General Relativity and Gravitation (2025) 57:24
https://doi.org/10.1007/s10714-025-03354-9

PERSPECTIVE

Simulating black hole imposters

Frans Pretorius[1]

Received: 19 November 2024 / Accepted: 7 January 2025 / Published online: 24 January 2025

Abstract
I briefly describe motivation for, and the current state of research into understanding the structure and dynamics of black hole "imposters": objects that could be misidentified as Kerr black holes given the current precision of LIGO/Virgo gravitational wave observations, or EHT accretion disk measurements. I use the term "weak imposter" to describe an object which is a black hole, i.e. it has an event horizon, but whose structure and dynamics is governed by a modified gravity theory. At the other end of the spectrum are "strong imposters": hypothetical horizonless, compact objects conjectured to form instead of black holes during gravitational collapse. To discover or rule-out imposters will require a quantitative understanding of their merger dynamics. This is hampered at present by a dearth of well-posed theoretical frameworks to describe imposters beyond perturbations of Kerr black holes and their general relativistic binary dynamics. That so little is known about non-perturbative modifications to dynamical, strongfield gravity is, I argue, due to a lamppost effect.

Keywords Gravitational waves · Black holes · Testing general relativity

Contents

1 Observing the dynamical, strong field regime of gravity

Einstein's theory of general relativity makes several remarkable predictions about the nature of spacetime in the strong field regime. First is the fact that there *is* a strong field regime in a theory without any (geometric) dimensionful scales or parameters. This is unlike any of the other fundamental forces we understand today, where new

✉ Frans Pretorius
fpretori@princeton.edu

1 Princeton University, Princeton, USA

physics arises at distinct energy scales associated with fundamental constants in the corresponding theories. For general relativity, this begs the question of what property of the theory governs the onset of the qualitatively distinctive strong versus weak field dynamics of spacetime. An answer, which is difficult to make precise due to the coordinate invariance of the theory, is strong field behavior is manifest when the nonlinear terms in the field equations become important or dominant in determining its solution. That there is no fundamental scale in the theory means that the nonlinear regime can, in principle, be reached at *any* length or energy scale.

A second remarkable fact about general relativity is how strikingly different its predictions of weak versus strong field spacetime structure is. The former is given by Minkowski spacetime and its perturbations, well described on Earth and in the solar system by Newtonian gravity with small relativistic corrections. The strong field most notably includes black hole solutions, where within their corresponding Schwarzschild radii spacetime undergoes *gravitational collapse*, becoming intrinsically dynamical, flowing to some kind of singularity or Cauchy horizon where general relativity breaks down as a sensible theory. Presumably some new theory, call it *quantum gravity*, will supercede Einstein's theory in providing a better description of nature then. General relativity putatively says this kind of breakdown occurs hidden behind an event horizon, and thus the best we can do to test this astonishing prediction of the theory is to observe the exterior structure and dynamics of black holes. Doing so has become possible in the last decade with LIGO/Virgo observations of binary black hole mergers [1], and the EHT observation of accretion disks about the supermassive black holes in M87 [2] and the Milky Way [3].

Given the uncertainties in modeling accretion disk astrophysics, there has been much debate in the literature about whether present day EHT observations can place quantitative bounds on how well the Kerr metric describes the corresponding spacetimes (see e.g. [4] and the citations therein). Moreover, given how massive M87* and SgrA* are relative to their accretion disks, outside the horizons these spacetimes are essentially stationary, and cannot be used to test the radiative degrees of freedom of strong field gravity. Black hole mergers observed by LIGO/Virgo are not (as far as we know) meaningfully influenced by a circumbinary environment, and certainly probe dynamical gravity, giving us at present the only direct tests of this regime of general relativity.[1] Current observations of black hole mergers are consistent with the predictions of general relativity (GR), though are not yet at the level of "precision" tests: lack of any detectable residual following subtraction of "best-fit" GR waveforms give consistency at the O(10%) level [5]; of this order or slightly better constraints can be placed for parameters governing various specific modified gravity theories (see e.g. [6]).

This then still leaves some room for the candidate black holes we have observed to be "imposters", where their structure and dynamics is governed by some modified or beyond-GR theory. Most proposed examples of imposters can be called "weak"

[1] Of course, the ground-based detectors are sensitive to mergers involving black holes of order $1-10^2$ solar masses, compared to the order 10^6 and 10^9 solar masses of SgrA* and M87*, respectively, that the EHT has observed. Thus there are conceivable scale-dependent modifications to general relativity that could, for example, not affect the LIGO/Virgo merger events yet be detectable in the EHT images, despite the latter not being sensitive to the radiative degrees of freedom of the theory.

imposters, for example as coming from Einstein–dilaton–Gauss–Bonnet (EdGB) or Chern–Simons (CS) modified gravity, where the solutions are still black holes but with only perturbative corrections compared to the structure and dynamics predicted by GR. Proposed "strong" imposters, such as gravastars, fuzzballs, or AdS black shells, do not have horizons. The purpose of this note is not to describe any of these imposters—see e.g. [7, 8] for more comprehensive reviews—rather, it is to briefly discuss some motivation for studying them, and what the present difficulties are in simulating them with the intent of making quantitative merger waveform predictions.

One motivation is as already discussed: testing general relativity in the dynamical strong field regime. Though the residual test is theory agnostic, somewhat better constraints for a specific imposter can be made if we have waveform models for it. Moreover, if an anomaly is ever detected, the residual by itself may give little information on the physics behind it; a better understanding of the gamut of theoretically viable beyond-GR phenomenology will be crucial to decipher the underlying cause.

A second motivation, discussed more after describing the difficulties in simulating imposters, is what can be called the "lamppost" problem in beyond-GR theories. Essentially it is that, at present, we are limited to studying theories that are perturbatively close to GR, not because observational data is telling us this is adequate, but rather because we do not know how to go beyond the perturbative description. Of course, since there is no detected waveform anomaly yet, the data *is* telling us we do not need anything but GR at present, let alone perturbative corrections, and solving the lamppost problem might only ever be of pure theoretical interest.

2 Difficulties simulating black hole imposters

The main difficulty at present in simulating many black hole imposters is coming up with a well-posed set of partial differential equations (PDEs) governing their structure and dynamics. For weak imposters, the issues stem from the fact that most beyond-GR theories introduce their modifications by adding higher order curvature scalars to the Einstein–Hilbert action, which typically leads to the principle parts of the PDEs having higher than second derivatives, and also complicates the nonlinear structure of the PDEs. One approach people have taken to deal with the problems this causes is to only treat these beyond-GR terms perturbatively, e.g. [9–11]. The equations then retain the mathematical structure of GR, and all the modern methods for numerically solving the Einstein equations for black hole mergers can be applied essentially verbatim. One problem with this approach is it is susceptible to spurious secularly growing solutions that require care to mitigate [12].

For the Horndeski class of theories [13], including EdGB gravity, the equations of motion are still second order, though have more complicated non-linear structure than GR, and one needs to introduce an additional scalar degree of freedom to give non-trivial modifications to GR in four spacetime dimensions. Using standard generalized harmonic coordinates, EdGB gravity is not well posed in generic settings [14, 15], however recently Kovács and Reall introduced a "modified harmonic" scheme that is well posed in the weakly-coupled limit [16], and was applied to binary merger simulations in [17–19]. A problem here though still arises in the strongly-coupled limit when the non-linear corrections to GR become large (which for binary mergers is achieved by making the black holes smaller than the length scale set by the coupling constant of the EdGB correction). Then the PDEs cease being hyperbolic, and the predictability of the theory fails form the perspective of the initial value problem [20].

A novel approach to deal with ill-posedness introduced by the addition of derivatives beyond second order, yet still be able to study non-perturbative corrections, is to elevate the beyond-GR terms to new fields, and introduce new equations of motion that "relax" them to their desired values on some chosen timescale [21, 22].[2] This approach is not coordinate invariant, and somewhat ad-hoc, though early studies show it does work, both in the sense of mitigating the ill-posedness problem, and that when in the perturbative regime the ad-hoc relaxation parameters do not significantly affect the solution [23]. How far solutions can be pushed beyond the perturbative regime remains to be seen, questions aside as to how such solutions should be interpreted, or what putative well-posed theory they may correspond to.

A very recent work by Figueras et al. [24] seems to show that, remarkably, a broad class of higher-curvature metric Lagrangians can, through the introduction of regularizing terms and field redefinitions, have their equations of motion cast into a well-posed hierarchy of coupled, nonlinear wave equations.[3] This introduces a corresponding set of new massive degrees of freedom, but they can be chosen to not affect the dynamics below some desired cutoff scale.

In all then, quite significant progress has been made toward taming the structure of beyond-GR theories, making it feasible to solve them numerically for binary merger processes. This should allow for a more thorough understanding of weak black hole imposters, and set more stringent constraints on specific theories with current GW data than agnostic tests can.

The theoretical health of strong imposters, i.e. horizonless alternatives to black holes, is in a more precarious state. Motivations for proposing these objects are largely theoretical, including to avoid the information loss problem in semi-classical GR, and/or singularity formation in classical GR. However, arguably none of the proposed black hole alternatives have a complete theory behind them, in particular from which solvable equations of motion predicting their dynamics can be derived. One possible exception are AdS black shells (or black bubbles) [27]. They are string theory inspired objects, argued to form from a tunneling process prior to formation of a classical horizon. Once formed, for macroscopic shells, the effective four dimensional system is argued to be well described by classical physics: a thin shell of matter separating an interior AdS spacetime from an exterior asymptotically flat spacetime, where the spacetime dynamics is governed by GR, and the matter by a set of (possibly charged) dissipative fluids. To date, the spherically symmetry case with a Schwarzschild exterior has been shown to be stable to radial perturbations and accretion with an appropriate equation of state [28], slowly rotating solutions have been found [29, 30], and certain electromagnetic properties studied [31] that suggest AdS black shells could be compatible with EHT observations despite the shell being a macroscopically large distance

[2] This method was inspired by the Muller–Israel–Stewart approach of fixing similar ill-posedness problems in the relativistic Navier–Stokes equations.

[3] This is a generalization of the previously known case of quadratic gravity [25, 26].

outside of the would be black hole horizon (it is at the Buchdahl radius, $9M/4$, for a non-rotating black shell). It remains to be seen if they are stable to non-radial perturbations, and it is unclear how their collision in a merger should be treated. If the latter is a new tunneling process creating a larger shell, then this model also suffers from lack of a theoretical framework to calculate a merger waveform in the coalescence regime. If it can be treated as the collision of two classical shells of matter, a waveform calculation should be possible, though it will be challenging to simulate due to the effectively distributional source of matter.

3 The Lamppost problem in theoretical, strong field gravity

In principle, non-GR metric theories of gravity should allow for black hole imposters that are *significantly* different from Kerr black holes, however none are presently known. For example, stable geons, black holes with different horizon topologies (as occurs in higher dimensional GR), or black holes that can support "true hair". What I mean by the latter is a horizon with (say) spherical topology, but with a stable, non-trivial geometric structure that depends on its *formation history*. Many current examples of modified GR theories are often claimed to imbue a black hole with hair; however given that said hair has a structure uniquely set by the parameters of the underlying black hole and coupling constant of the theory, this does not violate the spirit of the GR no-hair theorems that do not allow a black hole horizon to support non-decaying, formation dependent structure.

This begs the question of why all known 4D, asymptotically flat black holes in modified-GR theories are Kerr-like. I suspect the answer is because we have not been able to successfully look outside of the *lamppost* of GR. This is partly because of the conceptual and mathematical difficulties in doing so, that I will discuss in the following paragraphs, but also that for black holes there is not yet an observational anomaly that requires we do so. It would be surprising if the answer were instead that there is some as-of-yet undiscovered property of strong field gravity that, regardless of the nature of modifications to the Einstein Hilbert action, the only viable black hole solutions are Kerr with small perturbations, and we do not need to rely on observation to vet amongst possible theories describing the leading order phenomenology of horizons in our universe. Contrast for example the analogous question of what radiative degrees of freedom are in principle allowed in the weak field in a metric based theory of gravity: here we know the two transverse/traceless polarizations of GR is *not* the unique answer, but in general *six* linearly independent modes are possible [7].

One aspect of the lamppost problem is simply the difficulty of finding solutions to the more complicated, non-linear PDEs arising from curvature based modifications to the Einstein–Hilbert action. This is not surprising given that in GR itself almost 50 years of research occurred between discovery of the Schwarzschild [32] and Kerr solutions [33], the former satisfying considerably simpler differential equations compared to the latter (in appropriate symmetry adapted coordinates). That we have analytical forms for Schwarzschild and Kerr, and that most modified-GR theories reduced to GR in some well-defined manner, implies it is straight-forward to seek perturbative corrections to these solutions going beyond-GR. Looking for a completely different classes of black hole solutions, or proving that they do not exist, is a considerably more challenging endeavor, and is difficult to justify undertaking if the only reason to do so is theoretical curiosity.

A second aspect of the lamppost problem relates to a common expectation that new physics will manifest at some new scale governed by a corresponding dimensionful constant. For example, that the putative quantum nature of spacetime will be revealed at a Planck or string scale, or that a Hubble scale correction to classical gravity could explain the mystery of dark energy. The lamppost here is in part this philosophical bias that modified gravity should behave in this manner, but also in part the difficulty in contemplating how to alter strong field gravity to retain the scale-free nature of GR without breaking the well tested weak field regime. Given the many similarities between cosmological and black hole horizons in classical gravity, most importantly that both reside in the strong-field regime of GR (see for e.g. the discussion in [34]), and the unknown nature of dark energy, there is significantly more justification to spend effort to solve this aspect of the lamppost problem. In particular, this would allow testing such theories with GW observations of stellar mass black hole mergers, either constraining such theories or discovering an anomaly that could then be argued to also pertain to dark energy.

Author Contributions All of it.

Data availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare no Conflict of interest.

References

1. Abbott, B.P., et al.: LIGO Scientific, Virgo. Phys. Rev. Lett. **116**, 061102 (2016). https://doi.org/10.1103/PhysRevLett.116.061102. arXiv:1602.03837 [gr-qc]
2. Akiyama, K., et al.: Event horizon telescope. Astrophys. J. Lett. **875**, L1 (2019). https://doi.org/10.3847/2041-8213/ab0ec7. arXiv:1906.11238 [astro-ph.GA]
3. Akiyama, K., et al.: Event horizon telescope. Astrophys. J. Lett. **930**, L12 (2022). https://doi.org/10.3847/2041-8213/ac6674. arXiv:2311.08680 [astro-ph.HE]
4. Lupsasca, A., Mayerson, D. R., Ripperda, B., Staelens, S.: "A beginner's guide to black hole imaging and associated tests of general relativity. In: C. Bambi, A. Cardenas-Avendano (eds) Recent Progress on Gravity Tests Challenges and Future Perspectives, pp. 183–237 (2024). https://doi.org/10.1007/978-981-97-2871-8_6. arXiv:2402.01290 [gr-qc]
5. Abbott, R., et al.: LIGO Scientific. VIRGO, KAGRA (2021). arXiv:2112.06861 [gr-qc]
6. Yunes, N., Yagi, K., Pretorius, F.: Phys. Rev. D **94**, 084002 (2016). https://doi.org/10.1103/PhysRevD.94.084002. arXiv:1603.08955 [gr-qc]
7. Will, C. M.: Living Rev. Rel. **17**, 4 (2014), https://doi.org/10.12942/lrr-2014-4. arXiv:1403.7377 [gr-qc]
8. Cardoso, V., Pani, P.: Living Rev. Rel. **22**, 4 (2019). https://doi.org/10.1007/s41114-019-0020-4. arXiv:1904.05363 [gr-qc]
9. Witek, H., Gualtieri, L., Pani, P., Sotiriou, T.P.: Phys. Rev. D **99**, 064035 (2019). https://doi.org/10.1103/PhysRevD.99.064035. arXiv:1810.05177 [gr-qc]
10. Okounkova, M., Stein, L.C., Scheel, M.A., Teukolsky, S.A.: Phys. Rev. D **100**, 104026 (2019). https://doi.org/10.1103/PhysRevD.100.104026. arXiv:1906.08789 [gr-qc]

11. Okounkova, M.: Phys. Rev. D **102**, 084046 (2020). https://doi.org/10.1103/PhysRevD.102.084046. arXiv:2001.03571 [gr-qc]
12. Okounkova, M., Stein, L.C., Moxon, J., Scheel, M.A., Teukolsky, S.A.: Phys. Rev. D **101**, 104016 (2020). https://doi.org/10.1103/PhysRevD.101.104016. arXiv:1911.02588 [gr-qc]
13. Horndeski, G. W.: Int. J. Theor. Phys. **10**, 363 (1974). https://doi.org/10.1007/BF01807638
14. Papallo, G., Reall, H.S.: Phys. Rev. D **96**, 044019 (2017). https://doi.org/10.1103/PhysRevD.96.044019. arXiv:1705.04370 [gr-qc]
15. Papallo, G.: Phys. Rev. D **96**, 124036 (2017). https://doi.org/10.1103/PhysRevD.96.124036. arXiv:1710.10155 [gr-qc]
16. Kovács, A.D., Reall, H.S.: Phys. Rev. D **101**, 124003 (2020). https://doi.org/10.1103/PhysRevD.101.124003. arXiv:2003.08398 [gr-qc]
17. East, W.E., Ripley, J.L.: Phys. Rev. D **103**, 044040 (2021). https://doi.org/10.1103/PhysRevD.103.044040. arXiv:2011.03547 [gr-qc]
18. Corman, M., Ripley, J.L., East, W.E.: Phys. Rev. D **107**, 024014 (2023). https://doi.org/10.1103/PhysRevD.107.024014. arXiv:2210.09235 [gr-qc]
19. Aresté Saló, L., Clough, K., Figueras, P.: Phys. Rev. D **108**, 084018 (2023). https://doi.org/10.1103/PhysRevD.108.084018. arXiv:2306.14966 [gr-qc]
20. Ripley, J.L., Pretorius, F.: Phys. Rev. D **99**, 084014 (2019). https://doi.org/10.1103/PhysRevD.99.084014. arXiv:1902.01468 [gr-qc]
21. Cayuso, J., Ortiz, N., Lehner, L.: Phys. Rev. D **96**, 084043 (2017). https://doi.org/10.1103/PhysRevD.96.084043. arXiv:1706.07421 [gr-qc]
22. Cayuso, R., Lehner, L.: Phys. Rev. D **102**, 084008 (2020). https://doi.org/10.1103/PhysRevD.102.084008. arXiv:2005.13720 [gr-qc]
23. Corman, M., Lehner, L., East, W. E., Dideron, G.: (2024). arXiv:2405.15581 [gr-qc]
24. Figueras, P., Held, A., Kovács, A. D.: (2024). arXiv:2407.08775 [gr-qc]
25. Noakes, D. R.: J. Math. Phys. **24**, 1846 (1983). https://doi.org/10.1063/1.525906
26. Stelle, K. S.: Gen. Rel. Grav. **9**, 353 (1978). https://doi.org/10.1007/BF00760427
27. Danielsson, U.H., Dibitetto, G., Giri, S.: JHEP **10**, 171 (2017). https://doi.org/10.1007/JHEP10(2017)171. arXiv:1705.10172 [hep-th]
28. Danielsson, U., Lehner, L., Pretorius, F.: Phys. Rev. D **104**, 124011 (2021). https://doi.org/10.1103/PhysRevD.104.124011. arXiv:2109.09814 [gr-qc]
29. Danielsson, U., Giri, S.: JHEP **07**, 070 (2018). https://doi.org/10.1007/JHEP07(2018)070. arXiv:1712.00511 [hep-th]
30. Danielsson, U., Giri, S.: Phys. Rev. D **109**, 024038 (2024). https://doi.org/10.1103/PhysRevD.109.024038. arXiv:2310.12148 [gr-qc]
31. Giri, S., Danielsson, U., Lehner, L., Pretorius, F.: (2024). arXiv:2405.08062 [gr-qc]
32. Schwarzschild, K.: Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) **1916**, 189 (1916), arXiv:physics/9905030
33. Kerr, R. P.: Phys. Rev. Lett. **11**, 237 (1963). https://doi.org/10.1103/PhysRevLett.11.237
34. Pretorius, F.: In: International Congress of Mathematicians (2023). arXiv:2306.03797 [gr-qc]