Analytical Quantum Full-Wave Solution of Transmon Qubits in a 3D Waveguide Cavity

Soomin Moon[(1)] and Thomas E. Roth[(1),(2)]

(1) Elmore Family School of Electrical and Computer Engineering, Purdue University, Indiana 47906, USA (rothte@purdue.edu)

(2) Purdue Quantum Science and Engineering Institute, Purdue University, Indiana 47907, USA

Circuit quantum electrodynamics (cQED) is one of the most promising platforms for building quantum information processors. As quantum technology evolves the hardware is becoming increasingly complex, so there is a need for a general-purpose, high-fidelity numerical modeling method for an accurate analysis. However, there is a lack of analytical solutions that can serve as a validation method to support the development of efficient and accurate numerical models in this new field. In this study, we develop an analytical solution using microwave cavity perturbation and antenna theories within a field-based description of cQED devices (T. E. Roth and W. C. Chew, "Macroscopic circuit quantum electrodynamics: A new look toward developing full-wave numerical models," *IEEE Journal on Multiscale and Multiphysics Computational Techniques*, vol. 6, pp. 109-124, 2021). The system analyzed corresponds to one or two transmon qubits inside a rectangular waveguide cavity, where the transmons are formed by connecting a Josephson junction across the terminals of a small dipole antenna. We validate our analytical solution against our same formalism evaluated with numerical eigenmodes, the energy participation ratio quantization approach (Z. K. Minev, Z. Leghtas, S. O. Mundhada, L. Christakis, I. M. Pop, and M. H. Devoret, "Energy-participation quantization of Josephson circuits," *npj Quantum Information*, vol. 7, no. 1, pp. 1-11, 2021), and an impedance-based method (Solgun, Firat, David P. DiVincenzo, and Jay M. Gambetta. "Simple impedance response formulas for the dispersive interaction rates in the effective Hamiltonians of low anharmonicity superconducting qubits." *IEEE Transactions on Microwave Theory and Techniques*, vol. 67, no. 3, pp. 928-948, 2019).

To validate the approach, we form a matrix representation of our Hamiltonian operator and calculate the coupled system's eigenvalues and eigenvectors. Using these new eigenvalues and eigenvectors, we compute experimentally-relevant system parameters like the AC-Stark shift or ZZ-interaction rate. Results validating our analytical solution are shown in Fig. 1. The "corrected" analytical solution corresponds to using a more accurate prediction of the dipole capacitance from a simple numerical method rather than using an analytical formula for that particular parameter.

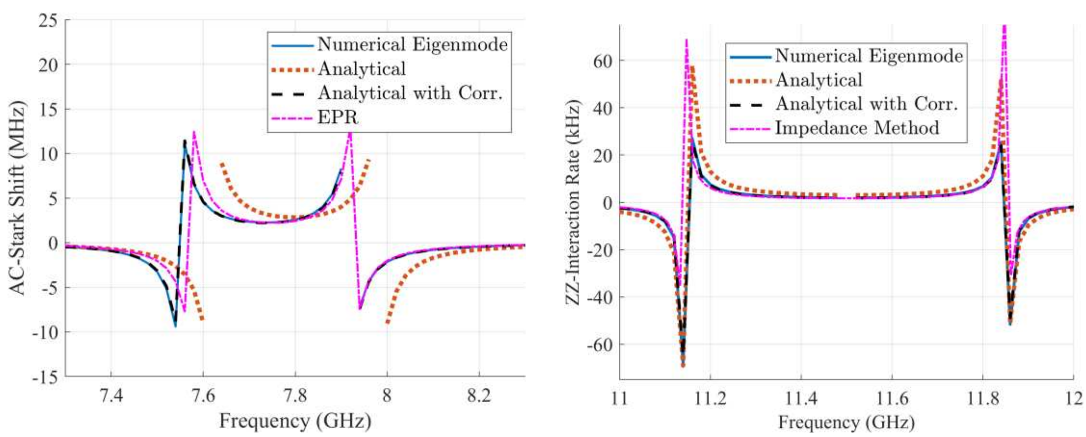

Figure 1. AC-Stark shift (left) and ZZ-interaction rate (right) computed using various methods.

This material is based upon work supported by the National Science Foundation under Grant No. 2202389 and Purdue ECE Elmore Emerging Frontiers Center "The Crossroads of Quantum and AI."