the communication performance while *minimizing* the impact of the sensing interference. It is important to note here that an interesting difference between the communication and sensing optimization problems is that while the sensing signals could cause interference that degrades the communication performance, the communication signals could generally be leveraged to further enhance the sensing performance. Next, we present two communication beamforming design solutions for the cases when (i) the sensing target is not present and when (ii) it is present.

Regularized Zero-forcing Beamforming: When the sensing target is not present, i.e., $Q = 0$, a near-optimal communication beamforming design is the regularized zero-forcing (RZF) [10]. This solution allows a trade-off between the multi-user interference and noise terms of the SINR through a regularization parameter λ, that is added to the ZF beamforming:

$$\tilde{\mathbf{f}}_u^{\mathrm{RZF}} = \left(\lambda \mathbf{I} + \sum_{u' \in \mathcal{U}} \mathbf{h}_{u'} \mathbf{h}_{u'}^H\right)^{-1} \mathbf{h}_u, \tag{13}$$

which then can be normalized to satisfy the power constraints, i.e., $\mathbf{f}_{mu}^{\mathrm{RZF}} = \sqrt{p_{mu}}(\tilde{\mathbf{f}}_{mu}^{\mathrm{RZF}}/|\tilde{\mathbf{f}}_{mu}^{\mathrm{RZF}}|)$. We here again adopt $p_{mu} = P_m/U$ with an equal power between the beams. For the RZF, it is preferable to have a higher regularization parameter in the scenarios with a higher noise and smaller regularization parameter in scenarios with more interference.

Sensing-Prioritized Optimal Communication Solution: For the case when the sensing beam is designed a priori, we derive a max-min fair rate optimal communication beamforming solution. First, this max-min problem can be written as

$$\max_{\{\mathbf{f}_{mu}\}} \min_u \quad \mathrm{SINR}_u^{(\mathrm{c})} \quad \text{s.t.} \sum_{u \in \mathcal{U}} \|\mathbf{f}_{mu}\|^2 \leq P_m^c, \quad \forall m \in \mathcal{M}_t, \tag{14}$$

where the objective is quasiconvex and shows a similar structure to the optimal beamforming formulation for the cell-free massive MIMO networks with only the communication objective [11]. For a given minimum SINR constraint γ, the problem can be written as the following feasibility problem

$$\begin{aligned} \text{find} \quad & \{\mathbf{f}_{mu}\} && (15a)\\ \text{s.t.} \quad & \mathrm{SINR}_u^{(\mathrm{c})} \geq \gamma, \quad \forall u \in \mathcal{U}, && (15b)\\ & \sum_{u \in \mathcal{U}} \|\mathbf{f}_{mu}\|^2 \leq P_m^c, \quad \forall m \in \mathcal{M}_t. && (15c) \end{aligned}$$

Here, we note that the SINR constraint (15b) is in a fractional form. This, however, can be converted to a second-order cone constraint. We can re-write the constraint as

$$\begin{aligned} &\left(1 + \frac{1}{\gamma}\right) \left| \sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu} \right|^2 \\ &\quad \geq \sum_{u' \in \mathcal{U}} \left| \sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu'} \right|^2 + \sum_{q \in \mathcal{Q}} \left| \sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mq} \right|^2 + \sigma_u^2. \end{aligned} \tag{16}$$

Now, taking the square root of both sides, we can convert the given form to a second-order cone constraint. The square root, however, leaves an absolute on the left-hand side, which is a non-linear function. This can be simplified as the real part of the variable [10], since any angular rotation ($e^{-j\psi}$) to the expression inside the absolute does not change the value. This can be seen as selecting the optimal solution with a specific angular rotation from the set of infinite rotations $\psi \in [0, 2\pi)$. Finally, we can write the constraint (15b) as a second-order cone as follows

$$\sqrt{\left(1 + \frac{1}{\gamma}\right)} \operatorname{Re}\left\{ \sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu} \right\} \geq \left\| \begin{matrix} \sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{m1} \\ \vdots \\ \sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mS} \\ \sigma_u \end{matrix} \right\|. \tag{17}$$

When (15b) is replaced with (17), it results in a second-order cone problem and can be solved by the convex solvers [12]. Using the bisection algorithm, the maximum SINR value, $\gamma^\star$, can be obtained by solving the convex feasibility problem (15) for different values of γ within a predetermined range $[\gamma_{\min}, \gamma_{\max}]$. This computes the optimal solution to (14).

V. Joint Sensing and Communication Beamforming Optimization

A more desirable approach for cell-free joint sensing and communication MIMO systems is to jointly optimize the beamforming vectors for the sensing and communication functions. Specifically, our objective is to maximize the sensing SNR together with the communication SINR of the UEs. Towards this objective, we reformulate (15) as a sensing SNR maximization problem with constraints on the minimum communication SINRs:

$$\begin{aligned} \max_{\{\mathbf{f}_{ms}\}} \quad & \mathrm{SNR}^{(\mathrm{s})} && (18a)\\ \text{s.t.} \quad & \mathrm{SINR}_u^{(\mathrm{c})} \geq \gamma, \quad \forall u \in \mathcal{U}, && (18b)\\ & \sum_{s \in \mathcal{S}} \|\mathbf{f}_{ms}\|^2 \leq P_m, \quad \forall m \in \mathcal{M}_t, && (18c) \end{aligned}$$

where the objective, i.e., the maximization of the convex SNR expression $\mathrm{SNR}^{(\mathrm{s})}$, is non-convex. Hence, a similar approach to the beamforming optimization in the previous section can not be adopted. The problem in (18), however, can be cast as a semidefinite program, which allows applying a semidefinite relaxation for the non-convex objective [13]. With the relaxation, the problem becomes convex, and it can be solved with the convex solvers. The result obtained from the relaxed problem can then be cast to the original problem's space with a method designed specifically for the problem. In the following, we present the details of our approach.

To reformulate (18) as an SDP, we first re-define the beamforming optimization variables as matrices: $\mathbf{F}_s = \mathbf{f}_s\mathbf{f}_s^H$, $\forall s \in \mathcal{S}$. Writing (18) in terms of $\mathbf{F}_s$ instead of $\mathbf{f}_s$ eliminates the quadratic terms in the sensing SNR and communication SINR expressions. This SDP formulation, however, by construction introduces two new constraints: (i) The convex hermitian positive semi-definiteness constraint $\mathbf{F}_s \in \mathbb{S}^+$, where $\mathbb{S}^+$ is the set of hermitian positive semidefinite matrices, and (ii) the non-convex rank-1 constraint $\mathrm{rank}(\mathbf{F}_s) = 1$. Further, we need to write the problem in terms of these newly introduced variables, $\{\mathbf{F}_s\}$. For this purpose, we define the selection matrix $\mathbf{D}_{m_t} = \mathrm{diag}(\mathbf{d}_{m_t}) \otimes \mathbf{I}^{N_t\times N_t}$, where $\mathbf{d}_m = [d_{m1}, \dots, d_{mM_t}]$ is an indicator vector, i.e., $d_{mm} = 1$ and $d_{mm'} = 0$ $\forall m, m' \in \mathcal{M}_t$ with $m \neq m'$. In addition, we define $\mathbf{A} = \tilde{\mathbf{a}}\tilde{\mathbf{a}}^H$ with $\tilde{\mathbf{a}} = [\mathbf{a}(\theta_1)^T, \dots, \mathbf{a}(\theta_{M_t})^T]^T$. Now, we can write the objective of (18) in terms of these variables as

$$\mathrm{SNR}^{(\mathrm{s})} = \frac{\sum\limits_{m_r\in\mathcal{M}_r}\sum\limits_{m_t\in\mathcal{M}_t} \zeta_{m_t m_r}^2 \,\mathrm{Tr}\left(\mathbf{D}_{m_t}\mathbf{A}\mathbf{D}_{m_t}\sum\limits_{s\in\mathcal{S}}\mathbf{F}_s\right)}{\sum\limits_{m_r\in\mathcal{M}_r}\varsigma_{m_r}^2}. \tag{19}$$

For the constraints of the problem in (18), we define $\mathbf{Q}_u = \mathbf{h}_u\mathbf{h}_u^H$ and re-write the SINR in (3) in terms of the new variables as

$$\mathrm{SINR}_u^{(\mathrm{c})} = \frac{\mathrm{Tr}\left(\mathbf{Q}_u\mathbf{F}_u\right)}{\sum\limits_{u'\in\mathcal{U}\backslash\{u\}}\mathrm{Tr}\left(\mathbf{Q}_u\mathbf{F}_{u'}\right) + \sum\limits_{q\in\mathcal{Q}}\mathrm{Tr}\left(\mathbf{Q}_u\mathbf{F}_q\right) + \sigma_u^2}. \tag{20}$$

With this, we can write the constraint in (18b) and the power constraint in (18c) as

$$\left(1+\gamma^{-1}\right)\mathrm{Tr}\left(\mathbf{Q}_u\mathbf{F}_u\right) - \mathrm{Tr}\left(\mathbf{Q}_u\sum_{s\in\mathcal{S}}\mathbf{F}_s\right) \geq \sigma_u^2, \tag{21}$$

$$\sum_{s\in\mathcal{S}}\mathrm{Tr}\left(\mathbf{D}_m\mathbf{F}_s\right) \leq P_m, \quad \forall m \in \mathcal{M}_t. \tag{22}$$

By collecting these expressions together, we can write the SDP form of our problem in (18) as

$$\begin{aligned} \max_{\{\mathbf{F}_s\}} \quad & \mathrm{SNR}^{(\mathrm{s})} \\ \text{s.t.} \quad & \text{(21) and (22)}, \\ & \mathrm{rank}(\mathbf{F}_s) = 1, \quad \mathbf{F}_s \in \mathbb{S}^+, \quad \forall s \in \mathcal{S}. \end{aligned} \tag{23}$$

which can be relaxed by removing the rank-1 constraint to be solved via CVX and convex SDP solvers [12]. Then, if the matrices obtained by this solution, denoted by $\{\mathbf{F}'_s\}$, are rank-1, then they are optimal for (23). The optimal beamforming vectors, $\mathbf{f}_s$, in this case, can be obtained as the eigenvector of $\mathbf{F}'_s$. For the case the matrices are not rank-1, we make the following proposition.

Proposition 1. *There exists a solution to the problem* (23)*, denoted by* $\{\mathbf{F}''_s\}$*, that satisfies* $\mathrm{rank}(\mathbf{F}''_u) = 1$, $\forall u \in \mathcal{U}$ *and*

$$\sum_{q\in\mathcal{Q}}\mathbf{F}''_q = \sum_{s\in\mathcal{S}}\mathbf{F}'_s - \sum_{U\in\mathcal{U}}\mathbf{F}''_u. \tag{24}$$

The communication beamforming vectors of this solution can be given as

$$\mathbf{f}''_u = (\mathbf{h}_u^H\mathbf{F}'_u\mathbf{h}_u)^{-\frac{1}{2}}\mathbf{F}'_u\mathbf{h}_u. \tag{25}$$

Further, if $\mathrm{rank}(\sum_{q\in\mathcal{Q}}\mathbf{F}''_q) \leq Q$*, the sensing beamforming vectors of this solution can be constructed by*

$$\mathbf{f}''_q = \sqrt{\lambda_{q-U}}\,\mathbf{u}_{q-U}, \tag{26}$$

with λ_i *and* $\mathbf{u}_i$ *being the* i*-th largest eigenvalue of* $\sum_{q\in\mathcal{Q}}\mathbf{F}''_q$ *and the corresponding eigenvector.*

The proof slightly extends the solution in [4, Theorem 1].

In the case $\mathrm{rank}(\sum_{q\in\mathcal{Q}}\mathbf{F}''_q) > Q$, we can still use the solution of the proposition, however, it is approximate. With the solution completed, we next evaluate our results.

VI. Results

In this section, we evaluate the performance of the proposed beamforming solutions for cell-free ISAC MIMO systems In particular, we consider a scenario where $\mathcal{M}_t = \mathcal{M}_r$ with two APs placed at $(25, 0)$ and $(75, 0)$ in the Cartesian coordinates. Each AP is equipped with a uniform linear array (ULA) along the x axis of $N_t = N_r = 16$ antennas. At $y = 50m$, we randomly place one sensing target and the $U = 5$ communications users along the x-axis. Specifically, the x coordinates of these locations are drawn from a uniform distribution in $[0, 100]$. For the communication channels, we adopt a LOS channel model and take $\sigma_u^2 = 1$. For the sensing channels, we adopt the parameters $\varsigma_{m_r}^2 = 1$ and $\zeta_{m_t m_r} = 0.1$. The transmit power of the APs is $P_m = 0$dBW and the number of sensing streams $Q = 1$. In the following, we average the results over 1000 realizations. For this setup, we compare the following solutions: (i) *NS Sensing - RZF Comm:* (12) and (13), (ii) *NS Sensing - OPT Comm:* (12) and (15), (iii) *CB Sensing - OPT Comm:* (11) and (15), (iv) *JSC beam optimization:* Proposition 1.

Sensing and Communication Power Allocation: We first investigate the sensing and communication performance for different power allocation ratios. Specifically, in Fig. 2, we show the sensing SNR and minimum communication SINR of UEs achieved by the different beamforming solutions for different values of $\rho \in (0, 1)$. It is important to note here that for the beamforming solutions (i)-(iii), the communication and sensing beams are separately designed, and we directly allocate the communication and sensing powers based on the ratio ρ. For the JSC beam optimization solution (iv), it implements the beamforming design in Proposition 1, which optimizes both the structure of the beams and the power allocation. Therefore, and for the sake of comparing with the other approaches, we plot the JSC optimization curve in Fig. 2 by setting the communication SINR threshold to be equal to the achieved SINR by solution (ii). This still respects the total power constraint, which is taken care of by (22). As seen in the figure, the first two solutions, (i) and (ii), achieve better communication SINR and less sensing SNR compared to solution (iii). This is expected as solution (iii) aims to maximize the sensing performance, irrespective of the communication, and hence, it causes some interference to the communication users. *Interestingly, while achieving the*

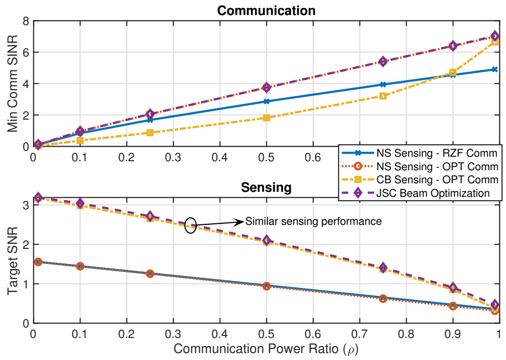

Fig. 2. Performance of the solutions for different power allocation ratios for the communications and sensing. The proposed JSC optimization provides a significant gain for sensing while satisfying the best communication SINR.

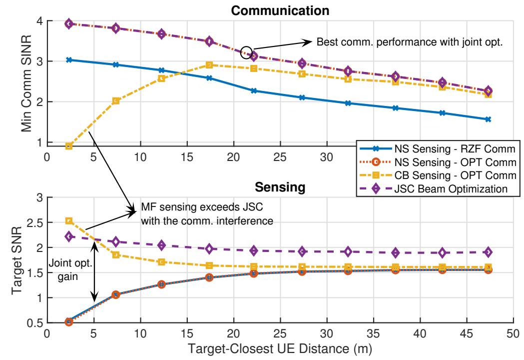

Fig. 3. Performance of the solutions versus the distance between the target and closest AP. The proposed JSC optimization provides almost a constant sensing SNR for different distances, with a significant gain over the NS solutions.

best communication performance of the separate solutions, the joint solution provides very similar sensing performance to the MF sensing. This highlights the gain of the developed JSC beamforming design.

Target distance to closest UE: To further investigate how the different beamforming approaches impact the trade-off between the sensing and communication performance, we evaluate this performance versus the distance between the sensing target and closest communication UE in Fig. 3. Note that, intuitively, as the sensing target gets closer to the communication users, the overlap between the communication and sensing channels' subspaces increases, which can benefit or penalize the communication and sensing performance depending on the beamforming design. In Fig. 3, we set the power ratio as 0.5 for the communication and sensing operation. This figure shows that for the smaller distances/separation between the sensing target and communication users, the conjugate beamforming sensing solution (solution (iii)) optimizes the sensing performance but causes non-negligible interference to the communication, which significantly degrades its performance. On the other side, solutions (i) and (ii), which prioritize the communication and keep the sensing beamforming in the null-space of the communication channels, optimize the communication SINR and degrade the sensing SNR. For the SINR constraint of the JSC optimization, we again adopt the SINR obtained by solution (ii), which achieves the best communication performance. Hence, the achieved communication SINR of this solution and JSC beam optimization are the same. *The sensing SNR, however, enjoys the advantage of the joint beam optimization. Specifically, it provides almost a constant sensing performance for different target-closest UE distances: Achieving a close sensing performance to solution (i) when the separation between the sensing target and communication users is small and exceeds the performance of all the other three solutions when this separation is large, which highlights the potential of the joint beamforming design.*

VII. Conclusion

In this paper, we investigated downlink beamforming for the joint sensing and communication in cell-free massive MIMO systems. Specifically, we designed communication-prioritized sensing beamforming and sensing-prioritized communication beamforming solutions as the baseline. Further, we have developed an optimal solution for the JSC beamforming. The results showed the advantage of the joint optimization, where the developed JSC beamforming is capable of achieving nearly the SINR that of the communication-prioritized sensing beamforming solutions with almost the same sensing SNR of the sensing-prioritized communication beamforming approaches.

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