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Letter

Quark counting, Drell-Yan West, and the pion wave function

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The relation between the pion's quark distribution function, $q(x)$, its light-front wave function, and the elastic charge form factor, $F(\Delta^2)$, is explored. The square of the leading-twist pion wave function at a special probe scale, ζ_H, is determined using models and Poincaré covariance from realistic results for $q(x)$. This wave function is then used to compute form factors with the result that the Drell-Yan-West and quark counting relationships are not satisfied. A new relationship between $q(x)$ and $F(\Delta^2)$ is proposed.

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Introduction. The structure of the pion continues to be of interest to many physicists. There are plans to measure the pion electromagnetic form factor at JLab and at the planned Electron-Ion Collider (EIC) [1]. There are also plans to remeasure the quark distribution of the pion, $q(x)$, via a new Drell-Yan measurement [2]. Much recent and older theoretical attention has been devoted to determining and understanding the behavior of the valence pion quark distribution function, $q_v(x)$, at high values of Bjorken x; see, e.g., the review [3].

Much of the recent interest stems from efforts to understand the behavior at high x. While many use the parametrization $q_v(x) \sim x^\alpha(1-x)^\beta$, there is a controversy over the value of β and its dependence on the variables x and the resolution scale, Q^2. See, for example, the differing approaches of [4–7]. Reference [4] finds that $\beta = 1$ at low resolution scales, rising to 1.5 at $Q^2 = 27$ GeV^2, while [7] finds that $\beta = 2 + \gamma(Q^2)$ with γ positive and increasing at Q^2 rises. Both sets of authors claim agreement with the available data set. The small values of β result from perturbative QCD and the larger values from nonperturbative techniques. Indeed, [5] finds that the value of β can lie between 1 and 2.5 depending on the technique used to resum the contributions of large logarithms in computing the relationship between $q(x)$ and the measured Drell-Yan cross section data. It would be beneficial to find the relation (if any) between the behavior at large values of x and the underlying dynamics.

The wide interest in the form factor and distribution function originates in the early hypotheses of the connection between the two observable quantities. Drell and Yan [8] and West [9] suggested a relation between $q(x)$ for large values of x and the elastic proton's Dirac form factor $F_1(\Delta^2)$, where Δ^2 is the negative of the square of the four-momentum transfer to the target hadron, at large values of Δ^2. Different aspects of the wave function are used to compute distribution functions and form factors, so the relation is very striking, namely

$$\lim_{x\to 1} q(x) = (1-x)^{n_H} \tag{1}$$

leads to the result

$$\lim_{\Delta^2\to\infty} F_1(\Delta^2) \propto \frac{1}{(\Delta^2)^{(n_H+1)/2}}, \tag{2}$$

with n_H the number of partons in the hadron. The F_1 form factor is the matrix element of the plus component of the electromagnetic current operator between proton states of the same spin. So the proof would provide the same relations for the pion. This relation between inclusive and exclusive processes points to a deep underlying connection between two observables resulting from the underlying light front wave function.

The Drell-Yan derivation was based on light-front perturbation theory in the infinite momentum frame, and in the pre-QCD era the degrees of freedom were hadronic. The result was obtained from assumptions regarding the values of energy denominators near the endpoints. In particular, the dominant contributions to the form factor came from the integration region of high x. West assumed that the wave function has an asymptotic power-law behavior to obtain his result, while commenting that the result could be different for other forms of the wave function. The original papers are heavily quoted now despite the ancient nature of these relations. Understanding the validity of the underlying assumptions will inform us about the nature of the wave function and therefore about confinement.

Another relationship between structure functions and form factors is obtained from the use of perturbative QCD and leads to quark counting rules for the proton and pion obtained by

Farrar and Jackson [10,11] and others, reviewed in [12]. These are

$$\lim_{x\to 1} q(x, Q^2) \propto (1-x)^{2n_H-3+2|\Delta_s|+\Delta\gamma}, \tag{3}$$

$$\lim_{\Delta^2\to\infty} F_H(\Delta^2) \propto \frac{1}{(\Delta^2)^{n_H-1}}, \tag{4}$$

with $\Delta\gamma$ a correction accounting for evolution that vanishes at a starting scale ζ_H^2, n_H is the minimum number of elementary constituents of the hadron, and we have taken the number of spectators to be $n_H - 1$. The quantity Δ_s is the difference between the z components of the quark and hadron spin. Thus for a proton the dominant term at high x has $|\Delta S_z| = 0$ and for a pion $|\Delta S_z| = 1/2$. The two sets of relations, Eqs. (1) and (2) and Eqs. (3) and (4), are approximately the same for the proton at ζ_H^2: namely, $q(x) \sim (1-x)^3$ and $F_1 \sim 1/\Delta^4$ with $n_H = 3$. There are two sets of predictions for the pion $q(x) \sim (1-x)^2$, $F(\Delta^2) \sim 1/\Delta^3$ for Drell-Yan West, and $q(x) = (1-x)^2$, $F(\Delta^2) \sim 1/\Delta^2$ for the quark counting rules.

These storied and valuable quark-counting rules were derived using perturbative QCD using the best techniques of the 1970s. But it is important to recall controversies regarding their applicability for experimentally available kinematic situations. See, for example the papers by Isgur and Llewellyn Smith [13–15] and Radyushkin [16]. Furthermore, computations were made that were based on assumptions regarding the non-perturbative quark distribution amplitudes. Contributions from transverse momenta of exchanged gluons were neglected, and contributions from the endpoints of integrals over longitudinal momenta were assumed to be small. Although more advanced work has been done since that time (see, e.g., the review [3]), we believe that it is worthwhile to examine these rules from a different nonperturbative perspective.

We now focus on the pion. The current literature tells us that the relation between the high-x behavior of $q(x)$ and the pion form factor is interesting. We aim to study the connection between $q(x)$ and the square of the pion valence light-front wave function.

Light-front analysis. Hadronic wave functions depend on a factorization scale ζ at which the hadron is probed. It has been widely argued that [7,17,18] there is a scale at which the hadron consists of only valence quarks. These quarks are linked to quarks of the quark-parton model as objects dressed by quark-gluon QCD interactions obtained from the quark gap equation. Gluon emission from valence quarks begins at ζ_H [19]. Thus, at ζ_H the dressed valence u and $\bar{d}$ quarks carry all of the momentum of the π^+ and each constituent (of equal mass) carries 1/2 of the pion momentum. The result of every calculation of pionic properties that respects Poincaré covariance, and the Ward-Green-Takahashi identities along with the consequences of dynamical symmetry breaking inherent in the quark gap-equation, has these features. See, e.g., Ref. [20].

In general the pion Bethe-Salpeter amplitude depends upon four different relativistic pseudoscalar terms, each multiplied by its own scalar function, see, e.g., [21]. However, the behavior of the form factor at large values of Δ^2 is dominated by the "leading-twist" amplitude proportional to γ^5 in which the quark and antiquark spins combine to zero and there is no angular momentum. For example, this is the amplitude used in the classic paper [22] to compute the pion electromagnetic form factor. Therefore, we analyze only the leading twist amplitude, the only one that enters at ζ_H.

The relation between the light-front wave function, evaluated at the hadron scale ζ_H^2, is given by

$$q(x) = \frac{1}{\pi}\int \frac{d^2k_\perp}{x(1-x)}|\Phi(x, k_\perp)|^2, \tag{5}$$

where $\Phi(x, k_\perp)$ is the wave function of the $q\bar{q}$ component. The function Φ represents the leading-twist component of the pion wave function, in which the quark and antiquark spins combine to 0. This component dominates computations of the high-momentum transfer form factor and the high x behavior of $q(x)$. The normalization is $\int_0^1 dx\, q(x) = 1$. We drop the explicit dependence on ζ_H^2 to simplify the equations.

There is a special feature of the wave function at ζ_H. Rotational invariance requires that $\Phi(x, k_\perp)$ is a function of a single variable, $\Phi(M_0^2)$, with $M_0^2 \equiv \frac{k_\perp^2+M^2}{x(1-x)}$, and M is the constituent quark mass [23,24]. The key point is that in the two-body sector one may construct a self-consistent representation of the Poincaré generators. Both $k_\perp^2$ and M^2 are dimensionless variables measured in terms of an appropriate intrinsic momentum scale, Λ^2. Then changing variables to $z = M_0^2$ leads to the exact result,

$$q(x) = \int_{\frac{M^2}{x(1-x)}}^{\infty} dz|\Phi(z)|^2. \tag{6}$$

A curious feature is that if $M = 0$, $q(x) = 1$ in disagreement with realistic extractions of $q(x)$ at the hadronic scale [7]. Moreover, the idea that there is a scale ζ_H goes along with the feature that spontaneous symmetry breaking causes M to be significantly larger than its current quark value. Thus we do not expect that $q(x)$ is constant when evaluated at ζ_H.

The next step is to take x to be near unity so that

$$q_{x\to 1}(x) \approx \int_{\frac{M^2}{1-x}}^{\infty} dz|\Phi(z)|^2. \tag{7}$$

The lower limit is large, $M^2/(1-x) \gg 1$. This shows immediately the connection between the large x behavior and the high-momentum part of the light front wave function. An interesting relation can be obtained by differentiating Eq. (7) with respect to x:

$$q'_{x\to 1}(x) = -\frac{M^2}{(1-x)^2}\left|\Phi\left(\frac{M^2}{1-x}\right)\right|^2. \tag{8}$$

Given a model wave function, one can obtain the high x behavior of $q'(x)$ and thus also that of $q(x)$ at ζ_H^2. Then Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution can be used to obtain the structure functions at larger values of probe scales.

Moreover, the finite nature of $q'(x)$ at $x \to 1$ immediately gives information about the high momentum behavior of the pion wave function. Namely,

$$\lim_{x\to 1}\left|\Phi\left(\frac{M^2}{1-x}\right)\right|^2 = c(1-x)^2 f(1-x), \tag{9}$$

where c is a finite number and $f(1-x)$ is finite as $x \to 1$. Thus the high x behavior of $q(x)$ tells us about specific features of the pion wave function. Can one say more?

Form factors are matrix elements of a conserved current and so are independent of the factorization scale [25], so that one may evaluate the form factor using the constraint at ζ_H. Then the form factor is given by the expression

$$F(\Delta^2) = \frac{1}{\pi}\int_0^1 dx \int \frac{d^2k_\perp}{x(1-x)}\Phi\left(\frac{k_\perp^2+M^2}{x(1-x)}\right) \times \Phi\left(\frac{(\mathbf{k}_\perp+(1-x)\boldsymbol{\Delta})^2+M^2}{x(1-x)}\right), \tag{10}$$

where the plus component of the spacelike momentum transfer to the proton is taken as zero, so that the momentum transfer ($\boldsymbol{\Delta}$) is in a transverse ($\perp$) direction.

To see if there is a connection between $F(\Delta^2)$ and $q(x)$ we use model wave functions to compute both quantities. The connection between wave functions and $q(x)$ is given by Eq. (6).

It is convenient to use a flexible power law (PL) form:

$$|\Phi^{\mathrm{PL}}(z)|^2 \to \frac{K}{(z)^{n+1}}, \quad q^{\mathrm{PL}} \sim (1-x)^n \tag{11}$$

with $n \geqslant 1$. This form does not build in the asymptotic behavior predicted by using perturbative QCD. However, the applicability of perturbative QCD to exclusive processes at nonasymptotic, experimentally realizable values of the momentum transfer has been questioned [15,16,26–28] for a variety of reasons including lack of knowledge of the nonperturbative part of the wave function, convergence issues, higher twist effects, and those of Sudakov suppression. Radyushkin [16] wrote, "for accessible energies and momentum transfers the soft (nonperturbative) contributions dominate over those due to the hard quark rescattering subprocesses." Many of the problems in computing form factors are related to the importance of the high-x region that Feynman argued [29] was dominant. We also note that our current procedure is very similar to that used in the seminal works of Drell-Yan and West. The main difference is that tools are available to do exact integration with results in closed form. Despite progress in understanding nonperturbative aspects using lattice QCD (see, e.g., [30]) and Dyson-Schwinger techniques (see, e.g., [7,31], we believe that it is worthwhile to examine models of nonperturbative wave functions.

With $n = 1$, $F(\Delta^2) \sim 1/\Delta^3$ with Drell-Yan West and $F \sim 1/\Delta^2$ with quark counting. These predictions can be checked by doing the exact model calculation. We thus expect the asymptotic form factor to behave as $\sim 1/\Delta^2$, Eq. (4), if quark counting is correct. We now check to see if the quark counting relations are respected if Eq. (11) describes the wave function.

Note that in the nonrelativistic limit that the integral appearing in Eq. (10) is dominated by values of x near $1/2$, and if $\Delta^2 \gg (M^2)$ then $F_{\mathrm{NR}}(\Delta^2) \sim \Phi(1/2\Delta^2) \sim (1/\Delta^2)^{(n+1)/2}$ in accord with Eq. (2). This result is similar to the nonrelativistic arguments presented by Brodsky and Lepage [32]. However, the region of x near unity is very important because the effects of a large value of Δ are mitigated.

TABLE I. Asymptotic behavior of F_m. The two leading terms are kept, and $n = 2m - 1$.

n	$\lim_{\Delta^2\to\infty} F_m(\Delta^2)$
1	$6\left(\frac{\ln^2(\Delta^2)-4\ln(\Delta^2)+8}{2\Delta^2} - \frac{2(\ln(\Delta^2)+2)}{\Delta^4}\right)$
2	$180\sqrt{\pi}\left(\frac{(\Delta^2-6)\ln\left(\sqrt{\Delta^2}+\sqrt{\frac{\Delta^2}{4}+1}\right)}{(\Delta^2)^{5/2}} + \frac{16-5\sqrt{\Delta^2+4}}{2\Delta^4}\right)$
3	$840\left(\frac{3(\ln^2(\Delta^2)-3\ln(\Delta^2)+7)}{\Delta^6} + \frac{3\ln(\Delta^2)-14}{6\Delta^4}\right)$

To understand this, let us compute the form factor using Eq. (11) in Eq. (10) with $m \equiv (n+1)/2$. Combining denominators using the Feynman parametrization and integrating over the transverse momentum variable leads to the result

$$F_m(\Delta^2) = CK_m\int_0^1 dx\int_0^1 du\frac{[x(1-x)]^{2m-1}[u(1-u)]^{m-1}}{[1+\Delta^2(1-x)^2u(1-u)]^{2m-1}}, \tag{12}$$

with Δ^2 expressed in units of M^2, and $CK_m = \frac{\Gamma(4m)\Gamma(m+1)}{\Gamma(m)^2\Gamma(2m)}$. A brief look at the integrand of Eq. (12) shows why it is difficult to determine the asymptotic behavior of $F(\Delta^2)$. The value of Δ^2 can be taken to be large, but the multiplying factor, $(1-x)^2u(1-u)$, can be very small. One must do the integral first and then take Δ^2 to be large. Closed form expressions for F_m can be obtained for values of m between 1 and 3, and the asymptotic forms of F_m for $n = 1, 2, 3$ are shown in Table I.

The results in Table I and Eq. (11) show that the Drell-Yan West relations (1) and (2) are violated by the logarithms, which are not related to those of perturbative QCD that involve the strong coupling constant α_S. If one uses the quark counting relations Eqs. (3) and (4) with $\Delta\gamma = 0$ from using the hadronic scale, and $n = 2n_s$, $n_H = n_s + 1 = n/2 + 1$, then the powers of Δ^2 do not match. In particular, if $n = 2$ quark counting rules would say $F \sim 1/\Delta^2$; instead we observe that $F \sim \ln\Delta/\Delta^3$. Moreover, the appearance of logarithms in Table I shows that the approach to asymptotic limit is extremely slow. Power law wave functions are not consistent with quark counting rules, but nevertheless are relevant. This is because terms like $\Delta^2(1-x)^2u(1-u)$ appear in the integrals resulting from the evaluation of Feynman diagrams, and the values of x and u approach unity when evaluating integrals.

Realistic form factors. The next step is to see if the power law form has any phenomenological relevance. To this end, we note that $q(x)$ at ζ_H^2 is described as a parameter-free prediction of the pion valence-quark distribution function in Ref. [7,31]:

$$\begin{aligned} q(x) &= 375.32x^2(1-x)^2[1-2.5088\sqrt{x(1-x)} \\ &\quad + 2.0250x(1-x)]^2, \\ &\equiv \sum_{N=4}^{8} C_N[x(1-x)]^{N/2}. \end{aligned} \tag{13}$$

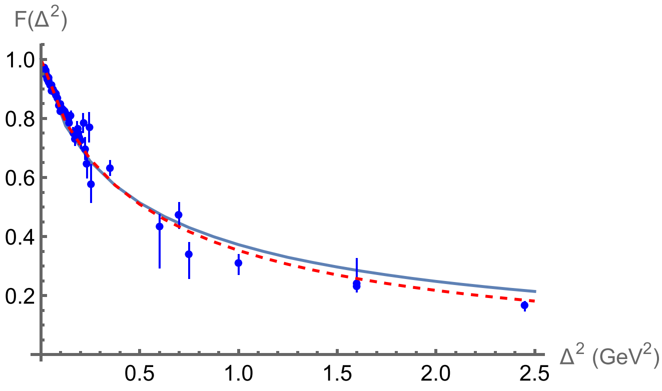

FIG. 1. $F(\Delta^2)$ results. Solid curve: model 1, $F(\Delta^2)$ of Eq. (10). Dashed curve: model 2, $\tilde{F}(\Delta^2)$ of Eq. (22). The data for $\Delta^2 \leqslant 0.253$ GeV^2 are from CERN [33]. The data for higher values are from JLab [34].

This distribution is defined as model 1. The corresponding pion wave function can then be written in a more general form than Eq. (11) as

$$\Phi(z) = \frac{1}{\sqrt{\pi}} \sum_{n=3}^{5} \frac{A_n}{z^{n/2}}. \tag{14}$$

Then, using Eq. (5),

$$q(x) = = \sum_{n=3}^{5} \sum_{N=4}^{8} \frac{2}{N} A_n A_{N+2-n} [x(1-x)]^{N/2}. \tag{15}$$

Then A_n is determined by equating Eq. (15) with Eq. (13). The result is

$$q(x) = \sum_{n=3}^{5} \sum_{N=4}^{8} \widetilde{C}(n, N+2-n) [x(1-x)]^{N/2}, \tag{16}$$

with $\widetilde{C}(3,3) = C_4$, $\widetilde{C}(3,4) = C_5$, $\widetilde{C}(3,5) = 3C_6 - (7/4)^2 C_7^2/C_8$, $\widetilde{C}(4,4) = (7/4)^2 C_7^2/C_8$, $\widetilde{C}(4,5) = 7/4 C_7$, $\widetilde{C}(5,5) = 4C_8$, and $\widetilde{C}(n,m) = \widetilde{C}(m,n)$.

The form factor is obtained from Eq. (10) and is given by

$$F(\Delta^2) = \sum_{m,n=3}^{5} \widetilde{C}(n,m) I_{nm}(\Delta^2), \tag{17}$$

$$I_{nm} \equiv \frac{\delta_{n+m,N+2}}{B(n/2, m/2)} \int_0^1 dx [x(1-x)]^{N/2-1} \times \int_0^1 du \frac{u^{n/2-1}(1-u)^{m/2-1}}{[1-\Delta^2(1-x)^2 u(1-u)]} \tag{18}$$

with B the beta function. The results are shown in Fig. 1. The units of Δ^2 are converted to GeV^2 by introducing a mass scale. We use $M = 134$ MeV to reproduce measured data.

An alternative model, model 2, is presented in Ref. [20]:

$$\tilde{q}(x) = 213.32[x(1-x)]^2 (1 - 2.9342\sqrt{x(1-x)} + 2.2911 x(1-x)). \tag{19}$$

This quark distribution can be rewritten in a form consistent with $\Phi^2(M^2/x(1-x))$:

$$\tilde{q}(x) = \frac{C}{\left(\Lambda^2 + \frac{M^2}{x(1-x)}\right)^{\alpha}}. \tag{20}$$

The constants are given by $\frac{M^2}{\Lambda^2} = 0.0550309$ and $\alpha = 3.266\,547\,03$, and C is for normalization. Note that the end-point behaviors of the two expressions (19) and (20) are very different, with the latter $\sim(1-x)^{3.27}$ instead of an exponent of 2. Nevertheless, the first 11 moments are reproduced to better than 1%, and the next 5 to better than 2%. The two distributions are experimentally indistinguishable, showing the elusive behavior of the endpoint behavior of $q(x)$.

Using Eq. (6) yields the square of the wave function to be

$$\Phi^2(z) = \frac{\alpha C}{\Lambda^{2\alpha}(1+z)^{1+\alpha}}. \tag{21}$$

Then using Eq. (10) the form factor is found to be

$$\tilde{F}(\Delta^2) = K \int_0^1 dx \int_0^1 du \frac{[x(1-x)]^{\alpha} [u(1-u)]^{\frac{1}{2}(\alpha-1)}}{[x(1-x) + M^2 + \Delta^2 (1-x)^2 u(1-u)]^{\alpha}}. \tag{22}$$

Integration over u leads to the generalized parton distribution $H(x, \Delta^2)$:

$$H(x,\Delta^2) \propto \frac{[(1-x)x]^{2\beta-1}\, {}_2F_1\left(\frac{1}{2}, 2\beta-1; \beta+\frac{1}{2}; \frac{(1-x)^2\Delta^2}{(1-x)^2\Delta^2 + 4(M^2+(1-x)x)}\right)}{\{4[M^2+(1-x)x] + \Delta^2(1-x)^2\}^{1-2\beta}}, \tag{23}$$

where $\beta = (1+\alpha)/2$ and ${}_2F_1$ is the hypergeometric function and $H(x, 0) = \tilde{q}(x)$. Both M^2 and Δ^2 are given in units of Λ^2. We choose $\Lambda^2 = 0.36$ GeV^2 to reproduce data. This corresponds to $M = 140$ MeV.

The results for both form factors are shown in Fig. 2. There are significant differences in the region that is not yet experimentally explored.

Conclusions. If the nonperturbative pion wave function can be modeled as a power-law form, or if the high-x behavior is important for computing the form factor, both the Drell-Yan West relations and quark counting rules for the pion are not

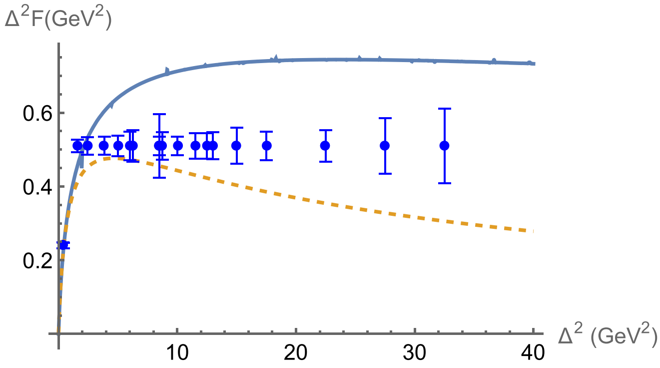

FIG. 2. $\Delta^2 F(\Delta^2)$ in units of GeV^2. Solid curve: model 1, $F(\Delta^2)$ of Eq. (10). Dashed curve: model 2, $\tilde{F}(\Delta^2)$ of Eq. (22). The projected error bars for the data points between $\Delta^2 = 0.375$ and 6 GeV^2 are from [35] and Huber (private communication). The projected error bars for the data points between $\Delta^2 = 8.50$ and 15 GeV^2 are from [36] and show what might be possible at a 22 GeV facility at JLab. The projected error bars for higher values of Δ^2 are from Huber (private communication) and [37]. In each case the values of $F(\Delta^2)$ are arbitrary.

completely correct. Based on our calculations we propose that the Drell-Yan West relations should be modified to

$$\lim_{x\to 1} q(x) = (1-x)^{n_H} \to F_1(\Delta^2) \sim \frac{\ln(\Delta^2)}{(\Delta^2)^{(n_H+1)/2}}, \tag{24}$$

with $q(x)$ evaluated at the hadron scale ζ_H^2, and with the logarithm, which is the new feature, not accompanied by a factor involving the strong coupling constant, α_S.

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