Modeling the Impact of Passive Ventilation Systems on Multi-Zone Thermal Dynamics

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Abstract

Heating, Ventilation, and Air Conditioning (HVAC) systems account for 36% of energy use in the United States (U.S) and significant portion of greenhouse gas emissions. Passive ventilation, which utilizes natural airflows without electro-mechanical systems, offers a sustainable alternative to traditional HVAC systems. Currently, however, passive conditioning systems are rarely implemented in U.S. buildings due to a lack of understanding of their effects on indoor thermal dynamics. In this paper, we attempt to address this gap by modeling and analyzing the impact of passive ventilation systems on the thermal dynamics of multi-zone buildings. We introduce a Locally interactive Bilinear Flow (LiBF) model, which extends existing linear models to evaluate the thermal influence of passive ventilation elements, such as doors and windows, in multi-zone spaces. Additionally, we propose a two-step method for parameter estimation and provide a detailed case study using a three-zone building model in EnergyPlus. Simulation results validate both our model and parameter identification method, demonstrating their utility in optimizing passive ventilation settings, such as window and door openings, to enhance energy efficiency.

CCS Concepts

• **Computing methodologies → Model verification and validation**.

Keywords

Passive ventilation, Bilinear flow model, Model identification, HVAC.

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1 Introduction

Globally, buildings are a significant contributor to energy consumption, accounting for approximately one-third of total usage, with this trend being particularly pronounced in the United States. In 2017, Heating, Ventilation, and Air Conditioning (HVAC) systems accounted for approximately 36% of the energy consumed in the commercial building sector in the United States [6], with inefficiencies leading to roughly 30% of this energy being wasted. Given that these systems predominantly rely on fossil fuels, they contribute to nearly 700 million metric tons of CO_2 emissions annually [4], underscoring their importance in ongoing decarbonization efforts.

Against this backdrop, passive ventilation emerges as a compelling alternative, leveraging external enviromental resources including natural airflows, and reducing the reliance on electromechanical systems. Passive ventilation systems can complement – or even serve as an alternative to – traditional conditioning systems, and is extensively utilized in areas with a Mediterranean climate. Embracing such systems not only cuts down on energy use but also enhances indoor air quality, offering a viable solution to the challenges posed by current active conditioning systems.

Currently, however, passive heating is used in fewer than 0.2% of U.S. buildings, primarily due to the lack of understanding of their operation [1]. While recent literature have highlighted the potential of passive heating, cooling, and ventilation elements (such as windows, doors, vents, movable insulation, shading etc.) in providing significant energy savings [8, 9], the lack of concrete models on how indoor temperature dynamics are impacted by these passive element operations pose an impediment to developing effective control solutions. Implementation of control strategies, such as Model Predictive Control (MPC), critically depend on models that can accurately capture the complex thermal dynamics of multi-zone buildings as a function of the operable passive elements.

In this paper, we aim to bridge this gap in the understanding of the impact of passive systems on building thermal dynamics, by focusing specifically on connected multi-zone indoor spaces. We utilize the graph (network) structure of multi-zone spaces to propose and evaluate a *Locally-interactive Bilinear Flow (LiBF)* model that is suitable for the study of the impact and control of passive ventilation elements on multi-zone thermal dynamics. While prior studies [2, 7] have proposed and evaluated multi-zone thermal models, they have not considered the impact of controlling passive ventilation systems such as windows and doors on the zonal thermal dynamics. We show that our LiBF model, with model parameters learned through data-driven estimation, is able to capture and predict the impact of the opening and closing of windows and doors on the thermal dynamics of a multi-zone building.

The specific contributions of this paper are as follows, structured into several sections. In Section 2 we describe the LiBF model that naturally generalizes prior linear networked models for multi-zone thermal dynamics under active heating/cooling sources to passive

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ventilation systems. In Section 3, we outline a two-step methodology for estimating the parameters of the model, along with the dependencies on the operational settings of the passive elements. Section 4 presents the results that validate our model through simulations, and demonstrates how the the dependencies on passive element settings (window and door opening factors in our case study) can be determined experimentally in an efficient manner. We conclude in Section 5, summarizing our results and providing directions for future work.

2 LiBF Model for Passive Ventilation Systems

In our model, each zone within the multi-zone building with N zones is represented as a distinct node in a graph. The state of each zone (node), which is the average zonal temperature in our case, is denoted by $T_i,\ i = 1, \cdots, N$. The edges of the graph capture the physical adjacency relationships between the zones, and between each zone and the ambient, through which heat is exchanged. The state variable for the external environment (ambient temperature) is denoted by T_∞. Thermal interactions across the M edges of the graph are parameterized by the heat flow coefficients K_{ij}, representing the thermal conductance between adjacent nodes i and j, which can be controlled through passive elements such as window and door openings. Additional heat into the zones can come from solar insolation, and internal heat gains (from humans, computing equipment, lighting etc.), represented by S_i and Q_i respectively for zone i. The solar heat gain can possibly be controlled through passive elements such as blinds or other shading elements, and characterized by solar gain coefficient D_i. Finally, the the presence of an active heating/cooling source (controllable) is represented by U, the heat gain from which could further be modulated in a zone-specific manner through passive element controls such as vents, represented by parameters B_i. Let Θ represent the vector of all passive element parameters (controllable). Then, based on the above discussion, the parameters K_{ij}, D_i and B_i are all functions of Θ. If N_i denote the set of adjacent zones of zone i, the thermal dynamics of the zone can be written as:

$$
\begin{aligned}
C_i \frac{dT_i(t)}{dt} = & \sum_{j \in N_i} K_{ij}(\Theta)\left(T_j(t) - T_i(t)\right) \\
& + K_{i\infty}(\Theta)\left(T_\infty(t) - T_i(t)\right) \\
& + B_i(\Theta)\, U(t) + D_i(\Theta)\, S_i + Q_i, \qquad (1)
\end{aligned}
$$

where C_i is the thermal capacitance of zone i. The *bilinearity* of the above model comes from the multiplication of the passive element parameters (controllable) $K_{ij}, K_{i\infty}$ with the state variables T_i, T_j; and the multiplication of the passive element parameters B_i with the active heat control variable U. The expression in (1) assumes a single active heat source U (controlled by a single thermostat, for example) as is typical for smaller multi-zone buildings (and in our case study described later in Section 4). If separate zonal controls exist (such an zone-specific thermostats or zonal AC/heating units) then the term U in (1) needs to be replaced by U_i.

In the above model, we further impose an assumption of *local dependency* of the system states on the passive elements, as explained next. We assume that a passive element associated with a zone directly impacts the temperature of that zone only. This implies that the terms $K_{ij}(\Theta)$, $K_{i\infty}(\Theta)$ can simply be written as $K_{ij}(\theta_{ij})$, $K_{i\infty}(\theta_{i\infty})$, where $\theta_{ij}, \theta_{i\infty}$ represent the corresponding door and window opening factors. Similarly, the terms $B_i(\Theta)$, $D_i(\Theta)$ can be written as $B_i(\hat{\theta}_i)$, $D_i(\tilde{\theta}_i)$, where $\hat{\theta}_i$ and $\tilde{\theta}_i$ are the corresponding vent and shading opening factors. While this *locally-interactive* nature of the model seems natural, its worth noting that it may not strictly hold in certain practical scenarios. For example, opening a window between a zone and the ambient may increase the airflow between the zone and neighboring zones, triggering more heat exchange for that zone with other zone. A similar phenomena could occur even with vent openings in forced-air based HVAC systems. Further, opening a window shading associated with a zone could not only let more sunlight into that zone, but in other adjacent zones as well that may fall in the path of the incoming sunlight. Such "higher order" dependencies between the passive element controls and zonal thermal states are more difficult to estimate, however, and greatly increases the complexity of the parameter estimation process of the underlying spatio-temporal thermal dynamics. Further these additional non-local dependencies can also be more transient in nature, and yet may require more data to estimate reliably. Our evaluation results (see Section 4) seem to indicate that our LiBF model is still quite effective in capturing the multi-zonal thermal dynamics of the building, despite ignoring these longer range dependencies. Our bilinear dynamical model, while novel in the context of passive ventilation controls, is based on the widely accepted linear heat flow model (see [2, 3], for example, and references therein) that assumes fixed passive ventilation controls. Such linear models are developed using fundamental energy flow exchange (balance) principles between adjacent zones and with the ambient environment. Our bilinear model follows naturally from these linear models when we consider the passive ventilation elements (such as windows and vents) to be controllable, represented by the heat exchange coefficients K_{ij} and $K_{i,\infty}$ being a function of the passive element control vector, Θ.

In this paper, we only focus on assessing the impact of window and door openings on multi-zone thermal dynamics. Therefore, for the purpose of the rest of the paper, we ignore the terms S_i in our model. Further, since the vent openings are not controlled, the terms B_i can be assumed to be constant values (but need to be estimated however). This reduces our LiBF model to:

$$
\begin{aligned}
C_i \frac{dT_i(t)}{dt} = & \sum_{j \in N_i} K_{ij}\left(\theta_{ij}\right)\left(T_j(t) - T_i(t)\right) \\
& + K_{i\infty}\left(\theta_{i\infty}\right)\left(T_\infty(t) - T_i(t)\right) + B_i U + Q, \qquad (2)
\end{aligned}
$$

where the constant parameters C_i, B_i, and parameter functions associated with the passive elements, $K_{ij}(\theta_{ij}), K_{i\infty}(\theta_{ij})$, need to be estimated through measurements. It is worth noting that the order of importance of the parameters in (2) can vary depending on the specific scenario being modeled or investigated. In our case, the most important parameters are the conductance terms $(K_{ij}, K_{i\infty})$, as they capture the heat exchange of a zone with the ambient environment and with neighboring zones. These are the terms that are directly influenced by the passive ventilation elements (windows and doors) which regulate the exchange of heat between the zones and the outside environment. The other parameters are important but not directly controlled in our study, and therefore their impact is not analyzed in this work. We will use the reduced order model in (2) to describe our estimation methodology and evaluation results.

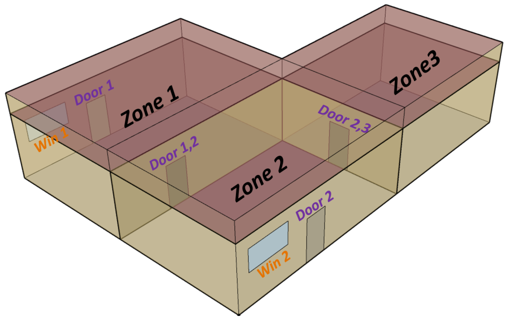

Figure 1: 3D architectural rendering of the multi-zone building used in the study.

3 Two-step Model Parameter Estimation

Our method to estimate the model parameters and their dependencies on the operational settings of the passive elements involves a two-stage process. In our two-step method, the initial phase involves using a discretized version of (1) to identify the parameters ($K_{ij}, K_{i_\infty}, C_i$) on measurements from running the EnergyPlus simulation model of the building. The parameter estimation requires two quantities, zonal temperatures (T_i), and heating power measurements (B_iU) which are measured from individual zones. We minimize the error between the observed indoor temperatures and those predicted by our discrete state-space LiBF model. This optimization is achieved using the non-negative least squares (NNLS) method, as detailed below:

$$\begin{aligned}\left\{K_{ij}^*, K_{i\infty}^*, C_i{}^*\right\} &= \underset{K_{ij},K_{i\infty},C_i}{\arg\min} \sum_{q=1}^{y} \left\|T[k] - \hat{T}[k]\right\|^2,\\ \text{s.t. } \hat{T}[k+1] &= \hat{T}[k] + \\ &\delta t \left(\sum_{j \in N_i} \frac{K_{ij}(\theta_{ij})(\hat{T}_j[k] - \hat{T}_i[k])}{C_i} \right.\\ &\left. + \frac{K_{i\infty}(\theta_{i\infty})(\hat{T}_\infty[k] - \hat{T}_i[k])}{C_i} + \frac{B_i\hat{U}[k]}{C_i} \right),\\ &K_{ij} \geq 0, K_{i\infty} \geq 0, C_i \geq 0.\end{aligned} \tag{3}$$

Further, to precisely model the impact of door and window openings on thermal conductance, we implement a constrained second-order polynomial fit (of order v). This approach can be viewed as a reduced order/complexity approach for estimating the thermal conductance (K_{ij},$K_{i\infty}$) for varying door and window openings, expressed as:

$$\begin{aligned}\{a_0^*, a_1^*, \ldots, a_v^*\} &= \underset{a_0,a_1,\ldots,a_v}{\arg\min} \left\| K(\theta_\star) - \sum_{l=0}^{v} a_{\star l}\theta^l \right\|^2,\\ \text{s.t. } &\sum_{l=1}^{v} l a_{\star l}\theta^{l-1} > 0,\end{aligned} \tag{4}$$

where $\star$ represents a pair (i, j) such that $i \neq j$; $i = 1, \ldots, N$; $j = 1, \ldots, N, \infty$.

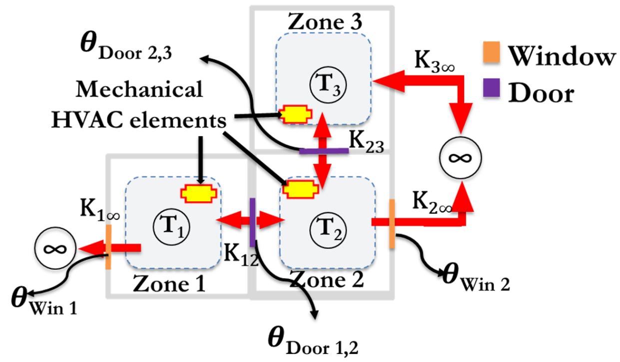

Figure 2: Illustration of the different model parameters in a 3-zone EnergyPlus building model used in our evaluations.

4 Evaluation Results

Case Study: In our evaluation, we use the EnergyPlus model of a small office commercial building provided by the Department of Energy (DOE) [5], with a few modifications as outlined below. The model is illustrated in Figs. 1 and 2. The layouts and thermal dynamics of such DOE prototype buildings are expected to be realistic representations of those in real buildings. This particular building is single-story, L-shaped and divided into three conditioned interior zones, covering a total floor area of 130.1 m² (1403 ft²). The building's layout is defined by its rectangular shape with 40 ft walls facing south and west, and a ceiling height of 10 feet. The model includes two doors, connecting zone 1 to zone 2 and zone 2 to zone 3, respectively, along with one window in each of the first two zones. The HVAC system features a centralized, three-zone configuration with a single air loop. It includes a DX cooling coil for space cooling, a gas heating coil for primary heating, and three terminal reheat coils that provide supplemental heat across the three zones of the multi-zone building. Additionally, the system is equipped with a variable air volume (VAV) supply fan, set to operate at a minimum of 20% of the design airflow rate for each terminal.

To better evaluate and isolate the impact of passive ventilation systems (windows and doors in particular), we made a few small modifications to the original model. We varied the size of the windows to evaluate how different window dimensions impact the building's thermal dynamics. The results reported in this paper was based on a 60% reduction from the original window sizes (which were quite large). Further, to better isolate and study the impact of ventilation on the thermal dynamics of the building, we removed the effects of solar radiation by eliminating sun exposure. This was accomplished by setting the "SunExposure" field in the building surfaces object of the EnergyPlus input file (IDF) to the "NoSun" boundary condition. In addition, we modeled air exchange through doors and windows using the zone infiltration and cross-zone mixing objects. We also utilized the Energy Management System (EMS) in EnergyPlus to control the zonal heating, ventilation, and air conditioning (HVAC) systems. Customized schedules were implemented to regulate the opening and closing of doors and windows at various opening factors, enabling a thorough evaluation of our bilinear model. The detailed setup shown in Fig. 2 allows for an

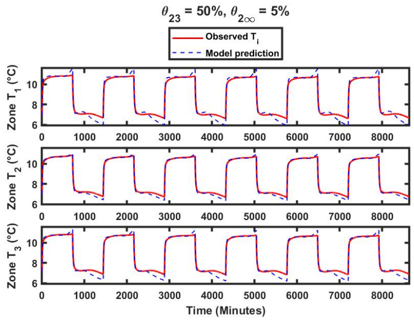

Figure 3: Visual comparison of observed temperatures (solid red lines) with model predictions (dashed blue lines) for each zone .

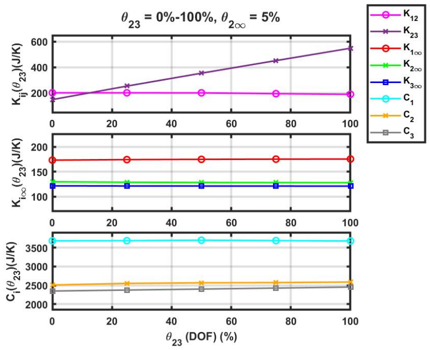

Figure 4: Parameter estimates for a fixed WOF with changing DOF.

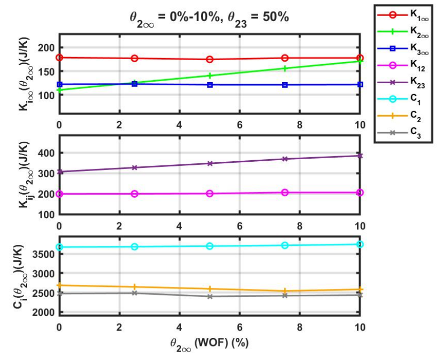

Figure 5: Parameter estimates for a fixed DOF with changing WOF.

extensive examination of multizone airflows driven by wind, providing a robust framework for our experimental study. We employ a weather profile for Bellingham, WA, derived from the Typical Meteorological Year (TMY) data.

Model Evaluation: The objective of our model evaluation process is to assess the accuracy of the model described in (1) using previously unseen data. Initially, we identify parameters using NNLS from data collected over a 30-day period in January. Further, we validate the model on new, unseen data by employing a set of measurements collected over a five-day period 5-day measurement period in February of the same season specifically excluded from the initial model identification phase. Fig. 3 presents a visual comparison between the observed temperatures (represented by solid red lines) and the predictions from the LiBF model (indicated by dashed blue lines) across different zones. To quantitatively assess the agreement between observed indoor temperatures and those predicted by the LiBF model, we calculate the Root Mean Square Error (RMSE). Specifically, the RMSE values are 0.53°C for Zone 1, 0.21°C for Zone 2, and 0.31°C for Zone 3. These results indicate that the LiBF model is reasonably accurate in capturing

the zonal thermal dynamics.

Effect of Door and Window Opening: Next we proceed to determine parameter estimates as a function of door opening factors (DOF). This involves incremental variation of the DOF between zones 2 and 3, ranging from a completely closed state (0%) to fully open (100%), with 25% increments. Concurrently, for each level of window opening factor (WOF)—starting at 0% and increasing in 2.5% increments up to 10%—the DOF was varied accordingly to assess their impact on the model's thermal dynamics. The rationale for limiting the WOF to 10% in our experiments stems from the fact that opening the window excessively during cold months can totally disrupt normal operations of the building.

In Fig. 4, we present the parameter estimates for a fixed WOF with changing DOF. Specifically, the WOF was maintained constant at 5%, ensuring that any observed changes in the parameter estimates could be directly attributed to variations in the door openings. On one hand, we observe that progressive increase in the DOF(θ_{23}) led to a corresponding increase in K_{23}, which implies an increased inter-zone coupling. On the other hand, K_{12} remains invariant as θ_{23} is increased, which is expected given that the door between these zones remains firmly closed throughout the experiment. Additionally, the thermal capacitances represented by C_1, C_2, and C_3 are observed to be constant despite changes in θ_{23}. This is anticipated since the capacitance values are primarily dependent on the physical characteristics of the zones, such as volume and construction materials, rather than on transient factors like door openings. In Fig. 5, we show the impact of varying WOF on the building's thermal dynamics while maintaining a constant DOF. For this study, the DOF is fixed at 50%, and the window between zone 2 and the outdoors is varied from 0 to 10%. This scenario is designed to isolate and quantify the effect of window openings on thermal interactions within the different zones. The trends are as as we expect, and broadly similar in nature to our observations made with increasing the DOF. In addition to the aforementioned results, experiments were conducted with varying window and door sizes, durations of opening, and the number of openings. From these experiments, we found that the model remains valid across different configurations. However, we observed that when the window size is larger, the maximum opening factor needs to be reduced accordingly. Based on our study, we conclude that the the LiBF model is adept at capturing the impact of passive controls such as door and window opening factors on the zonal thermal dynamics.

Polynomial Fit: Next we evaluate the the methodology described in Section 3 to approximate the thermal conductance across varying window and door opening factors. For a fixed DOF (50%), we estimate the thermal conductances across varying WOF (0-10%) with 2.5%. Similarly, for a fixed WOF (5%), we determine the thermal conductances across different DOF (0-100%) with 25% increments. For both setups, we fit the estimates with a constrained second-order polynomial.

Using the estimates K_{23} and K_{20} derived from the constrained second-order polynomial and the average values of other parameters , we estimate the model parameters for K_{23} and K_{20} for all other fixed configurations of DOF and WOF. Observations indicate that estimates derived from the fixed configuration of DOF = 50% and WOF = 5% provide reasonable approximations of thermal conductances for all other combinations of WOF and DOF, as evidenced by their RMSE values in Table 1. For clarity, we denote the RMSE

Table 1: RMSE Values for Different Door and Window Opening Factors. The table compares the RMSE values (y) from the model with those derived from the polynomial fit ($\hat{y}$) for various configurations of WOFs (rows) and DOFs (columns).

$\theta(\%)$	0		25		50★		75	
	y	$\hat{y}$	y	$\hat{y}$	y	$\hat{y}$	y	$\hat{y}$
0	0.65	0.65	0.64	0.64	0.63	0.64	0.63	0.64
	0.43	0.44	0.35	0.35	0.30	0.29	0.26	0.25
	0.59	0.61	0.45	0.45	0.36	0.35	0.30	0.30
2.5	0.57	0.59	0.58	0.59	0.58	0.58	0.58	0.58
	0.36	0.39	0.31	0.32	0.23	0.27	0.21	0.24
	0.59	0.62	0.44	0.45	0.29	0.36	0.25	0.30
5.0★	0.51	0.55	0.53	0.54	0.54	0.53	0.54	0.53
	0.33	0.34	0.28	0.29	0.25	0.25	0.22	0.22
	0.63	0.64	0.46	0.46	0.36	0.36	0.30	0.30
7.5	0.46	0.51	0.48	0.49	0.49	0.48	0.51	0.48
	0.29	0.29	0.26	0.26	0.23	0.21	0.21	0.19
	0.69	0.65	0.49	0.47	0.38	0.37	0.31	0.30
10.0	0.43	0.47	0.44	0.46	0.45	0.45	0.47	0.44
	0.27	0.25	0.24	0.23	0.22	0.21	0.20	0.19
	0.76	0.66	0.52	0.47	0.39	0.37	0.32	0.31

Table 2: Percentage deviation of K_{20} from Actual Parameter Estimates for different WOFs (rows) and DOFs (columns).

$\theta(\%)$	0	25	50	75	100
0.0	0.7	0.3	0.0	0.1	0.3
2.5	0.8	0.3	0.0	0.2	0.3
5.0	1.0	0.3	0.0	0.2	0.3
7.5	1.6	0.5	0.0	0.3	0.4
10.0	1.6	0.8	0.0	0.3	0.6

based on the model identification as y, and those derived from the polynomial fit as $\hat{y}$.

The RMSE values across the row with 5% WOF and the column with 50% DOF in Table 1 (marked by ★) are quite small, suggesting good model prediction accuracy for (DOF, WOF) settings on which the model was trained. What is more significant, however, is that even at the other (DOF, WOF) combinations (on which the model was not trained) the accuracy is quite good. This implies that with only $O(N + M)$ evaluations, we are able to evaluate the model for $O(NM)$ settings, where N (M) is the number of WOF (DOF, resp.) setpoints. This fact can be particularly useful in fitting a model to a large number of passive elements, as data collection for each combination of passive elements can be time-intensive.

The above observations are further supported by the data presented in Table 2, where we show the deviations of K_{20} obtained from the fit from those obtained through model identification.

5 Conclusion

In this paper, we proposed and evaluated a Locally-interactive Bilinear Flow (LiBF) model to assess and predict the impact of controlling passive ventilation elements on the thermal dynamics of multi-zone buildings. The model captures the impact of window and door openings (on the zonal thermal dynamics) through certain conductance parameters that can be easily estimated through measurements of zonal temperatures and per-zone power inputs. The model was demonstrated to predict the impact of window and door openings on the temperature provided it was trained on other days of the same season. We further demonstrated that the model can reliably estimate the impact of all possible door and window factor combinations with only a linear number of measurements.

Although we have conducted studies across multiple seasons, this paper focuses specifically on winter day results. To ensure the model's reliability, we validate it using data from different days within the same season, which helps maintain consistency in the parameter estimates. Given the significant variations in thermal dynamics across seasons, the model parameters need to be re-estimated for each season, as the current model cannot be directly applied to other periods without recalibration.

It is worth noting that the training and evaluation were all done on a DOE prototype building model (slightly modified, as described in Section 4) in EnergyPlus. The EnergyPlus model implements thermal dynamics of the building using a complex set of physics-based equations (details can be found be accessing the model in [5]) which is assumed to be a realistic representation of the thermal dynamics of an actual building. Our study shows that our simple bilinear flow model is able to accurately capture the thermal dynamics of the complex EnergyPlus building thermal model (from the perspective of passive ventilation control) with a much smaller set of parameters that are easy to train. This simpler model would allow design efficient control solutions, which could be explored in future work. Validation of our LiBF model, and the passive ventilation controls derived from the model, in a real building also remains to be investigated in future work. Further work is also needed in understanding the impact of solar gain and other passive elements like shading and vent control, and evaluating in larger and more complex buildings, both real and simulated.

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