Four Levels of Scientific Modeling Practices in Expert Learning [1]

John Clement, U. of Massachusetts, Amherst, MA

6/17/17

Presented at the annual meeting of the National Association for Research in Science Teaching, San Antonio, TX.

Abstract

This paper describes model construction practices used by scientifically trained experts. Our work on science experts has involved analyzing data from videotaped protocols of experts thinking aloud about unfamiliar explanation problems. These studies document the value of nonformal heuristic reasoning processes such as analogies, identification of new variables, Gedanken experiments, and the construction and running of visualizable explanatory models. Although theses processes are less formal than formal deduction or induction or statistical inference procedures, the case study analyzed here shows that they can lead to real insights and conceptual change. At a larger time scale, the subject went through model evolution cycles of model generation, evaluation, and modification that utilized the heuristic reasoning processes above. In addition, the prevalence of imagistic simulation as an underlying foundation in these episodes suggests that it may be important to pay greater attention to an imagistic level of processing in the analysis of expert thinking. Larger time scale modes of model evolution and model competition were also evidenced. The analysis leads to four levels of processes or practices:

IV. An overarching set of Model Construction Modes, primarily alternating between Model Evolution, in which a model is improved, and Model Competition, in which two or more models compete.

III. Modeling (GEM) Cycle process of Model Generation, Evaluation, and Modification at a Macro level, as shown in Figure 10.

II. Nonformal Reasoning Processes at a Micro level: e.g. analogy, running a model, identifying a new variable, and conducting a Gedanken experiment.

I. Underlying Imagistic process including Imagistic Simulation that may have been occurring within all of the above processes.

To our knowledge these four levels of processes have not been analyzed together in the past. They complement empirical processes of discovery, experimentation, and evaluative argumentation documented by others. Diagrams of how the above processes interact may give us some new ways to picture the roles of nonformal reasoning and imagistic processes during qualitative model construction. We call the set of processes at all four levels a 'Modeling Practices Framework'. Processes at a lower level serve as subprocesses for the level above it in this framework. Each level has multiple "things to try" to achieve tasks at the level above it. Thus the framework is an organized but flexible structure of heuristic processes. This lies between and contrasts with those who would describe theory making in science as either 'anarchistic', with no method structure, or 'algorithmic', with fairly standardized procedures.

---

[1] This material is based upon work supported by the U.S. National Science Foundation under Grants DRL- 1222709 and DRL-1503456, John J. Clement, PI. Any opinions, findings, conclusions or recommendations expressed in this paper are those of the author and do not necessarily reflect the views of the National Science Foundation.

INTRODUCTION

Aims

This study focuses on several questions related to the nature of scientific thinking: What scientific practices are used by experts during model construction? Are nonformal as well as formal thought processes used? Are imagistic processes used? Are these practices collectively organized into a 'scientific method' or not? Studies in history of science have paved the way for a model based learning approach to understanding science, e.g., Giere (1988) and Nersessian (2008). Previous studies have examined elements of the problem of how processes like analogy, imagery, and model construction can be used in science. Nersessian (1992), Trickett and Trafton (2002), Griffith, et al. (2000), Chan, et al. (2012), and Dunbar (1999), have described processes by which experts utilize analogies to construct models for conceptually difficult problems. The present study attempts to complement their approach by analyzing data from video-taped protocols of experts solving explanation problems (Clement,1989, 1994). Several levels of expert modeling practices will be identified.

Work in science education also contributes to a model based theoretical framework, such as Glynn & Duit (1995), Duschl & Osborne (2002), Gilbert (2004), Quintana, Reiser, Davis, Krajcik, et al (2004), Clement (2008), Osborne, Erduran, & Simon (2004) and others. In this framework, thinking about processes of model construction helps us organize and clarify the purpose of narrower, more specifically targeted teaching strategies. Our rationale for this expert study is the hope that comparisons to expert reasoning can sharpen our ways of describing the scientifically relevant reasoning of students and teachers during discussions, and help in describing important teaching strategies for supporting such reasoning.

The NGSS (2013) standards call for students to be engaged in scientific practices, including the practice of model construction. The standards are a big step forward in establishing a framework describing what competence in science should look like at different age levels but these are necessarily painted with a broad brush. We need much more detailed descriptions of scientific modeling practices.

Is there a Structured Scientific Method?

But there is a longstanding problem in how to describe the structure of science practice. That there is no such thing as 'Scientific method' (or 'mathematical method)." is a common position in science education. This goes along with the idea that: a circular flow pattern of five or six major practices may be useful for discussion with young students, but such a flow pattern is too rigid and does not allow for flexible movement between practices if we are really trying to understand and support scientific practice.
This could mean that there is no reason to search for practices, because there are no common practices. However, along with NGSS, many would probably agree that one can at least list some useful practices. So the real question is whether the following position is true:
"One can list many individual practices, but there is no systematic organization or ordering to the practices --no 'scientific method' for how they are used." In this paper, I attempt to outline a theoretical, organized structure for scientific practices, starting from a study of scientifically trained experts thinking aloud. The question is whether one can discern a structured framework of practices that lies in between the extreme positions of "no method' and a "consistent, deterministic, fully specified, algorithmic method"-- whether one can discern a framework with some structure, but not so much structure that it cannot account for creativity and flexibility within real scientific investigations.

It is true that there are certainly some historians of science (e.g. Feyerabend [1993]) that are skeptical that one can describe scientific thinking as a well defined, deterministic, serial procedure, even with a complex flow chart. But does this mean there is no structure in scientific or mathematical practice at all? By analogy, we note that pioneers such as Polya (1957) have proposed structured lists of heuristics for mathematical problem solving. Instead of a rigid procedure that should work deterministically, these are 'suggested things to try at different stages of problem solving', none of which are guaranteed to work. So Polya proposes some structure, but it is minimal: he simply divides the practices into several groups, to be tried at different stages of problem solving. But the collections of heuristics at different stages do represent his experience with practices that foster progress on mathematical problems. Many of these, such as breaking the problem into parts, are themselves nonformal in the sense that they do not conform to formal inductive or deductive logics.

Similarly, Darden (1991) has proposed semi-structured sets of heuristics for scientific thinking on the basis of analyzing the history of genetics. These again are 'suggested things to try' and only loosely organized collections for several major objectives. In this study we attempt to identify an organized framework for modeling practices, and it remains to be seen how much structure we should include in it.

Another very large hole in our field concerns the role of imagery and mental simulation. Finke (1990) has shown how lay subjects can combine images in novel ways to produce new images with new interpretations. Barsalou (1999) has described a theory of perceptual symbols which represent schematic elements of perceptual experience and that can be integrated to produce simulations. Although Lowe (2004), Stieff (2011), Trickett and Trafton (2002), Hegarty et al. (2003), and Schwartz, and Black (1996) have made important initial progress there is still very little information about how imagery and imagistic processes can support scientific modeling. Nersessian (2008) has suggested that mental simulation may be a central process scientists such as Maxwell used to construct and run scientific models. Tweney (1996) has given an historical account of Faraday's use of presymbolic processes of running experiments 'in the mind's eye'. But too little research exists on the collective relationships between scientific model construction, heuristic reasoning, and the use of imagery or mental simulation.

Purpose

Using data from expert think aloud protocols, I will attempt to construct a framework that describes expert reasoning and modeling practices at several levels. In developing a framework for modeling practices, I will attempt to use evidence from gestures and other protocol observations to build imagery and mental simulation into the framework as a lowest-level, foundational layer of modeling practices.

In the companion papers from our symposium cited at the end of this paper, the expert scientific practices (effective science learning processes) identified in this study will be compared to practices that experienced science teachers attempt to foster in whole class discussions, and some important similarities will be identified.

Method

The method in this study consists of a case study of expert reasoning to identify major scientific modeling practices at different levels. The study is qualitative, generative and descriptive and is not intended to project frequencies of strategies to a population. It is intended to (1) help us define new constructs for modeling practices to look for in model construction behavior of both experts and classrooms-- constructs that have their initial grounding in video tape case study data; (2) account for any patterns of use of the practices by hypothesizing a certain amount of structure or cycling. The episodes analyzed should also provide initial existence demonstrations for these phenomena. The study builds on an earlier study in

Clement (2008), but integrates more processes and levels into a more comprehensive framework. Eleven experts were asked to think aloud while working on unfamiliar explanation problems. Experts were professors or doctoral students who had passed comprehensives in technical fields. The interviewer used only minimal probing for clarification. Protocol analysis was conducted via a constant comparison method (Strauss and Corbin, 1998), used here to develop new observational and theoretical constructs for describing reasoning and learning processes. Extended individual case studies of problem solutions/explanations examined how several kinds of reasoning processes were combined together to support each other.

To develop the framework, I will provide initial documentation of each practice/process using examples from transcripts. Subjects were recorded while thinking aloud about the problem illustrated in Figure 1; I will call this the target problem or target case.

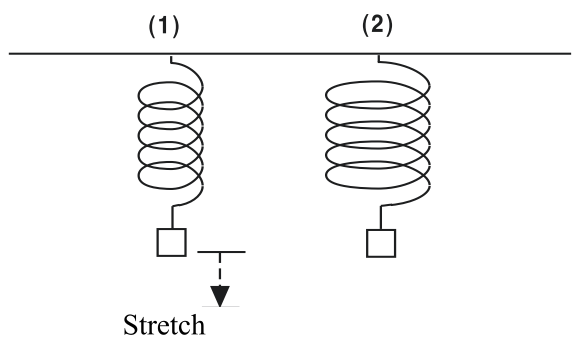

Figure 1: Spring Problem: *A weight is hung on a spring. The original spring is replaced with a spring made of the same kind of wire, with the same number of coils, but with coils that are twice as wide in diameter. Will the spring stretch from its natural length more, less, or the same amount under the same weight? (Assume the mass of the spring is negligible.) Why do you think so?*

Foreshadowing of Initial Findings: Nonformal Reasoning Strategies Used

Although observation and experiment are extremely important in science, this expert study focused mostly on the less studied rationalistic (as opposed to empirical) side of scientific thinking. Materials for experimental observation were not available in the interviews; nevertheless subjects used many rationalistic reasoning strategies to generate predictions, models, and explanations successfully, including Gedanken (thought) experiments, which are extremely interesting because they *feel* empirical to the scientist. The case studies of solutions also document the presence and import of *model construction cycles* and other *nonformal heuristic reasoning processes* such as *analogies*, *concept identification or differentiation*, *extreme cases*, as inputs to *the constructing and running of visualizable explanatory models.* The transcripts also provide existence demonstrations of many examples of the use of imagery and *imagistic simulation* occurring in conjunction with the heuristic reasoning processes above (Clement, 2008). Imagistic simulations were evidenced by subjects making a prediction about a system's behavior accompanied by one or more imagery indicators, such as spontaneous imagery reports and/or depictive gestures.

Case Study

Using an Analogous Case: Long and Short Bending Rods

In this paper due to space limitations I will focus on the solution of a single subject S2, who produced the most productive solution to the Spring Problem. Although the subjects were experts in technical fields, none were mechanical engineers, and they were working at the frontier of their own personal knowledge on an unfamiliar problem. From this it is plausible that their methods have some overlap with those used on the frontier of science. For the spring problem, S2 first generated an **analogous case** in which he predicted that a long horizontal rod fixed at one end would bend more than a short one (with the same weight attached to the other end of each rod), inferring that segments of the wider spring would bend more and therefore stretch more (It is true that the wide spring stretches more. Figures 2 to 6 below are cleaned-up versions of drawings made by the subject here. The full transcript is quite long; therefore verbatim excerpts are presented here. Brackets in transcript indicate action descriptions or my comments.) He says:

(1) "I have one good idea to start with; it occurs to me that a spring is nothing but a rod wound up uh, and therefore maybe I could answer the question for a rod."

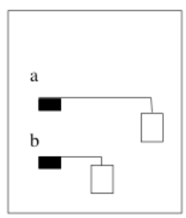

Figure 2: Bending Rods Analogy

and later:

(2) “I have a strong intuition--<u>a physical imagistic intuition</u>--that this [points to longer rod] will bend a lot more than that [shorter rod] will.”

A subject working on a target problem uses an analogy when they generate or recall a case that is significantly different from the target case, but that also may have structural similarities to the target, so that findings from it maybe applicable to the target case. In this instance, the analogous case of the bending rods is anchored in a physical intuition that appears to involve imagery. I will use underscored type to identify observations that provide some evidence for imagery (both kinesthetic and visual) use, such as the <u>spontaneous imagery report</u> in excerpt 2 above. Here the analogy gives S2 the correct prediction for the spring, but he still has doubts about his understanding of the system.

Running a Model and Checking it against Known Constraints: Bending Seen As Inconsistent

Once S2 began to take seriously the idea that bending could actually be occurring in the spring wire, we say that he begins to use bending as an *explanatory model* for understanding how the spring is stretching, rather than using it as just a playful, expedient analogy for getting a prediction. An explanatory model is a (usually hidden) mechanism that explains why the system behaves the way it does, by explicating the structure or dynamics of the system. However, S2 quickly became concerned about the appropriateness of bending as an explanatory model because of the apparent lack of a match between the fact that bending will produce *an increasing slope in the rod*, whereas *there is no increasing slope in the wire in a stretched spring*. One can visualize this discrepancy here by thinking of the increasing slope a bug would experience walking down a bending rod and the constant slope the bug would experience walking down the helix of a stretched spring. (This is my own descriptive analogy for purposes of clarity- not the subject's.) (Another way for the reader to see this problem is to note that the bending model predicts that the slope of the wire and the distance between coils will increase as one goes down the spring, as shown in Figure 3. Yet this does not happen in real springs.)

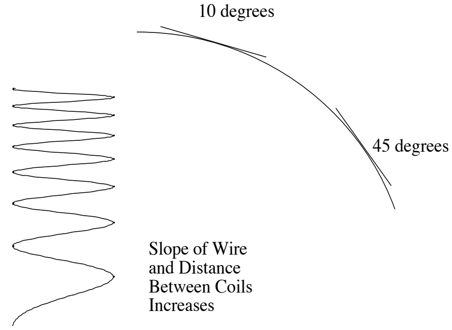

Figure 3: asymmetric spring

The discrepancy in slope led him to question whether the bending rod was an adequate explanatory model for the spring.

(3) "But then it occurs to me that there's something clearly wrong with that [bending rod] metaphor, because ..it would (raises hands together in front of face) droop (<u>moves r. hand to the right in a downward curve</u>) like that, its slope (<u>retraces curved path in air</u> with l. hand) would steadily increase, whereas... a real spring..would just stretch uniformly... And the slope would be constant all the way down

Later he says:

….. "You get a spring which stretches more and more at the bottom. The loops are wider apart there. But that isn't the case...they're uniform."

This appears to be a case where he imagines dynamically or "runs" the idea of bending taking place in the spring as it stretches, as shown in Fig. 3 above. That is, he examines the consequences of **running a model**-- the "bending model"-- in consecutive segments of the spring, inferring that you'd "get a spring which stretches more and more at the bottom." He then decides that this is in conflict with a property that he knows from prior observations: a spring stretches very nearly uniformly. We say that a subject runs an explanatory model when they use imagistic simulation to animate the model and make a prediction for an outcome of the model. Evidence of this would be relevant imagery indicators such as imagery reports or depictive gestures occurring near a prediction that comes from an expressed explanatory model. Examples of such gestures are underlined in Episode 3. (There is not space for a review here, but an increasing variety of studies of depictive gestures suggest that they are expressions of core meanings or reasoning strategies and not simply translations of speech. Others indicate that the same brain areas are active during real actions and corresponding imagined actions.)

This anomaly or mismatch with a prior observation appears to bother him considerably and drives further work on the problem. Certainly an important positive feature of the above section is the subject's ability to criticize his own initial model. Several other subjects who thought of the bending rod model did not make this interesting criticism of it.

Torsion Insight: Identifying a New Variable

After a (half hour) period of frustration in trying to make the bending model work, this subject finally produces an extremely productive **analogy** when he generates the idea of the hexagonally shaped coil in Figure 4 and moves from there to the idea of the square shaped coil in Figure 5.

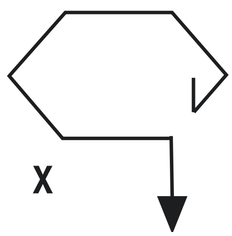

Figure 4

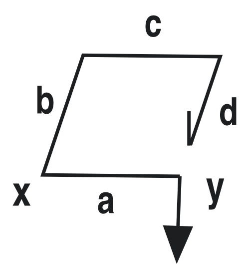

Figure 5

(4) "Aha! Now this is interesting. I imagined; ...the square is sort of like a circle and I wonder....what if I start with a rod and bend it once (places hands at each end of rod in Figure 2 and motions as if bending a wire) and then I bend it again.

What if I produce a series of successive approximations to... the circle by producing a series of polygons! Maybe that would clarify because maybe that, that's constructing a continuous bridge, or sort of a continuous bridge, between the two cases [the rod and the coil]. Clearly there can't be a hell of a lot of difference between the circle and say, a hexagon..."

These analogies lead him to a major breakthrough in the solution, which corresponds to the way engineering specialists view springs, as follows:

(5) "Now that's interesting. Just looking at this [hexagon] it occurs to me that when force is applied here, you not only get a bend on this segment, but because there's a pivot here (points to x in Figure 4), you get a torsion effect...

Aha! Maybe the behavior of the spring has something to do with twist (moves hands as if twisting an object) forces as well as bend forces (moves hands as if bending an object). That's a real interesting idea. That might be the key difference between this [bending rod] which involves no torsion forces, and this [hexagon]. Let me accentuate the torsion force by making a **square** where there's a right angle.

(6) Now [in Figure 5]...I have two forces introducing a stretch. I have the force that bends this...segment [a] and in addition I have a torsion force which twists [segment b] at vertex, um, X... (makes motion like turning a door knob with one hand)"

Here he appears to imagine the situation in Figure 5 as if side 'a' were a wrench acting at x to twist the end of side 'b' through an angle, while 'c' keeps the other end of 'b' from turning, resulting in a twisting deformation of the metal in 'b'. That is, pulling down at 'y' twists the metal throughout side 'b' like twisting a piece of taffee; except that unlike taffee, side 'b' is made of resilient metal so that it would spring back and untwist if one were to remove the downward force at point 'y'. (The same would be true for all other adjacent rod pairs.)

Twisting of the wire and the resulting torsional strain is in fact the most important source of stretching and restoring force, in the analysis of spring behavior as understood by engineers. Its discovery here represents a scientific insight in identifying a new variable or feature and causal mechanism for stretching. (See the appendix for a primer on the concepts of torsion and toque.) Later the subject draws Figure 6 to explain how a downward force F would produce torsion and twisting in segment 'w'.[2]

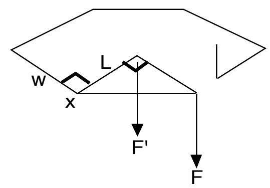

Figure 6: Torsion in w produced by torque from an adjacent segment in hexagonal coil

Using Imagery and Imagistic Simulation to Examine an Analogous Case

The subject still needs to determine what torsion in the square spring would predict for the answer to the original problem. The simpler analogous case he uses to consider this hypothesis is to compare a long and a short rod, each of which is twisted with the same torque (twisting force).

(7) "Now making the sides longer certainly would make the [square] spring stretch more.... the longer the segment (moves hands apart) the more the bendability (moves hands as if bending an object)...

(8) Now the same thing would happen to the torsion I think, because if I have a longer rod (moves hands apart), and I put a twist on it (moves hands as if twisting something in Figure 7), it seems to me--again physical intuition--that it will twist more, hush (looks to side and pauses 4 sec.) I'm- I think I trust that intuition… I'm (raises hands in same position as before and holds them there continuously) imagining holding something that has a certain twistyness to it, a-and twisting it…."

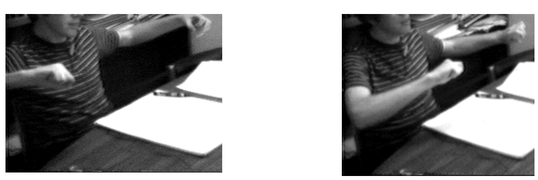

Figure 7: Evidence for imagistic simulation: Expert making spontaneous depictive gestures as he makes a prediction. "If I have a longer rod (moves hands apart), and I put a twist on it (moves hands as if twisting a rod), it seems to me--again, physical intuition--that it will twist more. "

This is an example of making a prediction from an **imagistic simulation,** evidenced by the subject making a prediction about a system's behavior accompanied by one or more imagery indicators, such as spontaneous imagery reports and/or depictive gestures.

[2] (Note: In a spring suspended from an arm to the center of the coil, the torque would be applied from there to point x.)

(Note: S2 also is encouraged by seeing that a spring made of square coils will stretch with an equal distance between the coils, unlike the false situation he imagined in Figure 3, a spring with increasing slope and an increasing distance between the coils toward the bottom. That is, when he "runs" the square coil being stretched (with gestures), there may be bending in each side, but because bending and slope "start over from zero" at each corner, the slope from the bends does not accumulate by adding. The same would be true for torsion effects. The square coil is a new case in which the increasing slope difficulty does not occur, suggesting it is a way to resolve his previous anomaly. This was another example of 'Running a Model to Evaluate It.')

Summary of Nonformal Heuristic Reasoning Processes Identified so Far

The above episodes include examples of the following Nonformal Reasoning processes: **Using an Analogy** (e.g. the bending rod, and twisting rod), **Running a Model to Evaluate it** (e.g. the bending model is run within the spring in transcript Episode 3; and he "runs" the square coil mentally to evaluate it for asymmetric stretching), and **Running a System and Recognizing a New Feature** in Episode 5, followed by **Adding a new Model Element** to his model of the spring. There is also evidence within these cases for the additional involvement of an **Imagistic Simulation** process as a subprocess operating within all of the processes above.

<u>Overall pattern of model evolution.</u> These processes can be seen as contributing to an overall pattern of model evolution as shown in Figure 8, where time runs from left to right. So far we have evidence for only the first two models shown in the top row, starting with an initial model where the spring wire is thought of as bending during stretching. This model is eventually modified to form a second model where the spring wire is both bending and twisting.

Below that, the next level shows Nonformal Reasoning processes mentioned above that help to evolve the model. This row contains both generative processes such as analogy that contributed to the models by suggesting elements of the mechanism in the spring, and evaluative processes such as thought experiments that support or conflict with the current model as it evolves. These are reasoning strategies that are less formal than deduction and so we refer to them collectively as Nonformal Reasoning Strategies.

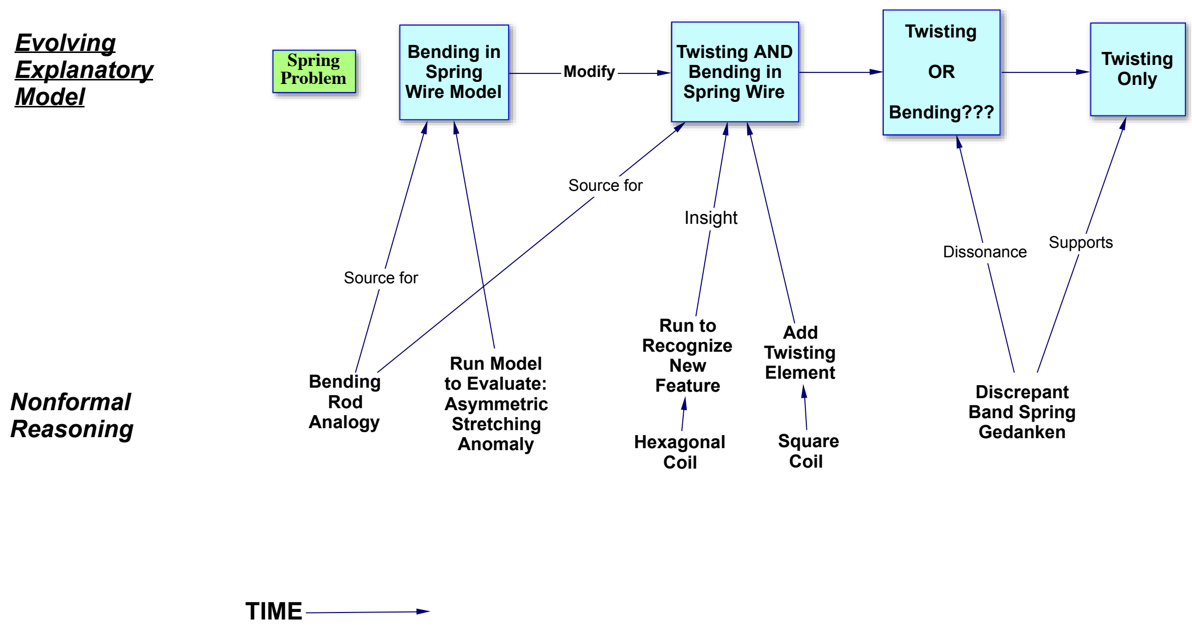

Fig. 8 Nonformal Reasoning Strategies used for Model Construction by an Expert

Evaluative Gedanken Experiment: The Band spring

The subject next asks whether the deformation in real springs could be all bending or all twisting. The next 'move' shown in Level 2 in Figure 8 is what I call an **evaluative Gedanken experiment** (Clement, 2002, 2008, 2009). There is no consensus on a precise definition for the term "Gedanken experiment," but I use the term "evaluative Gedanken experiment" here to mean a thought experiment especially designed or selected by the subject to help *evaluate* a concept, model or theory. S2 generates the case of a 'band spring' made of a vertically oriented band of material shown in Figure 9. (The reader might imagine a thin metal strip wound to make a spring, say, 3" wide.) This invented case allows him to test whether bending is as necessary as twisting as the primary mechanism at work in a spring.

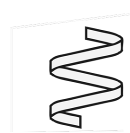

Figure 9: Gedanken experiment: A band spring that can twist but not bend

(9) " How about a spring made of something that can't bend. And if you showed that it still behaved like a spring you would be showing that the bend isn't the most important part. Or isn't particularly relevant at all maybe somehow…How could I imagine such a structure?... I'm thinking of something that's made of a band… we're trying to imagine configurations that wouldn't bend. Since it's cross section is like that (see Figure 9) ... it can't bend in the up-down (indicates up/down directions with hands) direction like that because it's too tall. But it can easily twist (motions as if twisting an object)."

Given the imagery report here, I interpret this to mean that the subject imagined that such a spring would still be quite stretchable even though the band "cannot bend in the up-down direction," challenging the necessity of bending as not "particularly relevant at all" to stretching. In this type of evaluatory

Gedanken experiment he designs a special case where the bending model yields a prediction, (predicts no stretch) but where he also has some other independent source of information that can evaluate that prediction (physical intuition predicts that it will stretch). This is an evaluative Gedanken experiment because it is designed by him to help him test a model. At this point S2 appears to be shifting from a model of the spring wire both bending and twisting to a model where it is undergoing twisting alone. This is shown in Figure 8 as the fourth model in the Evolving Explanatory Model sequence in the top row of Figure 8.

GEM Cycles

Looking over the whole solution so far, we can see the beginning of a pattern of a **Model Generation, Evaluation, and Modification** cycle (or **GEM cycle**), as shown in Figure 10. The initial bending model was generated, then evaluated negatively, then modified to include twisting, evidenced as follows.

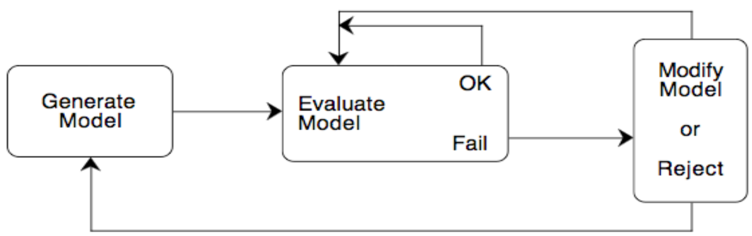

Figure 10: GEM cycle of model generation, evaluation, and modification

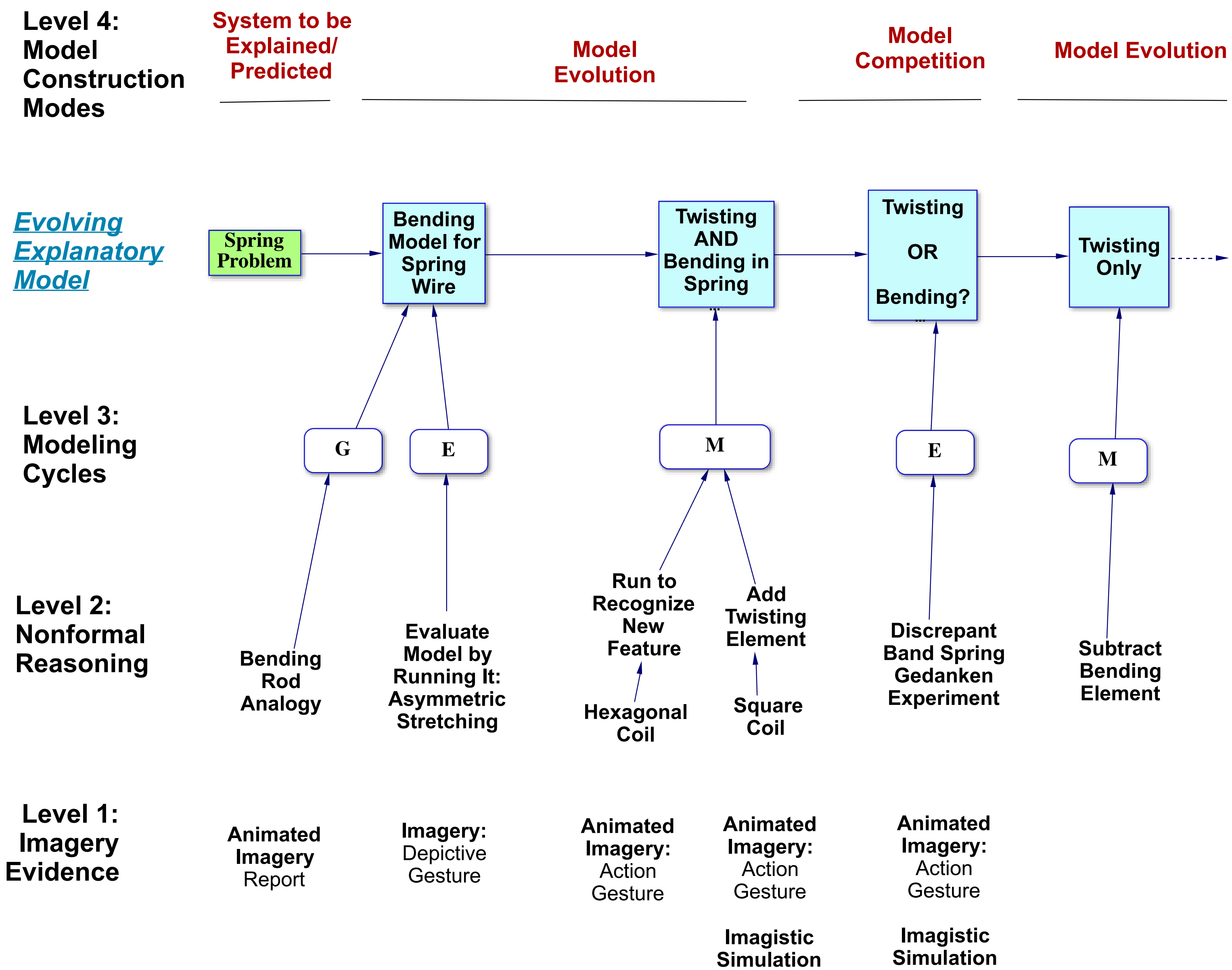

Figure 11: Time Sequence Diagram For Four Levels of Model Construction Practices Used by an Expert

Generation (G): The subject refers to a new: explanatory model; partial model; or conception.
Evaluation (E): The subject refers to an evaluation, judgment, refutation, criticism, support, or endorsement of a particular explanatory model or model element.
Modification (M): The subject refers to a change, adjustment, or modification to a theory or model. This may involve an alteration, variation, subtraction, or addition.

These processes are labeled G, E, M in Figure 11, in the row titled Level 3: Modeling Cycles. Figure 11 is an expanded version of Figure 8. It attempts to sort the variety of strategies S2 has used into four levels labeled on the left. Again time is moving from left to right.

At this point, because of the 'Band Spring' thought experiment, S2 appears to be shifting from a model of the spring wire both bending and twisting to a model where it is undergoing twisting alone. This is shown in Figure 11 as the fourth model in the Evolving Explanatory Model sequence in the second row. The new "Twisting Only" model is shown as the result of an additional cycle of evaluation and modification in the figure, as shown at Level 3. After that the subject makes further modifications and refinements that are more technical and that I do not have space for here.

Later the subject distinguishes between confidence in the *answer* to the spring problem, which has been quite high, and confidence in his *understanding* of it, and estimates that his torsion analysis has increased his understanding of the system from “way, way down” up to “like, 80%”. This completes the sections of protocol discussed in this paper.

Overall Pattern of Model Evolution Cycles and Levels of Processing

According to the analysis in Figure 11, the expert examined in this section appeared to use the following different levels of strategies :

IV. An overarching set of Model Construction Modes, primarily alternating between Model Evolution, in which a model is improved, and Model Competition, in which two or more models compete.

III. Modeling (GEM) Cycle process of Model Generation, Evaluation, and Modification at a Macro level, as shown in Figure 10.

II. Nonformal Reasoning Processes at a Micro level: e.g. analogy, running a model, identifying a new variable, and conducting a Gedanken experiment.

I. Underlying Imagistic processes including Imagistic Simulation that may have been occurring within all of the above processes.

Level I: Imagistic Processes

So far levels II and III have been discussed. Evidence for Imagery at Level I was seen through depictive gestures and other indicators throughout the protocol. “Mental imagery” has been defined as "the mental invention or recreation of an experience that in at least some respects resembles the experience of actually perceiving an object or an event" (Finke, 1989, 1990). Here we add the idea that this can include kinesthetic imagery of bodily forces or motions. Imagery may be useful in higher order cognition because it is capable of representing, in at least a skeletal manner, aspects such as: (a) the shapes of objects; (b) spatial relations among them , and also, (c) actions, object movements, changes, or interactions over time (in which case we refer to Animated Imagery as opposed to Static Imagery).

Major categories of imagery processes we have developed are (see Clement, 2008; Stephens and Clement, 2017):

Using (Static) Imagery
Using Animated Imagery
Using Imagistic Simulation
Using Drawings to Support Imagery
Using Imagery Enhancement

The use of imagery processes during protocol segments is recorded in Table 1. Imagistic processes are hidden so it is a challenge to develop criteria for evidence for them.

Static and Animated Imagery. Clement (2008) developed a set of observable indicators that can provide evidence for a subject using Static Imagery in protocols, including verbal reports of imagery ("I'm imagining a.."), and depictive gestures. When these indicate motion, changes, or interactions they provide evidence for Animated Imagery, as does describing projecting human actions into a situation as if they were conducted by a person ("if you were that molecule, you would be hitting all the others").

Imagistic Simulation is defined as the subject using imagery to make a prediction (or explanation). This is evidenced by the subject making a prediction about (or explanation of) a system's behavior accompanied by one or more imagery indicators, such as spontaneous imagery reports and/or depictive gestures.

Imagery Enhancement occurs when a subject chooses or modifies a case to make imagery of the system easier to think with-- as evidenced by the subject mentioning adjusting the size, perspective, simplicity, or alignment of a system or mentally adding 'markers' on it, in a way that would make it easier to imagine details that are not physically perceived. An example of Imagery Enhancement occurs in Episode 10 when the subject generates an extreme case of twisting a very short rod. This comes immediately after Episode 8 when he said "I'm i*magining holding something that has a certain twistyness to it, a-and twisting it (with hands held in air as if holding a rod 2 ft. long"….* "

(10) "Now I'm confirming (*moves clenched right hand toward clenched left hand*) that, by using this method of limits. As (*moves right hand to left hand until they almost touch at the first word "closer*") I bring my hand up closer and closer (keeps holding clenched right hand next to left hand, making slight vertical punctuating motions at the words "hold", "clearly", "harder", and "harder") to the original place where I hold it, *I realize very clearly that it will get harder and harder to twist*. So that confirms my intuition so I'm quite confident of that."

Here the subject adjusts the size of the rod he is twisting mentally and immediately experiences a rise in confidence in his prediction, while also giving evidence that he is using imagery to make the prediction. We infer that exaggerating the contrast between long and short rods has made the imagistic prediction more vivid and confident via Imagery Enhancement.

Transcript Line	**Specific Indicator of Imagistic Processing**	**Larger Imagistic Processing Category**
a physical imagistic intuition--that this [points to longer rod] will bend a lot more	Makes drawing Dynamic Imagery Report Describes non visible concrete feature in drawing	**Imagistic Simulation**
It will droop (moves r. hand to the right in a downward curve) like that	Depictive gesture	**Imagery**
bend it once (places hands at each end of rod in Figure 2 and motions as if bending a wire) and then I bend it again	Depictive Action gesture	**Animated Imagery**
5 looking at this [hexagon] it occurs to me that when force is applied here, you not only get a bend on this segment, but because there's a pivot here (points to x in Figure 4), you get a torsion effect...	Describes non visible concrete feature in drawing	**Drawing Supports (Animated) Imagery**
The spring has something to do with twist (moves hands as if twisting an object) forces	Depictive Action Gesture	**Animated Imagery**
as well as bend forces (moves hands as if bending an object).	Depictive Gesture	**Animated Imagery**
"Now making the sides longer certainly would make the [square] spring stretch more...	Describes non visible concrete feature in drawing	**Imagistic Simulation; Drawing Supports (Animated) Imagery**
the longer the segment (moves hands apart) the more the bendability (moves hands as if bending an object)..	Depictive action gesture Prediction	**Imagistic simulation**
I put a twist on it (moves hands as if twisting something in Figure 7), it seems to me--again physical intuition--that it will twist more	Depictive Gesture Prediction	**Imagistic Simulation**

Twisting an Extremely short rod	Adjusts size Prediction	**Imagery Enhance-ment, Imagistic Simulation**
we're trying to imagine configurations that wouldn't bend. Since it's cross section is like that (see Figure 9) ... it can't bend in the up-down (indicates up/down directions with hands) direction like that because it's too tall. But it can easily twist (motions as if twisting an object)."	Imagery report Action Gesture Action Gesture	**Imagistic simulation**

Table 1. Imagery Indicators and Major Types of Imagistic Processing in the Protocol.

Use of Drawings to Support Imagery. Figures 3 to 6 and 9 are cleaned up versions of the subject's drawings. Building on Stephens and Clement (2017), we use certain kinds of evidence to argue that they were used to support certain internal imagery processes and to focus attention. However, we are conservative in that we do not count someone simply looking at a drawing and talking about visible features in it as evidence for use of imagery. On the other hand, while drawings may show various shapes explicitly, other concrete properties such as 'twisting' are harder to show. When the subject speaks of such a *hidden* concrete feature over the drawing, we take it as evidence of imagery use. In particular, the drawings cannot show animation, and so in such cases we infer that animated imagery was used, as evidenced by gestures over a drawing in some cases, and by the subject's speaking of dynamics over the drawing in other cases. We also count initiating or modifying a drawing as an expression of imagery or an imagery support strategy, but I have not recorded that in Table 1 because the subject initiates so many drawings.

These Imagistic Process Categories are shown at Level 1 at the bottom of Figure 11 as occurring throughout the protocol. Furthermore, they occur contiguously with most of the nonformal reasoning episodes shown in Level 2 of Figure 11. We find this consistent with the hypothesis that Level 1 imagistic processes are being used as subprocesses of Level 2 non-formal reasoning processes during these reasoning episodes, and therefore that Level 1 is also underpinning and affecting the higher level processes at Levels 3 and 4.

Level IV: Model Construction Modes

We noted the importance of the GEM cycle earlier as a key pattern in this protocol. We call this overall pattern "Model Evolution" because a single model is gradually improved over a period of time. But what about the moment where the subject realizes that he has *two* models and that only one model (twisting or bending) may be correct? We cast this as the beginning of a different major mode of operating or modeling called 'Model Competition'. There are several possible modes other than Evolution, shown at Level 4 in Figures 11 and 13 and are defined as follows.

Description of Pattern to be Explained or Predicted. The subject describes or is presented with a question that calls for an explanation or prediction by developing an explanatory model.

Model Evolution. The subject goes through a process of improving a model, sometimes several times (via model Generation, Evaluation and Modification cycles). As we have seen, Evaluatory

Observations may also be involved in this mode.

Model Competition. The subject goes through a process in which two or more different model structures are focused on as alternative candidates for an explanatory model, motivating their competitive evaluation.

Application and Domain Extension. Subject applies a model to a new case for explanation or prediction. If the case is outside the initially perceived domain of application of the model, it may stretch or extend that domain.

Although the above is a plausible ordering for these modes, other orderings are possible, especially between Evolution and Competition, as indicated by the double arrow in Figure 13. For reasons of space Figure 13 actually shows subprocesses only for Model Evolution mode; in Nunez and Clement (2017) subprocesses for Model Competition are unpacked as well.

Theoretical Description of the Four Levels in the Modeling Practices Framework

By removing the sequence of cognitive acts in the protocol from Figure 11 we can construct a more abstract summary of the processes (practices) we are proposing. An initial partial representation for Levels 1-3 is shown in Figure 12. There is no representation of time running from left to right in this diagram. It simply shows which processes are used within other processes at a higher level.

In Figure 12, the three levels of processes are still arranged in order from larger time scale processes above to smaller time scale processes below. Once a concrete case for use in imagistic simulation is formulated and imagined, the simulation process is seen as happening quickly, within seconds, whereas Level II processes may take longer, and those at Level III even longer. This is reflected in the subprocess relationships pictured in Figure 12. By subprocesses, we mean a set of tactics that implement a larger strategy; each subprocess can contribute to a larger goal/process above it.

Figure 13 shows a more complete catalog of the processes hypothesized for this protocol, with all four levels shown. This diagram is then an abstract picture of a theoretical framework of modeling practices. It indicates that models can be evolved (improved) in the Model Evolution Mode at Level 4 via subprocesses of model Generation, Evaluation, and Modification, and that sometimes these form repeated cycles, as they did as shown in Figure 11 for the present protocol in the sequence G-E-M-E-M. In the framework in Figure 13, the 'GEM cycle' takes the form of a "suggested method to try". But humans have more going on in parallel than a simple rigid cycle algorithm can capture, and such a cycle can be interrupted by other activated ideas that rise in priority at any time. From the protocol it appears to be a powerful method, but it is only used as long as it appears to be productive. Sequences need not always have the form G-E-M-E-M... For example, there could be several Evaluations of different model elements in a row. Evaluation, in turn, can be implemented by any of the five subprocesses pointing to it, in any order, singly or with more than one being used. So there is a suggested algorithmic organization and subprocess organization in this framework, but it is flexible, rather than being a rigid procedure.

The imagery indicators underlined within the nonformal reasoning episodes in this case study provide some evidence that imagistic simulation at Level 1 in Figure 13 is a subprocess used within the Level 2 processes above it. And in turn the analyzed sequence shown in Figure 11 suggests that the processes at Level II appear in turn to serve as subprocesses for one of the processes at Level III in a nested way. These results of this case study from think aloud data support the historical accounts of Faraday's use of presymbolic processes of running experiments 'in the mind's eye' (Tweney, 1996), and of Maxwell's cycles of model evaluation and modification (Nersessian, 2008).

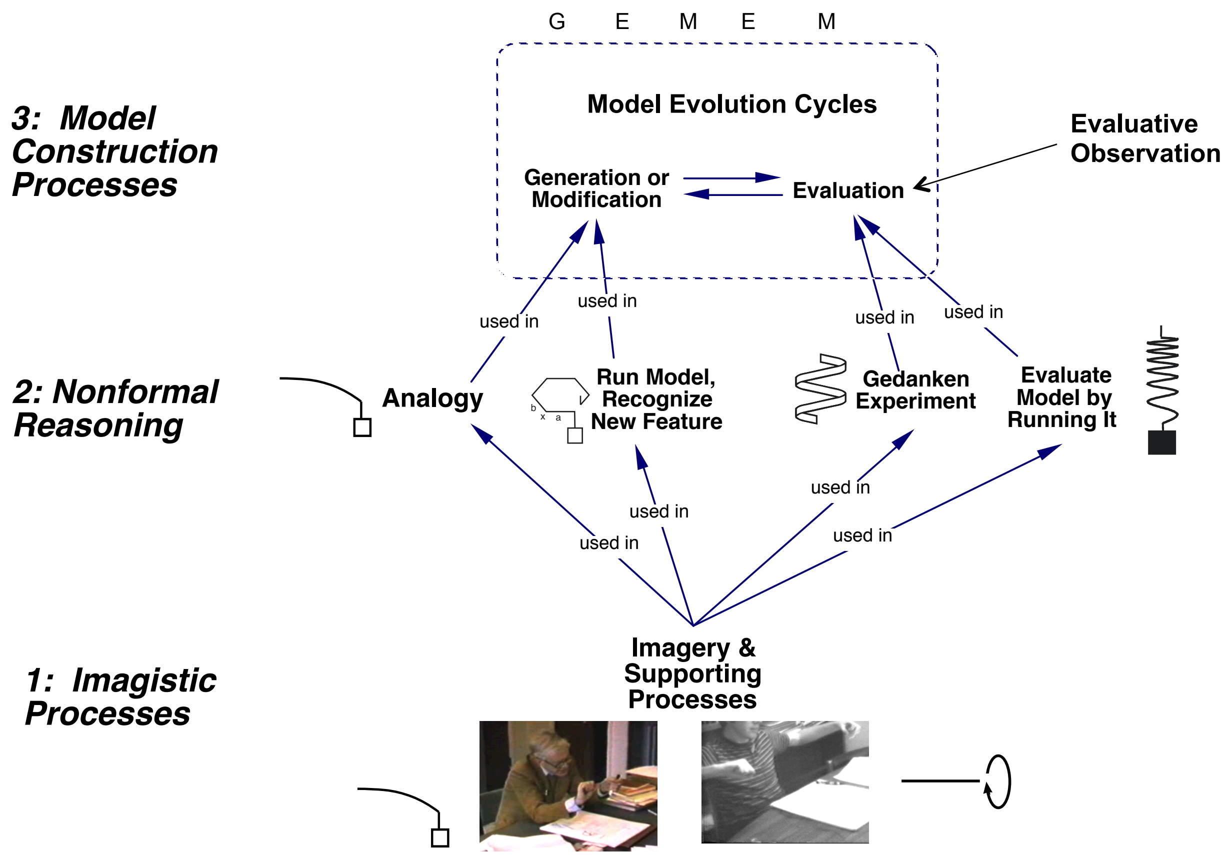

Figure 12: Hierarchical, partial view of three levels of processes and subprocesses in S2's protocol

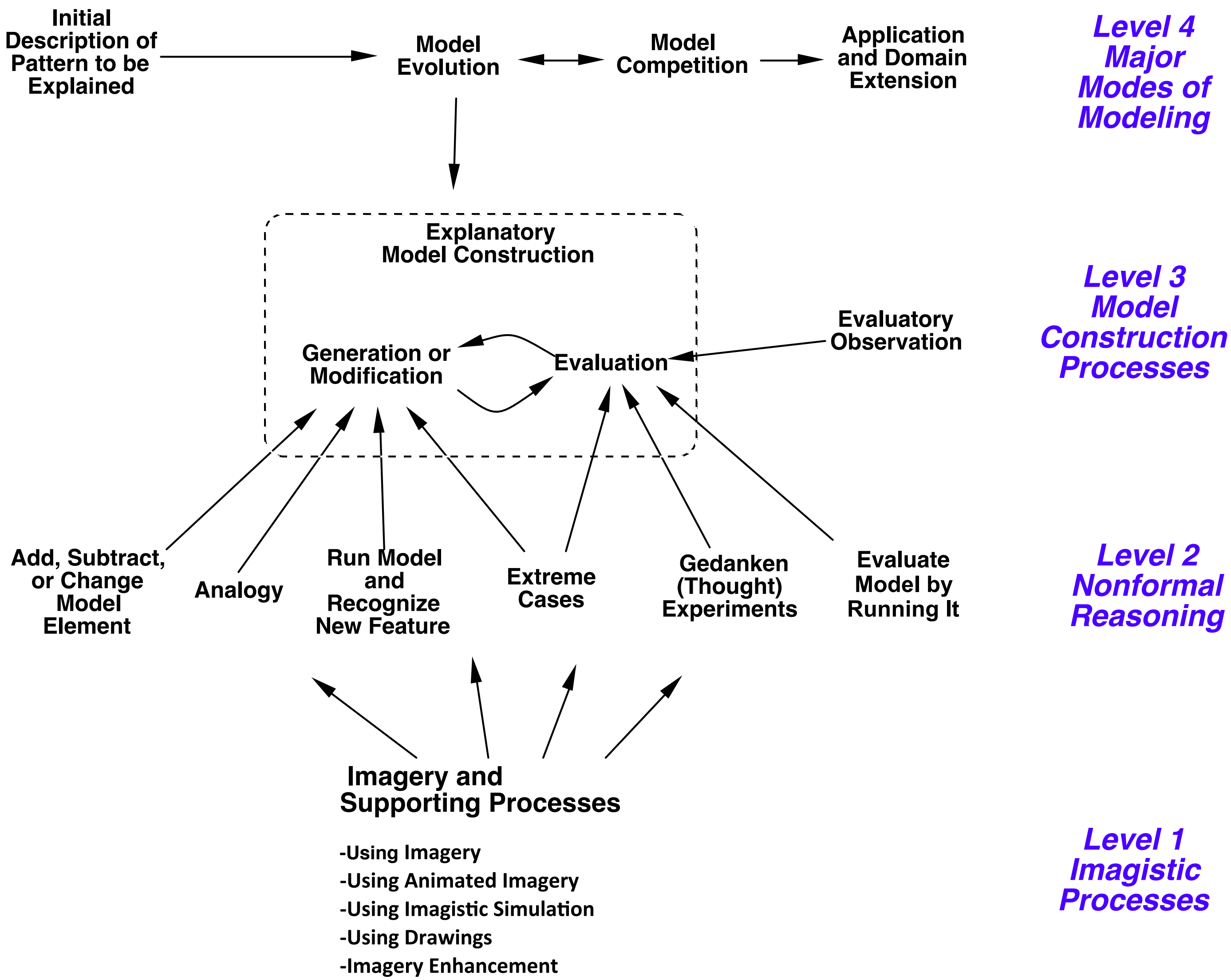

Figure 13. Modeling Practices Framework: Four Nested Levels of Processes.

Conclusion

Since each process in Figure 13 can utilize any one of a variety of processes below it; many choices are available, so the framework is ***not a rigidly structured procedure.*** There are always choices for the next step in the process, rather than a lockstep sequence. Processes are heuristic in the sense that they are not guaranteed to work and if one fails, others can be tried. But the ***modeling practices framework in Figure 13 is not 'anarchistic'-- it does have some organized structure***, in the levels and subprocess relationships, in the roughly ordered sequence to try at Level 4, and in GEM cycles suggested at Level 3. [3]

The prevalence of imagistic simulation as an underlying foundation in these episodes, as seen in Figure 13, suggests that it may be important to pay greater attention to this process in the analysis of learning than is commonly done. I do not claim that it is *always* involved as a subprocess for the processes at Level 2, but I have presented some evidence based on case study data, providing an existence demonstration, that it can be a very important subprocess. Similarly in going up one level, I do not claim that, say, Gedanken experiments are always used as a subprocess for evaluating a model, but we have seen some evidence that they can be powerful in that role.

Figures 11 and 13 give us new ways to picture the roles of imagistic and nonformal reasoning processes during qualitative model construction. These can be contrasted with more formal, procedural and traditional reasoning processes of deduction and induction by enumeration or statistical inference. The nonformal processes discussed here may be less procedural and carry less certainty than those traditional forms of reasoning, but they can be powerful engines for discovery if used within a self-correcting cycle of evaluation and modification. There are other processes at level 2 such as concept differentiation, integration, and evaluating a model for gaps; [Polya, 1954, 1957; Clement, 2008)] that I have not had space to deal with here, but these diagrams give us a starting framework that can be expanded. One interesting feature seen in Figure 11 depicting model evolution is that it did not matter that S2 began with a faulty model that was later rejected. Through the Level 3 processes of model evaluation and modification, he was able to use his initially faulty model as a useful starting point to engage in an ultimately productive and successful process of model construction.

Philosophers also point out that the whole model construction enterprise involves a process of 'abduction' in contrast to formal 'induction' by enumeration or deduction from premises. Unfortunately 'abduction' can be used by different scholars for different sized processes, including (from small to large in Figure 13) Model Generation or Modification, Explanatory Model Construction cycles, or Model Competition. (The model competition process as we have described it is closely related to what philosophers of science call 'Inference to the Best Explanation.') Here we use three different names for these processes in hopes of disambiguating them. These different uses for the term ‘abduction’ share in common the idea that explanatory model generation is conjectural and heuristic rather than being a form of high-certainty inference from either data patterns or premises. However, such conjectures can be successively improved in GEM cycles to a high level of certainty as they become more coherent with other established ideas and with more and more data. With regard to the first meaning, Model Generation or Modification, the case study provides some evidence that these processes can be based in imagistic, perceptual motor processes.

In sum, in contrast to model and theory building as a formal, algorithmic process, the present framework

---

[3] Technically, the intent of this framework is closer to the AI metaphor of a production system rather than a strict algorithm; that is, higher level processes can remain active as goals (at different strengths of activation) while a subprocess below them is operating. This is akin to saying that expert scientists and teachers can operate with multiple goals and operate at more than one level at a time in parallel, especially at the lower three levels.

depicts a set of non-formal, largely imagery-based processes that are loosely organized to make the system flexible. The power of each individual process in this framework may be modest, but together they can operate to produce real insights and conceptual change as new models are formed and improved.

Educational Applications

Are the learning processes identified here similar to those fostered by exemplary teachers in conceptual classroom discussions? We have been engaged in an effort to specify what scientific modeling practices are-- at a more detailed level than that specified in NGSS. By starting from the Framework of Practices presented here, along with other sources, other researchers in our group are attempting to apply these constructs to teaching and learning strategies observed in classrooms in the papers listed below. Important similarities to the expert practices have been found. Their classes also went through cycles involved in model generation, evaluation, and modification at level 3. Similar practices as seen in the experts were used at levels 1, 2, and 4 as well. These studies show that the four levels of processes described here can help to organize and understand the purpose of observed, cognitively targeted teaching strategies. Diagramming techniques used in this paper have been adapted for diagramming classroom discussions, and this has helped to illuminate these processes as well as the role of teacher scaffolding and co-construction of the models being learned. Those efforts have also identified other practices, leading to the expanded Modeling Processes Framework in Figure 14. We anticipate that it, in turn, will provide new ideas for analyzing expert thinking in the future.

The companion papers below from our symposium at the conference apply these processes to classrooms and provide more detailed criteria for identifying processes at each level. The other papers in our 2017 symposium at the National Association for Research in Science Teaching, San Antonio, TX in April, available at **https://tinyurl.com/knymh39**, are:

Grant Williams and John J. Clement
Co-Construction via Modeling Cycles in High School Physics: Comparing Degrees of Teacher and Student Participation in Whole Class Discussions
Discusses examples of the Level 2 Nonformal Reasoning Processes occurring in classroom discussions, and links them to Level 3 Model Construction processes

Lynn Stephens, John Clement, Norman Price, Maria Nunez-Oviedo
Identifying Teaching Strategies that Support Thinking with Imagery during Model-Based Discussions
Discusses examples of the Level 1 Imagistic Processes occurring in classroom discussions, and how these occur contiguously with Level 2 Nonformal Reasoning processes. (See also Price, et al., in press)

Maria Nunez and John Clement
Large Scale Scientific Modeling Practices that can Influence Science Instruction at the Unit and Lesson Levels
Discusses examples of the Level 4 Major Modeling Modes occurring in classroom discussions, and links them to Level 3 Model Construction processes. The Level 4 Modes are large time scale modes that may apply to how to structure different types of discussions over a lesson, set of lessons, or unit of instruction.

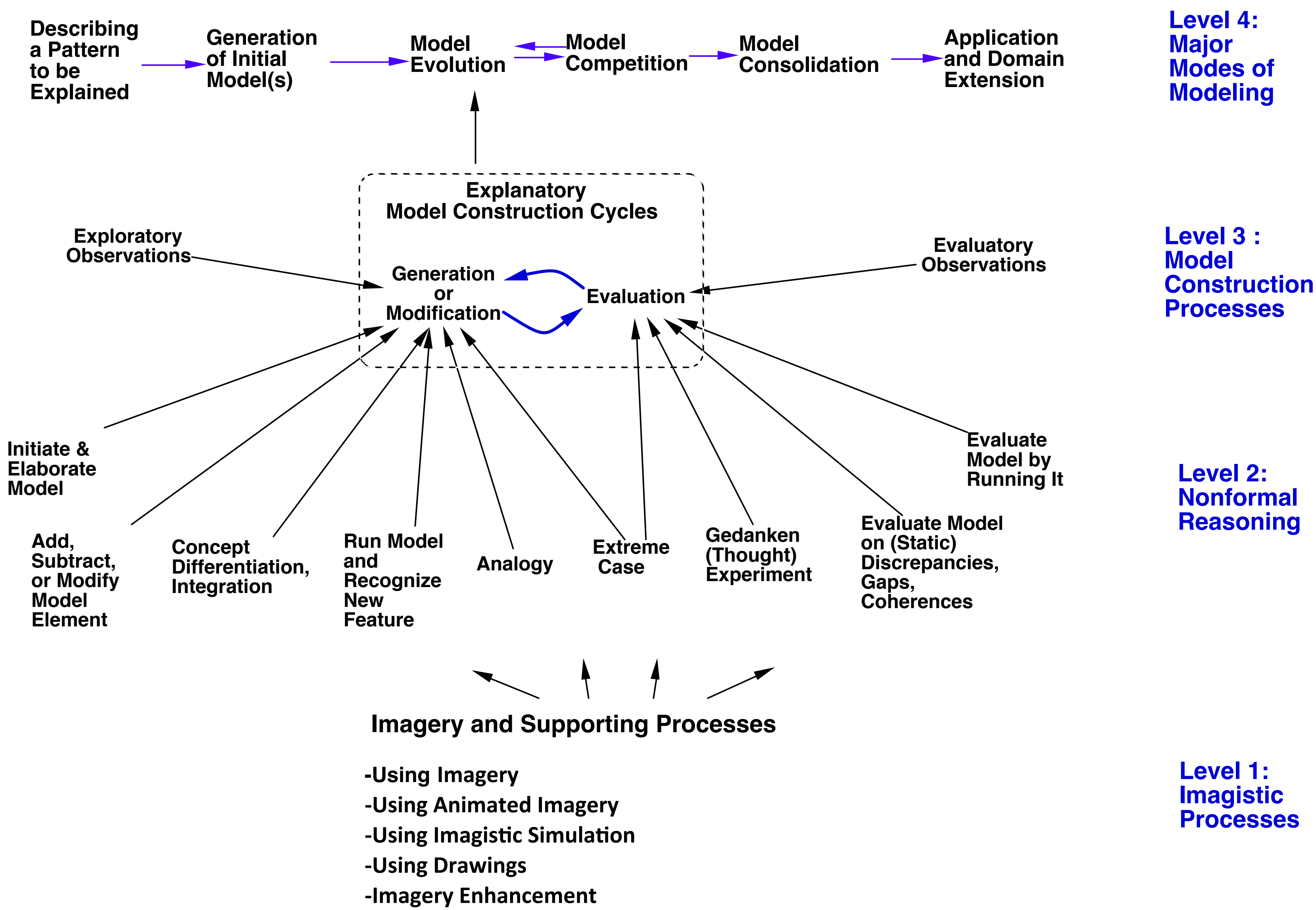

Figure 14: Expanded Modeling Practices Framework: Four Nested Levels of Processes. Each process also identifies a corresponding teaching strategy of scaffolding that particular process.

Appendix

Introduction to Concepts of Torque and Torsion

The concepts of torque and torsion can be introduced via Figure 15. For torque, if we ignore segment *hb* and think of segment *ab* as a pipe and segment *ga* as a pipe wrench that we are using to turn the pipe clockwise, so that the pipe goes into a tight, threaded socket at *b*, then **torque** can be thought of roughly as the "twisting force" applied by the wrench to the end of the pipe at *a* to turn it. The torque will be greater in proportion to the length of the wrench, *r* , since longer wrenches provide more leverage and more "twisting force". When the force *F* is perpendicular to *r*, the torque *T* applied to the end of the pipe is equal to the applied force *F* times the length of the wrench *r*.

$$T = F \times r$$

To define **torsion**, we need a different scenario. Imagine that *ab* is a steel rod only 1/8" thick with the end *bh* fixed in concrete so that the far end of the rod at *b* cannot turn. Then if we clamp a vise grip wrench *ga* to the near end of the rod at *a*, applying the same torque will end up only twisting (deforming) the metal somewhat, in every element of the entire rod *ab*, so that the near end at *a* turns through the angle ß shown in Figure 15 (called the angular displacement, or, informally, total amount of twist in the rod) and stops. If the rod is made of resilient metal, it will be elastic, meaning that if we remove the force *F*, the metal in the rod will untwist and spring back to its original orientation where ß was zero. **Torsion** refers to an action that twists a material, resulting in stresses and strains that make the rod want to spring back to its original shape. If the rod is twice as long, but *r* and *F* are the same, the angle ß will double. That is because the torque and resulting torsion stress will be the same as before, but there will be twice as much metal to deform under that stress, producing twice the total twist. In the protocols, subjects sometimes use the word "torsion" as defined above, but also sometimes slightly misuse the term torsion to mean torque, so the term must be read in context.

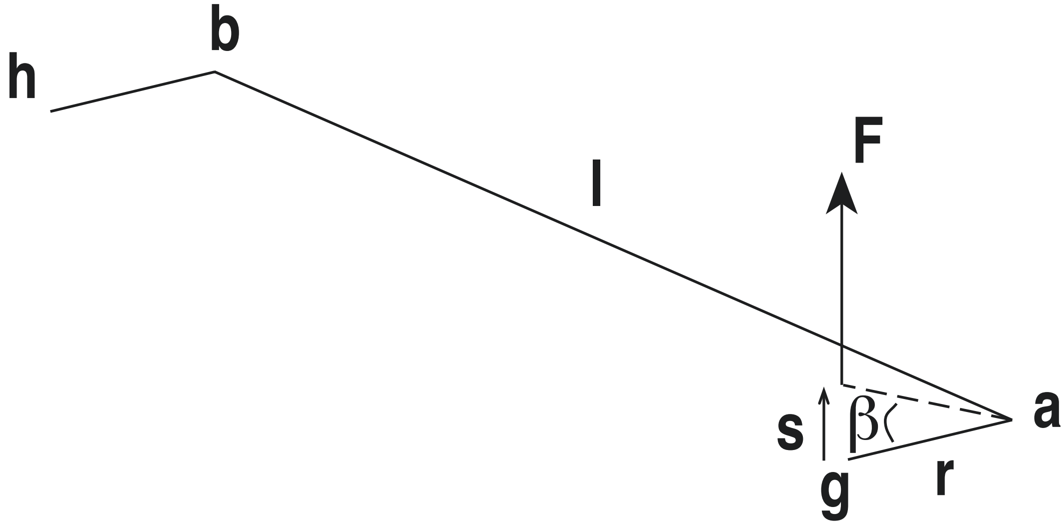

Figure 15: Torque and torsion

REFERENCES

Barsalou, L.W. (1999). Perceptual symbol systems (with commentaries and author's reply). *Behavioral and Brain Sciences* (22) 577-660.

Chan, J., Paletz, S. B., & Schunn, C. D. (2012). Analogy as a strategy for supporting complex problem solving under uncertainty. *Memory & cognition*, *40*(8), 1352-1365.

Chin, C. (2007). Teacher questioning in science classrooms: Approaches that stimulate productive thinking. *Journal of Research in Science Teaching*, *44*(6), 815-843.

Clement, J. (1988). Observed methods for generating analogies in scientific problem solving. *Cognitive Science, 12*: 563-586.

Clement, J. (1989). Learning via model construction and criticism: Protocol evidence on sources of creativity in science. Glover, J., Ronning, R., and Reynolds, C. (Eds.), *Handbook of creativity: Assessment, theory and research*. NY: Plenum.

Clement, J. (1994). Use of physical intuition and imagistic simulation in expert problem solving. Tirosh, D. (Eds.), *Implicit and explicit knowledge.* Norwood, NJ: Ablex Publishing Corp.

Clement, J. (2002). Protocol evidence on thought experiments used by experts. Gray, W., and Schunn, C. (Eds.), *Proceedings of the Twenty-Fourth Annual Conference of the Cognitive Science Society*. Mahwah, NJ: Erlbaum.

Clement, J. (2003). Imagistic simulation in scientific model construction. *Proceedings of the Twenty-Fifth Annual Conference of the Cognitive Science Society*, 25. Mahwah, NJ: Erlbaum.

Clement, J., (2008). *Creative model construction in scientists and students: The role of imagery, analogy, and mental simulation.* Dordrecht: Springer.

Clement, J. (2009). The role of imagistic simulation in scientific thought experiments. *TOPICS in Cognitive Science, 1*: 686–710.

Clement, J., Zietsman, A., Monaghan, J. (2005). Imagery in science learning in students and experts. Gilbert, J. (Ed.) *Visualization in Science Education.* Dordrecht, The Netherlands: Springer.

Darden, Lindley (1991). *Theory Change in Science: Strategies from Mendelian Genetics.* Oxford UP: New York.

Dunbar, K. (1999). The scientist in vivo: How scientists think and reason in the laboratory. In L. Magnani, ' N. Nersessian, &P. Thagard (Eds.), *Model-based reasoning in scientific discovery.* New York: Plenum Press.

Duschl, R. A. & Osborne, J. (2002). Supporting and promoting argumentation discourse in science education. *Studies in Science Education, 38,* 39-72.

Feyerabend, P. (1993). *Against method*. Verso.

Finke, R. (1990). *Creative imagery: Discoveries and inventions in visualizations*. Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Giere, R. (1988). *Explaining science: A cognitive approach*. Chicago: Chicago University Press.

Gilbert, J. (Ed.), (2005). *Visualization in science education.* Dordrecht, The Netherlands: Springer.

Glynn, S. M., & Duit, R. (1995). Learning science meaningfully: Constructing conceptual models. In S. M. Glynn & R. Duit (Eds.), *Learning science in the schools: Research reforming practice* (pp. 3-33). Mahwah, NJ: Erlbaum.

Gooding, D. (1992). The procedural turn: or, Why do thought experiments work? In Giere, R. (Ed.) *Cognitive models of science.* Minneapolis: U. of Minnesota Press.

Griffith, T. W., Nersessian, N. J., and Goel, A. (2000). Function-follows-form transformations in scientific problem solving. In *Proceedings of the Cognitive Science Society*, 22 (pp. 196–201). Mahwah, NJ: Lawrence Erlbaum.

Hegarty, M. (2002). Mental visualizations and external visualizations. In Wayne Gray and Christian Schunn, Eds., *Proceedings of the Twenty-Fourth Annual Conference of the Cognitive Science Society* 22, 40. Mahwah, NJ: Erlbaum.

Hegarty, M., Kriz, S. & Cate, C. (2003). The roles of mental animations and external animations in understanding mechanical systems. *Cognition and Instruction, 21*(4), 325-360.

Hogan, K., Nastasi, B.K. & Pressley, M. (2000). Discourse patterns and collaborative scientific reasoning in peer and teacher-guided discussions. *Cognition and Instruction, 17*(4), 379-432.

Kosslyn, S. (1980). *Image and mind*. Cambridge, MA: Harvard University Press.

Lowe, R. K. (2004). Interrogation of a dynamic visualization during learning. *Learning and Instruction, 14*(3), 257-274.

McNeill, K. and Krajcik, J. (2008). Scientific explanations: Characterizing and evaluating the effects of teachers' instructional practices on student learning. *Journal of Research in Science Teaching, 45* (1), 53-78.

Nersessian, N. (2008). *Creating scientific concepts*. Cambridge: MIT Press.

Nunez, M. & Clement, J. (2017). Large scale scientific modeling practices that can influence science instruction at the unit and lesson levels. Paper presented at the *2010 Annual Meeting of the National Association for Research in Science Teaching* (NARST), San Antonio, TX.

Nunez-Oviedo, J., Clement, J. and Ramirez, M. (2008). Developing complex mental models in biology through model evolution, in *Model based learning and instruction in science*, edited by J. Clement and M. A. Ramirez. Dordrecht: Springer, p.173-194.

Osborne, J. F., Erduran, S. & Simon, S. (2004). Enhancing the quality of argumentation in school science. *Journal of Research in Science Teaching, 41*(10), 994-1020.

Polya, G. (1954). *Mathematics and plausible reasoning.* Trenton, NJ: Princeton University Press.

Polya, G. (1957). *How to Solve It,* 2nd ed., Princeton: Princeton University Press.

Price, N., Stephens, L., Clement, J., & Nunez, M. (in press). How teachers can help students develop powerful imagistic models using imagery support strategies. *Science Scope*.

Quintana, C., Reiser, B., Davis, E. A., Krajcik, J., Fretz, E., Golan, R., et al. (2004). A scaffolding design framework for software to support science inquiry. *Journal of the Learning Sciences*, *13*(3), 337-386.

Schmidt, R. A. (1982). *Motor control and learning*. Champaign, IL: Human Kinetics Publishers.

Schwartz, D. L., & Heiser, J. (2006). Spatial representations and imagery in learning. *The Cambridge handbook of the learning sciences*, 283-298.

Stieff, M. (2011). When is a molecule three dimensional? A task-specific role for imagistic reasoning in advanced chemistry. *Science Education*, *95*(2), 310-336.

Stephens, L. & Clement, J. (2010). Documenting the use of expert scientific reasoning processes by high school physics students*. Physical Review Special Topics - Physics Education Research, 6(2): URL: http://link.aps.org/doi/10.1103/PhysRevSTPER.6.020122*

Stephens, A. L., Clement, J. J., Price, N., & Nunez-Oviedo, M. C. (2017). *Identifying teaching strategies that support thinking with imagery during model-based discussions*. Paper presented at the National Association for Research in Science Teaching (NARST), San Antonio, TX.

Strauss, A., & Corbin, J. (1998). *Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory*. Sage Publications. Thousand Oaks, CA., pp. 12 & 101.

Trickett, S. and Trafton, J. G. (2002) The instantiation and use of conceptual simulations in evaluating hypotheses: Movies-in-the-mind in scientific reasoning. In Wayne Gray and Christian Schunn, Eds., *Proceedings of the Twenty-Fourth Annual Conference of the Cognitive Science Society* 22, 878-883. Mahwah, NJ: Erlbaum.

Tweney, R. D. (1996). Presymbolic processes in scientific creativity. *Creativity Research Journal*, *9*(2-3), 163-172.

van Zee, E. & Minstrell, J. (1997). Reflective discourse: Developing shared understandings in a physics classroom. *International Journal of Science Education, 19*, 209-228.

Williams, E.G. (2011), Fostering high school physics students' construction of explanatory mental models for electricity: Identifying and describing whole-class discussion-based teaching strategies, Doctoral Dissertation. University of Massachusetts, Amherst.

Williams, E.G., and Clement, J. (2013). From research to practice: Fostering pre-service science teachers' skills in facilitating effective whole class discussions. Paper presented at NARST, 2013.

Williams, G., & Clement, J. (2017). Co-Constructing models in high school physics: Comparing degrees of teacher and student participation in whole class discussions. Paper presented at the *2010 Annual Meeting of the National Association for Research in Science Teaching* (NARST), San Antonio, TX.