Nexus Cognizant Pricing of Workplace Electric Vehicle Charging

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***Abstract*— This paper studies the problem of designing workplace electric vehicle (EV) charging tariffs (i.e., fee schedules) while considering their impact to the transportation-electricity nexus. In particular, we consider the morning commute problem where a collection of commuters who drive EV to work must go through a common traffic bottleneck. Individual commuters determine when to leave for work and whether to charge their EV at work by optimizing a payoff function accounting for their generalized travel cost and payoff from charging. As an arrival time dependent charging tariff can directly impact the commuter decisions at the user equilibrium, we tackle the problem of designing tariffs that optimize (a) only the transportation component of the social cost, (b) only the electricity component of the social cost, and (c) the total social cost for the coupled transportation and electricity system. Tariffs incentivizing user equilibria that achieve the same performance as centralized social cost minimization are derived for the first two settings. For the last setting, we establish a tight condition under which it is possible to decentralize social optimal decisions via tariffs, and design optimal and suboptimal tariffs when the condition holds and fails, respectively.**

I. Introduction

As the transportation system electrifies, it becomes more inter-connected with the electric power system than ever. Driving an electric vehicle (EV) from point A to point B contributes to (a) the flow of vehicles and traffic, (b) the flow of users of the transportation and power systems, and (c) the flow/relocation of energy stored in the EV battery and electricity demand induced by the user flow. These flows jointly comprise *the electrified traffic flow*, which is deeply tied to both transportation and power systems. *How to nudge this electrified traffic flow to optimize the transportation system, the power system, or the both systems simultaneously?*

This paper makes a first attempt in answering this question in the context of workplace charging pricing. In particular, we focus on *morning commute*, one of the most typical travel settings, and investigate how pricing of workplace EV charging can influence commuters' departure time choices and charging decisions, so to effectively improve the efficiency of transportation and power systems. At the heart of this work is the explicit modeling of the *closed-loop dynamics*, where the workplace charging tariff impacts travel patterns, the travel patterns determines EV arrivals and charging loads, and the cost for serving the charging loads (among other metrics) drives the tariff design.

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A. Contributions

This paper makes the following original contributions: (a) It first proposes the idea of leveraging EV charging pricing to improve the performance of transportation and power systems individually and jointly. This highlights the urgency of understanding EVs' impact to those two networks, and necessity of leveraging EV to achieve social optimum among two systems. (b) Commuters' disutility in departure time choices and charging decisions are formulated, where heterogeneity in commuters' valuation for charging service at work is explicitly considered. Using an arrival time dependent charging tariff, we demonstrate that the system-level non-atomic user equilibrium can be steered toward social optimum when diverse social cost metrics are considered. The implied time-varying traffic patterns, electricity use patterns, and social costs between the two systems are obtained analytically for policy implications. (c) As a starting point, we develop theories and models in a stylized single-bottleneck network with continuous departure time choices during morning commute. This reveals the efficiency of EV charging pricing as to influence both users' and system performance, which can be extended to large-scale general networks in the future.

B. Related literature

Modeling, analysis, and incentive design (primarily through tolls) for morning commute have been studied extensively in the transportation engineering literature. The starting point is the celebrated Vickrey's bottleneck model [1], which has been generalized in a number of directions to incorporate practical considerations such as heterogenous users, demand elasticity, and multiple bottlenecks. See [2] for a comprehensive review. This paper extends the classical bottleneck model [1], [3] by modeling EV drivers' heterogenous valuation in workplace charging, the induced charging decisions, and the impact on the user equilibrium.

Independently, there is a large literature on coordinating EVs to minimize the electricity cost of serving EV charging loads [4]–[6] (and see [7] for a review). In these studies, travel patterns of the EVs are usually treated as exogenous inputs to the model, impacting EV arrival and departure times. Only several papers [8], [9] have explicitly internalized how charging costs may impact travel patterns in static settings focusing on the spatial aspect of travel choices (e.g., route and charging station selection), without considering departure time choices or optimal dynamic charging tariff design. Moreover, related studies on jointly managing transportation and power networks fail to take user choices (e.g., in autonomous driving setups) into consideration; see [10] for a review.

The existing work that is closest to ours is by Cenedese et al [11], where charging pricing is optimized to manage transportation costs. The key differences with our work are: they do not model heterogeneous driving valuation for charging, require every EV to charge a constant time before reaching the bottleneck, and, most importantly, do not address diverse social cost metrics for power and transportation systems.

II. Model

A collection of EV drivers, labelled by $i \in \mathcal{I} := [0,1]$, commutes from home to work in the morning[1]. The time period for the entire morning commute and charging process is modeled as a continuous time interval $\mathcal{T} = [\underline{t}, \overline{t}] \subseteq \mathbb{R}$. The interval $\mathcal{T}$ is determined such that without loss of generality it will contain the earliest departure time of any commuter and the latest completion time of any EV charging sessions (see Section II-B for more details about the charging sessions). It is also assumed that $\mathcal{T}$ is shorter than a day so there is no overlap between the morning commute and charging process of consecutive days.

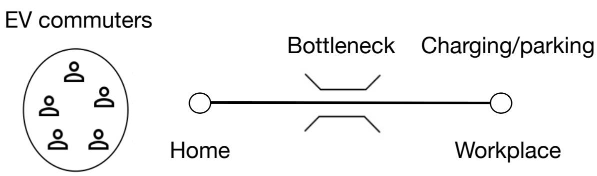

Fig. 1: Schematic of a simple transportation network with one bottleneck and workplace charging.

A. Transportation and charging infrastructure

As in the celebrated morning commute problem [1], commuters leave for work through a single route (Fig. 1). There exists a bottleneck on the route with flow capacity $s > 0$. Let the (endogenous) rate at which commuters depart from home be $d(t)$ and the length of the queue be $Q(t)$ at time $t \in \mathcal{T}$. Without loss of generality, we focus on variable travel time (i.e., waiting time in the queue) and assume zero fixed travel time as in [1], so a commuter arrives at the bottleneck immediately after departing from home and arrives at the workplace immediately after leaving the bottleneck. As a result, the rate at which commuters arrive at work is

$$a(t) = \begin{cases} d(t), & \text{if } Q(t) = 0 \text{ and } d(t) \leq s, \\ s, & \text{if } Q(t) > 0 \text{ or } d(t) > s. \end{cases}$$

In other words, if commuters arrive at the bottleneck at a rate faster than s, a *first-in-first-out queue* will form before the bottleneck and the service rate of the bottleneck is s. We can also define the cumulative departure and arrival functions as

$$D(t) = \int_{\underline{t}}^{t} d(\xi)\ \mathrm{d}\xi \quad \text{and} \quad A(t) = \int_{\underline{t}}^{t} a(\xi)\ \mathrm{d}\xi.$$

[1]Since we use an interval of unit length to model all commuters, we will refer to the fraction of the population of commuters and normalize all quantities by the actual size of the population accordingly.

Close to the workplace, the commuters can park and potentially charge their EVs. We consider a setting where any commuters who wish to charge their EV can get an EV charging connection. However, some commuters may choose to not charge the EV at the work depending on the existence of an outside option (e.g., charging at somewhere else) and the cost of charging at work. In other words, our focus is on how the charging cost rather than the availability of charging infrastructure can impact commuters' travel decisions.

B. Commuter model

In our model, each commuter can decide when to leave home for work, and whether to charge her EV at work. As in the classical setting [1], all commuters' ideal arrival time to work is assumed to be identical, and is denoted by $t^\star \in \mathcal{T}$. For commuter i who departs home at t_i, the time spent waiting at the queue formed at the traffic bottleneck is:

$$w(t_i) = Q(t_i)/s = (D(t_i) - A(t_i))/s,$$

where the second identity follows from $Q(t) = D(t) - A(t)$. The actual arrival time of commuter i is denoted by $t_i^{\mathrm{a}} := t^{\mathrm{a}}(t_i) := t_i + w(t_i)$. With these notations, we can relate the cumulative departures and arrivals with departure and arrival times of individual commuters:

$$D(t) = \int_{\mathcal{I}} \mathbb{1}\{t_i \leq t\}\ \mathrm{d}i \quad \text{and} \quad A(t) = \int_{\mathcal{I}} \mathbb{1}\{t_i^{\mathrm{a}} \leq t\}\ \mathrm{d}i,$$

where $\mathbb{1}\{\cdot\}$ denotes the indicator function.

A commuter's generalized travel cost models the equivalent monetary cost of the travel time of the commuter and early or late arrival penalty. As common in the transportation literature [3], the generalized travel cost of commuter i with departure time t_i takes the form of

$$c_{\mathrm{tr}}(t_i) = \alpha w(t_i) + \beta(t^\star - t^{\mathrm{a}}(t_i))_+ + \gamma(t^{\mathrm{a}}(t_i) - t^\star)_+,$$

where $(z)_+ := \max(z, 0)$, α is the monetary value of time, and β and γ are the penalty for unit-time schedule delay of early arrival and late arrival, respectively. We also adopt the standard assumption (cf. [3] and references therein) that $\gamma > \alpha > \beta > 0$.

Upon arrival, each commuter can decide whether to charge her EV at a workplace charging spot. Given that the availability of outside charging options varies among the commuters, we model the valuation of workplace charging of commuter i by $\theta_i \in [\underline{\theta}, \overline{\theta}]$ with $\overline{\theta} > \underline{\theta} \geq 0$, where θ_i models the willingness to pay of commuter i for a charging session (also see Section II-D) at work. Let $F(\theta) := \int_{\mathcal{I}} \mathbb{1}\{\theta_i \leq \theta\} \mathrm{d}i$ be a function characterizing the fraction of commuters whose valuation is no larger than any $\theta \in [\underline{\theta}, \overline{\theta}]$. For simplicity, we assume that all chargers have the same power rating (e.g., 6.6 kW for AC level 2 chargers), and the energy requirement of all EVs is the same[2]. Let the decision of whether to charge

[2]The first assumption is typically valid for workplace charging. The second one is unlikely the case. However, since our focus is on the power consumption profile induced by the morning commute travel decisions, it will be clear later that this assumption does not qualitatively impact our results as long as each EV, if decided to charge, will stay being charged during the morning commute rush hours.

be $x_i \in \{0,1\}$, the payoff associated with the EV charging decision of commuter i is

$$u_{\text{ch}}(t_i, x_i) = (\theta_i - \pi(t^{\text{a}}(t_i)))x_i,$$

where $\pi(t)$ denotes what the commuter has to pay to the charging provider for the entire charging session if starting to charge at time t, and we assume that every commuter starts the charging session upon arrival if deciding to charge[3].

Given the decisions (t_i, x_i), commuter i's total payoff is

$$u(t_i, x_i) = u_{\text{ch}}(t_i, x_i) - c_{\text{tr}}(t_i). \tag{1}$$

Since the only difference among the set of commuters is that they may have different valuation of the charging service θ_i, the individual departure time and charging decisions are functions of the valuation θ_i for some functions $\tau : \mathbb{R}_+ \mapsto \mathcal{T}$ and $\sigma : \mathbb{R}_+ \mapsto \{0,1\}$, respectively, such that

$$t_i = \tau(\theta_i), \quad x_i = \sigma(\theta_i), \quad i \in \mathcal{I}.$$

C. User equilibrium

To this point, we have defined the *morning commute and charging game*, where the set of the *nonatomic* players is $\mathcal{I}$ and the payoff of each player i is defined according to (1). The payoff of player i depends implicitly on the decision of other players via the travel delay term $w(t)$, which will affect both the generalized travel cost and the charging payoff (if the charging cost is time-varying).

By extending the classical concept of *(dynamic) user equilibrium* [1], [12] to incorporate charging decisions, we arrive at the following solution concept for the morning commute and charging game. This solution concept also coincides with Nash Equilibrium for the considered nonatomic game.

Definition 1 (User equilibrium): Given any fixed charging tariff function π, for the collection of commuters $\mathcal{I}$, the departure time decision τ and the charging decision σ constitute a User Equilibrium (UE), if for all $i \in \mathcal{I}$, $t_i \in \mathcal{T}$ and $x_i \in \{0,1\}$, we have

$$u(\tau(\theta_i), \sigma(\theta_i)) \geq u(t_i, x_i). \tag{2}$$

The interpretation of (2) is straightforward: no user has incentive to unilaterally deviate from the decisions specified by (τ, σ), if (τ, σ) constitutes a UE. Such a UE may be understood as a day-to-day equilibrium. In other words, given the parameters of the morning commute problem fixed, all commuters will become aware of the traffic conditions after a sufficient amount of commute experience, and the eventual travel patterns in terms of departure time as well as the charging decisions will be such that individual commuters cannot further unilaterally improve their own payoff.

Crucially, Definition 1 in fact defines a family of equilibria parametrized by the charging tariff π. That is, we may obtain different UE decisions (τ, σ) when we vary π. For this reason, we will denote the UE decision associated with any particular charging tariff π by (τ_π, σ_π).

D. Serving the EV charging load

Let the power rating of each charger be p and the duration of each EV charging session be Δ. Denote the charging completion time of commuter $i \in \mathcal{I}$ with type θ_i by $t^{\text{e}}(\tau(\theta_i)) := t^{\text{a}}(\tau(\theta_i)) + \Delta$. Then, the electric load at time $t \in \mathcal{T}$ induced by the arrival of morning commuters is

$$\ell_t(\tau, \sigma) = \int_{\underline{\theta}}^{\overline{\theta}} p\sigma(\theta_i) \mathbb{1}\{t \in [t^{\text{a}}(\tau(\theta_i)), t^{\text{e}}(\tau(\theta_i))]\} \, \mathrm{d}F(\theta_i),$$

where $t \in [t^{\text{a}}(\tau(\theta_i)), t^{\text{e}}(\tau(\theta_i))]$ implies commuter i's EV is still being charged provided that the commuter decides to use workplace charging, i.e., $\sigma(\theta_i) = 1$. The continuous time process $\ell(\tau, \sigma) = \{\ell_t(\tau, \sigma) : t \in \mathcal{T}\}$ then captures the load for charging the EVs over the entire time horizon $\mathcal{T}$.

To secure power supply needed for serving this load, the charging service provider will incur some cost. The actual institutional arrangement (e.g., among the third-party charging service providers, utility companies, and wholesale electricity market operators) for the electricity industry is complex and analyzing the incentives of these players in details is beyond the scope of this study. Instead, we ignore the monetary transfers among these players and consider the following stylized cost function for serving the load motivated by the increasing penetration of solar power generation

$$c_{\text{pwr}}(\ell(\tau, \sigma)) = \lambda \int_{\underline{t}}^{t^{\text{sr}}} \ell_t(\tau, \sigma) \, \mathrm{d}t, \tag{3}$$

where $\lambda > 0$ is the constant marginal cost of conventional generators, and t^{sr} is the sunrise time.

Remark 1 (Choice of electricity cost model): For a commuter arriving at t_i^{a} and deciding to charge, her contribution to the electricity cost (3) is a piecewise linear function $\lambda p(t^{\text{sr}} - t_i^{\text{a}})_+$. Using this function in the electricity cost gives arguably the simplest model that still has two important qualitative features: (a) only the electricity cost has a direct dependence on the charging decision while the travel cost does not, and (b) the travel and electricity costs are optimized by different arrival profiles therefore need to be traded off when jointly optimized. Practically, the use of this function is motivated by solar integration challenges seen in states like California where the net electricity demand in the morning ramps down rapidly after sunrise. Our model is mathematically equivalent to a more realistic model where the electricity price is $\lambda_1 > 0$ before sunrise and $\lambda_2 \in (0, \lambda_1)$ after sunrise. Indeed, note that the charging cost of a commuter in this case is $(\lambda_1 - \lambda_2)p(t^{\text{sr}} - t_i^{\text{a}})_+ + \lambda_2 p\Delta$. Ignoring the constant term (without impacting the optimal tariff) and setting $\lambda = \lambda_1 - \lambda_2$ reduce this to our model.

E. Optimal charging tariff design

It is evident that a time-varying charging tariff π can impact user travel and charging behaviors in the UE, which in turn affect the EV charging load and therefore the cost of

[3]This may not be valid if commuters prefer to delay when their EVs are connected to a charger after arrival. However, delaying the connection time often requires extra costs and efforts from the commuters, including time spent to park at an alternative location before relocating to the charging spot (since most paid charging spots disallow free parking of unconnected EVs). This way of "gaming" the system may not be preferred due to the extra costs while considering it will add another layer of complexity to the model. We thus leave exploring this direction to future work.

powering such a charging load. *What is an optimal design for the time-varying charging tariff π when these impacts are taken into account?* The first step in answering this question is identifying the metrics to optimize. From a social planner's perspective, the following metrics are of interest.

1) Social cost, the transportation component: If the social planner is more concerned about the performance of the transportation system (e.g., traffic congestions and associated delays), the following metric, as commonly used in the classical morning commute literature, represents the total generalized travel cost of all the commuters considered:

$$J_{\mathrm{T}}(\tau,\sigma)=\int_{\underline{\theta}}^{\overline{\theta}} c_{\mathrm{tr}}(\tau(\theta_i))\ \mathrm{d}F(\theta_i).$$

2) Social cost, the electricity component: If the social planner is more concerned about the performance of the power system, the total cost associated with serving the electric load for charging the EVs less the total value created by the charging service will be of the primary concern:

$$J_{\mathrm{E}}(\tau,\sigma)=c_{\mathrm{pwr}}(\ell(\tau,\sigma))-\int_{\underline{\theta}}^{\overline{\theta}} \theta_i\sigma(\theta_i)\ \mathrm{d}F(\theta_i).$$

Including the second term is essential as otherwise the trivial solution where no one charges will minimize this cost metric.

3) Total social cost: If the social planner cares about the performance of this coupled transportation and electricity infrastructure system, the total social costs for both systems will be considered: $J_{\mathrm{T+E}}(\tau,\sigma)=J_{\mathrm{T}}(\tau,\sigma)+J_{\mathrm{E}}(\tau,\sigma)$. Note that since both the generalized travel cost and the cost of electricity for serving the charging load are in terms of monetary values, they can be added directly.

For simplicity, let the social cost under consideration be

$$J(\tau,\sigma;\kappa):=\kappa_{\mathrm{T}}J_{\mathrm{T}}(\tau,\sigma)+\kappa_{\mathrm{E}}J_{\mathrm{E}}(\tau,\sigma),$$

and when $\kappa:=(\kappa_{\mathrm{T}},\kappa_{\mathrm{E}})\in\{0,1\}^2$ take different values, J models different social cost metrics mentioned above. Using this definition, the problem of identifying the optimal charging tariff can be written as

$$\min_{\pi\in\Pi}\quad J(\tau_\pi,\sigma_\pi;\kappa), \tag{4}$$

where $\Pi:=\{\pi:\mathbb{R}_+\mapsto\mathbb{R}\}$ is the set of all arrival time dependent charging tariff functions and (τ_π,σ_π) is a UE induced by the charging tariff π.

This problem, despite its simple appearance, is very challenging. First, the decision space is infinite dimensional as we are optimizing over functions. Furthermore, we cannot directly control the departure time and charging decisions of individual commuters. Instead, we can only indirectly influence the commuter behaviors through the charging tariff, whose impact is limited since unlike tolls charging is not mandatory. Finally, characterizing UE behaviors under various charging tariff designs is in general a non-trival task.

Remark 2 (Existence and uniqueness of UE): The existence and uniqueness of UE (in terms of departure and arrival profiles) have been established for common settings in the transportation engineering literature [13]. However, for optimization (4) and with an arbitrary π, the existence and uniqueness of UE are not guaranteed. This does not lead to issues for us since our approach is to analyze specific tariffs and prove they are optimal by showing they match the corresponding social optimal cost lower bounds (Section III), rather than searching over the space of feasible tariffs.

III. Benchmarks and Structural Results

We will evaluate our charging tariff designs by comparing to the following two benchmarks.

1) Status quo: In the status quo, the workplace charging cost does not vary with the arrival time, i.e., the charging tariff is $\pi_0(t)\equiv\hat{\pi}_0$ for all $t\in\mathcal{T}$ for some constant $\hat{\pi}_0\geq 0$.

2) Centralized social cost minimization: Rather than individuals making their own decision of departure time and charging, in this ideal benchmark, a social planner determines the departure time and charging decisions for each commuter in a way that minimizes the social cost metric under consideration. In other words, the social planner solves the following optimization to "assign" individual commuters to ideal departure times and charging decisions

$$\min_{\tau\in\Gamma,\ \sigma\in\Sigma}\quad J(\tau,\sigma;\kappa), \tag{5}$$

where $\Gamma:=\{\tau:\mathbb{R}_+\mapsto\mathcal{T}\}$ and $\Sigma:=\{\sigma:\mathbb{R}_+\mapsto\{0,1\}\}$ are the sets of possible departure time and charging decision assignments based on commuters' charging valuation, respectively. Denote an optimal solution of (5) by $(\tau^\star,\sigma^\star)$ where the corresponding κ will be clear from context.

It is then evident that for any fixed κ, if we denote the charging tariff solving problem (4) by $\pi^\star$, we have

$$J(\tau_{\pi^\star},\sigma_{\pi^\star};\kappa)\geq J(\tau^\star,\sigma^\star;\kappa).$$

Therefore, the social cost minimization benchmark will serve as a lower bound for evaluating any tariff that we design. In particular, a tariff that achieves the lower bound is an optimal tariff for the social cost metric under consideration.

Even for the status quo benchmark, characterizing the UE is a non-trivial problem with the introduction of the heterogenous user valuation for charging service. A key step in addressing this challenge is establishing the following structural results about the UE induced by an arbitrary charging tariff $\pi(\cdot)$. **Proofs are omitted due to the page limit and can be found in the online report [14].**

Lemma 1 (Structural characterization of UE): Given any π, if (τ,σ) constitutes a UE, then following statements hold:

1) There exists a $\theta^\star\geq 0$ such that $\sigma(\theta)=\mathbb{1}\{\theta\geq\theta^\star\}$. Furthermore if $\theta^\star\in(\underline{\theta},\overline{\theta})$, then it is the solution to $\pi(t^{\mathrm{a}}(\tau(\theta)))=\theta$.
2) The generalized travel cost of all commuters within the *non-charging group* is identical, i.e., there exists a constant $C_{\mathrm{tr}}^{\mathrm{nc}}\geq 0$ such that
$$c_{\mathrm{tr}}(\tau(\theta_i))=C_{\mathrm{tr}}^{\mathrm{nc}},\ i\in\mathcal{I}_{\mathrm{nc}}=\{i\in\mathcal{I}:\theta_i<\theta^\star\}.$$
3) The sum of the generalized travel cost and the charging cost, defined as $\widetilde{c}_{\mathrm{tr}}(\tau(\theta_i))=c_{\mathrm{tr}}(\tau(\theta_i))+\pi(t^{\mathrm{a}}(\tau(\theta_i)))$, is a constant among all commuters within the *charging group*, i.e., there exists a constant $\widetilde{C}_{\mathrm{tr}}^{\mathrm{c}}\in\mathbb{R}$ such that,
$$\widetilde{c}_{\mathrm{tr}}(\tau(\theta_i))=\widetilde{C}_{\mathrm{tr}}^{\mathrm{c}},\quad i\in\mathcal{I}_{\mathrm{c}}=\{i\in\mathcal{I}:\theta_i\geq\theta^\star\}.$$

This result is intuitive and is established by showing that unless the said conditions are met, individual commuters can unilaterally improve their payoff.

Equipped with Lemma 1 and extending the UE characterization for the classical morning commute problem [1], [3], we can obtain the following results regarding the UE for the constant charging cost benchmark.

Lemma 2 (UE for the status quo benchmark): Any UE decisions (τ, σ) for the morning commute and charging game with tariff $\pi_0(t) \equiv \hat{\pi}_0 > 0$ satisfy following properties:

1) The charging group and non-charging group (see Lemma 1) are defined by the threshold value $\theta^\star = \hat{\pi}_0$.
2) The generalized travel cost of all commuters are identical at the UE, i.e.,

$$c_{\rm tr}(\tau(\theta_i)) = C^{\rm nc}_{\rm tr} = \widetilde{C}^{\rm c}_{\rm tr} - \pi_0 = \frac{\beta\gamma}{\beta+\gamma}\frac{1}{s}, \quad i \in \mathcal{I}.$$

3) The departure rate function at the UE for any $t \in \mathcal{T}$ is

$$d(t) = \begin{cases} s\alpha/(\alpha-\beta), & \text{if } t \in [t^{\rm q}_0, \tilde{t}\,), \\ s\alpha/(\alpha+\gamma), & \text{if } t \in [\tilde{t}, t^{\rm q\prime}_0], \end{cases}$$

where $t^{\rm q}_0$ is the earliest departure time among the commuters, $t^{\rm q\prime}_0$ is the latest departure time, and $\tilde{t}$ is the departure time such that the arrival time is $t^\star$. At the UE, these times are $\tilde{t} = t^\star - \beta\gamma/[\alpha(\beta+\gamma)s]$,

$$t^{\rm q}_0 = t^\star - \gamma/[(\beta+\gamma)s] \quad \text{and} \quad t^{\rm q\prime}_0 = t^{\rm q}_0 + 1/s. \tag{6}$$

Lemma 2 outlines the departure time decisions at the UE under the constant charging tariff status quo. Commuters will first depart at a rate higher than s and thus form a queue at the bottleneck, until the critical time $\widetilde{t}$, after which they depart at a rate lower than s. This leads to *morning commute rush/peak hours* $[t^{\rm q}_0, t^{\rm q\prime}_0]$, during which there is a queue at the bottleneck. As a result, the arrival rate during the rush hours is constant s. Fig. 2 depicts the departures and arrivals against time in this case, where we can show that the grey region has an area that is proportional[4] to the total traffic congestion delay for the population (i.e., the integration of the $\alpha w(t_i)$ term over $\mathcal{I}$). Given Lemma 2, it is easy for us to calculate the total schedule delay and thus the social cost for the transportation system.

The social cost for the power system is less clear. We know that the arrival rate will be s during $[t^{\rm q}_0, t^{\rm q\prime}_0]$. However, how this stream of arrivals during $[t^{\rm q}_0, t^{\rm q\prime}_0]$ are split between the charging group and non-charging group is unclear. In fact, *any* way to assign the charging/non-charging group departure (and the corresponding arrival) rates satisfying the following conditions constitutes a UE: (a) the total arrival rates of the charging and non-charging group sum up to s in $[t^{\rm q}_0, t^{\rm q\prime}_0]$, and (b) the total mass of non-charging group is $F(\hat{\pi}_0)$. In Fig. 2, the area of the yellow region is $c_{\rm pwr}(\ell(\tau_{\pi_0}, \sigma_{\pi_0}))/\lambda p$ (thus is proportional to the electricity cost) when we consider the worst-case way of splitting the arrivals where all early arrivals belongs to the charging group, assuming $1 - F(\hat{\pi}_0) \geq (t^{\rm sr} - t^{\rm q}_0)s$ so the charging group can fill up the bottleneck during the part of the rush hour before the sunrise. The total power system cost $J_{\rm E}(\tau_{\pi_0}, \sigma_{\pi_0})$ in this case is the electricity cost (i.e., the yellow area multiplied by λp) less the value served by charging service (i.e., $\int_{\hat{\pi}_0}^{\overline{\theta}} \theta_i \, {\rm d}F(\theta_i)$).

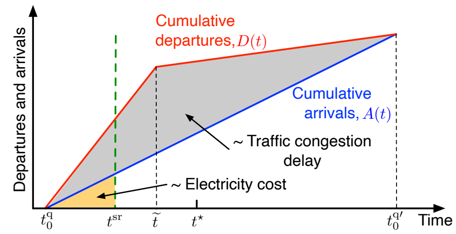

Fig. 2: Example of UE cumulative departures and arrivals under the status quo, where the yellow area that scales with the electricity cost represents UE charging decisions leading to the maximum electricity cost.

IV. Charging Tariff as a Toll

We start by analyzing the case where $\kappa = (1, 0)$. In this case, we are designing time-varying charging tariff to minimize the social cost for the transportation system. In some sense, this can be viewed as using charging tariff as a *more convenient toll*, whose implementation requires no traffic stops or dedicated new infrastructure. The downside of incentivizing commuters using a charging tariff instead of a toll is that not everyone has to pay the charging tariff, as some commuters may not drive an EV (which is not considered in this paper), and even if a commuter drives an EV, she may decide not to charge it at work. In other words, even though a time-varying charging tariff is easier to implement, its reach is limited and commuters have the option to strategically avoid it, based on their heterogeneous valuation for charging. This will be a recurrent theme throughout the rest of the paper when we optimize diverse social cost metrics.

When we are only optimizing the social cost for the transportation system, the solution to the centralized social cost minimization problem (5) is known in the transportation literature. Since the charging decision does not directly impact the generalized travel cost of commuters, it does not impact the social cost metric $J(\tau, \sigma; \kappa)$ when $\kappa = (1, 0)$. We reproduce the result from [3] here.

Lemma 3 (Transportation social cost minimization): The departure time assignment τ that solves the social cost minimization problem (5) with $\kappa = (1, 0)$ is such that the departure and arrival rates satisfy

$$d^\star_{\rm T}(t) = a^\star_{\rm T}(t) = \mathbb{1}\{t \in [t^{\rm q}_0, t^{\rm q\prime}_0]\}s, \quad t \in \mathcal{T}, \tag{7}$$

where $t^{\rm q}_0$ and $t^{\rm q\prime}_0$ are as defined in Lemma 2.

In other words, the social optimal assignment has the same rush hours $[t^{\rm q}_0, t^{\rm q\prime}_0]$ as in the status quo benchmark. During the rush hours, the commuters are coordinated such that they depart (and arrive) at rate s, maximizing the bottleneck capacity while not resulting in a queue. This completely eliminates the traffic delay term $\alpha w(t_i)$ in the generalized travel cost for every commuter i. However, since different commuters will have different schedule delay related cost at the solution, the solution by itself is not a UE as commuters

[4]Precisely, the area is the total traffic congestion delay divided by α.

have an incentive to unilaterally alter the departure time to minimize their schedule delays.

In the classical morning commute literature, the *optimal fine toll* [3] equalizes the schedule delays of different commuters and therefore decentralizes the social optimal solution (7). In our setting, we can use a time-varying charging tariff to mimic the optimal fine toll as long as we can incentivize every commuter to charge at work despite their heterogeneous valuation θ_i. This is achieved with the following charging tariff.

Lemma 4 (Transportation-optimal charging tariff): An optimal tariff solving (4) when $\kappa = (1, 0)$ is

$$\pi^\star_{\mathrm{T}}(t) = \pi_{\mathrm{oft}}\left(t;\, t^{\mathrm{q}}_0, t^{\mathrm{q}\prime}_0\right),$$

where the time-varying optimal fine toll based charging tariff with time interval $[t^{\mathrm{q}}, t^{\mathrm{q}\prime}]$ is defined as

$$\pi_{\mathrm{oft}}(t; t^{\mathrm{q}}, t^{\mathrm{q}\prime}) = \begin{cases} \underline{\theta} - \beta(t^\star - t)_+ - \gamma(t - t^\star)_+, & t \in [t^{\mathrm{q}}, t^{\mathrm{q}\prime}], \\ \overline{\theta}, & \text{otherwise.} \end{cases}$$

It is intuitive why this charging tariff is social cost optimal when only transportation costs are considered. As $\theta_i \geq \underline{\theta}$, it is optimal for all EV drivers to charge at work. Then, the form of the tariff ensures that the schedule delay terms in the travel cost are fully compensated by the negative terms in the charging tariff. As a result, departing following (7) is a UE. Denote such a departure time schedule by $\tau^\star_{\mathrm{T}}$ and the charging decision that all commuters charge at work by $\sigma^\star_{\mathrm{T}}$.

Fig. 3 depicts the cumulative departures and arrivals in this case. Comparing with Fig. 2, we can observe that the traffic congestion delay in the status quo is eliminated with the transportation-optimal tariff. Meanwhile, the morning commuter rush hours $[t^{\mathrm{q}}_0, t^{\mathrm{q}\prime}_0]$ stays the same as the status quo, so is the total schedule delay (as well as the transportation system cost) is obtained by integrating the schedule delay for all the commuters. On the other hand, since all commuters will charge at work with this tariff, the power system cost is the electricity cost (scaling with the area of the yellow region) less the charging value served, i.e., $\int_{\underline{\theta}}^{\overline{\theta}} \theta_i \,\mathrm{d}F(\theta_i)$.

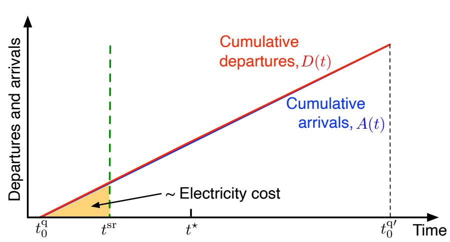

Fig. 3: Example of UE cumulative departures and arrivals under the transportation-optimal charging tariff.

V. Transportation-in-the-Loop Charging Pricing

We next analyze the setting where $\kappa = (0, 1)$. The idea here is to use a time-varying charging tariff to optimize the electricity cost of serving EV charging loads while recognizing and internalizing the extricable ties between commuters' travel and charging decisions. This is an important problem for power system operators as EV adoption increases.

With our stylized model for the power system cost, it is evident that $J_{\mathrm{E}}(\tau, \sigma)$ is optimized when all EVs decide to charge, and the cost of serving the charging load is 0. In other words, the power system cost is optimized when all the EVs arrive and charge after the sunrise time t^{sr}.

This can be achieved with many tariff designs.

Lemma 5 (Electricity-optimal charging tariffs): Any tariffs of the following form solve (4) with $\kappa = (0, 1)$:

$$\pi(t) = \mathbb{1}\{t < t^{\mathrm{sr}}\}\pi_-(t) + \mathbb{1}\{t \geq t^{\mathrm{sr}}\}\pi_+(t), \quad t \in \mathcal{T}, \quad (8)$$

where $\pi_-(t)$ and $\pi_+(t) \leq \underline{\theta}$ satisfy

$$\begin{aligned} &\beta(t^\star - t_-)_+ + \gamma(t_- - t^\star)_+ - (\underline{\theta} - \pi_-(t_-))_+ \geq \quad (9) \\ &\alpha/s + \beta(t^\star - t_+)_+ + \gamma(t_+ - t^\star)_+ - (\underline{\theta} - \pi_+(t_+))_+, \end{aligned}$$

for any $t_- < t^{\mathrm{sr}}$ and $t_+ \in [t^{\mathrm{sr}}, t^{\mathrm{sr}} + 1/s]$. In addition, the following tariff minimizes $J_{\mathrm{E}}(\tau_\pi, \sigma_\pi)$ while achieving the lowest $J_{\mathrm{T}}(\tau_\pi, \sigma_\pi)$ among all tariffs minimizing $J_{\mathrm{E}}(\tau_\pi, \sigma_\pi)$:

$$\pi^\star_{\mathrm{E}}(t) = \pi_{\mathrm{oft}}\left(t;\, t^{\mathrm{q}}_{\mathrm{E}}, t^{\mathrm{q}\prime}_{\mathrm{E}}\right), \quad (10)$$

where $t^{\mathrm{q}}_{\mathrm{E}} = \max\{t^{\mathrm{q}}_0, t^{\mathrm{sr}}\}$ and $t^{\mathrm{q}\prime}_{\mathrm{E}} = t^{\mathrm{q}}_{\mathrm{E}} + 1/s$.

Intuitively, tariffs of form (8) incentivize commuters to arrive and charge after t^{sr} by ensuring (a) commuters are better off to arrive during $[t^{\mathrm{sr}}, t^{\mathrm{sr}} + 1/s]$ with condition (9), and (b) commuters are better off to charge at work since $\pi_+(t) \leq \underline{\theta}$. In (9), the term α/s is an upper bound for the worst case traffic delay if arriving after t^{sr}. The particular tariff that we proposed in (10) in fact does not satisfy (8) and (9), which are sufficient conditions for optimal tariffs but not necessary. It is designed to nudge the departure and arrival times of all commuters to the interval $[t^{\mathrm{q}}_{\mathrm{E}}, t^{\mathrm{q}}_{\mathrm{E}} + 1/s]$, which is $[t^{\mathrm{q}}_0, t^{\mathrm{q}\prime}_0]$ if $t^{\mathrm{sr}} \leq t^{\mathrm{q}}_0$ and $[t^{\mathrm{sr}}, t^{\mathrm{sr}} + 1/s]$ otherwise. Furthermore, by compensating for the schedule delay terms as in the optimal fine toll defined in Lemma 4, it eliminates the traffic delay in the UE and it is not difficult to show that this tariff achieves the secondary goal of reducing the transportation system social cost given that the primary objective associated with the electricity system social cost is optimized.

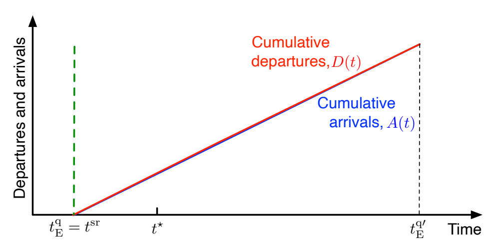

Fig. 4: Example of UE cumulative departures and arrivals under the electricity-optimal charging tariff (10).

VI. Nexus Cognizant Charging Pricing

In previous sections, we have seen that time-varying EV charging tariffs can be used to optimize the social cost metric for either the transportation system or the electricity system. In each of the cases, we are able to identify a charging tariff that achieves the same performance as the centralized social cost minimization benchmark for the metric under consideration. In this section, we move on to the setting where the social planner is cognizant to overall cost for the electricity-transportation nexus, i.e., when $\kappa = (1, 1)$. *Can the social costs for both the transportation and electricity systems be simultaneously optimized with one charging tariff? If not, can we identify a tariff that achieves an optimal trade-off between these two metrics?*

To answer these questions, we recognize one key common feature of the optimal tariffs that have been designed so far: the charging tariffs are such that all commuters are incentivized to charge their EVs at work. This is essential. Otherwise, commuters who do not charge at work will not *directly* face the incentives of the tariff and therefore their decisions may not follow the social cost minimization assignment. Indirectly nudging non-charging commuter via the decisions of charging commuters at the UE is possible. But it turns out that this is not sufficient for achieving the first best outcome as in social cost minimization benchmark. Motivated by this observation, we first characterize the social cost minimization solution when it is optimal to charge all EVs at work. Due to the space limit, we focus on the most typical scenario specified by the following two assumptions:

A1 *Sunrise time*. Sunrise time is earlier than the desired arrival time, i.e., $t^{\text{sr}} < t^\star$.

A2 *Charging rate*. AC level 2 chargers are used for work-place charging such that the charging duration is longer than the rush hours, i.e., $\Delta > 1/s$.

Assumption **A**1 usually holds when the desired arrival time is set to 9 a.m.. It may fail for certain regions with latitudes relatively close to the poles. For Assumption **A**2, using a common AC level 2 charger and considering the average commute distance in the U.S. and EV electricity consumption per mile, it usually takes more than the length of rush hours (e.g. 2 hours) to fully charge an EV used for commute.

A. All-charging case

Under these assumptions, we can obtain the following solution for the centralized social cost minimization when it is optimal for all commuters to charge at work.

Lemma 6 (Nexus cost minimization, all charging case): It is optimal for all commuters to charge in the centralized social cost minimization problem (5) with $\kappa = (1, 1)$ if and only if either one of the following two conditions hold:

1) *Early sunrise case*: If
$$t^{\text{sr}} \leq t_0^{\text{q}}, \tag{11}$$
where t_0^{q} is defined as in (6), then $\sigma^\star(\theta_i) = 1$ for all $i \in \mathcal{I}$, and the optimal departure time assignment $\tau^\star$ is such that the departure and arrival rates are $d_{\text{N}}^\star(t) = a_{\text{N}}^\star(t) = \mathbb{1}\{t \in [t_0^{\text{q}}, t_0^{\text{q}\prime}]\}s$, $t \in \mathcal{T}$.

2) *Late sunrise case*: If
$$t^{\text{sr}} > t_0^{\text{q}} \quad \text{and} \quad \underline{\theta} \geq \lambda p(t^{\text{sr}} - t_{\text{N1}}^{\text{q}}), \tag{12}$$
where $t_{\text{N1}}^{\text{q}} = t^{\text{sr}} - \frac{\gamma/s - (\beta+\gamma)(t^\star - t^{\text{sr}})}{\beta+\gamma+\lambda p}$, then $\sigma^\star(\theta_i) = 1$ for all $i \in \mathcal{I}$, and the optimal departure time assignment $\tau^\star$ is such that the departure and arrival rates are $d_{\text{N}}^\star(t) = a_{\text{N}}^\star(t) = \mathbb{1}\{t \in [t_{\text{N1}}^{\text{q}}, t_{\text{N1}}^{\text{q}\prime}]\}s$, $t \in \mathcal{T}$, where $t_{\text{N1}}^{\text{q}\prime} = t_{\text{N1}}^{\text{q}} + 1/s$.

Equations (11) and (12) offer a tight characterization for when it is optimal for all commuters to charge. Under these conditions, we can decentralize such social optimal solutions with the following tariffs.

Lemma 7 (Nexus-optimal tariff, all charging case): Under the all-charging condition (11) or (12), there is a charging tariff $\pi_{\text{N1}}^\star$ that decentralizes the social cost minimization decisions with $\kappa = (1, 1)$.

1) *Early sunrise case*: When (11) holds, $\pi_{\text{N1}}^\star(t) = \pi_{\text{T}}^\star(t)$.
2) *Late sunrise case*: When (12) holds, $\pi_{\text{N1}}^\star(t) = \pi_{\text{oft}}\left(t; t_{\text{N1}}^{\text{q}}, t_{\text{N1}}^{\text{q}\prime}\right)$.

In the early sunrise case, it turns out that the transportation-optimal tariff optimizes the performance of both systems. Indeed, when the sunrise is earlier than the earliest departure time under the transportation-optimal tariff, the cost of charging all EVs are zero and therefore there is no conflict between the cost metrics of the two systems. In the late sunrise case, this observation is no longer true. The goal to reduce the electricity cost in this case results in a later commute time window than that under the transportation-optimal tariff. The tradeoff between the cost metrics of the two systems also means that the earliest departure time is not t^{sr}; it is actually earlier than t^{sr} so it will incur non-zero cost for charging some EVs. The second inequality in (12) ensures the charging valuations are such that it is optimal to charge every EVs even at a non-zero cost.

B. General case

Moving beyond the all-charging conditions, the problem becomes much more challenging. The next result provides a complete characterization of the social optimal solution.

Theorem 1 (Nexus cost minimization, general case): Assume θ has probability density function $f(\theta) = \frac{\mathrm{d}}{\mathrm{d}\theta}F(\theta)$. When neither (11) nor (12) holds, the solution to the centralized social cost minimization with $\kappa = (1, 1)$ is characterized by a threshold in charging service valuation $\theta^\dagger$, which is the unique solution to the equation
$$\frac{[(\lambda p\gamma/2) - \lambda p(\beta+\gamma)F(\theta)]/s - \lambda p(\beta+\gamma)(t^\star - t^{\text{sr}})/2}{\beta+\gamma+\lambda p} = \theta.$$
The optimal charging decisions are $\sigma^\star(\theta_i) = \mathbb{1}\{\theta_i \geq \theta^\dagger\}$, $i \in \mathcal{I}$. The optimal departure time decisions are such that commuters depart at constant rate s during the interval $[t_{\text{N2}}^{\text{q}}, t_{\text{N2}}^{\text{q}\prime}]$, where $t_{\text{N2}}^{\text{q}\prime} = t_{\text{N2}}^{\text{q}} + 1/s$, and
$$t_{\text{N2}}^{\text{q}} = t^{\text{sr}} - \frac{[\lambda pF(\theta^\dagger) + \gamma]/s - (\beta+\gamma)(t^\star - t^{\text{sr}})}{\beta+\gamma+\lambda p}.$$
The non-charging group $\mathcal{I}_{\text{nc}}$, which accounts for $F(\theta^\dagger)$ fraction of the population, departs in $[t_{\text{N2}}^{\text{q}}, t_{\text{N2}}^{\text{q}} + F(\theta^\dagger)/s]$. The charging group $\mathcal{I}_{\text{c}}$ departs in $[t_{\text{N2}}^{\text{q}} + F(\theta^\dagger)/s, t_{\text{N2}}^{\text{q}\prime}]$.

Theorem 1 suggests that when the all-charging conditions fail, it is optimal for a $F(\theta^\dagger)$-portion of commuters, with lower valuations, to not charge. These commuters depart and arrive before the sunrise, in fact before $t_{\text{N2}}^{\text{q}} + F(\theta^\dagger)/s$ which can be shown to be strictly earlier than t^{sr}. All other commuters with higher valuations will depart later and charge their EV at work. Fig. 5 illustrates the solution.

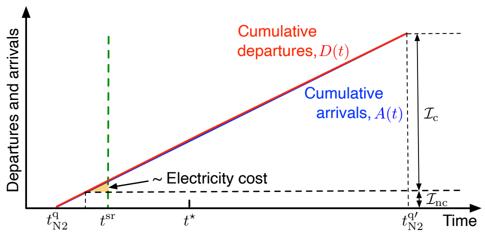

Fig. 5: Demonstration of social optimal cumulative departures and arrivals when all-charging conditions fail.

Since a portion of commuters will not charge in the social optimum, we can establish the following negative result.

Proposition 1 (Impossibility theorem): When neither (11) nor (12) holds, there is no charging tariff that can decentralize the social cost minimization solution for $\kappa = (1, 1)$.

Indeed, for the portion of commuters not to charge at work, charging tariffs will not be able to incentivize their travel behaviors. Trying to nudge the behaviors of the lower $F(\theta^{\dagger})$ portion via charging tariff is possible, but this comes at the cost that we need to incentivize them to charge first, which will result in sub-optimal electricity system cost:

Theorem 2 (All-charging tariff, general case): When neither (11) nor (12) holds, and with $\kappa = (1, 1)$, among the all-charging tariffs of the form $\pi_{\rm oft}\left(t;\, t^{\rm q}, t^{\rm q\prime}\right)$ with $t^{\rm q\prime} = t^{\rm q} + 1/s$, the tariff that optimizes the social cost metric is $\pi^{\star}_{\rm N1}(t) = \pi_{\rm oft}\left(t;\, t^{\rm q}_{\rm N1}, t^{\rm q\prime}_{\rm N1}\right)$.

Remarkably, it is optimal for the commuters to travel during $[t^{\rm q}_{\rm N1}, t^{\rm q\prime}_{\rm N1}]$ as in Lemma 7 (even when the all charging conditions (11) and (12) fail) if we restrict to all-charging tariffs. While the alternative tariff $\pi_{\rm oft}\left(t;\, t^{\rm q}_{\rm N2}, t^{\rm q\prime}_{\rm N2}\right)$ will incentivize the same departure time decisions as in the social optimum (cf. Theorem 1), it leads to a strictly higher total cost when the electricity cost is also considered.

VII. Comparison, Implications, and Conclusions

Collecting all our results, we can compare the social cost metrics in diverse settings. To simplify notation, let $J^{\pi}_{k} := J_k(\tau_\pi, \sigma_\pi)$, for any π and $k \in \{{\rm T}, {\rm E}\}$. Further, let $J^{\star}_{T} := \beta\gamma/[2(\beta+\gamma)s]$ (cf. [3]) and $J^{\star}_{E} := -\int_{\underline{\theta}}^{\overline{\theta}} \theta_i {\rm d}F(\theta_i)$ be the social optimal lower bounds obtained from centralized optimization. Under **A**1-2, we have:

Theorem 3 (Cost comparison):

1) If $t^{\rm sr} \le t^{\rm q}_0$ (i.e., early sunrise), then

$$J^{\pi_0}_{\rm T} > J^{\pi^{\star}_{\rm E}}_{\rm T} = J^{\pi^{\star}_{\rm N1}}_{\rm T} = J^{\pi^{\star}_{\rm T}}_{\rm T} = J^{\star}_{\rm T},$$

$$J^{\pi_0}_{\rm E} \ge J^{\pi^{\star}_{\rm T}}_{\rm E} = J^{\pi^{\star}_{\rm N1}}_{\rm E} = J^{\pi^{\star}_{\rm E}}_{\rm E} = J^{\star}_{\rm E}.$$

2) If $t^{\rm sr} > t^{\rm q}_0$ (i.e., late sunrise), then

$$J^{\pi_0}_{\rm T} \gtreqless J^{\pi^{\star}_{\rm E}}_{\rm T}, \quad J^{\pi^{\star}_{\rm E}}_{\rm T} > J^{\pi^{\star}_{\rm N1}}_{\rm T} > J^{\pi^{\star}_{\rm T}}_{\rm T} = J^{\star}_{\rm T},$$

$$J^{\pi_0}_{\rm E} \gtreqless J^{\pi^{\star}_{\rm T}}_{\rm E}, \quad J^{\pi^{\star}_{\rm T}}_{\rm E} > J^{\pi^{\star}_{\rm N1}}_{\rm E} > J^{\pi^{\star}_{\rm E}}_{\rm E} = J^{\star}_{\rm E}.$$

Theorem 3 packs both what is expected and what is surprising. First, in the early sunrise case, since the travel pattern optimizing the transportation costs also optimizes the electricity cost, there is no trade-off between the two cost metrics that we are considering. Therefore, we can reach both the lower bounds for $J_{\rm T}$ and $J_{\rm E}$ with the proposed tariffs, and they strictly improve upon the status quo performance. However, this is no longer the case for the late sunrise setting. The strings of strict inequalities are expected as typical in multi-objective optimization contexts, where the performance of the system measured by the optimized metric is strictly better than that measured by the metric that is not optimized for. Somewhat unexpected are the extents to which optimizing one metric may have on the performance of the other metric. For example, optimizing the electricity cost only could lead to a transportation system performance strictly worse than the status quo, which (as not hard to show) is already twice the optimal performance. This leads to the policy implication that *given the EV-induced strong coupling between the two systems, optimizing one system without considering its impact to the other can lead to severe negative externalities for the system not considered.*

This paper is only the first step in leveraging EVs to jointly optimize the coupled transportation and electricity systems. Our qualitative results highlight the importance of jointly modeling and managing the two systems. In future work, we plan to incorporate more realistic power system costs, address the institutional constraints (e.g., incentives and budget constraints of charging providers), explore joint toll and charging tariff design, and perform numerical evaluations with more complex power and transportation networks.

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