

A Traffic Control Framework for Uncrewed Aircraft Systems

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Abstract—The exponential growth of Advanced Air Mobility (AAM) services demands assurances of safety in the airspace. This research presents a Traffic Control Framework (TCF) for developing digital flight rules for Uncrewed Aircraft System (UAS) flying in designated air corridors. The proposed TCF helps model, deploy, and test UAS control agents, regardless of their hardware configurations. This paper investigates the importance of digital flight rules in preventing collisions in the context of AAM TCF is introduced as a platform for developing strategies for managing UAS traffic towards enhanced autonomy in the airspace. It allows for assessment and evaluation autonomous navigation, route planning, obstacle avoidance, and adaptive decision-making for UAS. It also allows for the introduction and evaluation of advanced technologies including Artificial Intelligence (AI) and Machine Learning (ML) in a simulation environment before deploying them in the real world. TCF can be used as a tool for comprehensive UAS traffic analysis, including KPI measurement. It offers flexibility for further testing and deployment, laying the foundation for improved airspace safety—a vital aspect of UAS technology advancement. Finally, this paper demonstrates the capabilities of the proposed TCF in managing UAS traffic at intersections and its impact on overall traffic flow in air corridors, noting the bottlenecks and the inverse relationship between safety and traffic volume.

Index Terms—Uncrewed Aircraft Systems, Air Corridors, Traffic Management, Key Performance Indicators

I. INTRODUCTION

AAM is a rapidly emerging area of research and development in the aviation industry that aims to provide effective, sustainable, and efficient means for transportation of people and goods. The demand for AAM services is on the rise around the world in terms of both commercial and industrial aspects however, AAM brings with it a pre-requisite of highest levels

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safety in the airspace. AAM can not become a reality without the highest level of safety assurance from the aviation industry.

In order to successfully ensure the safety of AAM platforms including UAVs, it is important to establish clear guidelines and flight rules in the airspace in which UAS are permitted to fly. This will prevent collisions and ensure that UAV do not interfere with other aircraft or people's privacy. These guidelines can also allow for efficient airspace use and help reduce congestion.

Developing regulations for air control management is much harder to implement due to the lack of resources. Controlled Structural Airspaces are yet to be built, and as such researchers are not given the opportunity to analyze patterns and behaviors of UAV in a physical setting. To combat this issue, this paper explains an efficient and reliable TCF to analyze traffic in the airspace. The framework provides Traffic Performance Indicator (TPI) to help operators analyze the airspace. The TCF outlined in this paper builds on an Air Corridor-based model.

Cutting-edge technologies like autonomous navigation, AI-driven route optimization, real-time obstacle avoidance, and adaptive decision-making enhance UAV operations [1]. Reinforcement Learning (RL) further boosts UAV performance. The AI/ML Assurance framework facilitates simulation and real-world deployment of AI-backed control policies. The TCF outlined in this paper allows for both simulation and real-world deployment of such AI backed control policies.

A. Contributions

The primary objective of this paper is to present a TCF as a tool for analyzing air traffic in a controlled environment among UAVs. The proposed TCF has also been implemented as a simulation tool. The capability of this tool in assessing key traffic performance metrics has been demonstrated. The TCF allows for modification and extension to test out more complex models or attempt to deploy the UAV control models used in

the simulations on real drones in a controlled environment. The research and analysis can be used as a foundation for enhanced safety in the airspace, essential for the future of UAV technology. The simulation environment is derived from the existing literature on air corridors [2], [3]. Traffic rules for safety in the airspace were implemented on the TCF. The impact of these rules on the traffic volume was studied. It was noted that higher levels of safety required higher levels of restrictions in the airspace, which in turn reduces traffic rate in the airspace.

B. Organization of this paper

Section II provides a comprehensive analysis of the recent studies and advancements within Air Traffic Management for UAV. Section III is an informative analysis of the structured spaces required for Air Traffic Management. The safety and regulatory aspects of air corridors, sky lanes, and air cells are presented in this section. Section IV introduces the TCF of the research paper. This section introduces control fundamentals, various traffic rules such as intersection handling, and Traffic Analytics. The results of the TCF simulation are shown in Section V and Section VI provide a conclusive review of the entire paper.

II. LITERATURE REVIEW

UAS have been in use for military purposes for over a century, but their use has spread to commercial and civilian applications in recent years. However, the increased use of UAS has created a need for managing the growing number of UAS in the sky to ensure their safe and efficient integration into airspace, similar to how Air Traffic Control (ATC) regulates crewed aircraft.

This need has led to the development of UAS Traffic Management (UTM) [4], which is required to guarantee the safe, secure, and efficient integration of UAS in the National Airspace System. UTM is intended for traffic management (monitoring, and enforcing traffic rules) of small UAS flying under 400 ft Above Ground Level (AGL).

AAM platforms such as electric Vertical Takeoff and Landing (eVTOL) vehicles fly between 500 ft - 3000 ft AGL and are intended for transporting people and large cargo. AAM architecture includes [5] federated traffic management system also known as Provider of Services for AAM and Supplemental Data Service Providers. Regulations are evolving to safely and efficiently integrate UTM and AAM air traffic.

The idea of air corridors is crucial in the context of AAM. Air corridors [6] are specialized paths inside the airspace that are intended for AAM vehicles, such as eVTOL planes, to move between locations. These corridors are different from regular manned aircraft routes and serve as specialized paths for AAM vehicles.

One of the major issues that air corridors address is the possibility of conflict and safety concerns brought on by the integration of AAM vehicles into conventional airspace systems. Potential confrontations with other airspace users, including manned and unmanned aircraft, can be reduced

by designating specific air corridors. The usage of corridors can be helpful in resolving challenges of airspace availability in urban settings, which are constrained by building height, weather effects, privacy requirements, and current air traffic flows [7].

Air corridors also deal with the issue of route planning optimization and ensuring effective operations for AAM vehicles [6]. AAM vehicles can use optimized routes that streamline operations, cut down on travel time, and improve overall efficiency by designating specified corridors. Air corridors offer passengers and AAM operators a reliable and predictable framework, which enhances the appeal and feasibility of AAM systems.

Moreover, by defining specific corridors and associated operating procedures, air corridors provide a clear framework for managing AAM traffic within the broader airspace ecosystem. This integration allows ATC and UTM systems to effectively monitor and regulate AAM operations, ensuring compliance with necessary regulations and maintaining the overall safety and reliability of the airspace system.

Various researchers propose approaches for traffic control frameworks. As discussed in [7], one such technique proposes a strategy to consider airspace constraints, UAS performance, and safety requirements, using simulation tools and advanced technologies to assess their efficacy. New policies and regulations specific to AAM operations are needed, and factors such as airspace structure, terrain, weather, environmental factors, and UAS capabilities must be considered. Safety requirements such as collision avoidance and emergency procedures must also be met.

A lane-based method, introduced in [8], proposes a standardized approach to coordinate UAS traffic and enhance predictability and safety. It utilizes clearly defined flight paths to ensure separation from other UAS and manned aircraft. Based on designated lanes, the Lane-Based Strategic Deconfliction (LBSD) system enables efficient and secure UAS operations while maintaining safety. The study also discusses the FAA-NASA Strategic Deconfliction (FNSD) method, which uses "block rules" to regulate UAS traffic. However, LBSD is considered more adaptable and effective. The regulatory framework for UAS traffic management may require further development to address the complexities and improve safety and effectiveness.

The Joint Control Framework [9] describes UTM as a collaboration between man & machine. It introduces Level of Autonomy in Cognitive Control (LACC) which delineates the 6 levels of control present in human-robot interaction. A traffic simulation in [10] illustrates an advantage of LACC within the context of UTM. This simulation utilizes 3D real-world maps and geofences. However, these papers prove a practical hypothesis. They do not specify any methodology or implementation details in managing air traffic.

In urban air freight operations within the vertiport environment, [11] emphasizes the vital role of 4D flight trajectories in AAM. These trajectories, integrating spatial coordinates and time, enable precise route planning and obstacle avoidance,

facilitated by Detect and Avoid (DAA) capabilities. DAA employs advanced sensors and algorithms to detect and track obstacles, allowing UAVs to adjust their flight paths in real time. This integration enhances coordination between aircraft and ATC, improving operational efficiency and safety.

The Federal Aviation Administration (FAA) plan [3] for AAM integration including the Innovate28 (I28) program, holds significance for UAS traffic control. The FAA's crawl-walk-run approach, coordination with ATC, adherence to Communication, Navigation, and Surveillance regulations, and tailored routing constructs for AAM resonate with UAS route planning. Infrastructure considerations, such as charging stations and parking zones, align with operational needs. This strategic alignment underscores how the AAM plan, along with the I28 program, informs UAS traffic control.

III. STRUCTURED AIRSPACES

Structured airspace can be designed through the establishment of air corridors, which serve as well-defined three-dimensional pathways for enabling safe navigation for UASs [2]. As depicted in Figure 1 air corridors are composed of air lanes & air cells and can form complex structures such as intersections.

A. Air Cells

Air cells represent the smallest unit of discretization within the airspace and are rectangular prisms with dimensions determined by the specific characteristics of the UASs being operated. These air cells facilitate the fine-grained partitioning of the airspace, enabling precise control and management of air traffic. By discretizing the airspace into these air cells, the surrounding environment becomes structured, allowing for efficient navigation and avoidance of potential conflicts. This paper assumes that each air cell measures 200 m x 100 m x 50 m (LxBxH). Air lanes are linear collections of air cells that form unidirectional pathways within the airspace. These lanes enable the controlled movement of UAVs, limiting their traffic flow to a single direction. By confining UAVs to designated air lanes, the risk of collisions and congestion is minimized, ensuring the safe and efficient flow of air traffic. Each air lane is composed of a series of air cells, and UAVs navigate through these lanes to reach their intended destinations.

B. Air Corridors

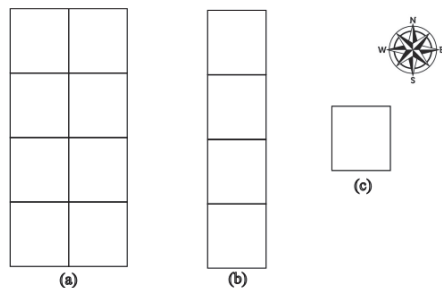


Figure 1. (a) Air corridors are composed of (b) air lanes which in turn contain (c) air cells

Air corridors refer to collections of multiple air lanes that have UASs moving parallel or anti-parallel in reference to each other. Air corridors allow for the coordinated movement of UAVs in a specific area along a specific line. Increasing the number of air lanes in an air corridor will increase the volume of traffic flowing along a given direction in the area. These corridors can accommodate different types of air traffic, facilitating efficient and organized UAV operations. Air corridors are particularly useful for managing UAV networks in complex environments with diverse traffic flows.

C. Intersections

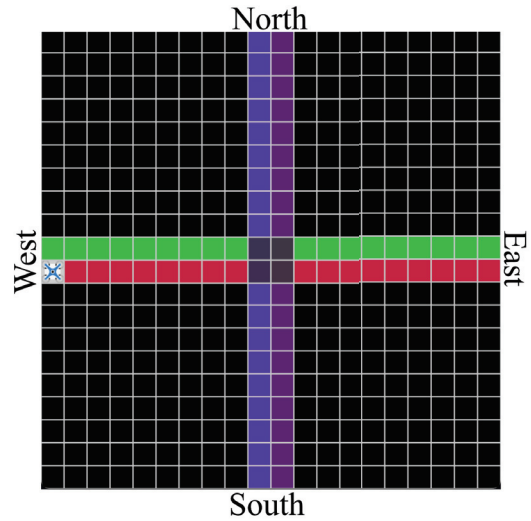


Figure 2. Green lane is Eastbound, Red lane is Westbound, Purple lane is Northbound & Violet lane is Southbound. Black grids constitute a non-traversal area. Intersections are shown in grey color.

Intersections form when two or more air lanes different directions overlap. Intersections are an inevitable feature of any transportation network that hosts a large number of vehicles. At these intersections, special protocols and coordination mechanisms are required to manage the flow of UAVs to prevent conflicts.

By employing a combination of air cells, air lanes, air corridors, and intersections, structured airspace provide a clear framework for organizing and managing UAV traffic, enhanced safety, seamless traffic, efficient in UAV operations, and optimal & allocation of resources.

IV. TRAFFIC CONTROL FRAMEWORK

The TCF for AAM is designed utilizing the concept of structured airspace. TCF facilitates autonomous traffic management amongst independent actors. This framework helps visualize high volumes of air traffic in real time. Additionally, TCF can simulate variety of air traffic conditions and estimate traffic parameters. This is achieved by enforcing a set of traffic rules on all the UAS that operate under this framework; providing a unified method for traffic management. TCF modularizes UAV behavior through the implementation

of UAS control agents providing a high-level control that enables instant modification of the vehicles' task. The Traffic Analytics feature of the TCF details essential metrics of the traffic. An operator can then use these statistics to control the traffic further. TA establishes a feedback mechanism in this framework.

A. Conformance Guidelines

Every UAV must conform to the following guidelines which ease large-scale air traffic management.

- i) **Unique Identification:** FAA also requires each UAV to have a remote identification [12]. An equivalent and simplified version of this is facilitated through a unique identification (UID) given to each UAV.
- ii) **Broadcasting Information:** All UAVs are obligated to broadcast certain environmental information and a unique identifier at predetermined intervals. This rule aligns with existing regulations set by the FAA for UAVs flying in most airspace. This data must also be captured by each UAS to gain insights into other dynamic systems present in their environment.
- iii) **Single Occupancy of Air Cells:** No air cell is allowed to accommodate more than one UAS simultaneously. This rule ensures that there is no congestion or conflict within each discrete unit of airspace.
- iv) **Single Occupancy of Intersection:** At most one UAV is allowed to use more than an intersection at any given point in time. This rule reduces the chances of collisions.
- v) **Safety Gap between UAVs:** It is required that each UAV maintains a minimum gap of at least 1 air cell when traveling within the same air lane. This rule guarantees that each vehicle has sufficient space to stop and recalibrate in the event of a malfunction or stoppage of the UAV ahead.

B. UAS Control Agents

UAS control agents define the behaviour of the UAV. Technically, they are the programs that process information sensed by a UAS to devise subsequent actions. Within the structure of the TCF, they establish an abstraction layer that facilitates interaction between multiple UASs and real-time sensor measurements, such as location, position, distance, etc. Different UAS manufacturers may employ different hardware configurations, resulting in variations in the operating systems exposed to the UAV controller. However, these UAS control agents allow for the extension of these abstractions, enabling support for a wide range of UAV controllers, thus unifying UAS management.

Control agents receive the current state of the UAS and the sensed environmental data as inputs. The agents analyze it in order to formulate subsequent actions. The method employed in this paper involves a rules-based approach; a predetermined set of rules or procedures dictate the agent's behavior. The agent assesses the current state of the UAS alongside its operating environment and determines the most suitable course of action as guided by its programming rules. Table I shows

the rules implemented. In addition to the rules shown, a Not-Operating (NOP) state is also defined in which no operation is performed. This system of discrete actions simplifies the decision-making process while maintaining sufficient flexibility for effective UAV navigation within the discrete and fully observable environment, and breaks down complex UAS operations into manageable and understandable units.

Action: RIGHT	LEFT	REVERSE
$N \rightarrow E$	$N \rightarrow W$	$N \rightarrow S$
$E \rightarrow S$	$E \rightarrow N$	$E \rightarrow W$
$S \rightarrow W$	$S \rightarrow E$	$S \rightarrow N$
$W \rightarrow N$	$W \rightarrow S$	$W \rightarrow E$

Table I
ACTION TO MOVEMENT CONVERSION TABLE.
N = NORTH, S = SOUTH, E = EAST & W = WEST

C. Communication Fabric

The proposed TCF abstracts this implementation of vehicle-to-vehicle communication by defining a communication fabric through which packets of data can be dispatched; conforming to prerequisites defined in section IV-A

D. Traffic Performance Indicators

This paper is focused on building a traffic management model and therefore it is important to identify some traffic performance indicators that quantize the effectiveness and usefulness of a traffic management model. TPIs can also be used to compare different models of traffic generation, UAS Control Agents as well as different environment parameters. TPI facilitate a feedback mechanism into the Framework; whereby operators receive critical information about the traffic behavior required to manage it. A comprehensive list of TPI is presented below.

- 1) **Mean UAV Velocity (v_{mean}):** Mean UAV velocity is the velocity that each UAV is expected to attain in smooth traffic flow. Given N_{uav} is the number of UAVs and v_i is the velocity of the i^{th} UAV, then the mean velocity [13] is defined as,

$$v_{\text{mean}} = \frac{1}{N_{uav}} \sum_{i \in N_{uav}} v_i.$$

- 2) **Traffic Smoothness (σ_v):** Traffic smoothness is the measure of the small-scale variations in traffic flow. Numerically, it can be defined as the standard deviation of UAV velocity vector within the observation space and is defined as,

$$\sigma_v = \sqrt{\frac{1}{N_{uav}} \sum_{i \in N_{uav}} (v_i - v_{\text{mean}})^2}.$$

- 3) **Traffic Delay (T_{id}):** Traffic delay is the amount of time an UAV waits until the gap in front of it is greater than the safety gap.
- 4) **Congestion Delay (T_{cong}):** Congestion is the situation when a vehicle is unable to move forward. Congestion delay is the amount of time an UAV waits until the gap in front of it is greater than the safety gap.

V. SIMULATIONS AND RESULTS

The TCF is primarily a tool that can be used to check whether a given strategy on a UAV controller is effective under ideal conditions. This in turn allows us to work on the control logic of the UAV whilst being indifferent to the hardware. We use this framework to develop a UAV agent that can handle a single air corridor intersection. This section highlights the simulation capabilities of TCF and the features therein. The simulation is performed on a single intersection of the air corridor. In order to overcome the “who has the right to cross the intersection”, a novel distributed queue model is described. Finally, UAV traffic simulations with two different traffic densities have been performed showcasing the capabilities of this simulation.

A. Single Intersection Model

The simulation model used in this paper consists of an intersection of air corridors as the one shown in Figure 2. TCF allows us to define the environment of this simulation with the help of a map file, where we specify the position of the corridors on the map as well as the vertiports that will dispatch UAVs to the environment. The intersection of the 2 air corridors we created will result in a “directionless” space that does not impose any restrictions on which directions the UAV can travel, as indicated in grey in the simulation visualization. This also gives UAVs a chance to change their directions and enter a different air lane, which may also be in a different air corridor. This intersection is analogous to a crossroad seen on the ground. To illustrate the use of TCF, we have programmed a simple distributed queue algorithm that determines the right of way for each UAS when it is at the intersection. The algorithm guarantees a collision-less sharing of the intersection, and the TCF allows us to only program the algorithm and the map environment to analyze the behavior of UAV traffic. Our model also tries to gauge the effect of weather on the UAV traffic. For the same, we assume that the weather at each cell can be monitored and hence is known to the UAV. At each of the edges of the lanes, there is assumed to be a Vertiport and the UAVs already possess the knowledge about their path and destination.

B. Traffic Generation

The current TCF model follows a binary probability distribution mechanism for traffic generation, however, more complex models will be addressed in future versions. At a given coordinate, dispatch vertiports generate UAVs in a particular direction and with a particular priority. Dispatch vertiports can have different probabilistic distributions powering them, and currently use the binary periodic distribution. The unit will attempt to generate a UAV 60% of the time every 5 epochs (the probability and the time period can be changed), and if the binary periodic distribution returns true it will dispatch a UAV on the map.

C. Congestion

Congestion refers to the scenario where traffic flow is not continuous. In the model used in this paper, congestion is caused when UAVs stop in order to maintain the safety gap. As traffic generation rate is the only factor affecting congestion, the threshold rate was investigated. Additionally, the traffic characteristics were analysed using the TPIs. It was observed that traffic generation rate of 0.25 is the threshold rate for congestion. When the traffic generation is set to less than 0.25, traffic congestion is never present. Figure 3 shows a visual representation of non-congested traffic. Figure 6 displays the same as there is no traffic delay. Other important inferences of this scenario is that intersection delay is uniform.

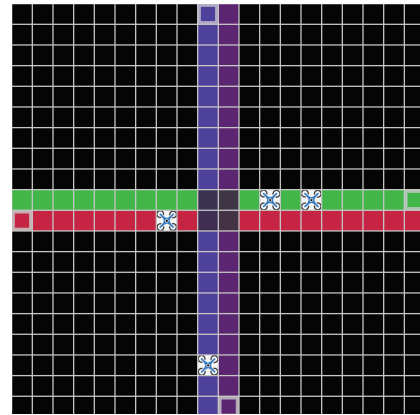


Figure 3. Simulation: No Traffic Congestion

D. Congestion Traffic Generation

When the traffic generation is set to more than 0.25, traffic congestion is never present. Figure 4 shows a visual representation of non-congested traffic. This is still a low traffic generation rate, however it is congesting since the bottleneck is 0.25 and the generation rate can be maximum up to 0.3. This will gradually cause a build up of drones in the system till there is a congestion.

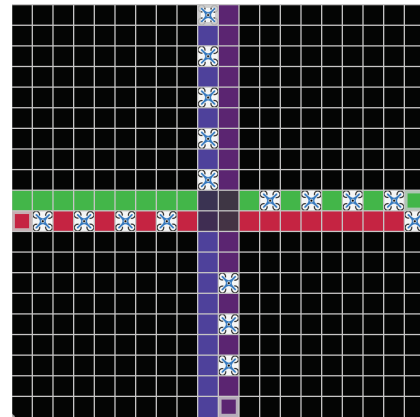


Figure 4. Simulation: Low Congestion

E. Results

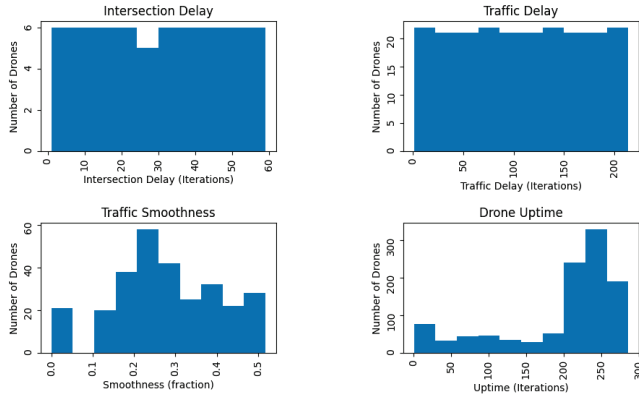


Figure 5. Performance Analysis: Low Congestion

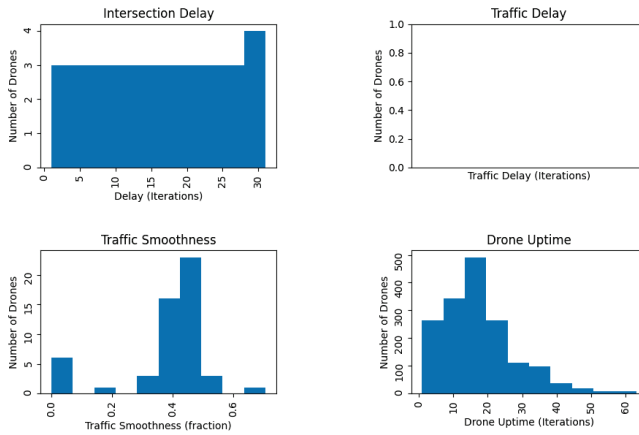


Figure 6. Performance Analysis: No Congestion

VI. CONCLUSION

Structured airspaces impose certain restrictions, enabling safer and more efficient deployment of a large variety of UAV. The idea of air corridors - with its constituent air lanes, and air cells - defines one such structure for airspaces. This concept was further enhanced by augmenting intersections and vertiports into it. Intersections are a natural consequence of structured transportation networks as any collection of UAVs will eventually cross each other's path. In the case of vertiports, they are the terminal points of a UAV's path. Thus, these two features make the air corridors model more substantial.

However, these structures are only pragmatic when they have an accompanying set of guidelines. To this extent, a novel TCF is proposed that seeks to abstract and unify the deployment, tracking, and controlling of autonomous air traffic. TCF defines UAS control agents which computerises a real-life UAV along with its control logic. This digital identity can then be used for a varied set of operations whilst being agnostic to hardware. TPI, described within TCF, disseminates certain information about the traffic facilitating decision-making, should

the need arise. The simulation feature of TCF enables ahead-of-time visualization of the aerial traffic which is essential in dispelling any deployment-related concerns. Using the very same feature, two scenarios were simulated as a demonstration of its capabilities.

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