

# Ferromagnetism triggered by band tripling in ruthenate $\text{Sr}_4\text{Ru}_3\text{O}_{10}$

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Itinerant ferromagnetism is understood in terms of a quasiparticle picture with renormalized many-body effects. While the ferromagnetic ground state is destabilized by thermal and quantum fluctuations leading to exotic states such as unconventional superconductivity, how the quasiparticles evolve across the ferromagnetic transition is a target of intensive debate. Here, we present a type of ferromagnetic transition that is accompanied by a drastic reconstruction of quasiparticle spectrum in a layered ferromagnetic ruthenate,  $\text{Sr}_4\text{Ru}_3\text{O}_{10}$ . Angle-resolved photoemission spectroscopy uncovered that the three-dimensional coherent states below the ferromagnetic transition temperature ( $T_C$ ) turn into two-dimensional incoherent electronic states slightly above  $T_C$  characterized by the disappearance of trilayer band splitting, ferromagnetic exchange splitting, and long-lived quasiparticles. Our findings suggest that the electronic coherence strongly modifies the fermiology and magnetic order, pointing to an intriguing coupling between quasiparticles and magnetic properties.

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## I. INTRODUCTION

Ferromagnetism is a quantum phenomenon in which exchange interaction between electron spins originating from the Coulomb repulsion and the Pauli exclusion principle [1] plays a role. In insulators, the exchange interaction between localized electrons is considered (Heisenberg picture), while in metals, itinerant quasiparticle bands dressed by the electron-electron interactions characterize the ferromagnetism (Stoner picture) [2–4]. In the latter case, since the exchange interaction is incorporated into a mean-field potential, spontaneous magnetization emerges depending on the difference in the occupancy of spin-up and spin-down bands. The exchange splitting is expected to decrease upon increasing temperature until spontaneous magnetization disappears at the Curie temperature ( $T_C$ ), whereas only a few materials were found to follow this simple Stoner picture. For instance, in a

typical ferromagnet such as Fe, the exchange splitting persists slightly above  $T_C$  due to residual short-range order [5,6]. In some kagome ferromagnets, no clear temperature evolution of the quasiparticle bands was observed across  $T_C$ , although the existence of flat bands (high density of states) around the Fermi level ( $E_F$ ) favors the Stoner picture [7]. Intriguingly, deviation from the Stoner picture was widely recognized in many other systems, such as two-dimensional (2D) van der Waals metallic ferromagnets [8,9], itinerant weak ferromagnets [10,11], and giant magnetoresistance (GMR) compounds [12,13]. While the ferromagnetic (FM) ground state is well captured by the density functional theory (DFT) calculations in many cases, the spectral evolution of the quasiparticles across the FM transition poses a long-standing theoretical challenge [14,15]. Experimentally unveiling the quasiparticle nature across the FM transition is of significant importance, since exotic phenomena such as unconventional superconductivity and quantum critical behavior appear in proximity to the FM phase [16,17].

Here we focus on the Ruddlesden-Popper-phase ruthenates  $\text{Sr}_{n+1}\text{Ru}_n\text{O}_{3n+1}$  (SRO;  $n$  is the number of  $\text{RuO}_2$  planes

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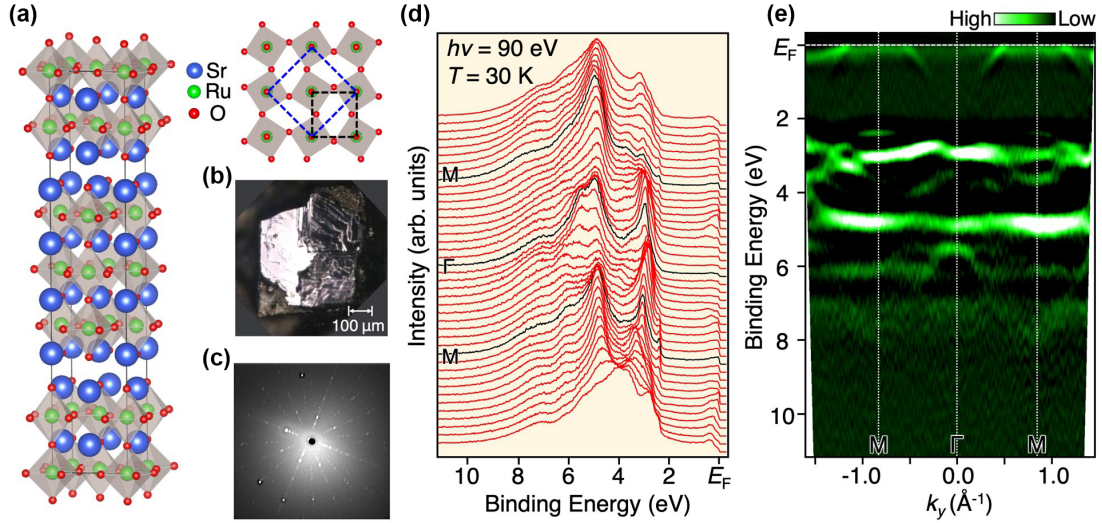


FIG. 1. (a) Crystal structure of Ruddlesden-Popper phase ruthenate  $\text{Sr}_{n+1}\text{Ru}_n\text{O}_{3n+1}$  ( $n = 3$ ) and (right) schematics of the rotation of  $\text{RuO}_6$  octahedra. Blue and black dashed squares correspond to the unit cell with and without the octahedral rotation, respectively. (b),(c) Typical photograph and x-ray Laue backscattering image of  $\text{Sr}_4\text{Ru}_3\text{O}_{10}$  single crystal, respectively. (d) EDCs in a wide binding-energy range for SRO ( $n = 3$ ) measured along the  $\Gamma\text{M}$  cut at  $T = 30\text{ K}$  with  $h\nu = 90\text{ eV}$ . (e) Corresponding ARPES intensity plot obtained by taking the second derivative of EDCs in (d).

in a unit cell) consisting of periodically stacked conducting  $\text{RuO}_2$  planes and  $\text{SrO}$  block layers to address the quasiparticles in the FM state. The SRO family serves as a good test-bed system in which one can study the interplay between quasiparticles and ferromagnetism under strong electron correlation. Infinite-layer  $\text{SrRuO}_3$  ( $n = \infty$ ) with cubic structure is a ferromagnet with  $T_C = 160\text{ K}$  [18,19], and the ferromagnetism is still kept down to  $n = 3$  although the  $T_C$  is reduced to 105 K [20,21].  $n = 2$  is located near the boundary of the FM phase, since metamagnetism and a quantum critical point appear [22]. The Fermi surface of SRO consists of multiple  $\text{Ru } 4d t_{2g}$  orbitals, and low-energy excitations are characterized by mass-renormalized quasiparticles as revealed by angle-resolved photoemission spectroscopy (ARPES) and thermodynamic measurements [23–26].

So far, the experimental investigations on electronic states of SRO have been mainly focused on  $n = 1$  and 2 [23–35], with a few studies for  $n = \infty$  [36–39]. In particular, the electronic states of  $n = 1$  and 2 have been intensively studied by ARPES [25,26,28–30,32,34,35], quantum oscillations [23,31], and DFT calculations [27,32,33]. These studies commonly pointed out the importance of multiple orbitals and band reconstruction associated with the rotation of an  $\text{RuO}_6$  octahedron [30,32,34,40]. On the other hand, for  $n = 3$  showing ferromagnetism [ $\text{Sr}_4\text{Ru}_3\text{O}_{10}$ ; see Fig. 1(a) for the crystal structure], only a limited number of experimental studies have been hitherto reported [33,41–43]. While the existence of quasiparticle peaks in the excitation spectra has been also suggested in  $n = 3$  [43], the interplay between quasiparticles and ferromagnetism, which can be investigated at  $n = 3$ , has yet to be clarified.

In this study, we have fabricated high-quality  $\text{Sr}_4\text{Ru}_3\text{O}_{10}$  single crystal using the floating-zone technique, and we succeeded in selectively probing a small but flat and clean region of a cleaved surface [typical photographs of the cleaved surface and the x-ray Laue image are shown in Figs. 1(b) and

1(c), respectively] by utilizing a micro ( $\mu$ ) focused beam spot in ARPES measurements that achieves a drastically improved momentum ( $k$ ) resolution. The  $\mu$ -ARPES plays a key role in resolving a band splitting associated with the interlayer coupling and the FM exchange interaction.

## II. EXPERIMENT AND CALCULATION

$\text{Sr}_4\text{Ru}_3\text{O}_{10}$  single crystals were grown using the floating-zone technique, and detailed growth conditions were reported in Ref. [44].  $T_C$  of the sample was estimated as 105 K from the magnetization measurement.  $\mu$ -ARPES measurements were performed with a Scienta-Omicron DA30 electron analyzer with microfocused synchrotron light at BL-28A in Photon Factory [45]. We used circularly/linearly polarized light of  $h\nu = 90\text{ eV}$ . This photon energy was found to be useful to clearly resolve the band splitting. The energy resolution was set to be 10–30 meV. Samples were cleaved *in situ* in an ultrahigh vacuum of  $1 \times 10^{-10}$  Torr and kept at  $T = 20$ –160 K during the measurements. First-principles band-structure calculations were carried out by using a projector augmented wave method implemented in Vienna Ab initio Simulation Package (VASP) code [46] with the generalized gradient approximation (GGA). Starting with the experimentally observed *Pbam* crystal structure [20], the internal atomic coordinates were optimized until forces acting on atoms were smaller than  $1 \times 10^{-5}\text{ eV/\AA}$ . The total energy was calculated self-consistently with the tetrahedron sampling of a  $8 \times 8 \times 1$   $k$ -point mesh. Spin-orbit coupling was taken into account in the calculation. The Fermi lines were drawn by using the FERMISURFER program [47].

## III. RESULTS AND DISCUSSION

At first, we present the overall valence-band structure of  $\text{Sr}_4\text{Ru}_3\text{O}_{10}$ . Figure 1(d) displays the energy distribution

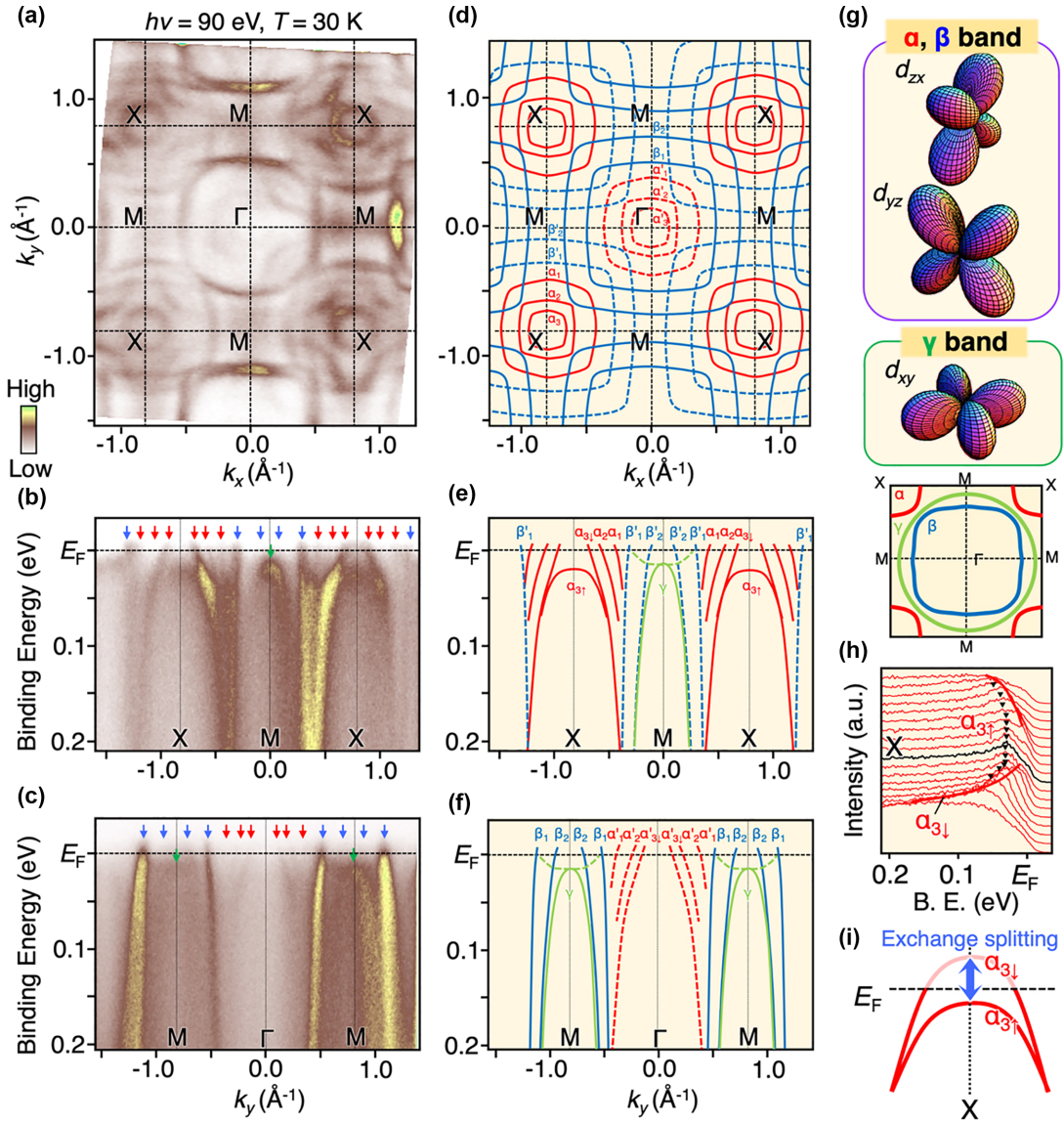


FIG. 2. (a) ARPES intensity at  $E_F$  for SRO ( $n=3$ ) measured at  $T=30$  K plotted as a function of in-plane wave vector  $k_x$  and  $k_y$  obtained with  $h\nu=90$  eV. (b), (c) Near- $E_F$  ARPES intensity along the  $\Gamma M$  and  $\Gamma M$  cuts, respectively. (d) Schematic experimental Fermi surface obtained by tracing the  $k_F$  points in (a). (e), (f) Schematic experimental band dispersions along the  $\Gamma M$  and  $\Gamma M$  cuts, respectively, obtained by tracing the peak position of EDCs/MDCs. Red, blue, and green curves in (d)–(f) correspond to the Fermi surfaces/band dispersions of the  $\alpha$ ,  $\beta$ , and  $\gamma$  bands, respectively. (g) Schematic wave functions for Ru  $d_{zx}$ ,  $d_{yz}$ , and  $d_{xy}$  orbital. The bottom panel shows the schematic Fermi surface for  $n=1$ . (h) EDCs near  $E_F$  around the X point. Triangles and red curves trace the dispersion of  $\alpha_{3\uparrow}$  and  $\alpha_{3\downarrow}$  bands, respectively. (i) Schematic band diagram of exchange-split  $\alpha_3$  bands near  $E_F$ .

curves (EDCs) in the wide binding-energy ( $E_B$ ) region measured along the  $\Gamma M$  cut of the  $1 \times 1$  (without octahedral rotation) Brillouin zone (BZ) in the FM phase ( $T=30$  K). One can identify several dispersive bands with a dominant intensity at  $E_B \sim 2$ – $8$  eV, together with weaker features dispersing within  $\sim 1$  eV of  $E_F$ . They are attributed to the O  $2p$  orbital and Ru  $4d$  quasiparticle bands, respectively [27–29], and they are better visualized in the second-derivative intensity plot in Fig. 1(e).

In Fig. 2(c), one can also recognize two highly dispersive bands outside the  $\Gamma$ -centered  $\alpha'_1$  pocket. They are attributed to the two  $\beta$  bands,  $\beta_1$  and  $\beta_2$  (blue arrows). As shown in Fig. 2(a), the  $\beta_1$  band forms a large squarelike Fermi surface centered at the  $\Gamma$  point outside the outer pocket  $\alpha'_1$ , and the  $\beta_2$

band forms an even larger square pocket also centered at the  $\Gamma$  point [see also Fig. 2(d)]. Corresponding replica bands due to the band folding appear as the  $\beta'_1$  and  $\beta'_2$  bands (blue dashed curves) in the ARPES intensity along the  $\Gamma M$  cut [Figs. 2(b) and 2(e)] and the Fermi-surface mapping [Figs. 2(a) and 2(d)]. The multiple character of  $\alpha$  and  $\beta$  bands and corresponding replica bands, as represented by the presence of three  $\alpha$  bands, suggests that they are caused by the trilayer band splitting associated with the inter-RuO<sub>2</sub>-layer coupling. The third  $\beta$  band is not clearly resolved, probably because it is too weak in the present experimental condition. The observation of trilayer splitting is also corroborated with our first-principles band-structure calculations for  $n=3$  (see Appendix A), supporting that the quasiparticle picture is a good starting point



to describe the band structure of  $\text{Sr}_4\text{Ru}_3\text{O}_{10}$  in the FM phase. Since the trilayer band splitting in layered compounds has been observed so far only in cuprate ( $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$ ) [48], the present result adds a second concrete material case. A key to resolve the trilayer splitting is the utilization of  $\mu$ -ARPES, which greatly improves the  $k$  resolution (see Appendix B).

A careful look at the ARPES intensity in Fig. 2(b) reveals the existence of an additional band topped at 15 meV below  $E_F$  at the  $M$  point (marked by a green arrow). This band forms a van Hove singularity at the  $M$  point (see Appendix C) and is assigned to the  $\gamma$  band whose singularity point is located above  $E_F$  for  $n = 1$  [25,26,30,34,35] and slightly below  $E_F$  for  $n = 2$ . This has been discussed as the origin of metamagnetism [32]. Interestingly, we do not resolve a clear trilayer splitting for the  $\gamma$  band unlike the case of the  $\alpha$  and  $\beta$  bands. The  $\gamma$  band has the  $d_{xy}$  orbital character [Fig. 2(g)] and its electronic wave function is elongated parallel to the  $\text{RuO}_2$  plane. Thus, the effective interlayer hopping is reduced compared to the  $\alpha$  and  $\beta$  bands with the  $d_{zx}$  and  $d_{yz}$  character, and this would lead to the very small trilayer splitting in the  $\gamma$  band (note that orbital assignment of the experimental bands is supported by our light-polarization-dependent ARPES measurements and first-principles calculations; see Appendix D). Such an orbital-dependent trilayer splitting is a unique feature of ruthenates with a multiorbital character, distinct from cuprates with a single-orbital ( $d_{x^2-y^2}$ ) character. As seen in Fig. 2(b), there exists a weak feature at the  $X$  point inside the  $\alpha_1$ – $\alpha_3$  bands with the top of dispersion at  $E_B \sim 20$  meV. This is better visualized in the EDCs around the  $X$  point in Fig. 2(h). Since this band shows a holelike dispersion centered at the  $X$  point as in the case of  $\alpha_1$ – $\alpha_3$  bands, it is also attributed to the  $\alpha$  band. Taking into account the fact that (i) the trilayer splitting is expected to maximally produce three  $\alpha$  bands, and (ii) the system is in the FM phase, this holelike band is likely produced by the FM exchange splitting of the  $\alpha_3$  band with the majority-spin state, called here  $\alpha_{3\uparrow}$ . Then, another  $\alpha_3$  band crossing  $E_F$  is attributed to the minority-spin state  $\alpha_{3\downarrow}$  [see a schematic band diagram for the  $\alpha_3$  bands in Fig. 2(i)]. Thus, besides the trilayer splitting, the  $\alpha$  bands undergo an additional splitting due to the FM exchange coupling. A reason why the exchange splitting of the  $\alpha_1$  and  $\alpha_2$  bands is not clearly resolved in the present experiment may be because their band top at which the splitting should be most clearly visible in the EDC is located much above  $E_F$  and is not accessible by ARPES. It is noted that spin-resolved ARPES measurements with a microfocused beam spot are necessary to further validate the FM exchange splitting.

The present result thus uncovers the fine band structure of  $n = 3$  and provides a unified picture for the mechanism of ferromagnetism and magnetic phase diagram in SRO through a comparison of quasiparticle bands among different  $n$ 's. Observation of the exchange splitting at  $n = 3$  suggests itinerant Stoner-type ferromagnetism, shedding light on the debates in bulk SRO regarding the localized versus itinerant nature of ferromagnetism [36,37]. Since an essential ingredient of ferromagnetism in the Stoner model is a high density of states (DOS) around  $E_F$ , it would be reasonable to attribute the electronic states around the  $M$  and  $X$  points to the origin of the ferromagnetism, because the van Hove singularity in the  $\gamma$  band at the  $M$  point and the shallow pocket with a low Fermi

velocity in the  $\alpha$  band (the  $\alpha_3$  band for  $n = 3$ ) at the  $X$  point are expected to largely contribute to the enhancement of the DOS around  $E_F$ . In this context, the  $\gamma$  band at the  $M$  point forms a van Hove singularity around  $E_F$  for both  $n = 2$  [32] and 3, whereas it is away from (much above)  $E_F$  for  $n = 1$  [30,34]. Regarding the shallow pocket at the  $X$  point seen for  $n = 3$ , it does not appear for  $n = 1$  and 2 because the top of the  $\alpha$  bands is pushed much upward into the unoccupied region [30,35]. Taking into account the fact that the spontaneous ferromagnetism appears for the first time at  $n = 3$  on increasing  $n$  from  $n = 1$ , it is inferred that the existence of both the van Hove singularity at the  $M$  point and the shallow pocket at the  $X$  point is a key to stabilizing the ferromagnetism. In other words, besides the  $d_{xy}$  orbital ( $\gamma$  band), the  $d_{yz/zx}$  orbital ( $\alpha$  band) also largely contributes to the enhancement of the DOS at  $E_F$  for  $n = 3$ . This tendency is likely maximized for bulk cubic SRO where all the orbitals ( $d_{xy}$ ,  $d_{yz}$ ,  $d_{zx}$ ) equally contribute to the DOS at  $E_F$  more effectively to realize the highest  $T_C$  [39,49].

Now that the electronic structure in the FM phase is established, next we present the temperature-dependent evolution of the quasiparticle bands across  $T_C$ . Figures 3(a1)–3(a8) show the near- $E_F$  ARPES intensity along the  $XM$  cut measured at various temperatures across  $T_C$  ( $= 105$  K). At  $T = 20$  K [Fig. 3(a1)], one can recognize three  $\alpha$  bands crossing  $E_F$  ( $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_{3\downarrow}$ ) due to the trilayer splitting, as in Fig. 2. On the other hand, at  $T = 125$  K in the paramagnetic phase [Fig. 3(a8)], one can see only a single  $\alpha$  band with a broader intensity distribution. A detailed look at the temperature dependence of MDC at  $E_F$  in Fig. 3(b) signifies that the three peaks at  $T = 20$  K associated with the  $k_F$  points of the  $\alpha_1$ – $\alpha_3$  bands gradually approach each other upon increasing temperature, still survive at 105 K ( $= T_C$ ) and 115 K, and eventually merge into a single peak at  $T \sim 125$  K. This behavior is supported by the quantitative analysis of MDCs; for details, see Appendix E. We have confirmed that the observed temperature-dependent band evolution is reproducible upon temperature cycle; for details, see Appendix F. These results indicate that the trilayer splitting vanishes slightly above  $T_C$ ; this is unexpected from the case of a trilayer cuprate where the splitting persists in a wide temperature range including the normal state [50].

The observed unprecedented temperature evolution of the electronic structure is associated with the coherent versus incoherent behavior in low-energy excitations. According to the previous report for  $n = 1$  [25], a quasiparticle peak starts to develop at  $T \sim 140$  K in accordance with a sharp drop in the  $c$ -axis electrical resistivity. This was interpreted in terms of the development of coherency among  $\text{RuO}_2$  layers. In  $n = 3$ , on the other hand, we found a sharp quasiparticle peak at  $T = 20$  K which gradually reduces its weight and vanishes at around  $T_C$  with increasing temperature, as seen in the representative EDCs at the  $M$  point in Fig. 3(c). This supports the development of coherency at low temperature also for  $n = 3$ , basically in line with the resistivity data [51], which turned out to be more complex than that of  $n = 1$  due to the magnetic transition. This coherency plays a crucial role in the fermiology. Namely, at high temperature, the coherent hopping among  $\text{RuO}_2$  layers is prohibited and the effective dimensionality of the system is reduced to a 2D “single-layer”



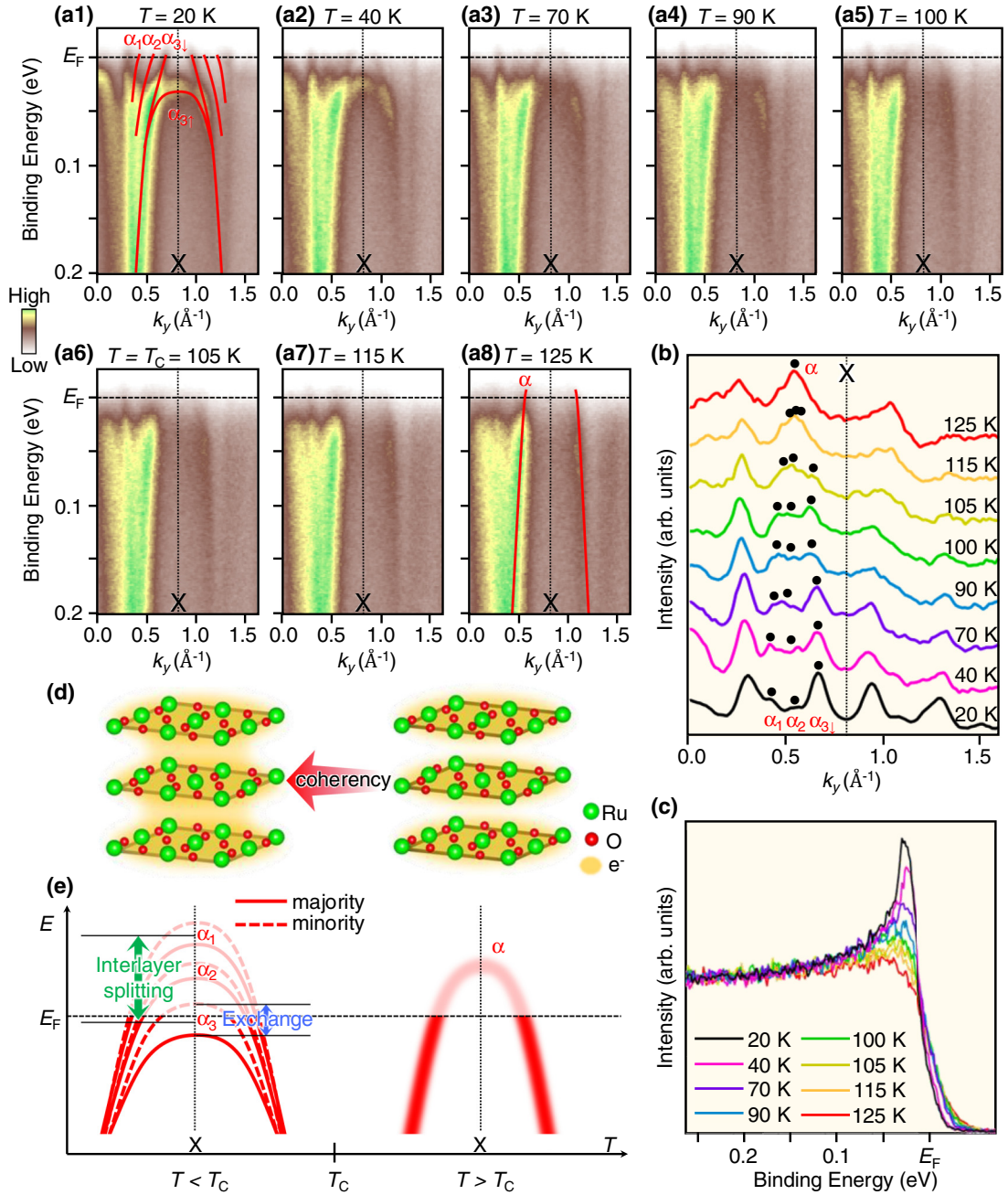


FIG. 3. (a1)–(a8) ARPES intensity along the  $XM$  cut obtained at various temperatures ( $T = 20$ – $125$  K) across  $T_C$  (105 K). Red curves in (a1) and (a8) trace the band dispersion of the  $\alpha$  bands. (b) MDCs at  $E_F$  along the  $MX$  cut obtained at various temperatures ( $T = 20$ – $125$  K). Black dots represent the  $k$  position of the  $\alpha$  bands estimated by the numerical fittings of MDCs with multiple Lorentzian peaks; for details, see Fig. 8 and Appendix E. (c) Temperature dependence of EDC at the  $X$  point that signifies the emergence of a quasiparticle peak below  $T_C$ . (d) Schematics of three  $\text{RuO}_2$  planes in the unit cell which depicts incoherent (coherent) behavior above (below)  $T_C$  without (with) interlayer electron hopping. (e) Schematics of the  $\alpha$ -band dispersion showing simultaneously the trilayer splitting due to the interlayer coupling and the exchange splitting due to the ferromagnetism (left panel). Such splitting disappears above  $T_C$  (right panel).

system in which only an incoherent (nonquasiparticle)  $\alpha$  band exists. The top of the  $\alpha$  band is located much above  $E_F$  [Fig. 3(e)] and does not contribute to the DOS at  $E_F$ . On the other hand, at low temperature, the development of coherency allows the interlayer hopping [Fig. 3(d)] to make the system more 3D-like, enabling the existence of quasiparticles. The resultant trilayer band splitting pushes one of the  $\alpha$  bands ( $\alpha_3$ ) downward to enhance the DOS at  $E_F$ , triggering the

ferromagnetism. Such electronic structure evolution is hard to explain in terms of a conventional Stoner picture where a large DOS at  $E_F$  that already exists well above  $T_C$  triggers the FM order. In contrast, our observation implies a more interesting situation associated with the development of the  $c$ -axis coherency [52], which simultaneously controls the interlayer hopping and ferromagnetism. We emphasize that our results are different from the Stoner ferromagnets such as Ni, where

the quasiparticle survives above  $T_C$  [53]. Our findings are also distinct from the non-Stoner-type ferromagnets such as Fe, the van der Waals magnet, and other itinerant ferromagnets [5,7,8,10,11] where the FM band splitting is unclear or persists even above  $T_C$ . Thus the present results provide a rare case of an FM transition that is correlated with the formation of quasiparticles and exchange splitting.

#### IV. CONCLUSION

In conclusion, we have reported ARPES results on  $\text{Sr}_4\text{Ru}_3\text{O}_{10}$  and clarified the following key spectral features: (i) evidence for the triple-layer band splitting associated with the interlayer coupling, (ii) observation of exchange band splitting, which supports the itinerant ferromagnetism, and most intriguingly, (iii) disappearance of the trilayer and exchange splitting above  $T_C$  in accordance with the weakening of  $c$ -axis electronic coherence. Our results lay the foundation for understanding the interplay among dimensionality, coherent quasiparticles, and magnetic order in strongly correlated electron systems.

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#### APPENDIX A: COMPARISON OF FERMI SURFACE BETWEEN ARPES AND CALCULATION

To validate that the observed band splitting is associated with the trilayer splitting, we have performed first-principles band-structure calculations for bulk SRO ( $n = 3$ ). The calculated Fermi surface is directly compared with the experimental Fermi surface in Fig. 4. One can recognize an overall agreement in the Fermi-surface topology between the experiment [Fig. 4(a)] and the calculation [Fig. 4(b)], in particular regarding the existence of several circular pockets centered at both  $\Gamma$  and  $X$  points, although the calculated Fermi surface shows very complicated fine structures. The multiple Fermi-surface pockets at the  $\Gamma$  and  $X$  points are produced by the trilayer splitting due to the inclusion of three  $\text{RuO}_2$  layers in the unit cell, supporting our experimental validation of the trilayer-split Fermi surface.

#### APPENDIX B: COMPARISON BETWEEN REGULAR AND $\mu$ -ARPES

To demonstrate the importance of a microbeam spot in resolving the fine structure of the Fermi surface, we directly

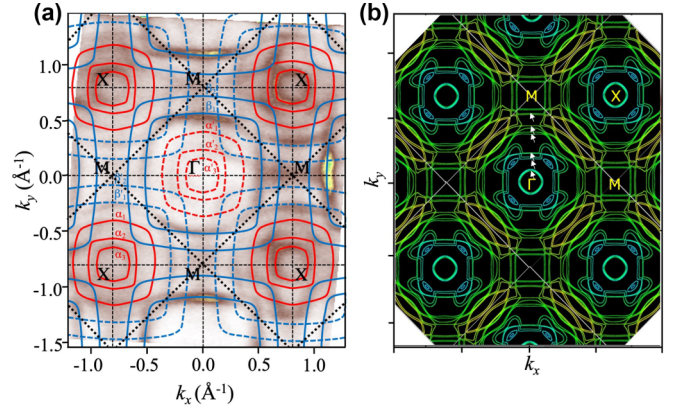


FIG. 4. (a) ARPES intensity mapping at  $E_F$  [same as Fig. 2(a)] overlapped with the experimental Fermi surface [solid curves; same as Fig. 2(d)]. (b) Calculated Fermi surface for bulk  $\text{Sr}_4\text{Ru}_3\text{O}_{10}$  at the  $k_z = 0$  plane in the nonmagnetic phase, obtained from the first-principles band-structure calculation, which includes the spin-orbit coupling. Possible trilayer-split  $\alpha$  and  $\beta$  bands are indicated by white arrows.

compare in Figs. 5(a) and 5(b) the ARPES intensity at  $E_F$  plotted as a function of in-plane wave vector at  $T = 30$  K obtained with a regular ( $200 \times 300 \mu\text{m}^2$ ) and a micro ( $12 \times 10 \mu\text{m}^2$ ) beam spot, respectively [the spot sizes are compared in the inset to Fig. 5(a)]. One can immediately recognize an apparent difference between them; ARPES intensity obtained with the microspot [Fig. 5(b)] resolves the fine structure of the Fermi surface in great detail in contrast to that with the regular spot [Fig. 5(a)]. In particular, the trilayer-split Fermi surface is clearly seen only when we utilize the microspot. Such a difference in the sharpness of Fermi-surface mapping is likely due to the improvement of effective  $k$  resolution by (i) the reduction of the spatial integration effect of photoelectron collection in the electron analyzer, and (ii) the reduction of the quasiparticle scattering rate by effectively avoiding crystal steps on the surface.

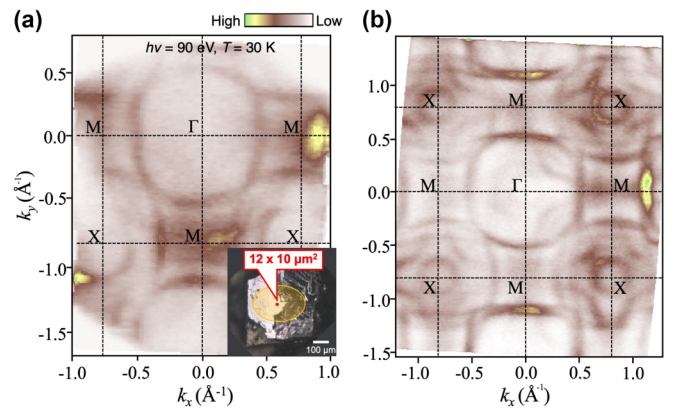


FIG. 5. (a), (b) Comparison of the Fermi-surface mapping for  $n = 3$  at  $T = 30$  K with  $h\nu = 90$  eV between a regular beam spot ( $200 \times 300 \mu\text{m}^2$ ) and a microbeam spot ( $12 \times 10 \mu\text{m}^2$ ). The regular and microbeam spots on the cleaved sample surface are shown by yellow and red circles in the inset to (a), respectively.



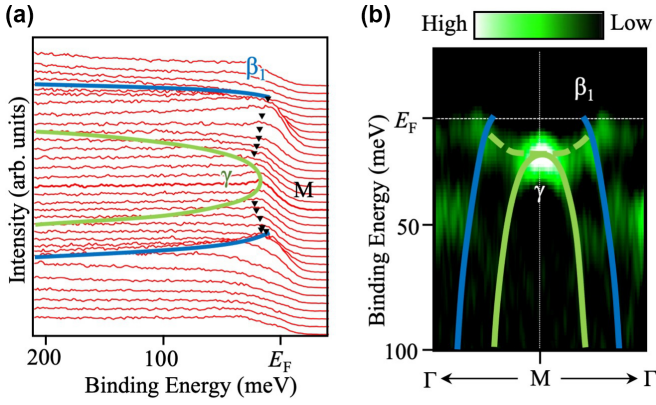


FIG. 6. (a) EDCs in the vicinity of  $E_F$  measured along the  $\Gamma M$  cut at  $T = 30$  K, obtained with  $h\nu = 90$  eV. (b) Corresponding second derivative intensity plot. Blue and green curves are a guide for the eyes to trace the band dispersion of the  $\beta_1$  and  $\gamma$  bands. The shallow electron band is also indicated by triangles and green dashed curves in (a) and (b), respectively.

### APPENDIX C: VAN HOVE SINGULARITY IN THE $\gamma$ BAND

To clarify the band dispersion of the  $\gamma$  band and its possible contribution to the mechanism of ferromagnetism, we show the energy distribution curves (EDCs) in the vicinity of  $E_F$  around the  $M$  point at  $T = 30$  K and the corresponding second derivative intensity plot in Figs. 6(a) and 6(b), respectively. Inside the  $\beta_1$  band, one can see a weak holelike feature topped at the binding energy ( $E_B$ ) of 15 meV [see also Figs. 2(c) and 2(f) in the main text] which is assigned to the  $\gamma$  band. One can also recognize a shallow electron pocket in the vicinity of  $E_F$ , whose bottom of dispersion at the  $M$  point appears to overlap with the top of the  $\gamma$  band to form a characteristic van Hove singularity at the  $M$  point. It is noted that a similar singular point was also seen for  $n = 2$  slightly below  $E_F$  [32], whereas it is absent below  $E_F$  for  $n = 1$  [30,34].

### APPENDIX D: ORBITAL CHARACTER OF ENERGY BANDS

To specify the orbital character of experimentally observed energy bands, we have carried out light-polarization-dependent ARPES measurements, and the result is shown in Fig. 7. In our experimental geometry, the emission plane of photoelectrons is in the  $x$ - $z$  plane (the  $x$ ,  $y$ , and  $z$  axes are defined in the figure) corresponding to the  $\Gamma M$  cut in the Brillouin zone which is along the  $a$  axis, as shown in Fig. 7(a). Here we focus on the  $\Gamma M$  cut, because the emission plane must be perpendicular to the  $\text{RuO}_2$  plane to simplify the discussion of the photoelectron matrix-element effect for the Ru  $4d$  orbital. In our geometry, linear horizontally (LH) polarized photons whose polarization vector lies in the emission plane enhance the photoelectron intensity from the orbitals with even parity (such as the  $d_{zx}$  orbital; see the top panel) with respect to the emission plane, when we take into account the photoelectron dipole matrix element term  $\langle \psi_f | A \cdot p | \psi_i \rangle$ , where  $A$ ,  $p$ ,  $\psi_f$ , and  $\psi_i$  represent polarization vector of incident photons, the momentum of photoelectrons, and the wave functions of the final and initial states, respectively. On the

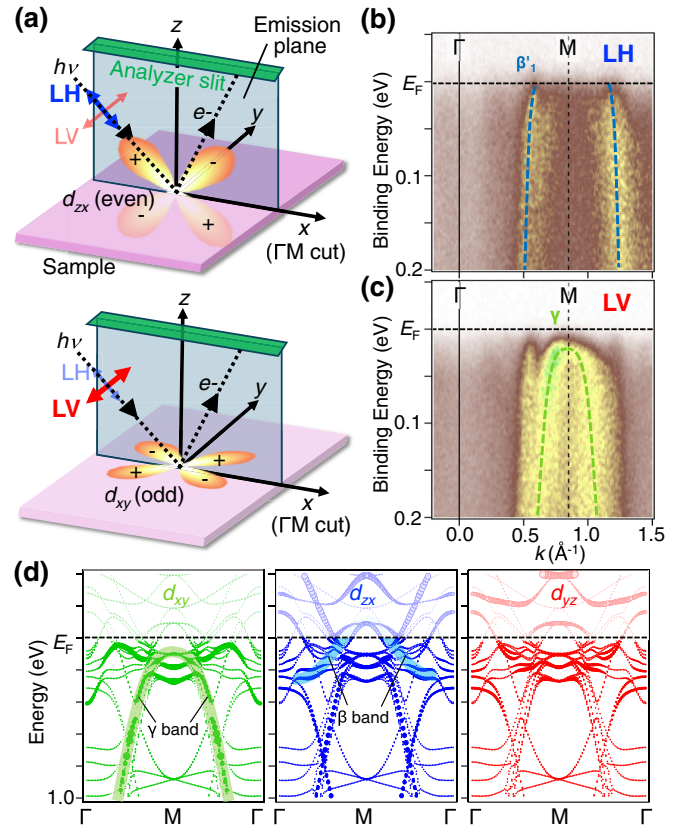


FIG. 7. (a) Schematics to show experimental geometry of the emission plane of photoelectrons, analyzer slit, light polarization (LH or LV), sample surface, and electron wave function for  $d_{zx}$  and  $d_{xy}$  orbitals. (b), (c) ARPES intensity at  $T = 20$  K along the  $\Gamma M$  cut measured with LH and LV photons, respectively. (d) Calculated band structure along the  $\Gamma M$  cut. Orbital projected weights for the  $d_{xy}$ ,  $d_{zx}$ , and  $d_{yz}$  orbitals are indicated by the size of circles.

other hand, linear vertically (LV) polarized photons enhance the photoelectron intensity from the orbitals with odd parity, i.e., the  $d_{xy}$  orbital (bottom panel). When the ARPES intensity along the  $\Gamma M$  cut is directly compared between the LH and LV photons in Figs. 7(b) and 7(c), the intensity of the  $\gamma$  band topped at the  $M$  point is relatively enhanced with the LV photons, whereas that of the  $\beta$  band outside the  $\gamma$  band is enhanced with the LH photons. This is consistent with our orbital assignment presented in Fig. 2 that attributed the  $\beta$  and  $\gamma$  bands to the  $d_{zx}$  and  $d_{xy}$  orbitals, respectively [note that the orbital character of the  $\beta$  band ( $d_{yz}$  or  $d_{zx}$ ) varies depending on the wave vector, and the  $d_{zx}$  component is expected to be dominant along the  $\Gamma M$  cut]. To further support these arguments from the calculations, we have calculated the orbital-projected band structure by the DFT calculation, taking into account that the Fermi-surface topology between the experiment and calculation shows a reasonable agreement (Fig. 5). As shown in Fig. 7(d), one may find dispersive features along the  $\Gamma M$  cut corresponding to the experimental  $\beta$  and  $\gamma$  bands (highlighted by shade) despite significant complication of the band structure due to the band folding and hybridization. Importantly, the  $\beta$  and  $\gamma$  bands show overall  $d_{zx}$  and  $d_{xy}$  weights,



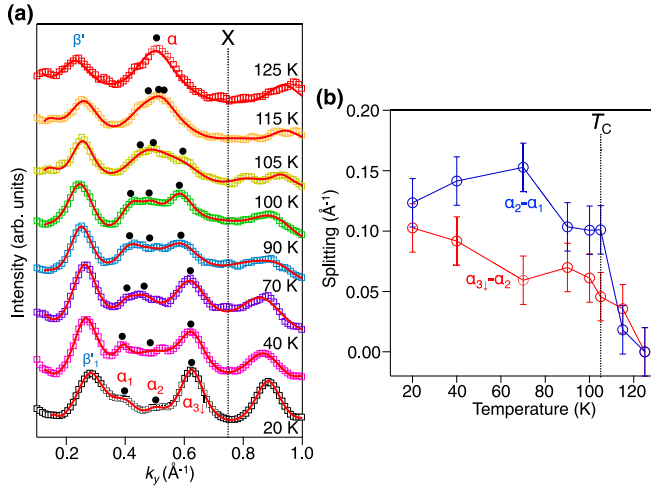


FIG. 8. (a) MDC at  $E_F$  for  $n = 3$  along the  $XM$  cut at various temperatures (open squares), together with the result of numerical fittings (red curves) by assuming multiple Lorentzian peaks and a moderate background. Black dots represent the  $k$  position of Lorentzian peaks. (b) Temperature dependence of the  $k$  separation (band splitting) between the  $\alpha_1$  and  $\alpha_2$  bands as well as the  $\alpha_{3\downarrow}$  and  $\alpha_2$  bands obtained from the numerical fittings.

respectively, consistent with the light-polarization-dependent ARPES measurements.

#### APPENDIX E: QUANTITATIVE ANALYSIS OF THE BAND SPLITTING

To show the presence of three  $\alpha$  bands and their evolution upon temperature variation, we have numerically fitted the momentum distribution curve (MDC) at  $E_F$  by assuming multiple Lorentzian peaks along a  $k$  cut (the  $XM$  cut) where the trilayer splitting was clearly observed at low temperatures. As shown in Fig. 8(a), the MDC at  $T = 20$  K is well reproduced by Lorentzian peaks for the  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_{3\downarrow}$  and folded  $\beta_1$  ( $\beta'_1$ ) bands with a moderate background, confirming trilayer splitting of the  $\alpha$  band. Upon increasing temperature, we found that the MDCs for the  $\alpha$  bands are well reproduced by assuming three Lorentzian peaks all the way up to 105 K (at  $T_C$ ). We also found that the MDC at  $T = 115$  K (10 K above  $T_C$ ) cannot be perfectly fitted with a single peak but could be better fitted with three peaks with smaller  $k$  separations compared to those at low temperatures. Finally, at  $T = 125$  K (20 K above  $T_C$ ), the MDC was reasonably fitted by assuming a single peak within experimental uncertainties (for the spectral component of each Lorentzian peak at representative temperatures, see Fig. 9). We plot in Fig. 8(b) the temperature dependence of  $k$  separation between the  $\alpha_1$  and  $\alpha_2$  bands as well as the  $\alpha_{3\downarrow}$  and  $\alpha_2$  bands, since they are a good measure of the magnitude of trilayer splitting. We found that the splitting seems to disappear not exactly at  $T_C$  but slightly above  $T_C$ . Based on this observation, it is expected that the counterpart

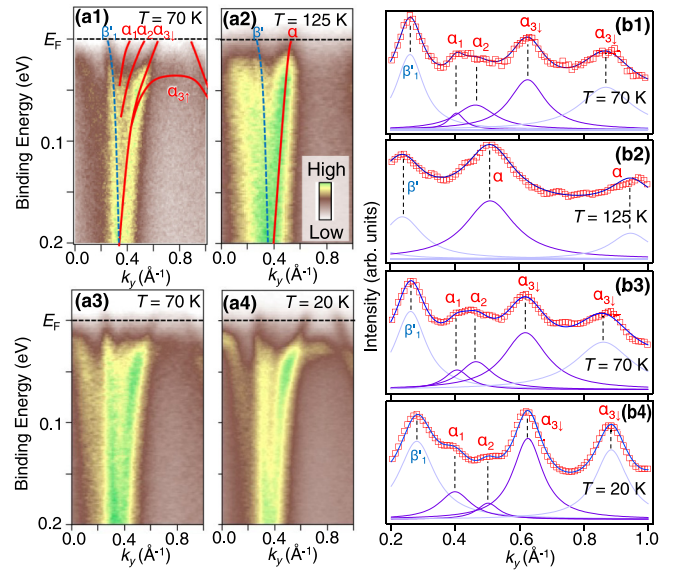


FIG. 9. (a1), (b1) ARPES intensity along the  $XM$  cut and corresponding MDC at  $E_F$ , respectively, obtained just after cleaving at  $T = 70$  K. Result of numerical fittings using multiple Lorentzian peaks (red curves) together with each Lorentzian peak are shown in (b). We carried out the ARPES measurements with the following cycle: (a1), (b1)  $T = 70$  K (just after cleaving)  $\rightarrow$  (a2), (b2)  $T = 125$  K  $\rightarrow$  (a3), (b3)  $T = 70$  K  $\rightarrow$  (a4), (b4)  $T = 20$  K.

of the  $\alpha_3$  band, i.e., the  $\alpha_{3\uparrow}$  band, located at  $E_B$  of  $\sim 40$  meV at the  $X$  point, disappears and merges into a single  $\alpha$  band slightly above  $T_C$ . This is indeed the case when we look at the temperature dependence of EDCs at the  $X$  point in Fig. 3(b) where the spectral weight of the  $\alpha_{3\uparrow}$  band monotonically reduces upon increasing temperature and finally disappears at around  $T = 115$  K. Our analysis thus implies that the trilayer splitting at first sets in slightly above  $T_C$ , and after the Stoner condition is satisfied by the splitting, the system undergoes a FM transition.

#### APPENDIX F: REPRODUCIBILITY OF THE TEMPERATURE-DEPENDENT BAND EVOLUTION

We have confirmed that the obtained data are reproducible upon temperature cycle. As shown by the ARPES intensity and corresponding MDC at  $E_F$  in Figs. 9(a1) and 9(b1), immediately after cleaving the crystal below  $T_C$  (70 K), one can recognize trilayer-split  $\alpha$  bands. After increasing temperature up to above  $T_C$  ( $T = 125$  K), the splitting disappears [Figs. 9(a2) and 9(b2)]. After cooling the sample again to  $T = 70$  K, the splitting recovers [Figs. 9(a3) and 9(b3)], and the splitting becomes more obvious at  $T = 20$  K [Figs. 9(a4) and 9(b4)]. This observation supports that the disappearance of the trilayer splitting well above  $T_C$  is reproducible and is not experimental artifacts such as thermal broadening and sample aging.

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