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Uniting low-energy semileptonic and hadronic anomalies within SMEFT

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Abstract: Two categories of four-fermion SMEFT operators are semileptonic (two quarks and two leptons) and hadronic (four quarks). At tree level, an operator of a given category contributes only to processes of the same category. However, when the SMEFT Hamiltonian is evolved down from the new-physics scale to low energies using the renormalization-group equations (RGEs), due to operator mixing this same SMEFT operator can generate operators of the other category at one loop. Thus, to search for a SMEFT explanation of a low-energy anomaly, or combination of anomalies, one must: (i) identify the candidate semileptonic and hadronic SMEFT operators, (ii) run them down to low energy with the RGEs, (iii) generate the required low-energy operators with the correct Wilson coefficients, and (iv) check that all other constraints are satisfied. In this paper, we illustrate this method by finding all SMEFT operators that, by themselves, provide a combined explanation of the (semileptonic) $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies and the (hadronic) $B \to \pi K$ puzzle.

Keywords: Rare Decays, Semi-Leptonic Decays, SMEFT

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Contents

1 Introduction

The standard model (SM) of particle physics has been enormously successful in describing the physics up to energy scales of $O(\text{TeV})$. It has made many predictions, almost all of which have been verified. Even so, it is not complete, as it cannot account for several observations (neutrino masses, dark matter, the baryon asymmetry of the universe, etc.). There must be physics beyond the SM.

No new particles have been observed at the LHC, so we are forced to conclude that this new physics (NP), whatever it is, must be heavy. The modern, model-independent approach to analyzing such NP uses effective field theories (EFTs). When the NP is integrated out, one obtains the SMEFT [1, 2],[1] an EFT that contains only the SM particles and obeys the SM gauge symmetry, $SU(3)_C \times SU(2)_L \times U(1)_Y$. The leading-order (dimension-4) terms are those of the SM; higher-order terms are suppressed by powers of the NP scale Λ.

These higher-order, non-SM terms can provide new contributions to low-energy processes. Their presence leads to an indirect signal of NP when the measurement of an observable in a given process disagrees with the prediction of the SM. Indeed, whenever such an anomaly is observed, one wants to identify the type of NP that could lead to this effect. One method is to build models. But the model-independent approach is to determine the SMEFT operator(s) that can modify the low-energy process. This is a complicated procedure because of operator mixing. A SMEFT operator is defined at the scale Λ. When the Hamiltonian is evolved down to low energies using the renormalization-group equations (RGEs), many operators are generated at one loop. This means that there are a number of SMEFT operators that could

[1]For a review, see ref. [3].

affect the process in question. But it also means that, if a given SMEFT operator creates a deviation in one low-energy process, it is likely to also lead to a discrepancy in another process (or more). Using the RGEs, we can determine which low-energy processes can be affected by each SMEFT operator (see, for example, refs. [4–12]).

At present, there are a number of anomalies in B decays. Some are in semileptonic decays,[2] while others are in hadronic decays [14–18]. Various NP solutions have been proposed as explanations of the individual anomalies, and as simultaneous explanations of all the semileptonic anomalies [19–28]. However, nobody has looked for a combined solution to one of each type of anomaly.[3] This involves finding NP that contributes to both semileptonic and hadronic $\bar{b} \to \bar{s}$ transitions. In this paper, we show that this can be done within SMEFT: when one takes a single SMEFT operator and runs it down to the scale m_b, both semileptonic and hadronic $\bar{b} \to \bar{s}$ operators can be generated. It is then necessary to check that the Wilson coefficients of these low-energy operators take the right values to explain the semileptonic and hadronic B anomalies. Other operators will also be generated, so that constraints from other processes must be taken into account. This procedure is quite general — it can be applied to any low-energy processes that exhibit a discrepancy with the SM (not just B decays).

In order to illustrate how this method works, here we focus on two specific B anomalies. First, for more than ten years, there have been a number of measurements of observables involving the semileptonic decay $\bar{b} \to \bar{s}\ell^+\ell^-$ ($\ell = \mu, e$) that are in disagreement with the predictions of the SM. Until 2021, these could be explained if only $\bar{b} \to \bar{s}\mu^+\mu^-$ receives NP contributions [13]. However, in late 2022, LHCb announced that it had remeasured the ratios R_K and R_{K^*}, which test lepton-flavour universality, and found that they agree with the SM [30, 31]. Now the most promising explanation is that the NP contributes equally to $\bar{b} \to \bar{s}\mu^+\mu^-$ and $\bar{b} \to \bar{s}e^+e^-$ [32–35].

The second B-decay anomaly has been around even longer, about 20 years: it is the $B \to \pi K$ puzzle (see refs. [14, 15] and references therein). Here the amplitudes for the four decays $B^+ \to \pi^+K^0$, $B^+ \to \pi^0K^+$, $B^0 \to \pi^-K^+$ and $B^0 \to \pi^0K^0$ obey a quadrilateral isospin relation. However, the measurements of the observables in these decays are not completely consistent with one another — there is a discrepancy at the level of $\sim 3\sigma$.

Because $\bar{b} \to \bar{s}\ell^+\ell^-$ and $B \to \pi K$ decays both involve $\bar{b} \to \bar{s}$ transitions, it is natural to look for a simultaneous explanation of the two anomalies. In this paper, we search for a single SMEFT operator that, when run down from the scale Λ to m_b, generates both $\bar{b} \to \bar{s}\ell^+\ell^-$ and $\bar{b} \to \bar{s}q\bar{q}$ ($q = u, d$) operators with Wilson coefficients of the right values to account for the $\bar{b} \to \bar{s}\ell^+\ell^-$ and $B \to \pi K$ anomalies. Constraints from other observables, such as $\bar{b} \to \bar{s}\nu\bar{\nu}$, ΔM_s, etc., must also be satisfied. As we will see, there are a handful of four-quark SMEFT operators that can do this.

We begin in section 2 with a review of the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies and the $B \to \pi K$ puzzle. In section 3, we examine the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies in the context of the SMEFT. We find that one semileptonic and six four-quark SMEFT operators can account for the $\bar{b} \to \bar{s}\ell^+\ell^-$ results, while satisfying all other constraints. The $B \to \pi K$ puzzle is studied in the context

[2] For example, see ref. [13].

[3] This is not completely true. In ref. [29], a Z' mode; is proposed to explain the $\bar{b} \to \bar{s}\mu^+\mu^-$ anomaly and the $B \to \pi K$ puzzle (and ε'/ε), but this was before the latest LHCb results regarding R_K and R_{K^*} [30, 31].

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of SMEFT in section 4. Of the seven SMEFT operators identified in section 3, we find that three four-quark operators can also give a good fit to the $B \to \pi K$ data, while one four-quark operator provides a passable fit. We conclude in section 5.

2 B-decay anomalies

2.1 $\bar{b} \to \bar{s}\ell^+\ell^-$

Over the years, there have been many analyses in which a global fit to all the $\bar{b} \to \bar{s}\ell^+\ell^-$ data was performed with the aim of determining which NP scenarios are preferred to explain the anomalies. The method is as follows.

At the scale m_b, the physics is described by the WET (weak effective theory), obtained by integrating out the heavy degrees of freedom. These include all SM and NP particles heavier than the b quark. The WET operators obey the $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{em}$ gauge symmetry. The effective Lagrangian describing $\bar{b} \to \bar{s}\ell^+\ell^-$ transitions is given by [36]

$$\mathcal{L}_{\rm eff} = \frac{4G_F}{\sqrt{2}}\, V_{tb}V_{ts}^*\, \frac{e^2}{16\pi^2} \sum_i \mathcal{C}_i \mathcal{O}_i \,, \tag{2.1}$$

where V_{ij} are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the WET operators are

$$\begin{aligned}
&\mathcal{O}_{9\ell}^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell)\,, && \mathcal{O}_{10\ell}^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)\,, \\
&\mathcal{O}_{7}^{(\prime)} = \frac{m_b}{e}\,(\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}\,, && \\
&\mathcal{O}_{S\ell}^{(\prime)} = (\bar{s} P_{R(L)} b)(\bar{\ell}\ell)\,, && \mathcal{O}_{P\ell}^{(\prime)} = (\bar{s} P_{R(L)} b)(\bar{\ell}\gamma_5\ell)\,,
\end{aligned} \tag{2.2}$$

with $P_{L(R)} = \frac{1}{2}(1 - (+)\gamma_5)$. In the above, only operators that are generated at dimension 6 in the SMEFT have been kept. This requirement also imposes the following conditions on the Wilson coefficients (WCs): $C_{S\ell} = -C_{P\ell}$ and $C'_{S\ell} = C'_{P\ell}$.

$\mathcal{H}_{\rm eff}$ is valid at the scale m_b. All information about the heavy particles that have been integrated out is encoded in the WCs of the operators, $\mathcal{C}_i$. Of the eight operators describing $\bar{b} \to \bar{s}\ell^+\ell^-$ transitions, the SM contributes mainly to $\mathcal{O}_7$, $\mathcal{O}_{9\ell}$ and $\mathcal{O}_{10\ell}$, but the NP can contribute to all of them.

There are a great many $\bar{b} \to \bar{s}\ell^+\ell^-$ observables. Some involve only the $\bar{b} \to \bar{s}\mu^+\mu^-$ transition. These include branching ratios (e.g., $\mathcal{B}(B^+ \to K^+\mu^+\mu^-)$, $\mathcal{B}(B_s \to \mu^+\mu^-)$, etc.) and angular observables in four-body decays (e.g., $B \to K^*(\to K\pi)\mu^+\mu^-$, $B_s \to \phi(\to K^+K^-)\mu^+\mu^-$). Others measure lepton-flavour-universality violation:

$$R_K = \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+e^+e^-)}\,, \quad R_{K^*} = \frac{\mathcal{B}(B \to K^*\mu^+\mu^-)}{\mathcal{B}(B \to K^*e^+e^-)}\,, \quad R_\phi = \frac{\mathcal{B}(B_s \to \phi\,\mu^+\mu^-)}{\mathcal{B}(B_s \to \phi\, e^+e^-)}\,. \tag{2.3}$$

The SM predicts that all of these ratios equal 1 (to a very good approximation).

All observables can be written as a function of the WCs in eq. (2.1). These WCs are separated into their SM and NP contributions: $\mathcal{C}_i = \mathcal{C}_i^{\rm SM} + \mathcal{C}_i^{\rm NP}$. By performing fits to the data, using subsets of the $\mathcal{C}_i^{\rm NP}$ as free parameters, it is possible to determine which (combinations of) $\mathcal{C}_i^{\rm NP}$ best explain the data, and what their best-fit values are.

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Decay	$\mathcal{B} \times 10^{-6}$	A_{CP}	S_{CP}
$B^+ \to \pi^+ K^0$	23.9 ± 0.6	-0.003 ± 0.015	
$B^+ \to \pi^0 K^+$	13.2 ± 0.4	0.027 ± 0.012	
$B^0 \to \pi^- K^+$	20.0 ± 0.4	-0.0831 ± 0.0031	
$B^0 \to \pi^0 K^0$	10.1 ± 0.4	0.00 ± 0.08	0.64 ± 0.13

Table 1. Branching ratios, direct CP asymmetries A_{CP}, and mixing-induced CP asymmetry S_{CP} (if applicable) for the four $B \to \pi K$ decay modes. The data are taken from the Particle Data Group 2024 [37].

Following the LHCb announcement in 2022 of the new results for R_K and R_{K^*} that agreed with the SM, four global-fit analyses appeared, refs. [32–35]. All found that the NP scenario that best fits the data involves a single WC, $\mathcal{C}_{9\mu}^{\rm NP} = \mathcal{C}_{9e}^{\rm NP} \equiv C_9^{\rm U}$. And they found similar best-fit values: $C_9^{\rm U} = -1.08 \pm 0.18$ [32], $C_9^{\rm U} = -0.77 \pm 0.21$ [33], $-1.17^{+0.16}_{-0.17}$ [34], -1.18 ± 0.19 [35]. In our analysis, we require that $C_9^{\rm U}$ equal the last of these best-fit values.

2.2 $B \to \pi K$

Here we follow and update the analysis of ref. [14]. There are four $B \to \pi K$ decays: $B^+ \to \pi^+ K^0$ (designated as $+0$ below), $B^+ \to \pi^0 K^+$ $(0+)$, $B^0 \to \pi^- K^+$ $(-+)$ and $B^0 \to \pi^0 K^0$ (00). Their amplitudes obey a quadrilateral isospin relation:

$$\sqrt{2} A^{00} + A^{-+} = \sqrt{2} A^{0+} + A^{+0} \,. \tag{2.4}$$

Using these decays, nine observables have been measured: the four branching ratios, the four direct CP asymmetries A_{CP}, and the mixing-induced indirect CP asymmetry S_{CP} in $B^0 \to \pi^0 K^0$. The latest data are shown in table 1.

Within the diagrammatic approach of refs. [38, 39], B-decay amplitudes are expressed in terms of six diagrams:[4] the color-favored and color-suppressed tree amplitudes T' and C', the gluonic penguin amplitudes P'_{tc} and P'_{uc}, and the color-favored and color-suppressed electroweak penguin amplitudes P'_{EW} and P'^{C}_{EW} (the primes on the amplitudes indicate $\bar{b} \to \bar{s}$ transitions). The $B \to \pi K$ amplitudes are given by

$$\begin{aligned}
A^{+0} &= -P'_{tc} + P'_{uc} e^{i\gamma} - \frac{1}{3} P'^{C}_{EW} \,, \\
\sqrt{2} A^{0+} &= -T' e^{i\gamma} - C' e^{i\gamma} + P'_{tc} - \; P'_{uc} e^{i\gamma} - \; P'_{EW} - \frac{2}{3} P'^{C}_{EW} \,, \\
A^{-+} &= -T' e^{i\gamma} + P'_{tc} - P'_{uc} e^{i\gamma} - \frac{2}{3} P'^{C}_{EW} \,, \\
\sqrt{2} A^{00} &= -C' e^{i\gamma} - P'_{tc} + P'_{uc} e^{i\gamma} - P'_{EW} - \frac{1}{3} P'^{C}_{EW} \,.
\end{aligned} \tag{2.5}$$

The weak-phase dependence (including the minus sign from $V_{tb}^* V_{ts}$ [in P'_{tc}]) is written explicitly, so that the diagrams contain both strong phases and the magnitudes of the CKM matrix

[4]The annihilation, exchange and penguin-annihilation diagrams are neglected, as they are expected to be very small in the SM.

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$\chi^2_{\rm min}/{\rm d.o.f.} = 16.9/5$, p-value $= 0.005$	
Parameter	Best-fit value
γ	$(64.74 \pm 2.86)^\circ$
β	$(22.12 \pm 0.69)^\circ$
$\lvert T'\rvert$	8.4 ± 0.9
$\lvert P'_{tc}\rvert$	51.2 ± 0.4
$\delta_{P'_{tc}}$	$(-13.2 \pm 1.7)^\circ$
$\delta_{C'}$	$(260.7 \pm 16.0)^\circ$

Table 2. $\chi^2_{\rm min}/{\rm d.o.f.}$ and best-fit values of unknown parameters in amplitudes of eq. (2.5). Constraints: $B \to \pi K$ data, measurements of β and γ, theoretical inputs $|C'/T'| = 0.2$, $P'_{uc} = 0$.

elements. The amplitudes for the CP-conjugate processes are obtained from the above by changing the sign of the weak phase γ.

For the fit to the data, we use the following conditions regarding the unknown theoretical parameters (we refer the reader to ref. [14] for explanations). (i) The diagrams P'_{EW} and P'^C_{EW} are not independent — to a good approximation, they can be related to T' and C' within the SM using flavor SU(3) symmetry [40–42]. (ii) We fix $|C'/T'| = 0.2$, its preferred theoretical value [43–47]. (iii) We neglect P'_{uc}.[5] (iv) We adopt the convention that the strong phase $\delta_{T'} = 0$ [49]. (v) For the weak phases, we include the constraints from direct measurements: $\beta = (22.2 \pm 0.7)^\circ$, $\gamma = (66.2^{+3.4}_{-3.6})^\circ$ [50].[6]

With the above conditions, there are four independent theoretical parameters in the amplitudes: $|T'|$, $|P'_{tc}|$, and the two strong phases $\delta_{P'_{tc}}$ and $\delta_{C'}$. With nine $B \to \pi K$ observables, a fit can be done; the results are given in table 2. The fit is poor: $\chi^2_{\rm min}/{\rm d.o.f.} = 16.9/5$, corresponding to a p-value of 0.5%. This is the $B \to \pi K$ puzzle: there is a discrepancy with the SM at the level of 2.8σ.

We now turn to NP. At the level of the WET effective Hamiltonian, the NP operators that contribute to the $B \to \pi K$ amplitudes take the form $\mathcal{O}^{ij,q}_{NP} \sim \bar{s}\Gamma_i b\, \bar{q}\Gamma_j q$ $(q = u, d)$, where $\Gamma_{i,j}$ represent Lorentz structures, and color indices are suppressed. The NP contributions to $B \to \pi K$ are encoded in the matrix elements $\langle \pi K | \mathcal{O}^{ij,q}_{NP} | B \rangle$. In general, each matrix element has its own NP weak and strong phases.

In ref. [14], it was argued that the NP strong phases are negligible (see also ref. [49]), so that one can combine many NP matrix elements into a single NP amplitude, with a single weak phase. There are two classes of such NP amplitudes, differing only in their color structure:

$$\sum \langle \pi K | \, \bar{s}_\alpha \Gamma_i b_\alpha \, \bar{q}_\beta \Gamma_j q_\beta \, | B \rangle \equiv \mathcal{A}'^{,q} e^{i\Phi'_q} \,,$$
$$\sum \langle \pi K | \, \bar{s}_\alpha \Gamma_i b_\beta \, \bar{q}_\beta \Gamma_j q_\alpha \, | B \rangle \equiv \mathcal{A}'^{C,q} e^{i\Phi'^C_q} \,, \quad q = u, d \,. \tag{2.6}$$

[5] For example, see ref. [48].
[6] See also online updates at [51].

Although there are four NP matrix elements that contribute to $B \to \pi K$ decays, only three combinations appear in the amplitudes: $\mathcal{A}'^{,comb} e^{i\Phi'} \equiv -\mathcal{A}'^{,u} e^{i\Phi'_u} + \mathcal{A}'^{,d} e^{i\Phi'_d}$, $\mathcal{A}'^{C,u} e^{i\Phi'^C_u}$, and $\mathcal{A}'^{C,d} e^{i\Phi'^C_d}$. The $B \to \pi K$ amplitudes can now be written in terms of the SM diagrams and these NP matrix elements:

$$\begin{aligned}
A^{+0} &= -P'_{tc} - \frac{1}{3} P'^C_{EW} + \mathcal{A}'^{C,d} e^{i\Phi'^C_d} , \\
\sqrt{2} A^{0+} &= P'_{tc} - T' \, e^{i\gamma} - P'_{EW} - C' \, e^{i\gamma} - \frac{2}{3} P'^C_{EW} + \mathcal{A}'^{,comb} e^{i\Phi'} - \mathcal{A}'^{C,u} e^{i\Phi'^C_u} , \\
A^{-+} &= P'_{tc} - T' \, e^{i\gamma} - \frac{2}{3} P'^C_{EW} - \mathcal{A}'^{C,u} e^{i\Phi'^C_u} , \\
\sqrt{2} A^{00} &= -P'_{tc} - P'_{EW} - C' \, e^{i\gamma} - \frac{1}{3} P'^C_{EW} + \mathcal{A}'^{,comb} e^{i\Phi'} + \mathcal{A}'^{C,d} e^{i\Phi'^C_d} .
\end{aligned} \tag{2.7}$$

We note in passing that a better understanding of these NP contributions can be found with a change of basis [52]:

$$\begin{aligned}
P'_{EW,NP} \, e^{i\Phi'_{EW}} &\equiv \mathcal{A}'^{,u} e^{i\Phi'_u} - \mathcal{A}'^{,d} e^{i\Phi'_d} , \\
P'_{NP} \, e^{i\Phi'_P} &\equiv \frac{1}{3} \mathcal{A}'^{C,u} e^{i\Phi'^C_u} + \frac{2}{3} \mathcal{A}'^{C,d} e^{i\Phi'^C_d} , \\
P'^C_{EW,NP} \, e^{i\Phi'^C_{EW}} &\equiv \mathcal{A}'^{C,u} e^{i\Phi'^C_u} - \mathcal{A}'^{C,d} e^{i\Phi'^C_d} .
\end{aligned} \tag{2.8}$$

In order, these three terms correspond to the inclusion of NP in the color-allowed electroweak penguin, the gluonic penguin, and the color-suppressed electroweak penguin amplitudes.

In the most general case, each of the three independent NP matrix elements has its own weak phase. There are therefore 10 parameters in the $B \to \pi K$ amplitudes: 5 magnitudes of diagrams, 2 strong phases, and 3 NP weak phases. This is greater than the number of observables (9), so a fit cannot be performed.

However, suppose that these NP matrix elements are the low-energy remnants of the same SMEFT operator. That is, we begin at the high-energy NP scale with a single SMEFT operator and evolve it down to the scale m_b with the RGEs. This will generate a variety of $\bar{b} \to \bar{s} q \bar{q}$ operators ($q = u, d$), and these can be used to compute the NP matrix elements (magnitudes and weak phases). A fit can then be performed to see if the addition of these NP contributions removes the discrepancy with the SM. As we will see in the next section, this is indeed possible. Not only that, but this same SMEFT operator will generate $\mathcal{O}_{9\ell}$ with $C_9^{\rm U} = -1.18 \pm 0.19$!

3 SMEFT and $\bar{b} \to \bar{s} \ell^+ \ell^-$

In the previous section, we saw that the most promising NP scenario to explain the $\bar{b} \to \bar{s} \ell^+ \ell^-$ anomalies involves a new contribution to the WET operator $\mathcal{O}_{9\ell} = (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \ell)$ with a value of the WC $C_9^{\rm U} = -1.18 \pm 0.19$. We also saw that any solution to the $B \to \pi K$ puzzle must involve NP contributions to WET $\bar{b} \to \bar{s} q \bar{q}$ operators ($q = u, d$). And we intimated that combined explanations can be found in which the WET operators all arise from the same SMEFT operator.

In order to see how this comes about, we need to examine (i) properties of the SMEFT, (ii) the SMEFT solution of the $\bar{b} \to \bar{s} \ell^+ \ell^-$ anomalies, and (iii) how this $\bar{b} \to \bar{s} \ell^+ \ell^-$ SMEFT solution connects to the $B \to \pi K$ puzzle. In this section, we focus on the first two points; the connection to $B \to \pi K$ will be discussed in section 4.

3.1 SMEFT

Up to dimension 6, the SMEFT Lagrangian can be written as

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{Q_a^\dagger = Q_a} C_a Q_a + \sum_{Q_a^\dagger \neq Q_a} \left(C_a Q_a + C_a^* Q_a^\dagger \right) . \tag{3.1}$$

Here $\mathcal{L}_{\rm SM}$ is the dimension-4 part of the SMEFT Lagrangian. The Q_a are dimension-6 operators; the C_a are their Wilson coefficients. All the Q_a were derived in refs. [2]; this set of operators is known as the Warsaw basis.

The SMEFT Lagrangian describes the physics from the scale Λ down to the weak scale. At the weak scale, the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry is broken to $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{em}$ and the WET Lagrangian is now applicable. One can work out which SMEFT operators generate a given WET operator. These are known as matching conditions.

But note that SMEFT operators are defined in the flavour (gauge) basis, whereas the WET operators use the physical mass basis. Thus, matching SMEFT and WET operators requires us to take into account the unitary transformations from the flavour to the mass basis. We adopt the convention that the flavour and mass eigenstates are the same for the charged leptons and down-type quarks. For the neutrinos, the difference between the two bases is unimportant, since they are not detected. However, for up-type quarks,

$$U_{Li} = V_{ij}^\dagger U_{Lj}^0 , \tag{3.2}$$

where $U = (u, c, t)^T$, V is the CKM matrix, and the superscript 0 (lack of superscript) indicates the flavour (mass) eigenstates.

Finally, both the SMEFT and WET are energy-dependent. That is, although the SMEFT is applicable from scale Λ down to the weak scale, the coefficients of the operators are not constant. Their values change due to operator mixing as the SMEFT evolves from one energy scale to another. This can all be computed using the RGEs. This also applies to the WET. RGEs can be used to calculate how the WET coefficients change as we evolve from the weak scale down to m_b.

3.2 $\bar{b} \to \bar{s}\ell^+\ell^-$ in SMEFT

As discussed above, the preferred NP solution to the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies is that there is a new contribution to the WET operator $\mathcal{O}_{9\ell} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)$ with a value of $\mathcal{C}_{9\mu}^{\rm NP} = \mathcal{C}_{9e}^{\rm NP} \equiv C_9^{\rm U} = -1.18 \pm 0.19$. The question is: what is the origin of this lepton-flavour-universal (LFU) effect? (Note that here "universal" refers to only e and μ.)

3.2.1 WET-SMEFT matching conditions

There are five vector SMEFT operators that involve two quarks and two leptons:

$$\begin{aligned} Q_{\ell q}^{(1)} &= (\bar{\ell}_i \gamma_\mu \ell_j)(\bar{q}_k \gamma^\mu q_\ell) , & Q_{\ell q}^{(3)} &= (\bar{\ell}_i \gamma_\mu \tau^I \ell_j)(\bar{q}_k \gamma^\mu \tau^I q_\ell) , \\ Q_{\ell d} &= (\bar{\ell}_i \gamma_\mu \ell_j)(\bar{d}_k \gamma^\mu d_\ell) , & Q_{qe} &= (\bar{q}_i \gamma_\mu q_j)(\bar{e}_k \gamma^\mu e_\ell) , \\ Q_{ed} &= (\bar{e}_i \gamma_\mu e_j)(\bar{d}_k \gamma^\mu d_\ell) . \end{aligned} \tag{3.3}$$

Here q and ℓ are LH $\mathrm{SU}(2)_L$ doublets, while d and e are RH $\mathrm{SU}(2)_L$ singlets. i, j, k, l are flavour indices — they indicate the generation of the fermion field. There are three vector SMEFT operators involving the Higgs field:

$$\begin{aligned}
&Q_{\varphi q}^{(1)} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_i \gamma^\mu q_j)\,, \qquad\qquad Q_{\varphi q}^{(3)} = (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{q}_i \tau^I \gamma^\mu q_j)\,, \\
&Q_{\varphi d} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_i \gamma^\mu d_j)\,.
\end{aligned} \tag{3.4}$$

There are four vector $\bar{b} \to \bar{s}\ell^+\ell^-$ WET operators: $\mathcal{O}_{9\ell}^{(\prime)}$ and $\mathcal{O}_{10\ell}^{(\prime)}$ [see eq. (2.2)]. The matching conditions between the WET and SMEFT operators are given by

$$\begin{aligned}
C_{9,\ell} &= \frac{1}{2\mathcal{N}} \left([C_{\ell q}^{(1)}]_{ll23} + [C_{\ell q}^{(3)}]_{ll23} + [C_{qe}]_{23ll} + \left[C_{\varphi q}^{(1)23} + C_{\varphi q}^{(3)23} \right] (-1 + 4\sin^2\theta_W) \right)\,, \\
C_{10,\ell} &= \frac{1}{2\mathcal{N}} \left([C_{qe}]_{23ll} - [C_{\ell q}^{(1)}]_{ll23} - [C_{\ell q}^{(3)}]_{ll23} + \left[C_{\varphi q}^{(1)23} + C_{\varphi q}^{(3)23} \right] \right)\,, \\
C'_{9,\ell} &= \frac{1}{2\mathcal{N}} \left([C_{\ell d}]_{ll23} + [C_{ed}]_{ll23} + C_{\varphi d}^{23}(-1 + 4\sin^2\theta_W) \right)\,, \\
C'_{10,\ell} &= \frac{1}{2\mathcal{N}} \left([C_{ed}]_{ll23} - [C_{\ell d}]_{ll23} + C_{\varphi d}^{23} \right)\,,
\end{aligned} \tag{3.5}$$

where $\mathcal{N} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2}$. For $\ell = e$, μ and τ, the indices ll are respectively 11, 22 and 33. Note that, since $\sin^2\theta_W = 0.223$, the factor $-1 + 4\sin^2\theta_W = -0.11$, so that the contribution of $C_{\varphi q}^{(1)23} + C_{\varphi q}^{(3)23}$ to $C_{9,\ell}$ is suppressed compared to its contribution to $C_{10,\ell}$.

These matching conditions are very informative. First, for a given ℓ (e or μ), in order to have only $C_{9,\ell}$ nonzero, we require particular relations among $[C_{ed}]_{ll23}$, $[C_{\ell d}]_{ll23}$ and $C_{\varphi d}^{23}$ (for $C'_{9,\ell}$ and $C'_{10,\ell}$), and among $[C_{qe}]_{23ll}$, $[C_{\ell q}^{(1)}]_{ll23}$, $[C_{\ell q}^{(3)}]_{ll23}$, $C_{\varphi q}^{(1)23}$ and $C_{\varphi q}^{(3)23}$ (for $C_{10,\ell}$). Second, in order to have $C_{9,\mu} = C_{9,e}$, an additional equality among WCs is needed, namrly $[C_{\ell q}^{(1)}]_{1123} + [C_{\ell q}^{(3)}]_{1123} + [C_{qe}]_{2311} = [C_{\ell q}^{(1)}]_{2223} + [C_{\ell q}^{(3)}]_{2223} + [C_{qe}]_{2322}$. In other words, we require many special relations among the WCs of operators that are *a-priori* independent. While this is logically possible, it is somewhat "fine-tuned."

3.2.2 One-loop RGE running

Fortunately, there is an alternative, more compelling explanation. Suppose we have a single SMEFT operator that contributes to $\bar{b} \to \bar{s} f \bar{f}$, where f is a light SM fermion. As the Hamiltonian is evolved down to low energies, renormalization-group running naturally generates $\bar{b} \to \bar{s}\ell^+\ell^-$ transitions at one loop via the exchange of an off-shell neutral gauge boson (GB) [53]: $f\bar{f} \to GB^* \to \ell^+\ell^-$. For the running from Λ to the weak scale, $GB = W_3^0, B^0$, while for the running from the weak scale to m_b, GB is dominantly a photon. This produces the operator $\mathcal{O}_{9\ell}$. And since the μ and e have the same quantum numbers (I_3, Y, Q_{em}), we automatically have $\mathcal{C}_{9\mu}^{\rm NP} = \mathcal{C}_{9e}^{\rm NP} \equiv C_9^{\rm U}$. This can then potentially account for the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies.

SMEFT operators that contribute to $\bar{b} \to \bar{s} f \bar{f}$ come in two categories: (i) semileptonic operators (two quarks, two leptons), and (ii) four-quark operators.[7] These can have scalar or vector Lorentz structures. However, we also require that the renormalization-group running

[7] Four-quark operators that contribute to $\bar{b} \to \bar{s}\ell^+\ell^-$ at one loop have also been considered within WET, see refs. [54–56].

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operator	definition	chirality	flavour structure
$Q_{\ell q}^{(1)}$	$(\bar{\ell}_i\gamma_\mu\ell_j)(\bar{q}_k\gamma^\mu q_\ell)$	$(\bar{L}L)(\bar{L}L)$	3323
$Q_{\ell q}^{(3)}$	$(\bar{\ell}_i\gamma_\mu\tau^I\ell_j)(\bar{q}_k\gamma^\mu\tau^I q_\ell)$		3323
$Q_{\ell d}$	$(\bar{\ell}_i\gamma_\mu\ell_j)(\bar{d}_k\gamma^\mu d_\ell)$	$(\bar{L}L)(\bar{R}R)$	3323
Q_{qe}	$(\bar{q}_i\gamma_\mu q_j)\ (\bar{e}_k\gamma^\mu e_\ell)$		2333
Q_{ed}	$(\bar{e}_i\gamma_\mu e_j)(\bar{d}_k\gamma^\mu d_\ell)$	$(\bar{R}R)(\bar{R}R)$	3323

Table 3. The list of semileptonic SMEFT operators that can potentially generate an LFU $\mathcal{O}_{9\ell}$ at scale m_b.

operator	definition	chirality	flavour structure
$Q_{qq}^{(1)}$	$(\bar{q}_i\gamma_\mu q_j)(\bar{q}_k\gamma^\mu q_\ell)$	$(\bar{L}L)(\bar{L}L)$	$ii23$
$Q_{qq}^{(3)}$	$(\bar{q}_i\gamma_\mu\tau^I q_j)(\bar{q}_k\gamma^\mu\tau^I q_\ell)$		$ii23$
$Q_{qd}^{(1)}$	$(\bar{q}_i\gamma_\mu q_j)(\bar{d}_k\gamma^\mu d_\ell)$	$(\bar{L}L)(\bar{R}R)$	$23ii$ and $ii23$
$Q_{qd}^{(8)}$	$(\bar{q}_i\gamma_\mu T^A q_j)\ (\bar{d}_k\gamma^\mu T^A d_\ell)$		$23ii$ and $ii23$
$Q_{qu}^{(1)}$	$(\bar{q}_i\gamma_\mu q_j)(\bar{u}_k\gamma^\mu u_\ell)$		$23ii$
$Q_{qu}^{(8)}$	$(\bar{q}_i\gamma_\mu T^A q_j)\ (\bar{u}_k\gamma^\mu T^A u_\ell)$		$23ii$
Q_{dd}	$(\bar{d}_i\gamma_\mu d_j)(\bar{d}_k\gamma^\mu d_\ell)$	$(\bar{R}R)(\bar{R}R)$	$ii23$
$Q_{ud}^{(1)}$	$(\bar{u}_i\gamma_\mu u_j)(\bar{d}_k\gamma^\mu d_\ell)$		$ii23$
$Q_{ud}^{(8)}$	$(\bar{u}_i\gamma_\mu\tau^I u_j)(\bar{d}_k\gamma^\mu\tau^I d_\ell)$		$ii23$

Table 4. The list of four-quark SMEFT operators that can potentially generate an LFU $\mathcal{O}_{9\ell}$ at scale m_b. Here $ii = 11, 22$ or 33.

of an operator generate an LFU $\mathcal{O}_{9\ell}$ at the scale m_b. This excludes the SMEFT operators with a scalar Lorentz structure.

The list of vector semileptonic SMEFT operators that can potentially generate an LFU $\mathcal{O}_{9\ell}$ is given in table 3 [see also eq. (3.3)]. Although the flavour indices are suppressed, we can easily deduce the possibilities. Because we have a $b \to s$ transition, the quark current must have indices 2 and 3. And because we want $\mathcal{C}_{9\mu}^{\rm NP} = \mathcal{C}_{9e}^{\rm NP}$, the indices for the leptonic current can only be 3 and 3. There are a total of 5 possible semileptonic operators.

Table 4 contains the list of possible vector four-quark SMEFT operators. One of the quark currents must have flavour indices 2 and 3, while the other quark current has indices i and i, $i = 1, 2, 3$. For the $Q_{qd}^{(1)}$ and $Q_{qd}^{(8)}$ operators, this flavour assignment can be done in two different ways. There are therefore a total of 33 possible four-quark operators.

3.2.3 $C_9^{\rm U} = -1.18 \pm 0.19$

Having identified the 5 semileptonic and 33 four-quark SMEFT operators that have the potential to generate an LFU $\mathcal{O}_{9\ell}$ operator with $C_9^{\rm U} = -1.18 \pm 0.19$, we must now determine which ones actually do this. To this end, we use the Wilson package [57] to perform the

$C_{\rm SMEFT}$ (TeV^{-2})		$C_9^{\rm U}$	$C_{10}^{\rm U}$	$C_9^{\prime\rm U}$	$C_{10}^{\prime\rm U}$
$[C_{lq}^{(1)}]_{3323}$	-0.23 ± 0.04	$-1.20 - i0.022$	-0.004	0	0
$[C_{lq}^{(3)}]_{3323}$	-0.23 ± 0.04	$-1.17 - i0.022$	-0.021	0	0
$[C_{qe}]_{2333}$	-0.22 ± 0.03	$-1.16 - i0.022$	-0.005	0	0

Table 5. For the subset of semileptonic SMEFT operators in table 3 that generate the desired $C_9^{\rm U}$ when run down to m_b: (i) central value and error of the WC that generates $C_9^{\rm U} = -1.18 \pm 0.19$ and (ii) central values of predictions for $C_9^{\rm U}$, $C_{10}^{\rm U}$, $C_9^{\prime\rm U}$, $C_{10}^{\prime\rm U}$. Operators producing all the desired WET WCs are highlighted in gray.

renormalization-group running from the NP scale Λ to the weak scale within SMEFT, and then from the weak scale to m_b within WET.

We begin with the semileptonic SMEFT operators and calculate the values of $C_{9,\ell}$, $C_{10,\ell}$, $C'_{9,\ell}$ and $C'_{10,\ell}$ for $\ell = e, \mu$ generated when we perform the running from scale Λ to m_b. Note that, because $Q_{\ell d}$ and Q_{ed} involve only RH down-type quarks (see table 3), they can never generate $C_{9,\ell}$ (or $C_{10,\ell}$). However, the other three semileptonic operators can generate $C_{9,\ell}$. Indeed, they all generate WCs that are universal, i.e., equal for $\ell = e, \mu$. In table 5, we list these operators, along with the central value + error of the SMEFT WC that generates the desired $C_9^{\rm U}$ and the central values of the generated WET WCs.

We now turn to the four-quark operators. The 15 operators $Q_{qd}^{(1)}$ and $Q_{qd}^{(8)}$ with flavour assignment $ii23$, Q_{dd}, $Q_{ud}^{(1)}$, and $Q_{qd}^{(1)}$ all involve only RH down-type quarks (see table 4), and so cannot generate $C_{9,\ell}$ (or $C_{10,\ell}$). However, the remaining 18 four-quark operators can generate $C_9^{\rm U}$ and $C_{10}^{\rm U}$, but here the universal index U means equal for $\ell = e, \mu, \tau$. In table 6, we list these operators, along with the central value + error of the SMEFT WC that generates the desired $C_9^{\rm U}$ and the central values of the generated WET WCs. Although all operators can generate the right $C_9^{\rm U}$, they are not all viable solutions of the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies. The point is that the preferred solution has NP contributions to $C_9^{\rm U}$, but $C_{10}^{\rm U}$ is small. However, in table 6 we see that four operators — $[C_{qq}^{(1)}]_{2333}$, $[C_{qq}^{(3)}]_{2333}$, $[C_{qu}^{(1)}]_{2333}$, $[C_{qu}^{(8)}]_{2333}$ — also generate large $C_{10}^{\rm U}$. This occurs because these operators mix strongly with $C_{\varphi q}^{(1)}$, which has a large contribution to C_{10} [see eq. (3.5)]. These operators are therefore excluded.

The upshot is that there are 3 semileptonic and 14 four-quark SMEFT operators that generate the required $C_9^{\rm U}$ WC when run down to the m_b scale. These are therefore potential solutions to the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies. However, this running may also generate other operators which may be constrained by different observables. For example, we see that the running generates a small imaginary piece of $C_9^{\rm U}$. While this is unimportant for $\bar{b} \to \bar{s}\ell^+\ell^-$ processes, imaginary pieces of the WCs of other operators may be generated. In principle, this may result in new, CP-violating contributions to other processes, and there may be constraints on such contributions. These other constraints are examined in the next subsubsection.

3.3 Other constraints

In general, through running, a given SMEFT operator will contribute to a variety of WET operators, and there may be important constraints on some of these. Thus, before declaring that the addition of a particular SMEFT operator can explain the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies, one

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C_{SMEFT} (TeV^{-2})		C_9^{U}	C_{10}^{U}	$C_9^{\prime\text{U}}$	$C_{10}^{\prime\text{U}}$
$[C_{qq}^{(1)}]_{1123}$	0.21 ± 0.03	$-1.17 - i0.022$	-0.004	0	0
$[C_{qq}^{(1)}]_{2223}$	0.25 ± 0.04	$-1.18 - i0.022$	0.028	0	0
$[C_{qq}^{(1)}]_{3323}$	2.5 ± 0.4	$-1.17 - i0.022$	$104.8 + i1.9$	0	0
$[C_{qq}^{(3)}]_{1123}$	-0.07 ± 0.01	$-1.15 - i0.022$	-0.019	0	0
$[C_{qq}^{(3)}]_{2223}$	-0.091 ± 0.014	$-1.17 - i0.022$	-0.023	0	0
$[C_{qq}^{(3)}]_{2333}$	-0.13 ± 0.02	$-1.17 - i0.022$	$2.86 + i0.05$	0	0
$[C_{qd}^{(1)}]_{2311}$	-0.22 ± 0.03	$-1.18 - i0.022$	-0.005	0	0
$[C_{qd}^{(1)}]_{2322}$	-0.22 ± 0.03	$-1.18 - i0.022$	-0.005	0	0
$[C_{qd}^{(1)}]_{2333}$	-0.22 ± 0.03	$-1.18 - i0.022$	-0.008	0	0
$[C_{qd}^{(8)}]_{2311}$	-3.10 ± 0.50	$-1.17 - i0.022$	$-0.040 - i0.0007$	0	0
$[C_{qd}^{(8)}]_{2322}$	-3.10 ± 0.50	$-1.17 - i0.022$	$-0.040 - i0.0007$	0	0
$[C_{qd}^{(8)}]_{2333}$	-3.10 ± 0.50	$-1.17 - i0.022$	$-0.041 - i0.0007$	0	0
$[C_{qu}^{(1)}]_{2311}$	0.11 ± 0.01	$-1.19 - i0.022$	-0.004	0	0
$[C_{qu}^{(1)}]_{2322}$	0.11 ± 0.01	$-1.19 - i0.022$	-0.005	0	0
$[C_{qu}^{(1)}]_{2333}$	0.50 ± 0.08	$-1.19 - i0.022$	$-20.97 - i0.39$	0	0
$[C_{qu}^{(8)}]_{2311}$	1.12 ± 0.18	$-1.18 - i0.022$	0.013	0	0
$[C_{qu}^{(8)}]_{2322}$	1.12 ± 0.18	$-1.18 - i0.022$	0.013	0	0
$[C_{qu}^{(8)}]_{2333}$	18.5 ± 3.0	$-1.17 - i0.022$	$-14.83 - i0.27$	0	0

Table 6. For the subset of four-quark SMEFT operators in table 4 that generate the desired C_9^{U} when run down to m_b: (i) central value and error of the WC that generates $C_9^{\text{U}} = -1.18 \pm 0.19$ and (ii) central values of predictions for C_9^{U}, C_{10}^{U}, $C_9^{\prime\text{U}}$, $C_{10}^{\prime\text{U}}$. Operators producing all the desired WET WCs are highlighted in gray.

has to be sure that all other constraints have been taken into account. In what follows we discuss the important constraints for each SMEFT operator under consideration.

Note that the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies are not the only measurements of semileptonic B decays that disagree with the SM. For many years, there have been anomalies in the measurements of $R_{D^{(*)}}$ that suggest NP in $b \to c\tau^-\bar{\nu}_\tau$ [58]. And recently, Belle II measured $\mathcal{B}(B^+ \to K^+\nu\bar{\nu}$, finding a value almost 3σ above the SM prediction [59]. We do not require our SMEFT operator to explain these anomalies. Instead, the constraint imposed is that the predictions for these observables should not be larger than the experimental value within 2σ, though it can be smaller. For the other constraints, we require that the prediction agree with the experimental value within 2σ.

3.3.1 $\bar{b} \to \bar{s}\nu\bar{\nu}$

We begin with $\bar{b} \to \bar{s}\nu\bar{\nu}$ transitions. The first step is to identify the WET operators that (i) contribute to $\bar{b} \to \bar{s}\nu\bar{\nu}$ and (ii) can be generated when we run the SMEFT operators down to the m_b scale. The SMEFT operators that have been identified above as having the

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potential to generate the required $C_9^{\rm U}$ WC are all vector operators. As a consequence, they will generate only vector $\bar{b} \to \bar{s}\nu\bar{\nu}$ WET operators. These are

$$\mathcal{N} C_L (\bar{s}\gamma_\mu P_L b)(\bar{\nu}_i \gamma^\mu (1-\gamma_5)\nu_i) + \mathcal{N} C_R (\bar{s}\gamma_\mu P_R b)(\bar{\nu}_i \gamma^\mu (1-\gamma_5)\nu_i) + h.c., \tag{3.6}$$

where $\mathcal{N}$ is defined below eq. (3.5).

For the first operator, the tree-level matching condition to the SMEFT operators is

$$C_L = \frac{1}{2\mathcal{N}}([C_{lq}^{(1)}]_{ii23} - [C_{lq}^{(3)}]_{ii23})\,. \tag{3.7}$$

A nonzero $[C_{lq}^{(1)}]_{3323}$ or $[C_{lq}^{(3)}]_{3323}$ are both potential solutions to the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies, see table 5. So there may be important constraints on these WCs from $\bar{b} \to \bar{s}\nu\bar{\nu}$. (Note that these constraints can be evaded in a specific model such as that of the U_1 leptoquark, in which $[C_{lq}^{(1)}]_{3323} = [C_{lq}^{(3)}]_{3323}$.) For the second operator, the WC C_R matches at tree level to C_{ld}. However, $Q_{\ell d}$ does not generate the required $C_9^{\rm U}$ WC, so that constraints on $C_{\ell d}$ are irrelevant. Of course, all the semileptonic WCs of table 5 may generate one or both of these WET operators through one-loop RGE running; if so, we must compute the constraints on these SMEFT WCs.

There are three $\bar{b} \to \bar{s}\nu\bar{\nu}$ processes that have been measured. Their current experimental values are

$$\begin{aligned}
\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) &= (2.3 \pm 0.5(\text{stat})^{+0.5}_{-0.4}(\text{syst})) \times 10^{-5}\,, \\
\mathcal{B}(B^+ \to K^{*+}(892)\nu\bar{\nu}) &< 4 \times 10^{-5} \text{ (90\% C.L.)}\,, \\
\mathcal{B}(B^0 \to K^0\nu\bar{\nu}) &< 2.6 \times 10^{-5} \text{ (90\% C.L.)}\,.
\end{aligned} \tag{3.8}$$

We take the first measurement from ref. [59] and the other two from ref. [37].

For the constraints on the SMEFT operators, we require only that the predicted branching ratios not exceed the measured values/limits to within 2σ (the theoretical and experimental (if applicable) errors are combined in quadrature).

3.3.2 $b \to c\tau^-\bar{\nu}_\tau$

Turning to the transition $b \to c\tau^-\bar{\nu}_\tau$, there are again several WET operators that contribute. However, only a single one can be generated by the running of a dimension-6 vector SMEFT operator. It is

$$\mathcal{N}'\, C_V^{33}\, (\bar{c}_L\gamma^\mu b_L)\,(\bar{\tau}_L\gamma_\mu\nu_{\tau L})\ , \tag{3.9}$$

where $\mathcal{N}' = \frac{4G_F}{\sqrt{2}}V_{cb}$. Its tree-level matching condition to the SMEFT operators is

$$C_V^{33} = \frac{1}{2\mathcal{N}'}\, V_{2k}\, [C_{lq}^{(3)}]^*_{33k3}\,, \tag{3.10}$$

where V_{2k} is a CKM matrix element [see eq. (3.2)]. In particular, $[C_{lq}^{(3)}]_{3323}$, which generates the required $C_9^{\rm U}$ when run down to m_b, also generates $b \to c\tau^-\bar{\nu}_\tau$ at tree level. We stress that the other semileptonic WCs of table 5 may also generate such a transition, albeit at loop level.

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The decays $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ involve the transition $b \to c\tau^-\bar{\nu}_\tau$. The ratios $R_{D^{(*)}}$ are defined as

$$R_D = \frac{\mathcal{B}(\bar{B} \to D\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D\ell^-\bar{\nu}_\ell)}\,, \quad R_{D^*} = \frac{\mathcal{B}(\bar{B} \to D^*\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^*\ell^-\bar{\nu}_\ell)}\,,\ \ell = e.\mu\,. \tag{3.11}$$

These have been measured by several different experiments. The latest world averages are [58]

$$R_D = 0.344 \pm 0.026\,, \quad R_{D^*} = 0.285 \pm 0.012\,, \tag{3.12}$$

with a correlation of -0.39. Both values are larger than the SM predictions, which are $R_D = 0.298(4)$ and $R_{D^*} = 0.254(5)$ [50, 51]. When the data of all the $b \to c\tau^-\bar{\nu}_\tau$ observables are combined, there is a 4.3σ disagreement with the SM.

As was the case with $\bar{b} \to \bar{s}\nu\bar{\nu}$, we do not require that our SMEFT operator explain the $b \to c\tau^-\bar{\nu}_\tau$ anomaly. (Indeed, only $[C_{lq}^{(3)}]_{3323}$, which generates $b \to c\tau^-\bar{\nu}_\tau$ at tree level, could possibly do so.) Instead, we only require that the predicted values of $R_{D^{(*)}}$ not exceed the measured values to within 2σ. That is, these predicted values can be smaller (like the SM).

3.3.3 $\Delta F = 2$ observables

The four-quark operators of table 6 violate quark flavour by one unit at tree level. All of them have $\Delta B = \Delta S = 1$, and those that involve up-type quarks can have $\Delta C = 1$ due to eq. (3.2). This means that, at one loop, i.e., through RGE running, they can contribute to $\Delta F = 2$ processes [6]. That is, there may be constraints on the SMEFT operators from low-energy measurements of the real and imaginary parts of K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing. This is examined in this subsubsection.

There are a variety of $\Delta F = 2$ observables that can, in principle, be used to constrain the SMEFT operators, but we find that only two of these — ΔM_s and ε_K — provide significant constraints. Here we focus on these two observables.

There are scalar and vector WET operators that contribute to $\Delta F = 2$ meson-mixing processes. As usual, since we have only vector SMEFT operators at the high-energy scale, only vector WET operators will be generated when we run down to the scale m_b. These WET operators are

$$\begin{aligned}
[O_{dd}^{V,LL}]_{ijij} &= (\bar{d}_i\gamma_\mu P_L d_j)(\bar{d}_i\gamma^\mu P_L d_j)\,, \\
[O_{dd}^{V,RR}]_{ijij} &= (\bar{d}_i\gamma_\mu P_R d_j)(\bar{d}_i\gamma^\mu P_R d_j)\,, \\
[O_{dd}^{V1,LR}]_{ijij} &= (\bar{d}_i\gamma_\mu P_L d_j)(\bar{d}_i\gamma^\mu P_R d_j)\,, \\
[O_{dd}^{V8,LR}]_{ijij} &= (\bar{d}_i\gamma_\mu P_L T^A d_j)(\bar{d}_i\gamma^\mu P_R T^A d_j)\,,
\end{aligned} \tag{3.13}$$

where $i \neq j$. Their tree-level matching conditions to SMEFT operators are

$$\begin{aligned}
[C_{dd}^{V,LL}]_{ijij} &= -([C_{qq}^{(1)}]_{ijij} + [C_{qq}^{(3)}]_{ijij})\,, \\
[C_{dd}^{V1,LR}]_{ijij} &= -[C_{qd}^{(1)}]_{ijij}\,, \\
[C_{dd}^{V8,LR}]_{ijij} &= -[C_{qd}^{(8)}]_{ijij}\,, \\
[C_{dd}^{V,RR}]_{ijij} &= -[C_{dd}]_{ijij}.
\end{aligned} \tag{3.14}$$

None of these SMEFT operators appear in table 6. This is good, as they would produce meson mixing at tree level, which is excluded. On the other hand, some operators in table 6 may mix at one loop with the above operators, and will thereby contribute to $\Delta F = 2$ meson mixing.

Consider first ΔM_s, which measures the real part of the B_s^0-$\bar{B}_s^0$ mixing amplitude. The operator $[C_{qq}^{(1)}]_{3323}$ (see table 6) contributes at tree level to $(\bar{b}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L b)$. But the $(\bar{b}\gamma_\mu P_L b)$ current can be turned into a $(\bar{s}\gamma_\mu P_L b)$ current at one loop via a box diagram similar to that describing B_s^0-$\bar{B}_s^0$ mixing. In the same way, other $\Delta F = 1$ operators in table 6 may also contribute to meson mixing at one loop, though the size of the effect depends on the exact flavour configuration [6].

In order to include the ΔM_s constraint, we run the $\Delta F = 1$ SMEFT WCs down to m_b and generate the $\Delta F = 2$ operators. These are then converted into a prediction for ΔM_s, which is compared with its experimental value [37]:

$$(\Delta M_s)_{\rm exp.} = 1.1683(13) \times 10^{-11}~\text{GeV}. \tag{3.15}$$

We require that the prediction and the experimental value agree to within 2σ, which places a constraint on the SMEFT WC.

The second important $\Delta F = 2$ observable is ε_K, which measures the imaginary part of the K^0-$\bar{K}^0$ mixing amplitude (i.e., indirect CP violation). As was the case for ΔM_s, when the $\Delta F = 1$ SMEFT operators are run down to m_b, they may generate at one loop $\Delta F = 2$ operators that lead to K^0-$\bar{K}^0$ mixing. Here the key point is that, even if the SMEFT WC is real, the running can produce an imaginary part of the WET WC, as was done with $C_9^{\rm U}$ (see table 6). This may lead to an important contribution to ε_K.

To place constraints on the SMEFT WCs from ε_K, we use [8]

$$(\varepsilon_K)^{NP} = \kappa_\varepsilon \times 10^{-3}\,, \quad -0.2 \le \kappa_\varepsilon \le 0.2. \tag{3.16}$$

3.3.4 CP violation in $K_L \to \pi\pi$

Another observable that can be used as a constraint is ε'/ε, which is a measure of direct CP violation in $K_L \to \pi\pi$ decays. This is a $\Delta F = 1$ process, which receives contributions from a variety of WET operators describing $s \to dq\bar{q}$ $(q = u, d)$ transitions.

Including isospin-breaking corrections, along with recent computations of the matrix elements from the RBC-UKQCD lattice group [60], the SM prediction is given by [61, 62]

$$(\varepsilon'/\varepsilon)_{\rm SM} = (13.9 \pm 5.4) \times 10^{-4}\,. \tag{3.17}$$

The experimental world average of measurements from the NA48 [63] and KTeV [64, 65] experiments is

$$(\varepsilon'/\varepsilon)_{\rm exp} = (16.3 \pm 2.3) \times 10^{-4}\,, \tag{3.18}$$

which is consistent with the SM prediction within the errors.

When our SMEFT operators are run down to m_b. they may generate some WET $s \to dq\bar{q}$ operators. And since the running may produce an imaginary piece for these WET WCs, there may be a contribution to ε'/ε. To compute this contribution, we use the master formula from ref. [66].

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As a constraint, we require that the value of ε'/ε found when one includes the SM and NP contribution agree with the experimental value within 2σ. However, we note that the error on the SM prediction is very large (40%). Furthermore, this error is almost entirely theoretical, so it is only an estimate. For this reason, we will not use the ε'/ε constraint to exclude any SMEFT operators; we will only note if there is a tension.

3.3.5 High-p_T searches

We began our analysis by looking for SMEFT operators that contribute to $\bar{b} \to \bar{s} f \bar{f}$, where f is a light SM fermion. In the literature, it has been suggested that one can search for, or put limits on, NP in $\bar{b} \to \bar{s} f \bar{f}$ by looking at the high-p_T distribution for $pp \to f\bar{f}X$ at the LHC.

The idea is as follows. The SM contribution to $pp \to f\bar{f}X$ comes mainly from $q\bar{q} \to f\bar{f}$ via the s-channel exchange of a γ, Z^0 or gluon, while the NP contribution is simply a four-fermion interaction. We can write

$$A = A_{SM} + A_{NP} = \frac{C_{SM}}{E^2} + \frac{C_{NP}}{\Lambda^2}\,, \tag{3.19}$$

where, for simplicity, we take C_{SM} and C_{NP} to be real. This leads to

$$|A|^2 = \frac{1}{E^4}\left[C_{SM}^2 + 2C_{SM}C_{NP}\frac{E^2}{\Lambda^2} + C_{NP}^2\frac{E^4}{\Lambda^4}\right]. \tag{3.20}$$

At low energies ($E \ll \Lambda$), this is dominated by the SM contribution. But as $E \to \Lambda$, the NP contributions become increasingly important. Thus, by looking at the distribution for $pp \to f\bar{f}X$ as a function of the $f\bar{f}$ invariant mass^2, and by subtracting the SM contribution, one will be sensitive to the NP contribution at high invariant mass^2. Perhaps a signal of NP will be seen, and if not, a limit can be placed on C_{NP}/Λ^2. This type of analysis has been done in refs. [67, 68] ($f = \tau$), [69, 70] ($f = q$) and [71] ($f = e, \mu$).

The problem here is that an EFT is really only applicable at $E \ll \Lambda$. As $E \to \Lambda$, the expansion in $1/\Lambda$ begins to break down. For example, in eq. (3.19) above, we included only the dimension-6 NP contribution. But say we add a dimension-8 term. This could be a four-fermion interaction with two derivatives. The coefficient is proportional to $1/\Lambda^4$, but by dimensional analysis there must be a factor of E^2 in the numerator (e.g., coming from the two derivatives). We then have

$$\begin{aligned} A &= \frac{C_{SM}}{E^2} + \frac{C_{NP}}{\Lambda^2} + \frac{C_{NP8}}{\Lambda^4} \\ |A|^2 &= \frac{1}{E^4}\left[C_{SM}^2 + 2C_{SM}C_{NP}\frac{E^2}{\Lambda^2} + C_{NP}^2\frac{E^4}{\Lambda^4} + 2C_{SM}C_{NP8}\frac{E^4}{\Lambda^4}\right]. \end{aligned} \tag{3.21}$$

Now, for $E \simeq \Lambda$, the last contribution is as important as the others. Even if $E < \Lambda$, the last term has the potential to be as important as the third term, since it is possible that $C_{NP8} > C_{NP}$. The point is that there is a certain amount of uncertainty in this type of analysis, depending on how close E is to Λ. For this reason, we do not require our SMEFT WCs to satisfy such constraints.

$C_{\rm SMEFT}$	$\mathcal{B}(B \to K\nu\bar{\nu})$	R_D	R_{D^*}
$[C_{lq}^{(1)}]_{3323}$	$(6.18 \pm 0.88) \times 10^{-4}$ $(\times)$	0.292 ± 0.007 $(\surd)$	0.242 ± 0.007 $(\surd)$
$[C_{lq}^{(3)}]_{3323}$	$(8.55 \pm 1.04) \times 10^{-4}$ $(\times)$	0.524 ± 0.013 $(\times)$	0.435 ± 0.010 $(\times)$
$[C_{qe}]_{2333}$	$(4.38 \pm 0.62) \times 10^{-6}$ $(\surd)$	0.295 ± 0.006 $(\surd)$	0.245 ± 0.006 $(\surd)$

Table 7. Semileptonic SMEFT WCs of table 5: predictions for $\mathcal{B}(B \to K\nu\bar{\nu})$, R_D and R_{D^*}, along with an indicator of whether the constraint is satisfied $(\surd)$ or violated $(\times)$. Operators that satisfy all the constraints are highlighted in gray.

3.3.6 SMEFT operators confront other constraints

There are two types of SMEFT operators that can generate the correct value of $C_9^{\rm U}$ when run down to the m_b scale, semileptonic operators (table 5) and four-quark operators (table 6). Similarly, the other constraints described above come in two categories, semileptonic constraints ($\bar{b} \to \bar{s}\nu\bar{\nu}$, $R_{D^{(*)}}$) and hadronic constraints (ΔM_s, ε_K, ε'/ε). It turns out that the constraints of a given type are important only for SMEFT operators of the same type. That is, the semileptonic (four-quark) operators satisfy all the hadronic (semileptonic) constraints.

In table 7, we show the predictions of the semileptonic SMEFT WCs of table 5 for $\mathcal{B}(B \to K\nu\bar{\nu})$, R_D and R_{D^*}. $[C_{lq}^{(1)}]_{3323}$ and $[C_{lq}^{(3)}]_{3323}$ both predict values for $\mathcal{B}(B \to K\nu\bar{\nu})$ that are much larger than the experimental measurements [eq. (3.8)], and $[C_{lq}^{(3)}]_{3323}$ predicts values for R_D and R_{D^*} that are also larger than the experimental values [eq. (3.12)]. Only $[C_{qe}]_{2333}$ satisfies all the constraints.

In table 8, we show the predictions of the four-quark SMEFT WCs of table 6 that generate the desired WET WCs for ΔM_s, ε_K and ε'/ε. Of the 14 WCs, only 6 of them — $[C_{qq}^{(1)}]_{1123}$, $[C_{qq}^{(3)}]_{1123}$, $[C_{qu}^{(1)}]_{2311}$, $[C_{qu}^{(1)}]_{2322}$, $[C_{qu}^{(8)}]_{2311}$, $[C_{qu}^{(8)}]_{2322}$ — satisfy the constraints. Also, there is a caveat that $[C_{qq}^{(1)}]_{1123}$ has a possible tension with ε'/ε.

4 SMEFT and $B \to \pi K$

In section 2.2, we saw that the $B \to \pi K$ amplitudes can be expressed in terms of diagrams. When the ratio $|C'/T'|$ is fixed to 0.2 (its preferred theoretical value) and a fit to the data is performed, we find that there is a 2.8σ discrepancy with the SM. This is the $B \to \pi K$ puzzle.

We also saw that all NP contributions to $B \to \pi K$ can be combined into three distinct matrix elements, each with its own weak phase. If all these phases are different, then there are too many unknown parameters to do a fit. However, if all three NP matrix elements are generated when a single SMEFT operator is run down to the scale m_b, then there will be only two NP parameters, the magnitude and the phase of the SMEFT WC. In this case, a fit can be performed, and we can see if the addition of the NP produces a good fit.

In the previous section, we found seven SMEFT operators that, when run down to the scale m_b, solve the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies by generating $C_9^{\rm U} = -1.18 \pm 0.19$, and are also consistent with all the other constraints. In this section, we examine whether any of these can also generate the right values of the NP $B \to \pi K$ matrix elements to also solve the $B \to \pi K$ puzzle.

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$C_{\rm SMEFT}$	ΔM_s ($\times 10^{11}$ GeV)	κ_ε	ε'/ε ($\times 10^4$)
$[C_{qq}^{(1)}]_{1123}$	(1.15 ± 0.06) ($\surd$)	-0.012 ($\surd$)	38.4 (?)
$[C_{qq}^{(1)}]_{2223}$	(2.72 ± 0.10) ($\times$)	0.11 ($\surd$)	15.8 ($\surd$)
$[C_{qq}^{(3)}]_{1123}$	(1.16 ± 0.06) ($\surd$)	-0.005 ($\surd$)	23.1 ($\surd$)
$[C_{qq}^{(3)}]_{2223}$	(0.59 ± 0.05) ($\times$)	-0.04 ($\surd$)	17.8 ($\surd$)
$[C_{qd}^{(1)}]_{2311}$	(1.16 ± 0.07) ($\surd$)	-0.75 ($\times$)	13.9 ($\surd$)
$[C_{qd}^{(1)}]_{2322}$	(1.55 ± 0.07) ($\times$)	0.75 ($\times$)	13.9 ($\surd$)
$[C_{qd}^{(1)}]_{2333}$	(0.76 ± 0.06) ($\times$)	0.0 ($\surd$)	13.9 ($\surd$)
$[C_{qd}^{(8)}]_{2311}$	(1.16 ± 0.06) ($\surd$)	-15.0 ($\times$)	12.8 ($\surd$)
$[C_{qd}^{(8)}]_{2322}$	(1.18 ± 0.05) ($\surd$)	14.3 ($\times$)	12.8 ($\surd$)
$[C_{qd}^{(8)}]_{2333}$	(10.6 ± 0.5) ($\times$)	-0.001 ($\surd$)	12.8 ($\surd$)
$[C_{qu}^{(1)}]_{2311}$	(1.15 ± 0.06) ($\surd$)	0.0 ($\surd$)	13.9 ($\surd$)
$[C_{qu}^{(1)}]_{2322}$	(1.16 ± 0.06) ($\surd$)	0.0 ($\surd$)	13.9 ($\surd$)
$[C_{qu}^{(8)}]_{2311}$	(1.15 ± 0.06) ($\surd$)	0.0002 ($\surd$)	13.9 ($\surd$)
$[C_{qu}^{(8)}]_{2322}$	(1.15 ± 0.06) ($\surd$)	-0.0003 ($\surd$)	13.9 ($\surd$)

Table 8. Four-quark SMEFT WCs of table 6 that generate the desired WET WCs: predictions for ΔM_s, κ_ε and ε'/ε,along with an indicator of whether the constraint is satisfied ($\surd$) or violated ($\times$), or if there is a tension (?). Operators that satisfy all the constraints are highlighted in gray.

4.1 New physics

Reminder: the three NP matrix elements that appear in the $B \to \pi K$ amplitudes are $\mathcal{A}^{\prime,comb} e^{i\Phi'} \equiv -\mathcal{A}^{\prime,u} e^{i\Phi'_u} + \mathcal{A}^{\prime,d} e^{i\Phi'_d}$, $\mathcal{A}^{\prime C,u} e^{i\Phi'^C_u}$, and $\mathcal{A}^{\prime C,d} e^{i\Phi'^C_d}$, where the individual contributions are defined in eq. (2.6) and are repeated here for convenience:

$$\sum \langle \pi K | \, \bar{s}_\alpha \Gamma_i b_\alpha \, \bar{q}_\beta \Gamma_j q_\beta \, | B \rangle \equiv \mathcal{A}^{\prime,q} e^{i\Phi'_q} \,,$$
$$\sum \langle \pi K | \, \bar{s}_\alpha \Gamma_i b_\beta \, \bar{q}_\beta \Gamma_j q_\alpha \, | B \rangle \equiv \mathcal{A}^{\prime C,q} e^{i\Phi'^C_q} \,, \quad q = u, d \,. \tag{4.1}$$

In the above, the $\bar{b} \to \bar{s} q \bar{q}$ WET operators can have any Lorentz structure. But in our case, since they are generated from the running of vector SMEFT operators, only the vector WET operators are relevant. Four of these are color-allowed:

$$C^q_{V_{LL}} \, \mathcal{N}'' (\bar{s}_L \gamma_\mu b_L)(\bar{q}_L \gamma^\mu q_L) \,, \qquad C^q_{V_{LR}} \, \mathcal{N}'' (\bar{s}_L \gamma_\mu b_L)(\bar{q}_R \gamma^\mu q_R) \,,$$
$$C^q_{V_{RL}} \, \mathcal{N}'' (\bar{s}_R \gamma_\mu b_R)(\bar{q}_L \gamma^\mu q_L) \,, \qquad C^q_{V_{RR}} \, \mathcal{N}'' (\bar{s}_R \gamma_\mu b_R)(\bar{q}_R \gamma^\mu q_R) \,, \tag{4.2}$$

where $\mathcal{N}'' = \frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^*$. Four are color-suppressed:

$$C^{q,C}_{V_{LL}} \, \mathcal{N}'' (\bar{s}^\alpha_L \gamma_\mu b^\beta_L)(\bar{q}^\beta_L \gamma^\mu q^\alpha_L) \,, \qquad C^{q,C}_{V_{LR}} \, \mathcal{N}'' (\bar{s}^\alpha_L \gamma_\mu b^\beta_L)(\bar{q}^\beta_R \gamma^\mu q^\alpha_R) \,,$$
$$C^{q,C}_{V_{RL}} \, \mathcal{N}'' (\bar{s}^\alpha_R \gamma_\mu b^\beta_R)(\bar{q}^\beta_L \gamma^\mu q^\alpha_L) \,, \qquad C^{q,C}_{V_{RR}} \, \mathcal{N}'' (\bar{s}^\alpha_R \gamma_\mu b^\beta_R)(\bar{q}^\beta_R \gamma^\mu q^\alpha_R) \,. \tag{4.3}$$

The matrix elements in eq. (4.1) have been computed for vector WET operators in ref. [43] using QCD factorization. With these results, we can write the three NP $B \to \pi K$

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matrix elements in terms of the WET WCs of eqs. (4.2), (4.3):

$$\begin{aligned} A'^{C,d} e^{i\phi'^{C}_u} &= \lambda_t \, C_V^{d,C} A_{\pi K} \,, \\ A'^{C,u} e^{i\phi'^{C}_d} &= \lambda_t \, C_V^{u,C} A_{\pi K} \,, \\ A'^{comb} e^{i\phi'} &= \lambda_t \, C_V^{ud} A_{K\pi} \,, \end{aligned} \tag{4.4}$$

where $\lambda_t = V_{tb}^* V_{ts}$. Here,

$$\begin{aligned} C_V^{d,C} &= C_{V_{LL}}^{d,C} + r_\chi^K C_{V_{LR}}^{d,C} - C_{V_{RR}}^{d,C} - r_\chi^K C_{V_{RL}}^{d,C} \,, \\ C_V^{u,C} &= C_{V_{LL}}^{u,C} + r_\chi^K C_{V_{LR}}^{u,C} - C_{V_{RR}}^{u,C} - r_\chi^K C_{V_{RL}}^{u,C} \,, \\ C_V^{ud} &= \left(-C_{V_{LL}}^{u} + C_{V_{LL}}^{d} + C_{V_{LR}}^{u} - C_{V_{LR}}^{d} \right) \\ &\quad + \left(C_{V_{RR}}^{u} - C_{V_{RR}}^{d} - C_{V_{RL}}^{u} + C_{V_{RL}}^{d} \right) \,, \end{aligned} \tag{4.5}$$

with

$$r_\chi^K(\mu) = \frac{2 m_K^2}{m_b(\mu)(m_q(\mu) + m_s(\mu))} \,, \tag{4.6}$$

which is formally of $O(1/m_b)$, but is numerically close to unity. Also,

$$\begin{aligned} A_{\pi K} &= i \frac{G_F}{\sqrt{2}} (m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_K^2) f_K \,, \\ A_{K\pi} &= i \frac{G_F}{\sqrt{2}} (m_B^2 - m_K^2) F_0^{B\to K}(m_\pi^2) f_\pi \,, \end{aligned} \tag{4.7}$$

where $F_0^{B\to M}(q^2)$ are semileptonic form factors, and f_π (f_K) is the pion (kaon) decay constant.

In order to illustrate the effect of NP, we (i) add the three NP matrix elements to the $B \to \pi K$ amplitudes [see eqs. (2.7) and (4.4)], (ii) assume that all three have the same NP weak phase, and (iii) redo the fit to the $B \to \pi K$ data. The results are shown in table 9. We see that, with the addition of NP, one obtains a good fit to the data. These results suggest that the $\mathcal{A}'^{C,u}$ NP amplitude is the most important, with $\mathcal{A}'^{C,d}$ being smaller, and $\mathcal{A}'^{,comb}$ irrelevant (it is consistent with zero).

4.2 Simultaneous explanations

We have identified seven SMEFT operators that can solve the $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies by generating $C_9^{\rm U} = -1.18 \pm 0.19$ when run down to the scale m_b. This running will also generate the WET operators of eqs. (4.2), (4.3). Given the values of the WET WCs, we can compute the real and imaginary values of $C_V^{d,C}$, $C_V^{u,C}$ and C_V^{ud} [eq. (4.5)]. We can then perform a fit to the $B \to \pi K$ data and see if the fit is good.

We follow this procedure for each of the seven candidate SMEFT operators. The results can be found in table 10. We see that, in fact, none of the SMEFT operators produce a good fit. The fits of two of them — $[C_{qq}^{(1)}]_{1123}$ and $[C_{qu}^{(8)}]_{2322}$ — have p-values of 0.12, which is passable, but we are looking for stronger explanations of the $B \to \pi K$ data.

It is clear what is going on here. When the $B \to \pi K$ fit was performed including NP, a good fit was found, see table 9. However, a small, nonzero NP weak phase was required. But the SMEFT WCs in table 10 are all real, which leads to $B \to \pi K$ fits that are passable at best.

$\chi^2_{\rm min}/{\rm d.o.f.} = 0.30/1$, p-value = 0.58	
parameter	best fit value
γ	$(66.46 \pm 3.44)^\circ$
β	$(22.12 \pm 0.69)^\circ$
$\lvert T'\rvert$	-5.2 ± 1.2
$\lvert P'_{tc}\rvert$	51.8 ± 0.5
$C_V^{d,C}$	0.012 ± 0.007
$C_V^{u,C}$	0.074 ± 0.017
C_V^{ud}	0.0 ± 1.4
$\delta_{P'_{tc}}$	$(198.2 \pm 4.0)^\circ$
$\delta_{C'}$	$(-28.7 \pm 34.4)^\circ$
ϕ	$(1.95 \pm 0.57)^\circ$

Table 9. $\chi^2_{\rm min}$/d.o.f. and best-fit values of unknown parameters in amplitudes of eq. (2.7) [see also eq. (4.4)], with the same NP weak phase ϕ for all NP diagrams. Constraints: $B \to \pi K$ data, measurements of β and γ, theoretical inputs $|C'/T'| = 0.2$, $P'_{uc} = 0$.

$C_{\rm SMEFT}$	$C_V^{d,C}$	$C_V^{u,C}$	C_V^{ud}	$\chi^2_{\rm min}$/d.o.f.	p-value
$[C_{qe}]_{2333}$	0	0	0.0001	16.7	0.005
$[C_{qq}^{(1)}]_{1123}$	$0.056 + 0.001i$	$0.056 + 0.001i$	-0.001	8.8/5	0.12
$[C_{qq}^{(3)}]_{1123}$	$-0.020 - 0.0004i$	$0.022 + 0.0004i$	$0.123 + 0.002i$	15.1/5	0.01
$[C_{qu}^{(1)}]_{2311}$	$0.0016 + 0.00003i$	$-0.043 - 0.001i$	$-0.071 - 0.0013i$	14.7/5	0.01
$[C_{qu}^{(1)}]_{2322}$	$0.0016 + 0.00003i$	$0.0017 + 0.00003i$	-0.0001	16.5/5	0.005
$[C_{qu}^{(8)}]_{2311}$	$0.056 + 0.001i$	$-0.915 - 0.017i$	$0.120 + 0.002i$	65.0/5	0
$[C_{qu}^{(8)}]_{2322}$	$0.056 + 0.001$	$0.056 + 0.001$	0	8.8/5	0.12

Table 10. Candidate SMEFT WCs: predictions for the NP $B \to \pi K$ parameters $C_V^{d,C}$, $C_V^{u,C}$ and C_V^{ud}, along with the result of the $B \to \pi K$ fit in terms of $\chi^2_{\rm min}$/d.o.f. and the p-value. None of the operators produce a good fit to the $B \to \pi K$ data.

To correct this problem, we add a small NP weak phase to the SMEFT WCs. The results are shown in table 11. For $\bar b \to \bar s \ell^+ \ell^-$, nothing has changed — all SMEFT WCs still generate the desired value of $C_9^{\rm U}$. But there is a marked difference in the $B \to \pi K$ fits: now three WCs — $[C_{qq}^{(1)}]_{1123}$, $[C_{qu}^{(1)}]_{2311}$ and $[C_{qu}^{(8)}]_{2322}$ — produce good $B \to \pi K$ fits. A fourth WC, $[C_{qq}^{(3)}]_{1123}$, has a passable fit.

The addition of a NP weak phase could potentially affect CP-violating observables such as κ_ϵ, ε'/ε and β_s (the imaginary part of B_s^0-$\bar B_s^0$ mixing). However, because the added NP weak phase is small, we do not expect significant contributions to the observables. Indeed, this is what we find. For all four of the WCs that produce good or passable $B \to \pi K$

$C_{\rm SMEFT}$ (TeV^{-2})	$C_9^{\rm U}$	$C_{10}^{\rm U}$
$[C_{qe}]_{2333}$ $-0.22e^{0.10i}$	$-1.15-0.14i$	$-0.005-0.0006i$
$[C_{qq}^{(1)}]_{1123}$ $0.21e^{0.05i}$	$-1.17-0.08i$	$-0.004-0.0003i$
$[C_{qq}^{(3)}]_{1123}$ $-0.07e^{0.10i}$	$-1.14-0.14i$	$-0.019-0.002i$
$[C_{qu}^{(1)}]_{2311}$ $0.11e^{0.10i}$	$-1.18-0.14i$	$-0.004-0.0005i$
$[C_{qu}^{(1)}]_{2322}$ $0.11e^{0.20i}$	$-1.16-0.26i$	$-0.005-0.001i$
$[C_{qu}^{(8)}]_{2311}$ $1.12e^{0.10i}$	$-1.17-0.14i$	$0.013-0.001i$
$[C_{qu}^{(8)}]_{2322}$ $1.12e^{0.05i}$	$-1.18-0.08i$	$0.012+0.0008i$

s]

$C_{\rm SMEFT}$ (TeV^{-2})	$C_V^{d,C}$	$C_V^{u,C}$	C_V^{ud}	$\chi^2_{\rm min}$/d.o.f.	p-value
$[C_{qe}]_{2333}$ $-0.22e^{0.10i}$	0	0	$0.0001e^{-3.02i}$	16.5/5	0.005
$[C_{qq}^{(1)}]_{1123}$ $0.21e^{0.05i}$	$0.056e^{-0.0001i}$	$0.056e^{0.07i}$	$0.001e^{2.78i}$	0.43/5	0.99
$[C_{qq}^{(3)}]_{1123}$ $-0.07e^{0.10i}$	$0.02e^{3.12i}$	$0.022e^{0.115i}$	$0.123e^{0.116i}$	8.8/5	0.12
$[C_{qu}^{(1)}]_{2311}$ $0.11e^{0.10i}$	$0.0016e^{0.03i}$	$0.043e^{-3.02i}$	$0.071e^{-3.02i}$	2.67/5	0.75
$[C_{qu}^{(1)}]_{2322}$ $0.11e^{0.20i}$	$0.0016e^{0.042i}$	$0.0017e^{0.22i}$	$0.0001e^{-2.97}$	14.5/5	0.01
$[C_{qu}^{(8)}]_{2311}$ $1.12e^{0.10i}$	$0.056e^{0.057i}$	$0.915e^{-3.02}$	$0.120e^{0.12i}$	62.4/5	0.0
$[C_{qu}^{(8)}]_{2322}$ $1.12e^{0.05i}$	$0.056e^{0.038i}$	$0.056e^{0.069i}$	$0.00016e^{1.24i}$	3.65/5	0.60

Table 11. Candidate SMEFT WCs: predictions for (i) $C_9^{\rm U}$ and $C_{10}^{\rm U}$ of $\bar b \to \bar s \ell^+\ell^-$ (upper table), and (ii) the NP $B \to \pi K$ parameters $C_V^{d,C}$, $C_V^{u,C}$ and C_V^{ud}, along with the result of the $B \to \pi K$ fit in terms of $\chi^2_{\rm min}$/d.o.f. and the p-value (lower table). Operators that produce a good fit to the $B \to \pi K$ data are highlighted in gray.

fits, the contributions to κ_ϵ and ε'/ε are virtually unchanged from table 8. As for β_s, we choose to compute instead $S_{\psi\phi}$, the indirect CP asymmetry in $B_s^0 \to \psi\phi$, which is a function of β_s. For all four WCs, the prediction for $S_{\psi\phi}$ is essentially the same as that of the SM, and this is consistent with experiment. Thus, except for the fact that $[C_{qq}^{(1)}]_{1123}$ still has a possible tension with ε'/ε, see table 8, the constraints from all CP-violating observables on the NP weak phase are satisfied.

4.3 Discussion

We have found three four-quark SMEFT operators which can provide a combined explanation of the $\bar b \to \bar s \ell^+\ell^-$ anomalies and the $B \to \pi K$ puzzle. One obvious question is: is it possible to distinguish these scenarios? Each SMEFT operator contributes at tree level to a set of four-quark WET transitions, and these sets are not the same for all three operators, so the answer is potentially yes.

To be specific, from table 4 we see that the operators (i) $[Q_{qq}^{(1)}]_{1123}$, (ii) $[Q_{qu}^{(1)}]_{2311}$ and (iii) $[Q_{qu}^{(8)}]_{2322}$ respectively include (i) colour-allowed $\bar b \to \bar s d \bar d$, $\bar b \to \bar s u \bar u$, $\bar b \to \bar s c \bar c$, $\bar b \to \bar s u \bar c$ and $\bar b \to \bar s c \bar u$ transitions, (ii) only colour-allowed $\bar b \to \bar s u \bar u$ transitions, and (iii) both colour-allowed and colour-suppressed $\bar b \to \bar s c \bar c$ transitions. All operators also contribute to processes in which

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the $\bar{b} \to \bar{s}$ is replaced by $\bar{t} \to \bar{c}$. The point is that the three operators will affect different types of hadronic B decays. The measurements of a variety of such decays may reveal which ones are affected by NP or not, which will allow us to distinguish the three solutions.

Another question one might ask is: what model of NP can explain $\bar{b} \to \bar{s}\ell^+\ell^-$ and $B \to \pi K$? This is complicated. We have found solutions with a single SMEFT operator. But realistic models typically contain many SMEFT operators, so our solutions only provide a starting point. The full NP model would presumably include one of our SMEFT operators, but then it would be necessary to compute the contributions of the other SMEFT operators to $\bar{b} \to \bar{s}\ell^+\ell^-$ and $B \to \pi K$, in order to see if the model still provides an explanation of the two anomalies.

One important ingredient in our analysis was the use of both semileptonic and four-quark operators within SMEFT. As we have stressed, due to RGE running, each type of SMEFT operator can potentially generate the other type of WET operator at one loop. This was key in identifying the SMEFT operators that could generate the desired value of $C_9^{\rm U}$ in $\bar{b} \to \bar{s}\ell^+\ell^-$. In the literature, most analyses generally focus on the semileptonic WET operator $\bar{b} \to \bar{s}\tau^+\tau^-$ (for example, see ref. [72]). However, we showed that there are more four-quark SMEFT operators (i.e., $\bar{b} \to \bar{s}q\bar{q}$ WET operators) that can do this.

We also found that, while some hadronic SMEFT operators can explain both the semileptonic $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies and the hadronic $B \to \pi K$ puzzle, the semileptonic SMEFT operator that explains $\bar{b} \to \bar{s}\ell^+\ell^-$ cannot also explain $B \to \pi K$. In this particular case, the RGE running of the semileptonic operator, which involves only electroweak gauge bosons, did generate the $\bar{b} \to \bar{s}q\bar{q}$, $q = u, d$ WET operators, but the WCs were not sufficiently large. The four-quark SMEFT operators did not have this problem because their RGE running involves both gluons and the Higgs in addition to electroweak gauge bosons. This suggests that it may be difficult to explain hadronic anomalies with semileptonic SMEFT operators. However, this cannot be concluded definitively, so it is worthwhile to continue to explore this possibility.

Finally, we note that this is the first time anyone has attempted to explain the $B \to \pi K$ puzzle within SMEFT. In our treatment of the $B \to \pi K$ puzzle, we minimized the theoretical hadronic input. We fit to the SM parameters (no form-factor calculations were needed), and we treated the NP effects with factorization. (While one must worry about non-factorizable effects in performing SM calculations, these are just second-order corrections when dealing with NP.)

5 Conclusions

Despite its success in explaining almost all experimental data to date, we know that the standard model is not complete — there must be physics beyond the SM. Since no new particles have been seen at the LHC, we also know that this new physics, whatever it is, must be heavy, with masses greater than O(TeV). When these NP particles are integrated out, one obtains the SMEFT, which includes the (dimension-4) SM and higher-order non-SM operators.

Because the NP is very heavy, it is likely that the first signs of NP will be indirect: the measured value of an observable in a low-energy process disagrees with the prediction of the SM. Whenever such an anomaly is seen, we want to know what kind of NP could account for it. In SMEFT language, this amounts to asking which SMEFT operator(s) can generate the low-energy WET operator that describes the observable.

If the anomaly is in a semileptonic or hadronic process, the first thought is to look for the semileptonic (two quarks and two leptons) or hadronic (four quarks) SMEFT operators that contain the desired WET operator. One thing we have stressed in this paper is that this is not enough. When one uses the renormalization-group equations to evolve the SMEFT Hamiltonian down to low energies, due to operator mixing semileptonic (hadronic) SMEFT operators can generate hadronic (semileptonic) WET operators at one loop. The point is that, if one wants to find an explanation of any low-energy anomaly, or combination of anomalies, within SMEFT, one must (i) identify the candidate semileptonic and four-quark SMEFT operators, (ii) run them down to low energy with the RGEs, (iii) generate the required WET operators with the correct WCs, and (iv) check that all other constraints are satisfied.

In this paper, we have illustrated this method by applying it to two anomalies in the B system. We have found three four-quark SMEFT operators which, when run down to the scale m_b, can simultaneously explain the semileptonic $\bar{b} \to \bar{s}\ell^+\ell^-$ anomalies and the hadronic $B \to \pi K$ puzzle. A key ingredient in our analysis was considering both semileptonic and hadronic SMEFT operators. Note that, while different NP scenarios have been proposed to explain each of these anomalies, this is the first time a combined explanation has been found.

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