Cell-Free ISAC MIMO Systems: Joint Sensing and Communication Beamforming

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***Abstract*—This paper considers a cell-free integrated sensing and communication (ISAC) MIMO system, where distributed MIMO access points (APs) jointly serve the communication users and sense the target. For this setup, we derive a sensing SNR for multi-static sensing where both joint communication and sensing signals transmitted by different APs are utilized. With this sensing objective, we develop two baseline approaches that separately design the sensing and communication beamforming vectors, namely communication-prioritized sensing beamforming and sensing-prioritized communication beamforming. Then, we consider the joint sensing and communication (JSC) beamforming design and derive the optimal structure of these beamforming vectors based on a max-min fairness formulation. In addition, considering any pre-determined JSC beam design, we devise a power allocation approach. The results show that the developed JSC beamforming is capable of achieving nearly the same communication signal-to-interference-plus-noise ratio (SINR) of the communication-prioritized sensing beamforming solution with almost the same sensing SNR of the sensing-prioritized communication beamforming approach. The proposed JSC beamforming optimization also provides a noticeable gain over the power allocation with regularized zero-forcing beamforming, yielding a promising strategy for cell-free ISAC MIMO systems.**

I. INTRODUCTION

The integration of sensing functions into the communication systems is envisioned to be an integral part of the 6G and future communication systems [2]–[5]. If the hardware and wireless resources are efficiently shared, this can enable the communication infrastructure to have sensing capabilities at minimal cost and make the sensing frequency bands available for wireless communication. The sensing capabilities may also be utilized to aid the communication system and improve its performance [6]–[9]. These capabilities can enable innovative applications in security, healthcare, and traffic management. Achieving efficient joint sensing and communication operation, however, requires the careful design of the various aspects of the integrated sensing and communication (ISAC) system, including the transmission waveform, the post-processing of the received signals, and the MIMO beamforming. While these problems have recently attracted increasing research interest, the prior work has mainly focused on the single ISAC basestation case. In practice, however, multiple ISAC basestations will operate in the same geographical region, frequency band, and time, causing interference on each other for both the sensing and communication functions. This motivates the coordination among these distributed nodes to improve both communication and sensing performance. This ultimately leads to *cell-free ISAC MIMO* systems, where distributed ISAC basestations jointly serve the same set of communication users and sense the same targets. With this motivation, this paper investigates the joint sensing and communication beamforming design of these cell-free ISAC MIMO systems.

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A. Prior Work

Distributed antenna systems and interference management in the multi-cell MIMO networks have been extensively studied in the literature [10]–[12]. With the possibility of more extensive coordination among the basestations, coordinated multi-point transmissions [13], and, more recently, with the densification of the networks, cell-free massive MIMO [14] have attracted significant interest. Cell-free massive MIMO is a concept where multiple access points (APs) jointly serve the user equipments (UEs) by transmitting messages to every user. Note that it is a distributed multi-user MIMO approach, and there are no limitations of cell boundaries. Due to its potential, various aspects of cell-free massive MIMO have been extensively investigated for further improvements [15]–[20]. For example, precoding techniques for cell-free massive MIMO are studied in [15], energy minimization in [16], fronthaul limitations in [17], scalability aspects in [19], and wireless fronthaul in [20]. Most of these studies, however, did not include the unification of the sensing and communication functions in cell-free massive MIMO networks.

The literature for joint sensing and communication (JSC), also called dual-functional radar-communication (DFRC), has mainly focused on the single node (basestation) scenarios [21]–[25]. For example, the design of the JSC waveform is studied in [21]. Specifically, the author investigated the JSC waveform design for correlated and uncorrelated waveforms and the trade-offs between communication and sensing. The authors in [22] proposed sensing post-processing for JSC systems. For beamforming, the work in [23] investigated the JSC beamforming design of a co-located MIMO system with monostatic radar that serves multiple users. The hybrid beamforming design for OFDM DFRC system is studied in [24]. The optimal beamforming solution for JSC with and without sensing signal's successive interference cancellation is provided in [26]. Along a similar direction, [25] formulated an outage-based beamforming problem and provided the optimal solution.

More relevantly, JSC with distributed nodes (basestations) has been investigated in a few papers [27]–[30] for power allocation and beamforming. In most of them, however, each user is served by a single AP [27]–[29]; hence, these studies focused on the interference and not considered a fully cell-free MIMO setup. For example, [28] proposes a transmit and receive beamforming optimization for JSC in a single basestation multi-user scenario, where the signals from a different cell are used to improve the sensing performance without communication interference. In [27], a power allocation problem for JSC is formulated. The problem, however, assumes a single antenna system, and every UE is served by a single AP. In [29], the authors proposed a JSC beamforming optimization for maximizing the detection, however, this work only relied on each AP serving a single UE.

As the more relevant studies, the optimization of the JSC power allocation for cell-free massive MIMO has been investigated in [30]. The authors in this work adopted fixed beam designs, i.e., regularized zero beamforming for the communication with the sensing beamforming in the nullspace of the communication channels without further optimization, and focused on optimizing the power allocated to these beams. For the solution, they proposed a convex-concave procedure. In [31], the authors extended the work in [30], and included multiple targets, cluttering, and the effect of direct paths from different APs in sensing. For the beamforming in cell free ISAC systems, in our prior work [1], we investigated joint beamforming design, presented the optimal solution, and compared it with different beamforming strategies. In the similar direction, [32] formulated a beam pattern matching problem and presented an optimization solution. In [33], the authors optimized the beamforming for the detection probability in synchronous and asynchronous systems. [34] investigated beam design with security, and [35] provided more on the cell-free analysis with beam design, AP selection, and asymptotic analysis. Since cell-free ISAC MIMO systems rely mainly on beamforming in their dual-function operation, it is very important to investigate the design of these JSC beams. With this motivation, we propose and compare various beamforming and power allocation strategies, including beamforming optimization, for the cell-free ISAC MIMO systems, and present a deeper analysis on the optimization and sensing streams.

B. Contributions

To investigate the JSC transmit beamforming in cell-free massive MIMO systems, in this paper, we consider a system model with many APs and UEs, where the APs jointly serve the UEs and sense the targets in the environment. APs transmit communication streams to serve users and sense the target, one stream for each user, and a generic number of sensing streams, whose number is to be investigated in the paper. With this model, we formulate beamforming and power allocation problems, develop various solutions, and investigate the number of sensing streams required to achieve the full potential. Our contributions in this paper can be summarized as follows:

- For beamforming, we first present two baseline strategies that we call *communication-prioritized sensing* and *sensing-prioritized communication beamforming*. In these strategies, either the sensing or the communication beamforming is prioritized, hence, designed without accounting for the other function (e.g., sensing beams are designed without any consideration of communication, maximizing the sensing performance). The beamforming of the other function is designed in a way that does not affect the performance of the higher-priority function.
- For the sensing-prioritized beamforming, we develop a communication beamforming optimization approach for the given sensing beamforming vector. In this approach, we cast the communication beamforming design problem as a convex second-order cone program and provide the optimal solution, which can be obtained using convex solvers.
- Next, we investigate the joint communication and sensing beamforming design. For this, we formulate a JSC beamforming problem that aims to maximize the sensing SNR while satisfying the communication SINR constraints. We then re-formulate this problem as a non-convex semi-definite problem (SDP) and apply semi-definite relaxation (SDR) to find the optimal beamforming structure for a large set of classes.
- As a point of interest, we analyze the number of sensing streams required for achieving the maximum sensing performance given the communication and power constraints. Specifically, we derive the dual problem of the SDR, and evaluate the rank constraints on the sensing beams. Our analysis revealed that the number of sensing streams is hard-limited by the number of APs, and this limit can be further refined on the specific assumptions on the channels.
- Finally, for comparison, we develop a power allocation problem for the given beamforming vectors. Specifically, with pre-determined beamforming coefficients, we convert our JSC beamforming problem to a power allocation problem. This problem is also an SDP, and we develop a solution using SDR.

We have extensively evaluated the proposed approaches and showed that the JSC beamforming design provides better sensing performance than the sensing-prioritized solution while achieving the communication rates provided by the communication-prioritized solution. This is thanks to the co-design of the communication and sensing functions. Further, the JSC beamforming design overperforms the JSC power allocation for the regularized zero-forcing beams with a significant sensing SNR gain while providing the same communication rates. This shows the advantage of the joint beamforming optimization, making it desirable for future cell-free ISAC MIMO systems.

Organization: In Section II, we present our system model with the communication and sensing objectives. In Section III and IV, we respectively present the communication- and sensing-prioritized beamforming approaches. Then, we develop the joint sensing and communication beamforming optimization in Section V, and the power allocation formulation and solution in Section VI. Finally, in Section VII, we provide the numerical results evaluating the developed solutions and present our conclusions of the paper in Section VIII.

Notation: We use the following notation throughout this

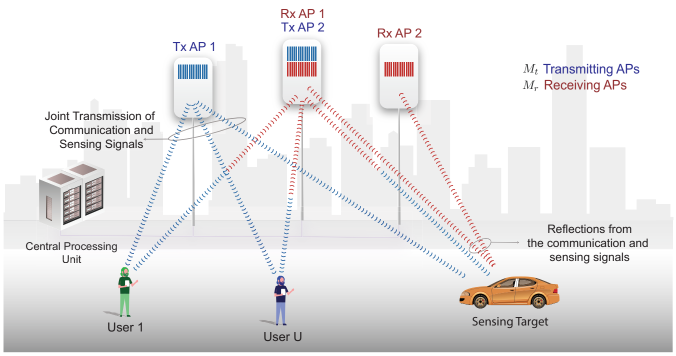


Fig. 1. The system model with the joint sensing and communication transmissions is illustrated. The APs serve multiple users while aiming to sense the target.

paper: $\mathbf{A}$ is a matrix, $\mathbf{a}$ is a vector, a is a scalar, $\mathcal{A}$ is a set. $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathbf{A}^*$, $\mathbf{A}^{-1}$, $\mathbf{A}^\dagger$ are transpose, Hermitian (conjugate transpose), conjugate, inverse, and pseudo-inverse of $\mathbf{A}$, respectively. $\|\mathbf{a}\|$ is the l_2-norm of $\mathbf{a}$ and $\|\mathbf{A}\|_\mathrm{F}$ is the Frobenius norms of $\mathbf{A}$. $\mathbf{I}$. $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ is a complex Gaussian random vector with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$. $\mathbb{E}[\cdot]$ and $\otimes$ denote expectation and Kronecker product, respectively. $\mathbb{S}^+$ is the set of Hermitian positive semidefinite matrices.

II. System Model

We consider a cell-free massive MIMO ISAC system with M access points (APs) and U communication users, as illustrated in Fig. 1. In the downlink, and without loss of generality, we assume that a subset $\mathcal{M}_t$ (out of the M APs) are transmitting communication and sensing waveforms to jointly serve the U users, where $|\mathcal{M}_t| = M_t$. Simultaneously, a subset $\mathcal{M}_r$ (out of the M APs) is receiving the possible reflections/scattering of the transmitted waveforms on the various targets/objects in the environment, with $|\mathcal{M}_r| = M_r$. It is important to note here that the subsets $\mathcal{M}_t$ and $\mathcal{M}_r$ may generally have no, partial, or full overlap, which means that none, some, or all the APs could be part of $\mathcal{M}_t$ and $\mathcal{M}_r$ and are simultaneously transmitting and receiving signals. It is important to note that the receiving APs have access to both communication and sensing waveforms, and use both to sense the target. The transmitting and receiving APs are equipped with N_t and N_r antennas. Further, for simplicity, all the APs are assumed to have digital beamforming capabilities, i.e., each antenna element has a dedicated radio frequency (RF) chain. The UEs are equipped with single antennas. The APs are connected to a central unit that allows joint design and processing, and they are assumed to be fully synchronized for both sensing and communication purposes. We further assume perfect CSI is available at at the APs.

A. Signal Model

In this subsection, we define the joint sensing and communication signal model for the downlink transmissions. The APs jointly transmit U communication streams, $\{x_u[\ell]\}_{u\in\mathcal{U}}$, and Q sensing streams, $\{x_q[\ell]\}_{q\in\mathcal{Q}}$, where $\mathcal{Q} = \{U+1, \ldots, U+Q\}$ and with ℓ denoting the ℓ's symbol in these communication/sensing streams [1]. For ease of exposition, we also define the overall set of streams as $\mathcal{S} = \mathcal{U} \cup \mathcal{Q} = \{1, \ldots, S\}$ with $S = U + Q$. If $\mathbf{x}_m[\ell] \in \mathbb{C}^{N_t \times 1}$ denotes the transmit signal from the transmitting AP m due to the ℓ-th symbol, we can then write

$$\mathbf{x}_m[\ell] = \underbrace{\sum_{u\in\mathcal{U}} \mathbf{f}_{mu} x_u[\ell]}_{\text{Communication}} + \underbrace{\sum_{q\in\mathcal{Q}} \mathbf{f}_{mq} x_q[\ell]}_{\text{Sensing}} = \sum_{s\in\mathcal{S}} \mathbf{f}_{ms} x_s[\ell], \quad (1)$$

where $x_s[\ell] \in \mathbb{C}$ is the ℓ-th symbol of the s-th stream, $\mathbf{f}_{ms} \in \mathbb{C}^{N_t\times 1}$ is the beamforming vector for this stream applied by AP m. The symbols are assumed to be of unit average energy, $\mathbf{E}[|x_s|^2] = 1$. The beamforming vectors are subject to the total power constraint, P_m, given as

$$\mathbf{E}[\|\mathbf{x}_m[\ell]\|^2] = \sum_{s\in\mathcal{S}} \|\mathbf{f}_{ms}\|^2 \leq P_m. \quad (2)$$

Further, by stacking the beamforming vectors of stream s of all the APs, we define the beamforming vector $\mathbf{f}_s$

$$\mathbf{f}_s = \begin{bmatrix} \mathbf{f}_{1s}^T & \ldots & \mathbf{f}_{M_t s}^T \end{bmatrix}^T \in \mathbb{C}^{M_t N_t}. \quad (3)$$

For each stream s, we denote the sequence of L transmit symbols as $\mathbf{x}_s = \left[x_s[1], \ldots, x_s[L]\right]^T$. Given this notation, we make the following assumption, which is commonly adopted in the literature [23]: The messages of the radar and communication signals are statistically independent, i.e., $\mathbf{E}[\mathbf{x}_s \mathbf{x}_s^H] = \mathbf{I}$ and $\mathbf{E}[\mathbf{x}_s \mathbf{x}_{s'}^H] = \mathbf{0}$ for $s, s' \in \mathcal{S}$ with $s \neq s'$. Note that the radar signal generation with these properties can be achieved through pseudo-random coding [36]–[39] and stochastic radar waveforms [40]. In addition to satisfying the described statistical properties, such waveforms provide further advantages such as improved range/Doppler-resolution, better detection probability and separability, thanks to the desirable cross-correlation and auto-correlation properties, lower co-channel interference (with communication messages and each other).

B. Communication Model

We denote the communication channel between UE u and AP m as $\mathbf{h}_{mu} \in \mathbb{C}^{N_t\times 1}$. Further, by stacking the channels between user u and all the APs, we construct $\mathbf{h}_u \in \mathbb{C}^{M_t N_t \times 1}$. Next, considering a block fading channel model, where the channel remains constant over the transmission of the L symbols, we can write the received signal at UE u as

$$\begin{aligned} y_u^{(\mathrm{c})}[\ell] &= \sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{x}_m[\ell] + n_u \\ &= \underbrace{\sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu} x_u[\ell]}_{\text{Desired Signal (DS)}} + \underbrace{\sum_{u'\in\mathcal{U}\setminus\{u\}} \sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu'} x_{u'}[\ell]}_{\text{Multi-user Interference (MUI)}} \\ &\quad + \underbrace{\sum_{q\in\mathcal{Q}} \sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mq} x_q[\ell]}_{\text{Sensing Interference (SI)}} + \underbrace{n_u[\ell]}_{\text{Noise}}, \end{aligned} \quad (4)$$

[1]Similar to [23], [26], [30], the communication streams primarily aim to serve users and one stream is allocated for each user. Meanwhile, the sensing streams only aim to sense the target.

where $n_u[\ell] \sim \mathcal{CN}(0, \sigma_u^2)$ is the receiver noise of UE u. Then, the communication SINR of UE u can be obtained as

$$\text{SINR}_u^{(\text{c})} = \frac{\mathbb{E}[|\text{DS}|^2]}{\mathbb{E}[|\text{MUI}|^2] + \mathbb{E}[|\text{SI}|^2] + \mathbb{E}[|\text{Noise}|^2]}. \quad (5)$$

This SINR can be expressed in terms of the individual beamforming variables, $\{\mathbf{f}_{ms}\}$, as given in (6). Additionally, we write this expression in terms of the stacked vector variables of each UE u as in (3), as

$$\text{SINR}_u^{(\text{c})} = \frac{\left|\mathbf{h}_u^H \mathbf{f}_u\right|^2}{\sum_{u' \in \mathcal{U} \setminus \{u\}} \left|\mathbf{h}_u^H \mathbf{f}_{u'}\right|^2 + \sum_{q \in \mathcal{Q}} \left|\mathbf{h}_u^H \mathbf{f}_q\right|^2 + \sigma_u^2}. \quad (7)$$

C. Sensing Model

For the sensing, as we assume the receiving APs have the knowledge of both communication and sensing signals, they are both utilized for sensing. As the sensing channel model, we consider a single-point reflector, as commonly adopted in the literature [30], [41]. Specifically, the transmit signal is scattered from the single-point reflector and received by the receiving APs in $\mathcal{M}_r$. With a single path model, the channel between the transmitting AP m_t and the receiving AP m_r through the reflector is defined as

$$\mathbf{G}_{m_t m_r} = \alpha_{m_t m_r} \mathbf{a}(\theta_{m_r}) \mathbf{a}^H(\theta_{m_t}), \quad (8)$$

where $\alpha_{m_t m_r} \sim \mathcal{CN}(0, \zeta_{m_t m_r}^2)$ is the combined sensing channel gain, which includes the effects due to the path-loss and radar cross section (RCS) of the target. In this model, we assume that the instantaneous value of $\alpha_{m_t m_r}$ is not available, but its statistics are known. This can be achieved by targeting a specific position (leading to known angles and distance/ToA/path-loss) with known zero-mean RCS distribution, similar to [29], [31]. $\mathbf{a}(\theta)$ is the array response vector. The angles of departure/arrival of the transmitting AP m_t and receiving AP m_r from the point reflector are respectively denoted by θ_{m_t} and θ_{m_r}. We consider the Swerling-I model for the sensing channel [42], which assumes that the fluctuations of RCS are slow, and the sensing channel does not change during the transmission of the L sensing and communication symbols in $\mathbf{x}_s$.

In addition to the channel due to the reflections from the target, direct channels between the transmitting and receiving APs also contribute to the received signal. This effect can be classified in two folds: (i) If the AP is both receiving and transmitting (i.e., full-duplex), the self-interference effects are observed. (ii) The direct channels between the transmitting AP and receiving AP at different locations. For these, we define the channel between the transmitting AP $m_t \in \mathcal{M}_t$ and receiving AP $m_r \in \mathcal{M}_r$ as $\mathbf{H}_{m_t m_r} \in \mathcal{C}^{N_r \times N_t}$. Note that if it corresponds to the self-interference channel if $m_t = m_r$. With these definitions, the signal received at AP m_r at instance ℓ can be written as

$$\tilde{\mathbf{y}}_{m_r}^{(\text{s})}[\ell] = \sum_{m_t \in \mathcal{M}_t} \left(\mathbf{G}_{m_t m_r} + \mathbf{H}_{m_t m_r}\right) \mathbf{x}_{m_t}[\ell] + \tilde{\mathbf{n}}_{m_r}[\ell] \quad (9)$$

where $\mathbf{n}_{m_r}[\ell] \in \mathbb{C}^{N_r}$ is the receiver noise at AP m_r and has the distribution $\mathcal{CN}(0, \tilde{\varsigma}_{m_r}^2 \mathbf{I})$. Here, we assume that the channels between transmitting and receiving APs, $\mathbf{H}_{m_t m_r}$, are known at the corresponding receiving APs (or their estimation is available). This can potentially be achieved through an additional estimation step, where the APs transmit their pilots, and the receiving APs estimate the channels. This step can possibly be incorporated into the channel estimation phase of cell-free massive MIMO systems, where the pilots from the APs are jointly transmitted for UEs to estimate their channels. With this assumption, and with the knowledge of the transmitted signals, it is now possible to apply interference cancellation (clutter cleaning) at each receiving AP.

We note that although from the mathematical perspective, it is possible to almost fully cancel the interference with the provided linear model, in actual systems, the interference is usually stronger than the desired radar signals. Therefore, the receiver front-end including the amplifiers and analog-to-digital converters may become saturated and introduce non-linear distortion effects. These effects could possibly be more pronoun in self-interference (full-duplex case) compared to separate transmitter/receiver links. In this paper, however, our model is generic, thus, we do not particularly distinguish between these cases and deepen the analysis[2]. We, however, assume that the residual interference can be modeled as zero mean additive white Gaussian noise, similar to [43]–[45] in the full-duplex literature. After the interference cancellation, we can write the received radar signal as

$$\begin{aligned} \mathbf{y}_{m_r}^{(\text{s})}[\ell] &= \sum_{m_t \in \mathcal{M}_t} \mathbf{G}_{m_t m_r} \mathbf{x}_{m_t}[\ell] + \mathbf{n}_{m_r}[\ell] \\ &= \sum_{m_t \in \mathcal{M}_t} \alpha_{m_t m_r} \mathbf{a}(\theta_{m_r}) \mathbf{a}^H(\theta_{m_t}) \mathbf{x}_{m_t}[\ell] + \mathbf{n}_{m_r}[\ell], \end{aligned} \quad (10)$$

where $\mathbf{n}_{m_r}[\ell] \sim \mathcal{CN}(0, \varsigma_{m_r}^2 \mathbf{I})$ is the joint noise including the receiver noise and residual. Next, to write the received radar signal due to the L symbols in a compact form, we introduce

$$\overline{\mathbf{F}}_m = [\mathbf{f}_{m1}, \dots, \mathbf{f}_{mS}] \in \mathbb{C}^{N_t \times S}, \quad (11)$$
$$\overline{\mathbf{X}} = [\mathbf{x}_1, \dots, \mathbf{x}_S]^T \in \mathbb{C}^{S \times L}. \quad (12)$$

Then, we can write the transmit signal from each AP m_t, in (1), due to the L symbols as

$$\mathbf{X}_{m_t} = \overline{\mathbf{F}}_{m_t} \overline{\mathbf{X}} \in \mathbb{C}^{N_t \times L}. \quad (13)$$

With that, we can re-write the sensing signal in (10) at each receiving AP m_r, due to the L symbols, in a compact form as

$$\mathbf{Y}_{m_r}^{(\text{s})} = \underbrace{\sum_{m_t \in \mathcal{M}} \underbrace{\alpha_{m_t m_r} \mathbf{a}(\theta_{m_r}) \mathbf{a}^H(\theta_{m_t}) \overline{\mathbf{F}}_{m_t}}_{\triangleq \overline{\mathbf{G}}_{m_t m_r}}}_{\triangleq \overline{\mathbf{G}}_{m_r}} \overline{\mathbf{X}} + \mathbf{N}_{m_r}, \quad (14)$$

with $\overline{\mathbf{G}}_{m_r}$ denoting the beam-space sensing channel of the receiving AP m_r and the receive noise matrix $\mathbf{N}_{m_r} = [\mathbf{n}_{m_r}[1], \dots, \mathbf{n}_{m_r}[L]]$.

[2]There are several works in the literature with the focus on these scenarios. For example, the target detection performance with inter-link interference was investigated in [31].

$$\text{SINR}_u^{(\text{c})} = \frac{\left|\sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu}\right|^2}{\sum_{u'\in\mathcal{U}\setminus\{u\}} \left|\sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu'}\right|^2 + \sum_{q\in\mathcal{Q}} \left|\sum_{m\in\mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mq}\right|^2 + \sigma_u^2}, \tag{6}$$

To define a general sensing objective that is correlated with the performance of various sensing tasks (e.g., detection, range/Doppler/angle estimation and tracking), we adopt the joint SNR of the received signals as the sensing objective. This objective aims to maximize ratio of the signal power reflected from the target to the noise, and essentially aims to have more power through the target, making it easier to distinguish from the noise. Although it is difficult to generalize its relationship to various sensing tasks with different pre- and post-processing, it is expected to exhibit monotonic behavior. For example, for the detection, it was shown in [29], [33] that the sensing SNR is monotonically increasing with the detection probability. Here, we note that the use of the joint SNR requires joint processing of the radar signal at the $\mathcal{M}_r$ sensing receivers. The sensing SNR can be given as

$$\begin{aligned}\text{SNR}^{(\text{s})} &= \frac{\mathbb{E}\left[\sum_{m_r\in\mathcal{M}_r} \left\|\overline{\mathbf{G}}_{m_r}\overline{\mathbf{X}}\right\|_\text{F}^2\right]}{\mathbb{E}\left[\sum_{m_r\in\mathcal{M}_r} \left\|\mathbf{N}_{m_r}\right\|_\text{F}^2\right]} \\ &= \frac{\sum_{m_r\in\mathcal{M}_r}\sum_{m_t\in\mathcal{M}_t} \zeta_{m_t m_r}^2 \left\|\mathbf{a}^H(\theta_{m_t})\overline{\mathbf{F}}_{m_t}\right\|^2}{\sum_{m_r\in\mathcal{M}_r} \varsigma_{m_r}^2},\end{aligned} \tag{15}$$

where the derivation is provided in Appendix A. Recall that $\zeta_{m_t m_r}^2$ denotes the variance of the combined sensing channel gain and $\varsigma_{m_r}^2$ is the variance of the radar receiver noise. The sensing SNR is scaled with the contribution of all the *communication and sensing* streams.

Our objective is then to design the cell-free communication beamforming $\{\mathbf{f}_u\}_{u\in\mathcal{U}}$ and the sensing beamforming $\{\mathbf{f}_q\}_{q\in\mathcal{Q}}$ to optimize the communication SINR and the sensing SNR defined in (7) and (15). It is important to note here that, in this paper, we focus on the beamforming design assuming that the communication channel and the sensing target angles are known to the transmit APs. Extending this work to include imperfect channels/angle knowledge is an interesting direction. In the next three sections, we present the proposed beamforming strategies for communication-prioritized sensing, sensing-prioritized communication, and joint sensing and communication.

III. Communication-Prioritized Sensing Beamforming Design

In this section, we investigate the scenario where communication has a higher priority. In this case, the communication beams are already designed apriori (which may be via commonly adopted methods, e.g., zero-forcing), and the sensing is designed to not affect the communication performance. Then, the objective in this section is to design the sensing beams to optimize the sensing performance while not affecting the communication performance (i.e., not causing any interference to the U communication users). Note that in this section and the next section, Section IV, we adopt $Q=1$ since we have a single sensing target [3]. We also assume that the total power is divided with a fixed ratio ρ, leading to $P_m^\text{c} = \rho P_m$ for the communication power and $P_m^\text{s} = P_m - P_m^\text{c}$ for the sensing power. This motivates further exploration of the joint optimization of beamforming and power allocation, as presented in Section VI. Next, we present two sensing beamforming design solutions for the cases (i) when the communication users are not present and when (ii) they are present.

Conjugate Sensing Beamforming: When the communication users are not present (i.e., $U=0$), for example, during downtimes, the system can completely focus on the sensing function. In this case, and given the single target sensing model, the conjugate sensing beamforming solution becomes optimal, as it directly maximizes the sensing SNR. With this solution, the sensing beamforming vectors can be written as

$$\mathbf{f}_{mq}^\text{CB} = \sqrt{\frac{p_{mq}}{N_t}}\,\mathbf{a}(\theta_m), \tag{16}$$

where $p_{mq} = P_m^s$ is the power allocated for the sensing beam.

Communication-Prioritized Optimal Sensing Solution: When the communication users exist (i.e., $U \geq 1$), and since the communication has a higher priority, a straightforward optimal sensing beamforming approach is to project the optimal sensing beams (constructed through conjugate beamforming) to the nullspace (NS) of the communication channels. This way, the interference contribution of the sensing beam to the communication channels is eliminated while the sensing SNR is maximized within the communication null space. Let $\mathbf{H}_m = [\mathbf{h}_{m1}, \ldots, \mathbf{h}_{mU}] \in \mathbb{C}^{N_t\times U}$ denote the full channel matrix from the transmit AP m to all the UEs, then the NS sensing beamforming can be constructed as

$$\mathbf{f}_{mq}^\text{NS} = \sqrt{p_{mq}}\frac{\left(\mathbf{I} - \mathbf{H}_m\left(\mathbf{H}_m^H\mathbf{H}_m\right)^\dagger\mathbf{H}_m^H\right)\mathbf{a}(\theta_m)}{\left\|\left(\mathbf{I} - \mathbf{H}_m\left(\mathbf{H}_m^H\mathbf{H}_m\right)^\dagger\mathbf{H}_m^H\right)\mathbf{a}(\theta_m)\right\|}, \tag{17}$$

where we again set the allocated power $p_{mq} = P_m^s$ as we consider a single sensing beam.

IV. Sensing-Prioritized Communication Beamforming Design

In this section, we consider the scenario where the sensing has a higher priority. In this case, the sensing beams are already designed a priori (which may be via commonly adopted methods, e.g., conjugate beamforming), and the communication does not degrade the sensing performance. Then, the objective is to design the communication beams to optimize the communication performance. It is important to note here that an interesting difference between the communication and sensing optimization problems is that while the sensing signals

[3]Our purpose in this section is to provide the baselines with communication/sensing priority. To that end, we leave further complicated designs beyond a single sensing beam for future work.

could cause interference that degrades the communication performance, the communication signals could generally be leveraged to further enhance the sensing performance. Next, we present two communication beamforming design solutions for the cases when (i) the sensing target is not present and when (ii) it is present.

Regularized Zero-forcing Beamforming: When the sensing target is not present, i.e., $Q = 0$, a near-optimal communication beamforming design is the regularized zero-forcing (RZF) [46]. This solution allows a trade-off between the multi-user interference and noise terms of the SINR through a regularization parameter λ, which is added to the ZF beamforming:

$$\tilde{\mathbf{f}}_u^{\text{RZF}} = \left(\lambda \mathbf{I} + \sum_{u' \in \mathcal{U}} \mathbf{h}_{u'} \mathbf{h}_{u'}^H \right)^{-1} \mathbf{h}_u, \tag{18}$$

which then can be normalized to satisfy the power constraints, i.e., $\mathbf{f}_{mu}^{\text{RZF}} = \sqrt{p_{mu}} (\tilde{\mathbf{f}}_{mu}^{\text{RZF}} / |\tilde{\mathbf{f}}_{mu}^{\text{RZF}}|)$. We here again adopt $p_{mu} = P_m/U$ with an equal power between the beams. For the RZF, it is preferable to have a higher regularization parameter in the scenarios with higher noise, and smaller in scenarios with more interference. For further details, we refer to [46].

Sensing-Prioritized Optimal Communication Solution: For the case when the sensing beam is designed a priori, we derive a max-min fair rate optimal communication beamforming solution. First, this max-min problem can be written as

$$\text{(P1.1):} \quad \max_{\{\mathbf{f}_{mu}\}} \min_{u} \quad \text{SINR}_u^{(\text{c})} \tag{19a}$$

$$\text{s.t.} \quad \sum_{u \in \mathcal{U}} \|\mathbf{f}_{mu}\|^2 \leq P_m^c, \quad \forall m \in \mathcal{M}_t, \tag{19b}$$

where the objective is quasiconvex [47] and shows a similar structure to the optimal beamforming formulation for the cell-free massive MIMO networks with only the communication objective [18]. For a given minimum SINR constraint γ, (P1.1) can be written as the feasibility problem

$$\text{(P1.2):} \quad \text{find} \quad \{\mathbf{f}_{mu}\} \tag{20a}$$

$$\text{s.t.} \quad \text{SINR}_u^{(\text{c})} \geq \gamma, \quad \forall u \in \mathcal{U}, \tag{20b}$$

$$\sum_{u \in \mathcal{U}} \|\mathbf{f}_{mu}\|^2 \leq P_m^c, \quad \forall m \in \mathcal{M}_t. \tag{20c}$$

Here, we note that the SINR constraint (20b) is in a fractional form. This, however, can be converted to a second-order cone constraint. For this purpose, we can re-write the constraint as $\left(1 + \frac{1}{\gamma}\right) \left|\sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu}\right|^2 \geq \sum_{u' \in \mathcal{U}} \left|\sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu'}\right|^2 + \sum_{q \in \mathcal{Q}} \left|\sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mq}\right|^2 + \sigma_u^2$. Now, taking the square root of both sides, we can convert the given form to a second-order cone constraint. The square root, however, leaves an absolute on the left-hand side, which is a non-linear function. This can be simplified as the real part of the variable [46], since any angular rotation ($e^{-j\psi}$) to the expression inside the absolute does not change the value, i.e., $\left|\sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu}\right| = \left|\sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu} e^{-j\psi}\right| = \text{Re}\left\{\sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu}\right\}$. This approach can be seen as selecting the optimal solution with a specific angular rotation from the set of infinite rotations $\psi \in [0, 2\pi)$. Finally, we can write the constraint (20b) as a second-order cone as follows

$$\left(1 + \frac{1}{\gamma}\right)^{\frac{1}{2}} \text{Re}\left\{\sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mu}\right\} \geq \left\| \begin{matrix} \sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{m1} \\ \vdots \\ \sum_{m \in \mathcal{M}_t} \mathbf{h}_{mu}^H \mathbf{f}_{mS} \\ \sigma_u \end{matrix} \right\|. \tag{21}$$

When (20b) is replaced with (21), it results in a second-order cone problem and can be solved by the convex solvers [48]. Using the bisection algorithm, the maximum SINR value, $\gamma^\star$, can be obtained by solving the convex feasibility problem (20) for different values of γ within a predetermined range $[\gamma_{\text{min}}, \gamma_{\text{max}}]$. This computes the optimal solution to (19).

V. Joint Sensing and Communication: Beamforming Optimization

A more desirable approach for cell-free joint sensing and communication MIMO systems is to jointly optimize the beamforming vectors for the sensing and communication functions. Specifically, our objective is to maximize the sensing SNR together with the communication SINR of the UEs. Towards this objective, we reformulate (20) as a sensing SNR maximization problem by (i) adding the sensing SNR maximization as an objective to the feasibility problem, and (ii) generalizing the minimum communication SINR limit, γ, individually for each UE with γ_u. Then, the JSC beamforming optimization problem can be written as

$$\text{(P2.1):} \quad \max_{\{\mathbf{f}_{ms}\}} \quad \text{SNR}^{(\text{s})} \tag{22a}$$

$$\text{s.t.} \quad \text{SINR}_u^{(\text{c})} \geq \gamma_u, \quad \forall u \in \mathcal{U}, \tag{22b}$$

$$\sum_{s \in \mathcal{S}} \|\mathbf{f}_{ms}\|^2 \leq P_m, \quad \forall m \in \mathcal{M}_t, \tag{22c}$$

where the objective, i.e., the maximization of the convex SNR expression, $\text{SNR}^{(\text{s})}$, is non-convex and the problem is a non-convex quadratically constrained quadratic program (QCQP). Hence, a similar approach to the beamforming optimization in the previous section can not be adopted. The problem in (22), however, can be cast as a semidefinite program, which allows applying a semidefinite relaxation for the non-convex objective [49]. With the relaxation, the problem becomes convex, and the optimal solution can be obtained with the convex solvers. Afterward, the solution to the relaxed problem can be projected back into the space of the original problem, with a method specifically designed for this purpose. In the following, we present the details of our approach.

To reformulate (22) as an SDP, we first re-define the beamforming optimization variables as matrices: $\mathbf{F}_s = \mathbf{f}_s \mathbf{f}_s^H$, $\forall s \in \mathcal{S}$. Writing (22) in terms of $\mathbf{F}_s$ instead of $\mathbf{f}_s$ eliminates the quadratic terms in the sensing SNR and communication SINR expressions. This SDP formulation, however, by construction introduces two new constraints: (i) The convex Hermitian positive semi-definiteness constraint $\mathbf{F}_s \in \mathbb{S}^+$, where $\mathbb{S}^+$ is the set of Hermitian positive semidefinite matrices, and (ii) the non-convex rank-1 constraint $\text{rank}(\mathbf{F}_s) = 1$. Further, we need to write the problem (P2.1) in terms of these newly introduced variables, $\{\mathbf{F}_s\}$. For this purpose, we define the AP selection matrix, $\mathbf{D}_m \in \mathbb{R}^{MN_t \times MN_t}$, where each element of this matrix is given by

$$[\mathbf{D}_m]_{ij} = \begin{cases} 1 & \text{if } (m-1)N_t + 1 \leq i \leq mN_t \text{ with } i = j, \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

where the only non-zero elements of the $\mathbf{D}_m$ is the identity matrix placed at the m-th cross diagonal $N_t \times N_t$ block matrix. To write the sensing SNR in a compact form, we define $\mathbf{A} = \sum_{m_t \in \mathcal{M}_t} \bar{\zeta}_{m_t} \mathbf{D}_{m_t}, \overline{\mathbf{A}} \mathbf{D}_{m_t}$, where $\overline{\mathbf{A}} = \overline{\mathbf{a}}\,\overline{\mathbf{a}}^H$ with $\overline{\mathbf{a}} = [\mathbf{a}(\theta_1)^T, \ldots, \mathbf{a}(\theta_{M_t})^T]^T$, and $\bar{\zeta}_{m_t} = \sum_{m_r \in \mathcal{M}_r} \zeta^2_{m_t, m_r}$. Now, we can write the objective of (22) (sensing SNR) in terms of $\mathbf{A}$ as

$$\text{SNR}^{(\text{s})} = \frac{\text{Tr}\left(\mathbf{A} \sum_{s \in \mathcal{S}} \mathbf{F}_s\right)}{\sum\limits_{m_r \in \mathcal{M}_r} \zeta^2_{m_r}}. \tag{24}$$

where the derivation is provided in Appendix B.

For the constraints of the problem in (22), we define $\mathbf{Q}_u = \mathbf{h}_u \mathbf{h}_u^H$ and re-write the SINR in (7) in terms of the new variables as

$$\text{SINR}_u^{(\text{c})} = \frac{\text{Tr}\left(\mathbf{Q}_u \mathbf{F}_u\right)}{\sum\limits_{u' \in \mathcal{U} \setminus \{u\}} \text{Tr}\left(\mathbf{Q}_u \mathbf{F}_{u'}\right) + \sum\limits_{q \in \mathcal{Q}} \text{Tr}\left(\mathbf{Q}_u \mathbf{F}_q\right) + \sigma_u^2}. \tag{25}$$

With this, we can write the constraint in (22b) and the power constraint in (22c) as

$$\left(1 + \gamma_u^{-1}\right) \text{Tr}\left(\mathbf{Q}_u \mathbf{F}_u\right) - \text{Tr}\left(\mathbf{Q}_u \sum_{s \in \mathcal{S}} \mathbf{F}_s\right) \geq \sigma_u^2, \tag{26}$$

$$\sum_{s \in \mathcal{S}} \text{Tr}\left(\mathbf{D}_m \mathbf{F}_s\right) \leq P_m, \quad \forall m \in \mathcal{M}_t. \tag{27}$$

Here, we notice that for the objective and constraints, we can simplify the sensing variables by defining $\mathbf{F}_\mathcal{Q} = \sum_{q \in \mathcal{Q}} \mathbf{F}_q$, since the sensing variables $\mathbf{F}_q$ only appear in the defined summation form. For the optimality of the problem, however, this variable needs to have at most rank Q, so that we can construct Q beamforming vectors[4]. Then, by collecting the expressions together, we can write the SDP form of our problem (P2-QCQP) as

$$\begin{aligned}
\text{(P2.1-SDP):} \quad & \max_{\{\mathbf{F}_u\}, \mathbf{F}_\mathcal{Q}} \quad \text{Tr}\left(\mathbf{A} \sum_{s \in \mathcal{S}} \mathbf{F}_s\right) && \text{(28a)} \\
& \text{s.t.} \quad \text{(26) and (27)}, && \text{(28b)} \\
& \qquad \mathbf{F}_u \in \mathbb{S}^+ \quad \forall u \in \mathcal{U}, \quad \mathbf{F}_\mathcal{Q} \in \mathbb{S}^+ && \text{(28c)} \\
& \qquad \text{rank}(\mathbf{F}_u) = 1 \quad \forall u \in \mathcal{U}, && \text{(28d)} \\
& \qquad \text{rank}(\mathbf{F}_\mathcal{Q}) \leq Q, && \text{(28e)}
\end{aligned}$$

which can be relaxed by removing the rank constraints, i.e., (28d)-(28e). This relaxed problem **(P2.1-SDR)**, defined as (28a)-(28c), can be solved via CVX and convex SDP solvers [48], [50]. Then, if the matrices obtained by this solution, denoted by $\{\mathbf{F}'_u\}$, are rank-1, and $\mathbf{F}'_\mathcal{Q}$ is at most rank-Q, then they are optimal for (28). The optimal user beamforming vectors, $\mathbf{f}_u$, in this case, can be obtained as the eigenvector of $\mathbf{F}'_u$. Similarly, $\{\mathbf{f}_q\}$ can be constructed as the Q eigenvectors of $\mathbf{F}'_\mathcal{Q}$. For the case the user matrices are not rank-1, we make the following proposition.

Proposition 1. *There exists a solution to the problem* (28)*, denoted by* $\{\mathbf{F}''_u\}$ *and* $\mathbf{F}''_\mathcal{Q}$*, that satisfies* $\text{rank}(\mathbf{F}''_u) = 1$, $\forall u \in \mathcal{U}$ *and*

$$\mathbf{F}''_\mathcal{Q} = \mathbf{F}'_\mathcal{Q} + \sum_{u \in \mathcal{U}} \mathbf{F}'_u - \sum_{u \in \mathcal{U}} \mathbf{F}''_u. \tag{29}$$

where $\mathbf{F}'_\mathcal{Q}$ *and* $\{\mathbf{F}'_u\}$ *are the solutions of the SDP problem in (P2.1-SDR). The communication beamforming vectors of this solution can be given as*

$$\mathbf{f}''_u = (\mathbf{h}_u^H \mathbf{F}'_u \mathbf{h}_u)^{-\frac{1}{2}} \mathbf{F}'_u \mathbf{h}_u. \tag{30}$$

Further, if $\text{rank}(\mathbf{F}''_\mathcal{Q}) \leq Q$*, the optimal sensing beamforming vectors of this solution can be constructed by*

$$\mathbf{f}''_q = \sqrt{\lambda_{q-U}}\, \mathbf{u}_{q-U}, \tag{31}$$

with λ_i *and* $\mathbf{u}_i$ *being the* i*-th largest eigenvalue of* $\mathbf{F}''_\mathcal{Q}$ *and the corresponding eigenvector.*

The proof extends the solution in [23, Theorem 1], which we provide in Appendix C. For $\text{rank}(\mathbf{F}''_\mathcal{Q}) \leq Q$, the solution obtained is from Proposition 1 optimal. In the case $\text{rank}(\mathbf{F}''_\mathcal{Q}) > Q$, however, (31) will not lead to the optimal solution. We will examine the performance of this approximation in Fig. 6. Next, we investigate the value of Q required to satisfy the optimality.

A. How Many Sensing Streams Do We Need?

In the formulations of (P2.1-SDP) and (P2.1-SDR) in Section V, the number of sensing streams is kept generic with the variable Q. In reality, however, it would be preferable to have as few as possible sensing streams. To that end, it is interesting to investigate how many sensing beams are needed to achieve optimal sensing performance. For this objective, we can further investigate (P2.1-SDR) to find the constraints on the optimal sensing solution. Specifically, we attempt to solve the problem (P2.1-SDR). Since this problem is convex, it satisfies the strong duality [47], [51]. Then, we can derive the dual problem as

$$\begin{aligned}
\text{(D2.1-SDR):} \quad & \min_{\{\lambda_u\}, \{\nu_m\}} \quad \sum_m \nu_m P_m - \sum_u \lambda_u \sigma_u^2 \\
& \text{s.t.} \quad \mathbf{B}_u \preceq 0 \quad \forall u \in \mathcal{U}, \quad \mathbf{B}_\mathcal{Q} \preceq 0
\end{aligned} \tag{32}$$

where $\{\lambda_u \geq 0\}$, $\{\nu_m \geq 0\}$ are the Lagrangian coefficients corresponding to the SINR and power constraints, respectively, and

$$\mathbf{B}_u = \mathbf{A} + \lambda_u \gamma_u^{-1} \mathbf{Q}_u - \sum_{u' \in \mathcal{U} \setminus \{u\}} \lambda_{u'} \mathbf{Q}_{u'} - \sum_m \nu_m \mathbf{D}_m, \tag{33}$$

$$\mathbf{B}_\mathcal{Q} = \mathbf{A} - \sum_{u' \in \mathcal{U}} \lambda_{u'} \mathbf{Q}_{u'} - \sum_m \nu_m \mathbf{D}_m. \tag{34}$$

[4]If the rank of this variable is less than Q, some of the beamforming vectors are not needed, and can be selected as zero.

The derivation of the dual function is provided in Appendix D. Further, we make the following remark on the definition of the new variables in the dual problem.

Remark 1. From the definition of the new variables, $\mathbf{B}_u$ and $\mathbf{B}_{\mathcal{Q}}$, we also have the relation

$$\mathbf{B}_u = \mathbf{B}_{\mathcal{Q}} + \lambda_u(1+\gamma_u^{-1})\mathbf{Q}_u. \tag{35}$$

Let us assume that there exists a feasible set of primal-dual optimal variables, i.e., $\{\mathbf{F}_u^\star\}$, $\mathbf{F}_{\mathcal{Q}}^\star$, $\{\lambda_u^\star\}$, $\{\eta_m^\star\}$, and the corresponding variables $\mathbf{B}_{\mathcal{Q}}^\star$ and $\{\mathbf{B}_u^\star\}$. With the Karush-Kuhn-Tucker (KKT) conditions [47], we obtain the complementary slackness for the semidefinite constraints

$$\mathbf{B}_u^\star \mathbf{F}_u^\star = \mathbf{0} \quad \text{and} \quad \mathbf{B}_{\mathcal{Q}}^\star \mathbf{F}_{\mathcal{Q}}^\star = \mathbf{0} \tag{36}$$

which shows that $\mathbf{F}_{\mathcal{Q}}^\star$ is in the nullspace of $\mathbf{B}_{\mathcal{Q}}^\star$. We can further refine this condition with the following proposition.

Proposition 2. *The sensing beamforming matrix is in the nullspace of $\mathbf{A} - \sum \nu_m^\star \mathbf{D}_m$. In addition, it is in the nullspace of $\mathbf{Q}_u$ for any user with $\lambda_u^\star > 0$.*

Proof: We have

$$\begin{aligned}\lambda_u^\star(1+\gamma_u^{-1})\mathrm{Tr}\left(\mathbf{Q}_u\bar{\mathbf{F}}_{\mathcal{Q}}^\star\right) &= \mathrm{Tr}\left((\mathbf{B}_u^\star - \mathbf{B}_{\mathcal{Q}}^\star)\bar{\mathbf{F}}_{\mathcal{Q}}^\star\right)\\ &= \mathrm{Tr}\left(\mathbf{B}_u^\star\bar{\mathbf{F}}_{\mathcal{Q}}^\star\right)\\ &\le \max_{\bar{\mathbf{F}}_{\mathcal{Q}}\succeq 0} \mathrm{Tr}\left(\mathbf{B}_u^\star\bar{\mathbf{F}}_{\mathcal{Q}}\right) = 0,\end{aligned} \tag{37}$$

where the first equality is due to definition (Remark 1), the second equality by the complementary slackness condition given in (36), and the latter inequality due to the multiplication of the negative and positive semidefinite matrices.

This proposition shows that a sensing matrix will be in the nullspace of stacked UE channels, $\mathbf{h}_u$, for every UE u that satisfies the SINR constraint at the equality. On the other hand, when the equality of the SINR constraint is not satisfied, we have $\lambda_u^\star = 0$ due to the complementary slackness condition of the SINR constraint. This leads to $\mathbf{B}_u^\star = \mathbf{B}_{\mathcal{Q}}^\star$ as shown in Remark 1. Combined with the complementary slackness conditions in (36), it results in $\mathbf{F}_u^\star$ and $\mathbf{F}_{\mathcal{Q}}^\star$ being in the same space defined as the nullspace of the channels of the UEs with $\lambda_u^\star > 0$ and the sensing direction via $\mathbf{A} - \sum \nu_m^\star \mathbf{D}_m$. There is, however, no enforcement towards the direction of the user itself because $\lambda_u^\star(1+\gamma_u^{-1})\mathbf{Q}_u = 0$. Then, this case is likely to appear only if there is sufficient SINR with the transmission towards the sensing direction from all the APs (e.g., the UE and target are at the same location). Hence, it is trivial and not of significant interest. With this observation, we focus on the case with $\lambda_u^\star > 0$ for every UE.

In Proposition 2, we also have the nullspace of $\mathbf{A} - \sum \nu_m^\star \mathbf{D}_m$ to define the sensing matrix. We note that $\sum \nu_m^\star \mathbf{D}_m$ is a diagonal matrix with the diagonal of each block having the same value $\nu_m^\star$.

Remark 2. The sensing SNR matrix, $\mathbf{A}$, is a block diagonal matrix of rank-1 blocks, i.e.,

$$\mathbf{A} = \mathrm{diag}(\bar{\zeta}_1\mathbf{a}(\theta_1)\mathbf{a}^H(\theta_1), \ldots, \bar{\zeta}_{M_t}\mathbf{a}(\theta_{M_t})\mathbf{a}^H(\theta_{M_t})). \tag{38}$$

This fact can be seen by the definition of $\mathbf{A} = \sum_{m_t\in\mathcal{M}_t}\bar{\zeta}_{m_t}\mathbf{D}_{m_t}\overline{\mathbf{A}}\mathbf{D}_{m_t}$, where the multiplication of a matrix from both sides with the selection matrix, $\mathbf{D}_{m_t}$, results in the block diagonal of the selected entries. Further, with a slight abuse of notation, each block of the diagonal $\mathbf{A}_{m_t} \triangleq \bar{\zeta}_{m_t}\mathbf{a}(\theta_{m_t})\mathbf{a}^H(\theta_{m_t}) = \mathbf{D}_{m_t}\mathbf{A}\mathbf{D}_{m_t}$ is a weighted outer product of an array response vector. Therefore, we have $\mathrm{rank}(\mathbf{A}_{m_t}) = 1, \forall m_t \in \mathcal{M}_t$ and $\mathrm{rank}(\mathbf{A}) = M_t$.

With Remark 2, we can further refine our space as $\mathbf{A} - \sum \nu_m^\star \mathbf{D}_m = \sum(\mathbf{A}_m - \nu_m^\star \mathbf{I}_m)$, where each component, $\mathbf{A}_m - \nu_m^\star \mathbf{I}_m$, is a diagonal block. For $\nu_m^\star > 0$, each of these components can have a single nullspace vector $\mathbf{f}_{mq} = \mathbf{a}(\theta_m)$ if $\nu_m^\star = \bar{\zeta}_m$. Note that this only constructs a part of the sensing matrix, not the full domain $\mathbf{f}_q$. If $\nu_m^\star \neq \bar{\zeta}_m$, the component is full rank, and the nullspace is empty. In other words, the existence of sensing beams is independently determined at each AP based on $\nu_m^\star$ and only available if $\nu_m^\star = \bar{\zeta}_m$. For any other $\nu_m^\star > 0$, there is no sensing stream. Then, the sensing beams can be written in the form of $\mathbf{f}_q = [\eta_{1q}\mathbf{a}(\theta_1), \ldots, \eta_{Mq}\mathbf{a}(\theta_M)]$ for some $\eta_{mq} \in \mathbb{C}$, and this vector is in the nullspace of $\mathbf{h}_u$ for every UE with $\lambda_u^\star > 0$. To that end, further investigating the performance of the nullspace sensing beam with the suboptimal beamforming solutions is interesting. As an alternative to beamforming optimization, in the next section, we develop a power allocation approach for the pre-determined beamforming vectors. Before moving on, we further conclude the limitations on the sensing streams.

Proposition 3. *For $\nu_m^\star > 0\ \forall m \in \mathcal{M}_t$, the maximum number of sensing streams is limited by*

$$\mathrm{rank}(\mathbf{F}_{\mathcal{Q}}^\star) \le M_t. \tag{39}$$

The proof follows Remark 2, where the minimum rank of $\mathbf{A} - \sum_m \nu_m^\star \mathbf{D}_m = (M_t - 1)N_t$, and its nullspace can have at most M_t dimensions. Further, we can refine the limit in the case of random Rayleigh channels as follows.

Proposition 4. *For $\nu_m^\star > 0\ \forall m \in \mathcal{M}_t$ and $\mathbf{h}_{mu} \sim \mathcal{CN}(0, \mathbf{I})$, the maximum number of sensing streams is limited by*

$$\mathrm{rank}(\mathbf{F}_{\mathcal{Q}}^\star) \le \max\{M_t - U, 0\}. \tag{40}$$

with probability 1.

The proof follows the fact that the probability of any $\mathbf{h}_u$ drawn from random Gaussian distribution can be spanned by the space constructed by $\mathbf{A} - \sum_m \nu_m^\star \mathbf{D}_m$ with probability 0. Hence, each UE reduces the available dimensions for the sensing stream with probability 1.

As shown in Proposition 3, the number of sensing streams upper bounded by the number of APs. More interestingly, in the case of Rayleigh channels, the number of sensing streams is limited to $M_t - U$, i.e., the difference between the number of transmitting APs and UEs. From this result, if the number of transmitting APs is smaller than the number of UEs, no sensing streams is required. However, in the cell-free massive MIMO regime where the number of APs is much larger than the number of UEs, almost a single stream for each AP may be required.

VI. Joint Sensing and Communication: Power Allocation with Fixed Beams

In this section, we investigate the power allocation along our framework to compare with the developed beamforming solutions. We emphasize that power allocation provides several advantages over beamforming optimization: (i) The power allocation problem needs to be solved over multiple coherence blocks as it is mainly affected by the large-scale coefficients. In contrast, changes in the channel due to small-scale fading effects require optimization of the beamforming vectors, leading to frequent updates. (ii) Power allocation only requires a single coefficient (beamforming gain) for each channel, and does not require the complex coefficients for each antenna. This reduces the amount of information exchanged at the fronthaul for the optimization, allowing a lower signaling overhead. Similarly, with smaller number of optimization variables, the computational complexity is reduced. These advantages, however, come with the potential performance loss from beamforming, which may make the beamforming optimization more preferable depending on the other system parameters.

Before moving on to the formulation, we note that the formulated beamforming optimization problem jointly optimizes the power along with the beams since the power constraints of the beams are set to satisfy the power constraints at each AP. Another interesting case with the cell-free massive MIMO, however, is to allocate the power for pre-determined suboptimal beams. For this purpose, in this section, we develop a power allocation formulation for given beams. Differently from the approach developed for power optimization in [30], which adopts an iterative convex-concave programming approach and does not guarantee optimality, we maintain the SDP framework of our paper and develop a power allocation approach with the SDR relaxation. Although this approach cannot provide an optimality guarantee with the relaxation, it can provide an upper bound with the relaxation.

Mathematically, let us denote the pre-determined unit-power beamforming vectors and power coefficients by $\{\bar{\mathbf{f}}_{mq}\}$ and $\{p_{mq}\}$. With this notation, the beamforming vectors in the previous formulation in (28) can be written as $\mathbf{f}_{mq} = \sqrt{p_{mq}}\bar{\mathbf{f}}_{mq}$. In this model, the fixed beamforming vectors can be selected by the approaches given in Section III and Section IV. With this definition, we can rewrite our JSC objective in terms of the power variables of the beams, p_{mq}. Before moving on, we also define the effective channel of UE u and AP m due to the stream u' as $\rho_{muu'} = \mathbf{h}_{mu}^H \bar{\mathbf{f}}_{mu'}$ and the sensing channel gain due to the stream s of AP m as $\varrho_{ms} = \left|\mathbf{a}^H(\theta_m)\bar{\mathbf{f}}_{ms}\right|^2 \sum_{m_r \in \mathcal{M}_r} \zeta_{mm_r}^2$. With this, we can define the power allocation problem as

$$\text{(P3.1a):} \quad \max_{\{p_{ms}\}} \quad \sum_{m\in\mathcal{M}_t}\sum_{s\in\mathcal{S}} p_{ms}\varrho_{ms} \tag{41a}$$

$$\text{s.t.} \quad {\gamma_u}^{-1}\left|\sum_{m\in\mathcal{M}_t}\sqrt{p_{mu}}\rho_{muu}\right|^2 \geq \sum_{s\in\mathcal{S}\setminus\{u\}}\left|\sum_{m\in\mathcal{M}_t}\sqrt{p_{ms}}\rho_{mus}\right|^2 + \sigma_u^2 \tag{41b}$$

$$\sum_{s\in\mathcal{S}} p_{ms} \leq P_m, \quad \forall m \in \mathcal{M}_t, \tag{41c}$$

This problem, however, is difficult to solve since (i) includes the square root of the power terms, i.e., $\{\sqrt{p_{ms}}\}$, and (ii) contains the summation inside the absolute terms. For (i), we can write the problem in terms of the square root power terms, $\{\sqrt{p_{ms}}\}$. For (ii), we define the per-stream vector form of the power coefficients by stacking the power coefficients of every AP for a given stream, similar to the one applied in (7), given as $\mathbf{p}_s = [\sqrt{p_{1s}}, \dots, \sqrt{p_{Ms}}]^T$. We note that this variable is defined in terms of the square-root of the power variables. To complement this variable in our new formulation, we also define the vectors $\boldsymbol{\rho}_{us} = [\rho_{1us}, \dots, \rho_{Mus}]$, and $\boldsymbol{\varrho}_s = [\varrho_{1s}, \dots, \varrho_{Ms}]$. In addition, we define the AP selection matrix for the power allocation formulation, $\widetilde{\mathbf{D}}_m \in \mathbb{R}^{M\times M}$, where each element of this matrix is given as $[\widetilde{\mathbf{D}}_m]_{ij} = 1$ if $i = j = m$, and 0 otherwise. This variable allows rewriting the power constraint in terms of the stacked variable. Then, we can rewrite the problem in terms of the newly defined variables as

(P3.1b):

$$\max_{\{\mathbf{p}_s\}} \quad \sum_{s\in\mathcal{S}} (\mathbf{p}_s \odot \mathbf{p}_s)^T \boldsymbol{\varrho}_s \tag{42a}$$

$$\text{s.t.} \quad \gamma_u^{-1}\left|\mathbf{p}_u^T\boldsymbol{\rho}_{uu}\right|^2 \geq \sum_{s\in\mathcal{S}\setminus\{u\}}\left|\mathbf{p}_s^T\boldsymbol{\rho}_{us}\right|^2 + \sigma_u^2, \quad \forall u \in \mathcal{U}, \tag{42b}$$

$$\sum_{s\in\mathcal{S}}\left\|\widetilde{\mathbf{D}}_m\mathbf{p}_s\right\|^2 \leq P_m, \quad \forall m \in \mathcal{M}_t, \tag{42c}$$

where the problem is a non-convex QCQP due to the maximization of the sensing SNR, which is a quadratic function of the variable $\mathbf{p}$. Similar to the Section V, we can transform it into an SDP and apply SDR. For this purpose, we define the new optimization variables for the SDP, i.e., $\mathbf{P}_s = \mathbf{p}_s\mathbf{p}_s^T$, which, by definition, introduces three constraints on the problem (i) The convex symmetric positive semi-definiteness constraint $\mathbf{P}_s \in \mathbb{S}^+$, where $\mathbb{S}^+$ is the set of symmetric positive semidefinite matrices, (ii) the non-convex rank-1 constraint $\text{rank}(\mathbf{P}_s) = 1$, (iii) The matrix $\mathbf{P}_s$ is non-negative since it is outer product of power values. To complement this variable in our formulation, we also define $\boldsymbol{\Gamma}_{us} = \boldsymbol{\rho}_{us}\boldsymbol{\rho}_{us}^H$, and $\boldsymbol{\Gamma}'_s = \text{diag}(\boldsymbol{\varrho}_s)$. Then, (42) can be written as an SDP in terms of these variables, given by

(P3.1-SDP):

$$\max_{\{\mathbf{P}_s\}} \quad \sum_{s\in\mathcal{S}} \text{Tr}\left(\mathbf{P}_s\boldsymbol{\Gamma}'_s\right) \tag{43a}$$

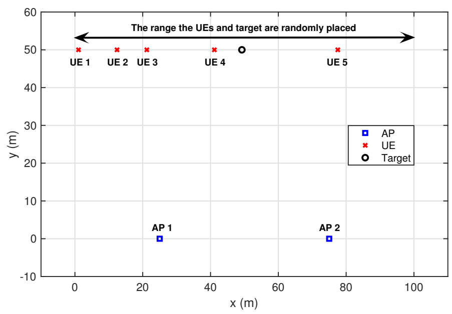
$$\text{s.t.} \quad \gamma_u^{-1}\text{Tr}\left(\mathbf{P}_u\boldsymbol{\Gamma}_{uu}\right) \geq \sum_{s\in\mathcal{S}\setminus\{u\}} \text{Tr}\left(\mathbf{P}_s\boldsymbol{\Gamma}_{us}\right) + \sigma_u^2, \tag{43b}$$

$$\sum_{s\in\mathcal{S}} \text{Tr}\left(\mathbf{P}_s\widetilde{\mathbf{D}}_m\right) \leq P_m, \quad \forall m \in \mathcal{M}_t, \tag{43c}$$

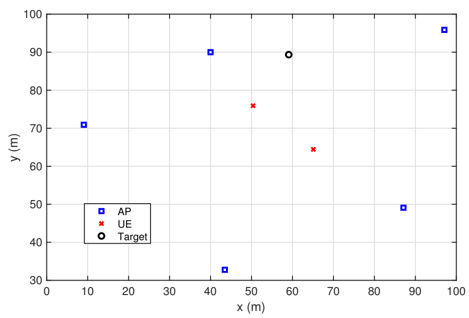
$$\mathbf{P}_s \in \mathbb{S}^+, \mathbf{P}_s \geq 0 \tag{43d}$$

$$\text{rank}(\mathbf{P}_s) = 1 \tag{43e}$$

The given problem is non-convex due to the rank-1 constraint. To obtain a convex problem, we apply SDR by removing this constraint. Then, the relaxed formulation for the power allocation, denoted as **(P3.1-SDR)**, can be given by the equations (43a)-(43d). The solution to (P3.1-SDR) can be obtained by



(a) Line setup



(b) Square setup

Fig. 2. The simulation placement is illustrated. For different realizations, the AP positions are fixed. In (a), the UEs and target are randomly placed over the y-axis, while in (b), the UEs and target are randomly placed over the square area of 100m$\times$100m.

the convex solvers. This, however, only results in the matrices $\{\mathbf{P}_s^\star\}$, which is not necessarily rank-1, and the reconstruction of the individual power variables, $\{\mathbf{p}_s^\star\}$, is required.

To reconstruct the solution, one can apply a heuristic approach inspired by the solution in Section V or develop different approaches with the randomization techniques [49]. For our purposes of evaluating the beamforming against the power optimization, we utilize a rank-1 heuristic approach, where we take the most significant eigenvector (multiplied by the square root of its eigenvalue) as the solution. All elements of this eigenvector is necessarily positive thanks to Perron-Frobenius theorem [52][5], and the eigenvalue is positive due to semi-definiteness of this matrix. With this rank-1 approach, the available power at the APs may not be fully utilized. For that, by taking advantage of the NS sensing solution[6], we allocate remaining power from communication to sensing without introducing any additional interference. In addition to this solution, we also utilize the solution obtained by the (P3.1-SDR), which provides an upper bound on the power optimization. With the solution completed, we next evaluate our results.

VII. Results

In this section, we evaluate the performance of the proposed beamforming solutions for cell-free ISAC MIMO systems. For this setup, we compare the following solutions:

(i) **NS Sensing - RZF Comm** which designs the sensing beam as conjugate beamforming projected on the null space of the communication channels as in (17) and implements the communications beams according to the RZF design in (18).

(ii) **NS Sensing - OPT Comm** which has the same sensing beam design as in (i) but designs the communication beam based on the max-min optimization in (20).

(iii) **CB Sensing - OPT Comm** which first designs the sensing beam as the conjugate beamforming in (16) and then designs the communication beams to solve the max-min optimization in (20).

(iv) **JSC Beam Optimization** which implements the communication and sensing beams based on the SDR problem (P2.1-SDR) and Proposition 1, which jointly optimizes the beamforming vectors based on the communication and sensing functions.

(v) **JSC Power Optimization** which implements the communication and sensing beam powers based on the SDR problem in (P3.1-SDR) that jointly optimizes the power coefficients for the given beams along with a rank-1 heuristic approach by taking the most significant eigenvectors as the solution. The pre-determined beamforming vectors are taken as in the (i) *NS Sensing - RZF Comm* approach.

(vi) **JSC Beam SDR UB** which applies the matrix solution for the communication and sensing beams based on the SDR problem in (P2.1-SDR). There is no rank constraint on the beams; hence, it is an upper bound.

(vii) **JSC Power SDR UB** which applies the matrix solution for the communication and sensing beam powers based on the SDR problem (P3.1-SDR). There is no rank constraint on the power variables; hence, it is an upper bound.

A. LoS Channels

In particular, we consider a scenario where $\mathcal{M}_t = \mathcal{M}_r$ with two APs placed at $(25, 0)$ and $(75, 0)$ in the Cartesian coordinates, as shown in Fig. 2(a). Each AP is equipped with a uniform linear array (ULA) along the x axis of $N_t = N_r = 16$ isotropic antennas. At $y = 50m$, we randomly place one sensing target and the $U = 5$ communications users along the x-axis. Specifically, the x coordinates of these locations are drawn from a uniform distribution in $[0, 100]$. For the communication channels, we adopt a LOS channel model and take $\sigma_u^2 = 1$. For the sensing channels, we adopt the parameters $\varsigma_{m_r}^2 = 1$ and $\zeta_{m_t m_r} = 0.1$. The transmit power of

[5]The Perron-Frobenius theorem states that a non-negative square matrix has an eigenvalue of its spectral radius, and the corresponding eigenvector is non-negative.

[6]Recall that the NS sensing does not introduce any interference to communication, since the sensing beam is in the nullspace of the communication channels, i.e., $\mathbf{h}_{mu}^H \mathbf{f}_{mq} = 0$.

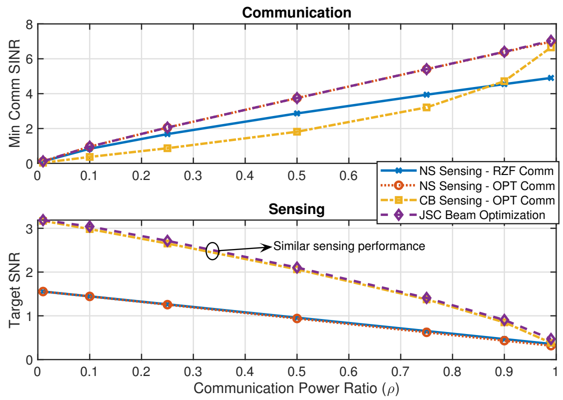


Fig. 3. Performance of the solutions for different power allocation ratios for the communications and sensing. The proposed JSC optimization provides a significant gain for sensing while satisfying the best communication SINR.

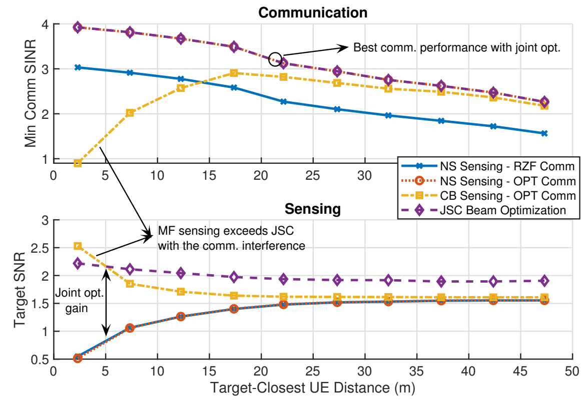


Fig. 4. Performance of the solutions versus the distance between the target and closest AP. The proposed JSC optimization provides almost a constant sensing SNR for different distances, with a significant gain over the NS solutions.

the APs is $P_m = 0$dBW and the number of sensing streams $Q = 1$. In the following, we average the results over 1000 realizations.

1) Providing NS Sensing - OPT Comm SINR for All UEs: With the defined setup, we first focus on an equal rate case, i.e., γ_u is the same for every UE. For the selection of this value, we adopt the minimum UE SINR obtained from solution (ii). Further, we do not include the power optimization solution as it is not able to satisfy the SINR constraints for most cases with $\gamma > 0.2$.

Sensing and Communication Power Allocation: We first investigate the sensing and communication performance for different power allocation ratios. Specifically, in Fig. 3, we show the sensing SNR and minimum communication SINR of UEs achieved by the different beamforming solutions for different values of $\rho \in (0, 1)$. It is important to note here that for the beamforming solutions (i)-(iii), the communication and sensing beams are separately designed, and we directly allocate the communication and sensing powers based on the ratio ρ. The JSC beam optimization solution (iv) implements the beamforming design in Proposition 1, which optimizes both the structure of the beams and the power allocation. Therefore, and for the sake of comparing with the other approaches, we plot the JSC optimization curve in Fig. 3 by setting the communication SINR threshold to be equal to the achieved SINR by solution (ii). This still respects the total power constraint, which is taken care of by (27). As seen in the figure, the first two solutions, (i) and (ii), achieve better communication SINR and less sensing SNR compared to the solution (iii). This is expected as the solution (iii) aims to maximize the sensing performance, irrespective of the communication, and hence, it causes some interference to the communication users. *Interestingly, while achieving the best communication performance of the separate solutions, the joint solution provides very similar sensing performance to the MF sensing.* This highlights the gain of the developed JSC beamforming design.

Target distance to closest UE: To further investigate how the different beamforming approaches impact the trade-off between the sensing and communication performance, we evaluate this performance versus the distance between the sensing target and closest communication UE in Fig. 4. Note that, intuitively, as the sensing target gets closer to the communication users, the overlap between the communication and sensing channels' subspaces increases, which can benefit or penalize the communication and sensing performance depending on the beamforming design. In Fig. 4, we set the power ratio as 0.5 for the communication and sensing operation. This figure shows that for the smaller distances/separation between the sensing target and communication users, the conjugate beamforming sensing solution (solution (iii)) optimizes the sensing performance but causes non-negligible interference to the communication, which significantly degrades its performance. On the other side, solutions (i) and (ii), which prioritize the communication and keep the sensing beamforming in the nullspace of the communication channels, optimize the communication SINR and degrade the sensing SNR. For the SINR constraint of the JSC optimization, we again adopt the SINR obtained by solution (ii), which achieves the best communication performance. Hence, the achieved communication SINR of this solution and JSC beam optimization are the same. *The sensing SNR, however, enjoys the advantage of the joint beam optimization. Specifically, it provides almost a constant sensing performance for different target-closest UE distances: Achieving a close sensing performance to solution (i) when the separation between the sensing target and communication users is small and exceeds the performance of all the other three solutions when this separation is large, which highlights the potential of the joint beamforming design.*

2) Providing NS Sensing - RZF Comm SINR for each UE: Now, to further investigate the performance of the joint optimization, we select each γ_u individually as their SINR obtained from solution (i). Differently from the previous approach (minimum RZF rate), we test the imbalanced rates and the gain concerning solution (i) while satisfying the same SINR values. In Fig. 5, we provide the closest distance figure with these SINR constraints. In the figure, the beamforming (with $Q = 0$) and power solutions with corresponding upper bounds, (iv) with (vi) and (v) with (vii), achieve the same results, hence only (iv) and (v) are illustrated. Compared to

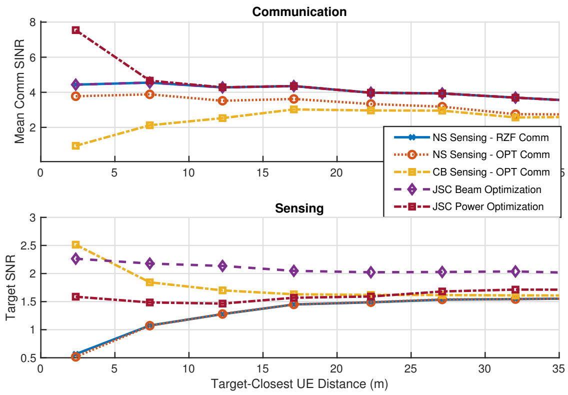
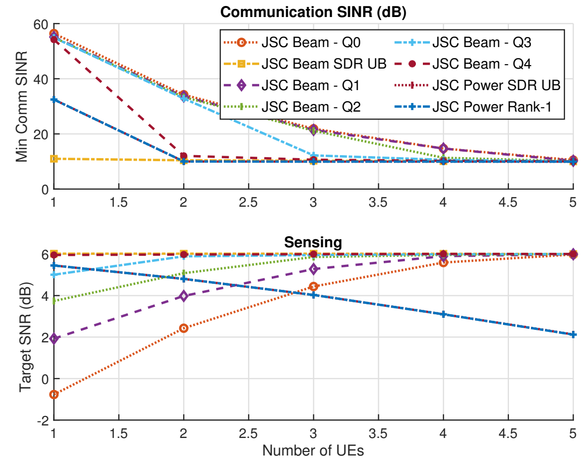
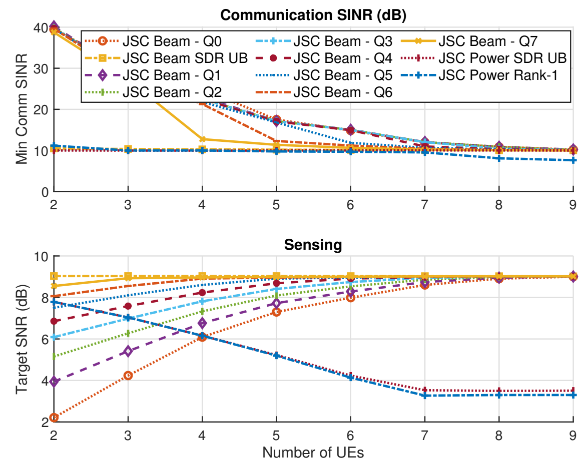


Fig. 5. Performance of the mean SINR versus the distance between the target and closest AP. The optimizations are carried out to satisfy individual rates achieved by RZF. A similar pattern to the previous case is observed.



(a) $M_t = 5$ APs



(b) $M_t = 10$ APs

Fig. 6. Performance of the SDR-based optimization solutions with varying number of sensing streams and number of UEs and $M_t =$ (a) 5, (b) 10 APs.

the previous figure, the mean SINR of solution (i) shows less degradation with larger distances, thanks to being able to exploit the imbalance for the mean rates. On the other hand, the beamforming optimization (iv) achieves the same SINR and provides a significant sensing SNR gain over all other solutions without any sensing beams, as indicated by Proposition 4. This shows a similar advantage to the previous case. For very small target-closest UE distances, (iv) the beamforming optimization is worse for sensing than (iii) CB Sensing - OPT Comm, which cannot achieve similar communication SINRs due to the high interference. At the communication part, (v) the power optimization solution achieves higher average communication SINR with very close distances since allocating the power onto the UE with the closest distance provides more gain for sensing than the NS sensing beam. In the general case, however, (iv) the beamforming optimization provides significant sensing gain over all the solutions, showing a similar pattern to the previous case.

B. Rayleigh Channels

As we have only investigated a simplified setup so far to examine the effects, we now provide a more realistic setup. In this setup, the AP and UEs are placed over a square area of 100m$\times$100m. We utilize the $f_c = 28$GHz band and place $M_t = M_r = \{5, 10\}$ APs, each equipped with a ULA of $N_t = 8$ isotropic antennas. To show the need for additional sensing streams, based on our observations in Section V, we take the minimum number of UEs as 2. The setup is illustrated in Fig. 2(b). For the path-loss, we adopt the 3GPP UMi path loss [53] given as

$$\mathrm{PL} = 22.4 + 35.3 \log_{10}(\text{distance}) + 21.3 \log_{10}(f_c) + \mathcal{X} \quad (44)$$

where $\mathcal{X}$ is the shadow fading effect determined by a Gaussian random variable zero mean and standard deviation 7.82dB. Further, we assume Rayleigh fading for the AP-UE channels. The receiver noise at the UEs is taken as -135dBm. To evaluate the performance in this setup, we set the minimum communication SINR threshold as 10dB for all UEs.

In Fig. 6, we show the achieved sensing SNR and communication SINR values with different numbers of UEs. As expected from Proposition 4, we need $Q \leq M - U$ streams to achieve the beamforming upper bound. To that end, the sensing SNR provided by any beamforming optimization (iv) curve with $Q = M - U$ achieves the same value, while satisfying the communication constraints. This boundary seems tighter for small number of APs ($M_t = 5$) in Fig. 6(a), while it is relaxed slightly with more APs ($M_t = 10$) in Fig. 6(b). The power optimization perform better for the small number of UEs for sensing, but the performance degrades as the number of UEs increases. This is primarily due to the increased power allocated to communication in order to satisfy the UE SINR requirements, which does not contribute sufficiently to sensing. In addition, in Fig. 6(b), we note that the provided heuristic rank-1 power optimization solution slightly violates the SINR constraints for large number of UEs, and better heuristic methods for the rank-1 constructions may be required. However, we leave this for future work, as the focus of this paper is to investigate JSC beamforming, and also compare it with the potential of power allocation, which is well demonstrated by the SDR solution (power optimization upper bound). As shown in the figure, adding sensing streams can provide significant advantages for the cell-free massive MIMO systems, and this can allow further gain over the suboptimal beams with power allocation.

VIII. Conclusion

In this paper, we investigated downlink beamforming for joint sensing and communication in cell-free massive MIMO systems. Specifically, we designed communication-prioritized sensing beamforming and sensing-prioritized communication beamforming solutions as the baseline. Further, we have developed an optimal solution for the JSC beamforming. The results showed the advantage of the joint optimization, where the developed JSC beamforming is capable of achieving nearly the SINR of the communication-prioritized sensing beamforming solutions with almost the same sensing SNR of the sensing-prioritized communication beamforming approaches.

Appendix

A. Derivation of the Sensing SNR

The expectation of the nominator can be simplified as

$$\mathbb{E}\left[\sum_{m_r\in\mathcal{M}_r}\left\|\overline{\mathbf{G}}_{m_r}\overline{\mathbf{X}}\right\|_{\mathrm{F}}^2\right] \tag{45}$$

$$= \sum_{m_r\in\mathcal{M}_r}\mathrm{Tr}\left\{\mathbb{E}\left[\overline{\mathbf{X}}\,\overline{\mathbf{X}}^H\overline{\mathbf{G}}_{m_r}^H\overline{\mathbf{G}}_{m_r}\right\}\right] \tag{46}$$

$$= \sum_{m_r\in\mathcal{M}_r}\mathrm{Tr}\left\{\underbrace{\mathbb{E}\left[\overline{\mathbf{X}}\,\overline{\mathbf{X}}^H\right]}_{=L\cdot\mathbf{I}}\mathbb{E}\left[\overline{\mathbf{G}}_{m_r}^H\overline{\mathbf{G}}_{m_r}\right]\right\} \tag{47}$$

$$\begin{aligned} &= L\sum_{m_r\in\mathcal{M}_r}\sum_{m_t\in\mathcal{M}_t}\Bigg(\mathrm{Tr}\,\mathbb{E}\left[\overline{\mathbf{G}}_{m_tm_r}\overline{\mathbf{G}}_{m_tm_r}^H\right] \\ &\quad + L\sum_{m_t'\in\mathcal{M}_t\setminus\{m_t\}}\mathrm{Tr}\,\mathbb{E}\left[\overline{\mathbf{G}}_{m_tm_r}\overline{\mathbf{G}}_{m_t'm_r}^H\right]\Bigg)\end{aligned} \tag{48}$$

where (46) and (47) are obtained by applying the expansion of the Frobenius norm, interchanging expectation and trace, and permutating the inner terms of the trace operation several times. To obtain (48), we apply the definition $\overline{\mathbf{G}}_{m_r} = \sum_{m_t}\overline{\mathbf{G}}_{m_tm_r}$ given in (14), and re-organize the multiplication terms. Further, for (48), due to the expectation over the random variables $\{\alpha_{m_tm_r}\}$ and independence of them, we have $\mathbb{E}\left[\overline{\mathbf{G}}_{m_tm_r}\overline{\mathbf{G}}_{m_t'm_r}^H\right] = 0$, which makes the latter line of (48) zero. For the former, we have

$$\mathrm{Tr}\,\mathbb{E}\left[\overline{\mathbf{G}}_{m_tm_r}\overline{\mathbf{G}}_{m_tm_r}^H\right] \tag{49}$$

$$= \mathrm{Tr}\,\mathbb{E}\left[\left|\alpha_{m_tm_r}\right|^2\mathbf{a}(\theta_{m_r})\mathbf{a}^H(\theta_{m_t})\overline{\mathbf{F}}_{m_t}\overline{\mathbf{F}}_{m_t}^H\mathbf{a}(\theta_{m_t})\mathbf{a}^H(\theta_{m_r})\right] \tag{50}$$

$$= \zeta_{m_tm_r}^2\mathrm{Tr}\,\mathbb{E}\left[\mathbf{a}^H(\theta_{m_t})\overline{\mathbf{F}}_{m_t}\overline{\mathbf{F}}_{m_t}^H\mathbf{a}(\theta_{m_t})\underbrace{\mathbf{a}^H(\theta_{m_r})\mathbf{a}(\theta_{m_r})}_{=N_r}\right] \tag{51}$$

$$= \zeta_{m_tm_r}^2 N_r\left\|\mathbf{a}^H(\theta_{m_t})\overline{\mathbf{F}}_{m_t}\right\|^2. \tag{52}$$

For the denominator, we can write

$$\begin{aligned}\mathbb{E}\left[\sum_{m_r\in\mathcal{M}_r}\left\|\mathbf{N}_{m_r}\right\|_{\mathrm{F}}^2\right] &= \mathbb{E}\left[\sum_{m_r\in\mathcal{M}_r}\sum_{\ell=1}^{L}\left\|\mathbf{n}_{m_r}[\ell]\right\|^2\right] \\ &= LN_r\sum_{m_r\in\mathcal{M}_r}\varsigma_{m_r}^2.\end{aligned} \tag{53}$$

Finally, combining (48), (52), and (53) in (15), we obtain the result.

B. Derivation of the Sensing SNR in SDP form

To simplify the sensing SNR expression given in (15) in the SDP form, we can write the nominator as

$$\mathbb{E}\left[\sum_{m_r\in\mathcal{M}_r}\left\|\overline{\mathbf{G}}_{m_r}\overline{\mathbf{X}}\right\|^2\right] \tag{54}$$

$$= N_rL\sum_{m_t\in\mathcal{M}_t}\sum_{m_r\in\mathcal{M}_r}\zeta_{m_t,m_r}^2\sum_{s\in\mathcal{S}}\left|\mathbf{a}^H(\theta_{m_t})\mathbf{f}_{m_ts}\right|^2 \tag{55}$$

$$= N_rL\sum_{m_t\in\mathcal{M}_t}\sum_{m_r\in\mathcal{M}_r}\sum_{s\in\mathcal{S}}\left|\zeta_{m_t,m_r}\overline{\mathbf{a}}^H\mathbf{D}_{m_t}\mathbf{f}_s\right|^2 \tag{56}$$

$$= N_rL\sum_{m_t\in\mathcal{M}_t}\sum_{m_r\in\mathcal{M}_r}\sum_{s\in\mathcal{S}}\mathrm{Tr}\left(\zeta_{m_t,m_r}^2\overline{\mathbf{a}}^H\mathbf{D}_{m_t}\mathbf{f}_s\mathbf{f}_s^H\mathbf{D}_{m_t}\overline{\mathbf{a}}\right) \tag{57}$$

$$= N_rL\sum_{m_t\in\mathcal{M}_t}\sum_{m_r\in\mathcal{M}_r}\sum_{s\in\mathcal{S}}\mathrm{Tr}\left(\zeta_{m_t,m_r}^2\mathbf{D}_{m_t}\overline{\mathbf{A}}\mathbf{D}_{m_t}\mathbf{F}_s\right) \tag{58}$$

$$= N_rL\mathrm{Tr}\left(\sum_{m_t\in\mathcal{M}_t}\mathbf{D}_{m_t}\overline{\mathbf{A}}\mathbf{D}_{m_t}\bar{\zeta}_{m_t}\sum_{s\in\mathcal{S}}\mathbf{F}_s\right) \tag{59}$$

$$= N_rL\mathrm{Tr}\left(\mathbf{A}\sum_{s\in\mathcal{S}}\mathbf{F}_s\right) \tag{60}$$

where we obtain (56) by the definitions of $\overline{\mathbf{a}}$, $\mathbf{D}_{m_t}$, and $\mathbf{f}_s$, (57) by $|\mathbf{X}|^2 = \mathrm{Tr}\left(\mathbf{X}\mathbf{X}^H\right)$, (58) by cyclic permutation property of the trace operation and the definitions of $\mathbf{F}_s$ and $\overline{\mathbf{A}}$, (59) by rearranging the summations and defining $\bar{\zeta}_{m_t} = \sum_{m_r\in\mathcal{M}_r}\zeta_{m_t,m_r}^2$, and (60) by defining $\mathbf{A} = \sum_{m_t\in\mathcal{M}_t}\bar{\zeta}_{m_t}\mathbf{D}_{m_t}\overline{\mathbf{A}}\mathbf{D}_{m_t}$.

C. Proof of Proposition 1

This extends the proof in [23]. For this purpose, we first note that in the problem formulation in (P2.1-SDR), the sensing variable, $\mathbf{F}_{\mathcal{Q}}$, are utilized together as a summation of all of the streams, both in the objective and constraints. Hence, if we define $\bar{\mathbf{F}} = \sum_{s\in\mathcal{S}}\mathbf{F}_s$, we can eliminate the sensing term $\mathbf{F}_{\mathcal{Q}}$, and apply the optimization in terms of the user streams $\mathbf{F}_u$ and $\bar{\mathbf{F}}$. To that end, we re-formulate the problem (P2.1-SDR) as

$$\max_{\{\mathbf{F}_u\},\bar{\mathbf{F}}}\quad \mathrm{Tr}\left(\mathbf{A}\bar{\mathbf{F}}\right) \tag{61a}$$

$$\left(1+\gamma_u^{-1}\right)\mathrm{Tr}\left(\mathbf{Q}_u\mathbf{F}_u\right) - \mathrm{Tr}\left(\mathbf{Q}_u\bar{\mathbf{F}}\right) \geq \sigma_u^2, \quad \forall u\in\mathcal{U} \tag{61b}$$

$$\mathrm{Tr}\left(\mathbf{D}_m\bar{\mathbf{F}}\right) = P_m, \quad \forall m\in\mathcal{M}_t, \tag{61c}$$

$$\mathbf{F}_u\in\mathbb{S}^+, \quad \forall u\in\mathcal{U}, \tag{61d}$$

$$\bar{\mathbf{F}} - \sum_{u\in\mathcal{U}}\mathbf{F}_u\in\mathbb{S}^+, \quad \bar{\mathbf{F}}\in\mathbb{S}^+. \tag{61e}$$

Let us denote the variables obtained by the solution of this problem by $\{\mathbf{F}'_u\}$ and $\bar{\mathbf{F}}'$. Using this solution, we aim to construct an alternative optimal solution of rank-1. For this purpose, we construct the following rank-1 set of solutions

$$\bar{\mathbf{F}}'' = \bar{\mathbf{F}}', \quad \mathbf{F}''_u = \mathbf{f}''_u(\mathbf{f}''_u)^H, \quad \mathbf{f}''_u = (\mathbf{h}_u^H \mathbf{F}'_u \mathbf{h}_u)^{-\frac{1}{2}} \mathbf{F}'_u \mathbf{h}_u. \tag{62}$$

whose optimality needs to be proved. For this, we need to check if (i) the value of the objective is the same and (ii) the constraints are satisfied. First, the objective only contains the summation variable and provides the optimal value by definition. For (61b), we define $v_u = (\mathbf{h}_u^H \mathbf{F}'_u \mathbf{h}_u)^{-\frac{1}{2}}$, and write $\mathrm{Tr}\left(\mathbf{Q}_u \mathbf{F}''_u\right) = \mathrm{Tr}\left(\mathbf{h}_u v_u^2 \mathbf{h}_u^H \mathbf{F}'_u \mathbf{h}_u \mathbf{h}_u^H \mathbf{F}'^H_u\right) = \mathrm{Tr}\left(\mathbf{Q}_u \mathbf{F}'_u\right)$, where we used the cyclic permutation property of the trace and $\mathbf{F}'^H_u = \mathbf{f}'_u \mathbf{f}'^H_u = \mathbf{F}'_u$. With the addition of $\bar{\mathbf{F}}'' = \bar{\mathbf{F}}'$, (61b) is satisfied. Similarly, the constraints (61c) and (61e) are already satisfied by $\bar{\mathbf{F}}'' = \bar{\mathbf{F}}'$. Further, (61d) and the solution being rank-1 are also satisfied by the definition of $\mathbf{F}''_u$ in (62). For (61e), we have $\mathbf{v}^H(\mathbf{F}'_u - \mathbf{F}''_u)\mathbf{v} = \mathbf{v}^H \mathbf{F}'_u \mathbf{v} - (\mathbf{h}_u^H \mathbf{F}'_u \mathbf{h}_u)^{-1} \left|\mathbf{v}^H \mathbf{F}'_u \mathbf{h}_u\right|^2$. From the Cauchy-Schwarz inequality, we also have $(\mathbf{v}^H \mathbf{F}'_u \mathbf{v})(\mathbf{h}_u^H \mathbf{F}'_u \mathbf{h}_u) \geq \left|\mathbf{v}^H \mathbf{F}'_u \mathbf{h}_u\right|^2$. Combining these two equations, we obtain $\mathbf{v}^H(\mathbf{F}'_u - \mathbf{F}''_u)\mathbf{v} \geq 0$, which leads to $\mathbf{v}^H \mathbf{F}''_u \mathbf{v} \geq 0$ since it is the summation of two semidefinite matrices, $\mathbf{F}'_u - \mathbf{F}''_u$ and $\mathbf{F}'_u$. Finally, (61e) can be shown via $\bar{\mathbf{F}}'' - \sum_{u\in\mathcal{U}} \mathbf{F}''_u = \bar{\mathbf{F}}' - \sum_{u\in\mathcal{U}} \mathbf{F}'_u + \sum_{u\in\mathcal{U}}(\mathbf{F}'_u - \mathbf{F}''_u)$ which again leads to the summation of semidefinite matrices.

Finally, for constructing the sensing matrices of the solution, we want to find Q rank-1 matrices whose summation is $\sum_{q\in\mathcal{Q}} \mathbf{F}''_q$. For this purpose, we can utilize the eigendecomposition, i.e., $\sum_{Q} \mathbf{F}''_q = \mathbf{U}\Lambda\mathbf{U}^H = \sum_{q'=1}^{Q'} \lambda_{q'} \mathbf{u}_{q'} \mathbf{u}_{q'}^H$, and take the largest Q eigenvectors as the beams via $\mathbf{f}''_q = \sqrt{\lambda_{u'}}\, \mathbf{u}_{q'}$. Here, it is important to note that it is only possible if the rank of the summation, $Q' = \mathrm{rank}(\sum_{q\in\mathcal{Q}} \mathbf{F}''_q)$, is smaller than or equal to the number of the sensing streams, Q.

D. Derivation of D2.1-SDR

For the dual of (P2.1-SDR), we first write the Lagrangian function as follows.

$$\begin{aligned}
&\mathcal{L}(\{\mathbf{F}_u\}, \mathbf{F}_\mathcal{Q}, \{\mathbf{Z}_u\}, \mathbf{Z}_\mathcal{Q}, \{\lambda_u\}, \{\nu_m\}) \\
&= \sum_u \mathrm{Tr}\left(\mathbf{A}\mathbf{F}_u\right) + \mathrm{Tr}\left(\mathbf{A}\mathbf{F}_\mathcal{Q}\right) + \sum_u \lambda_u \left(1 + \gamma_u^{-1}\right) \mathrm{Tr}\left(\mathbf{Q}_u \mathbf{F}_u\right) \\
&\quad - \sum_u \sum_{u'} \lambda_{u'} \mathrm{Tr}\left(\mathbf{Q}_{u'} \mathbf{F}_u\right) - \sum_{u'} \lambda_{u'} \mathrm{Tr}\left(\mathbf{Q}_{u'} \mathbf{F}_\mathcal{Q}\right) \\
&\quad - \sum_u \lambda_u \sigma_u^2 - \sum_u \sum_m \nu_m \mathrm{Tr}\left(\mathbf{D}_m \mathbf{F}_u\right) - \sum_m \nu_m \mathrm{Tr}\left(\mathbf{D}_m \mathbf{F}_\mathcal{Q}\right) \\
&\quad + \sum_m \nu_m P_m + \sum_u \mathrm{Tr}\left(\mathbf{Z}_u \mathbf{F}_u\right) + \mathrm{Tr}\left(\mathbf{Z}_\mathcal{Q} \mathbf{F}_\mathcal{Q}\right)
\end{aligned} \tag{63}$$

where $\{\lambda_u\} \geq 0$, $\{\nu_m\} \geq 0$, $\{\mathbf{Z}_u\} \succeq 0$, and $\mathbf{Z}_\mathcal{Q} \succeq 0$ are the Lagrangian variables corresponding to the SINR constraints, AP power constraints, and the semidefiniteness constraints for the user matrices and the sensing matrix. Collecting all the multiplications with $\mathbf{F}_u$, and $\mathbf{F}_\mathcal{Q}$, we can rewrite the Lagrangian function in a compact form as

$$\begin{aligned}
&\mathcal{L}(\{\mathbf{F}_u\}, \mathbf{F}_\mathcal{Q}, \{\mathbf{Z}_u\}, \mathbf{Z}_\mathcal{Q}, \{\lambda_u\}, \{\nu_m\}) \\
&= \sum_m \nu_m P_m - \sum_u \lambda_u \sigma_u^2 \\
&\quad + \sum_u \mathrm{Tr}\left((\mathbf{B}_u + \mathbf{Z}_u)\mathbf{F}_u\right) + \mathrm{Tr}\left((\mathbf{B}_\mathcal{Q} + \mathbf{Z}_\mathcal{Q})\mathbf{F}_\mathcal{Q}\right).
\end{aligned} \tag{64}$$

Then, we note that supremum of Lagrangian for $\mathbf{F}_u$ and $\mathbf{F}_q$ is only bounded if $\mathbf{B}_u + \mathbf{Z}_u = \underline{0}$. Thus, replacing the variable $\mathbf{Z}_u \geq 0$ with $\mathbf{B}_u \leq 0$, and similarly, for the sensing matrix, we can derive the dual problem via

$$\begin{aligned}
&\min \sup_{\{\mathbf{f}_u\}, \mathbf{f}_q} \mathcal{L}(\{\mathbf{f}_u\}, \bar{\mathbf{f}}_q, \{\lambda_u\}, \{\nu_m\}) = \\
&\begin{cases} \sum_m \nu_m P_m - \sum_u \lambda_u \sigma_u^2 & \text{if } \mathbf{B}_q \preceq 0 \text{ and } \mathbf{B}_u \preceq 0,\ \forall u \in \mathcal{U} \\ \infty & \text{otherwise.} \end{cases}
\end{aligned} \tag{65}$$

References

[1] U. Demirhan and A. Alkhateeb, "Cell-free joint sensing and communication MIMO: A max-min fair beamforming approach," in *2023 57th Asilomar Conference on Signals, Systems, and Computers*. IEEE, 2023.

[2] F. Liu, C. Masouros, A. P. Petropulu, H. Griffiths, and L. Hanzo, "Joint radar and communication design: Applications, state-of-the-art, and the road ahead," *IEEE Transactions on Communications*, vol. 68, no. 6, pp. 3834–3862, 2020.

[3] F. Liu, Y. Cui, C. Masouros, J. Xu, T. X. Han, Y. C. Eldar, and S. Buzzi, "Integrated sensing and communications: Towards dual-functional wireless networks for 6G and beyond," *IEEE Journal on Selected Areas in Communications*, 2022.

[4] U. Demirhan and A. Alkhateeb, "Integrated sensing and communication for 6G: Ten key machine learning roles," *IEEE Communications Magazine*, 2023.

[5] A. Alkhateeb, G. Charan, T. Osman, A. Hredzak, J. Morais, U. Demirhan, and N. Srinivas, "Deepsense 6G: A large-scale real-world multi-modal sensing and communication dataset," *IEEE Communications Magazine*, 2023.

[6] F. Liu, W. Yuan, C. Masouros, and J. Yuan, "Radar-assisted predictive beamforming for vehicular links: Communication served by sensing," *IEEE Transactions on Wireless Communications*, vol. 19, no. 11, pp. 7704–7719, 2020.

[7] U. Demirhan and A. Alkhateeb, "Radar aided 6G beam prediction: Deep learning algorithms and real-world demonstration," in *2022 IEEE Wireless Communications and Networking Conference (WCNC)*. IEEE, 2022, pp. 2655–2660.

[8] ——, "Radar aided proactive blockage prediction in real-world millimeter wave systems," in *ICC 2022-IEEE International Conference on Communications*. IEEE, 2022, pp. 4547–4552.

[9] A. Alkhateeb, S. Jiang, and G. Charan, "Real-time digital twins: Vision and research directions for 6G and beyond," *IEEE Communications Magazine*, vol. 61, no. 11, pp. 128–134, 2023.

[10] R. W. Heath Jr, T. Wu, Y. H. Kwon, and A. C. Soong, "Multiuser MIMO in distributed antenna systems with out-of-cell interference," *IEEE Transactions on Signal Processing*, vol. 59, no. 10, pp. 4885–4899, 2011.

[11] D. A. Schmidt, C. Shi, R. A. Berry, M. L. Honig, and W. Utschick, "Comparison of distributed beamforming algorithms for MIMO interference networks," *IEEE Transactions on Signal Processing*, vol. 61, no. 13, pp. 3476–3489, 2013.

[12] K. Zheng, L. Zhao, J. Mei, B. Shao, W. Xiang, and L. Hanzo, "Survey of large-scale MIMO systems," *IEEE Communications Surveys & Tutorials*, vol. 17, no. 3, pp. 1738–1760, 2015.

[13] G. Nigam, P. Minero, and M. Haenggi, "Coordinated multipoint joint transmission in heterogeneous networks," *IEEE Transactions on Communications*, vol. 62, no. 11, pp. 4134–4146, 2014.

[14] H. Q. Ngo, A. Ashikhmin, H. Yang, E. G. Larsson, and T. L. Marzetta, "Cell-free massive MIMO versus small cells," *IEEE Transactions on Wireless Communications*, vol. 16, no. 3, pp. 1834–1850, 2017.

[15] E. Nayebi, A. Ashikhmin, T. L. Marzetta, H. Yang, and B. D. Rao, "Precoding and power optimization in cell-free massive MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 16, no. 7, pp. 4445–4459, 2017.

[16] H. Q. Ngo, L.-N. Tran, T. Q. Duong, M. Matthaiou, and E. G. Larsson, "On the total energy efficiency of cell-free massive MIMO," *IEEE Transactions on Green Communications and Networking*, vol. 2, no. 1, pp. 25–39, 2017.

[17] G. Femenias and F. Riera-Palou, "Cell-free millimeter-wave massive MIMO systems with limited fronthaul capacity," *IEEE Access*, vol. 7, pp. 44 596–44 612, 2019.

[18] A. Zhou, J. Wu, E. G. Larsson, and P. Fan, "Max-min optimal beamforming for cell-free massive MIMO," *IEEE Communications Letters*, vol. 24, no. 10, pp. 2344–2348, 2020.

[19] E. Björnson and L. Sanguinetti, "Scalable cell-free massive MIMO systems," *IEEE Transactions on Communications*, vol. 68, no. 7, pp. 4247–4261, 2020.

[20] U. Demirhan and A. Alkhateeb, "Enabling cell-free massive MIMO systems with wireless millimeter wave fronthaul," *IEEE Transactions on Wireless Communications*, vol. 21, no. 11, pp. 9482–9496, 2022.

[21] F. Liu, L. Zhou, C. Masouros, A. Li, W. Luo, and A. Petropulu, "Toward dual-functional radar-communication systems: Optimal waveform design," *IEEE Transactions on Signal Processing*, vol. 66, no. 16, pp. 4264–4279, 2018.

[22] S. Fortunati, L. Sanguinetti, F. Gini, M. S. Greco, and B. Himed, "Massive MIMO radar for target detection," *IEEE Transactions on Signal Processing*, vol. 68, pp. 859–871, 2020.

[23] X. Liu, T. Huang, N. Shlezinger, Y. Liu, J. Zhou, and Y. C. Eldar, "Joint transmit beamforming for multiuser MIMO communications and MIMO radar," *IEEE Transactions on Signal Processing*, vol. 68, pp. 3929–3944, 2020.

[24] Z. Cheng, Z. He, and B. Liao, "Hybrid beamforming design for OFDM dual-function radar-communication system," *IEEE Journal of Selected Topics in Signal Processing*, vol. 15, no. 6, pp. 1455–1467, 2021.

[25] A. Bazzi and M. Chafii, "On outage-based beamforming design for dual-functional radar-communication 6G systems," *IEEE Transactions on Wireless Communications*, 2023.

[26] H. Hua, J. Xu, and T. X. Han, "Optimal transmit beamforming for integrated sensing and communication," *IEEE Transactions on Vehicular Technology*, 2023.

[27] Y. Huang, Y. Fang, X. Li, and J. Xu, "Coordinated power control for network integrated sensing and communication," *IEEE Transactions on Vehicular Technology*, vol. 71, no. 12, pp. 13 361–13 365, 2022.

[28] N. Zhao, Y. Wang, Z. Zhang, Q. Chang, and Y. Shen, "Joint transmit and receive beamforming design for integrated sensing and communication," *IEEE Communications Letters*, vol. 26, no. 3, pp. 662–666, 2022.

[29] G. Cheng and J. Xu, "Coordinated transmit beamforming for multi-antenna network integrated sensing and communication," in *ICC 2023-IEEE International Conference on Communications*. IEEE, 2023, pp. 3528–3533.

[30] Z. Behdad, Ö. T. Demir, K. W. Sung, E. Björnson, and C. Cavdar, "Power allocation for joint communication and sensing in cell-free massive MIMO," in *GLOBECOM 2022-2022 IEEE Global Communications Conference*. IEEE, 2022, pp. 4081–4086.

[31] ——, "Multi-static target detection and power allocation for integrated sensing and communication in cell-free massive MIMO," *IEEE Transactions on Wireless Communications*, 2024.

[32] W. Mao, Y. Lu, J. Liu, B. Ai, Z. Zhong, and Z. Ding, "Beamforming design in cell-free massive MIMO integrated sensing and communication systems," in *GLOBECOM 2023-2023 IEEE Global Communications Conference*. IEEE, 2023, pp. 546–551.

[33] G. Cheng, Y. Fang, J. Xu, and D. W. K. Ng, "Optimal coordinated transmit beamforming for networked integrated sensing and communications," *IEEE Transactions on Wireless Communications*, 2024.

[34] S. Rivetti, E. Björnson, and M. Skoglund, "Secure spatial signal design for ISAC in a cell-free MIMO network," in *2024 IEEE Wireless Communications and Networking Conference (WCNC)*. IEEE, 2024, pp. 01–06.

[35] M. Elfiatoure, M. Mohammadi, H. Q. Ngo, and M. Matthaiou, "Multiple-target detection in cell-free massive MIMO-assisted ISAC," *arXiv preprint arXiv:2404.17263*, 2024.

[36] G. Sharma and K. R. Rajeswari, "Four-phase orthogonal code design for MIMO radar systems," in *2012 National Conference on Communications (NCC)*. IEEE, 2012, pp. 1–4.

[37] E. H. Kim and K. H. Kim, "Random phase code for automotive MIMO radars using combined frequency shift keying-linear FMCW waveform," *IET Radar, Sonar & Navigation*, vol. 12, no. 10, pp. 1090–1095, 2018.

[38] D. V. Sarwate and M. B. Pursley, "Crosscorrelation properties of pseudorandom and related sequences," *Proceedings of the IEEE*, vol. 68, no. 5, pp. 593–619, 1980.

[39] S. W. Golomb and G. Gong, *Signal design for good correlation: for wireless communication, cryptography, and radar*. Cambridge University Press, 2005.

[40] M.-A. Govoni, *Linear frequency modulation of stochastic radar waveform*. Stevens Institute of Technology, 2011.

[41] F. Liu, Y.-F. Liu, A. Li, C. Masouros, and Y. C. Eldar, "Cramér-rao bound optimization for joint radar-communication beamforming," *IEEE Transactions on Signal Processing*, vol. 70, pp. 240–253, 2022.

[42] M. A. Richards, J. Scheer, W. A. Holm, and W. L. Melvin, *Principles of modern radar*. Citeseer, 2010, vol. 1.

[43] T. Riihonen, S. Werner, and R. Wichman, "Mitigation of loopback self-interference in full-duplex MIMO relays," *IEEE transactions on signal processing*, vol. 59, no. 12, pp. 5983–5993, 2011.

[44] B. P. Day, A. R. Margetts, D. W. Bliss, and P. Schniter, "Full-duplex MIMO relaying: Achievable rates under limited dynamic range," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 8, pp. 1541–1553, 2012.

[45] Z. Xiao and Y. Zeng, "Waveform design and performance analysis for full-duplex integrated sensing and communication," *IEEE Journal on Selected Areas in Communications*, vol. 40, no. 6, pp. 1823–1837, 2022.

[46] E. Björnson, M. Bengtsson, and B. Ottersten, "Optimal multiuser transmit beamforming: A difficult problem with a simple solution structure [lecture notes]," *IEEE Signal Processing Magazine*, vol. 31, no. 4, pp. 142–148, 2014.

[47] S. Boyd, S. P. Boyd, and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[48] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.

[49] Z.-Q. Luo, W.-K. Ma, A. M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20–34, 2010.

[50] K.-C. Toh, M. J. Todd, and R. H. Tütüncü, "SDPT3—a MATLAB software package for semidefinite programming, version 1.3," *Optimization methods and software*, vol. 11, no. 1-4, pp. 545–581, 1999.

[51] J. Xu, L. Liu, and R. Zhang, "Multiuser MISO beamforming for simultaneous wireless information and power transfer," *IEEE Transactions on Signal Processing*, vol. 62, no. 18, pp. 4798–4810, 2014.

[52] K.-C. Chang, K. Pearson, and T. Zhang, "Perron-frobenius theorem for nonnegative tensors," 2008.

[53] 3GPP TR 38.901 version 14.1.1 Release 14, "Study on channel model for frequencies from 0.5 to 100 GHz," , Tech. Rep., 2017.

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