Distributed Tracking Control of Multiple high-order Uncertain Nonlinear Systems with Guaranteed Performance

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Abstract— **This paper addresses the distributed tracking control of multiple uncertain high-order nonlinear systems with prescribed performance requirements. By introducing a performance function and a nonlinear transformation, the prescribed fixed-time performance tracking control problem is reformulated as a distributed tracking control problem for multiple special nonlinear systems. With the aid of the universal approximation theorem for continuous functions and algebraic graph theory, distributed robust adaptive controllers are designed using the backstepping technique. Simulation results are presented to demonstrate the effectiveness of the proposed algorithms.**

Index Terms— **Consensus; leader-following control; nonlinear systems; uncertain systems; distributed tracking control.**

I. Introduction

Over the past few decades, significant research efforts have been devoted to distributed cooperative control of multiple systems. This field has proven to be crucial across diverse domains, where multiple agents or components collaborate to achieve shared goals. Applications span a wide range of areas, including search-and-rescue operations, swarm robotics, autonomous vehicles, power distribution networks, wireless sensor networks, and satellite and UAV networks, among others.

The consensus control problem involves designing distributed control laws for a group of systems to ensure that their outputs converge to an agreement on a specific quantity of interest. It plays a crucial role in distributed cooperative control, with extensive research yielding significant results [1–5]. A key performance metric for consensus algorithms is the consensus rate, particularly in the context of multiple first-order linear systems. The consensus rate is determined by the second smallest eigenvalue of the Laplacian matrix of the communication graph, referred to as the algebraic connectivity. Strategies to enhance algorithm performance often involve manipulating the communication graph to increase algebraic connectivity [6]. Another approach focuses on designing finite-time consensus algorithms that drive consensus errors to zero within a finite time. For example, in [7], finite-time algorithms were proposed for single-integrator dynamic systems using Lyapunov techniques. Similarly, [2, 8] developed finite-time algorithms using terminal sliding-mode control for multi-robot systems. In [9], consensus algorithms for nonlinear dynamic systems were introduced, employing integral sliding-mode control and finite-time observers. Meanwhile, [10–12] explored the use of fuzzy logic control and neural networks to achieve practical finite-time convergence of consensus errors. However, traditional finite-time consensus algorithms are sensitive to initial conditions, leading to longer settling times when initial errors are large. To address this, researchers have explored fixed-time consensus algorithms in recent decades [3, 4, 13–20]. These algorithms ensure that the settling time is independent of initial conditions, although they rely heavily on the communication graph's topology, which may not be known in practice. To overcome this limitation, prescribed fixed-time consensus algorithms have been proposed. For instance, [21] addressed the consensus problem for first-order systems without a leader, developing practical prescribed fixed-time algorithms by estimating algebraic connectivity.

Fixed-time and prescribed finite-time control algorithms ensure the convergence of consensus errors to zero within a finite time. However, these algorithms may face limitations in transient performance, which might not satisfy specific requirements. To overcome this challenge, the prescribed performance-based controller design technique has proven effective [22]. For example, the study in [23] investigated the formation control of first-order

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and second-order systems with multiple leaders. Distributed tracking controllers were designed to ensure prescribed performance by utilizing the prescribed performance function (PPF). Similarly, [24] explored prescribed performance formation control for second-order multi-agent systems, proposing distributed controllers that meet predefined performance criteria using the PPF. Notably, both studies [23, 24] focused on linear systems with no uncertainties in their dynamics.

In this paper, we address the prescribed performance consensus control problem for multiple high-order nonlinear systems with uncertainties. Our objective is to design a distributed controller for each system, ensuring that the tracking error converges to a small neighborhood of the origin within a prescribed fixed time while meeting specific performance criteria. To achieve this, we adopt a multifaceted approach. First, we introduce a prescribed performance function (PPF) that integrates fixed-time, transient, and steady-state performance requirements into a unified framework. Next, using the Lyapunov technique, algebraic graph theory, and the universal approximation theorem of functions, we propose distributed virtual controllers that satisfy the required performance conditions. Finally, we design real controllers employing the backstepping technique to ensure the system outputs converge to the desired outputs within the prescribed fixed time and meet the defined performance standards. The contributions of this paper are as follows:

- This paper addresses the leader-following control problem for multiple nonlinear systems with performance requirements. In contrast, the study in [21] solves the leader-following control problem for first-order uncertain nonlinear systems without considering transient performance requirements. In this work, both transient and steady-state performance are incorporated by introducing a performance function in the controller design.
- This paper addresses the leader-following control problem for multiple uncertain high-order nonlinear systems with performance requirements. In contrast, the studies in [23, 24] solve the leader-following control problem with performance requirements for first-order and second-order systems without uncertainties. In this work, the leader-following control problem for high-order nonlinear systems under uncertainty is studied and a systematic controller design procedure is proposed.

The subsequent sections of this paper are organized as follows: Section 2 outlines the problem formulation. Section 3 presents the systematic design of the controllers. Section 4 provides the simulation results. Finally, the paper is concluded in Section 5.

II. Problem Statement and Preliminaries

Consider a group of m nonlinear systems described by the following equations

$$\begin{aligned} \dot{x}_{ij} &= x_{i+1,j} + f_{ij}(\bar{x}_{ij}) && (1)\\ &\quad i = 1, \ldots, n-1 \\ \dot{x}_{nj} &= u_j + f_{nj}(\bar{x}_{nj}) && (2)\\ y_j &= x_{1j} && (3) \end{aligned}$$

where $x_{ij} \in R$ denotes the state, $u_j \in R$ is the control input, y_j is the output, f_{ij} is a smooth function of $\bar{x}_{ij}$ and is unknown, and $\bar{x}_{ij} = [x_{1j}, \ldots, x_{ij}]^\top$. The initial condition $\bar{x}_{nj}(0) \in \mathcal{X}$ where $\mathcal{X} = \{\zeta | \|\zeta\| < r, \forall \zeta \in R^n\}$ is a compact set in R^n for some positive constant r.

It is assumed that there exists a virtual system whose dynamics are represented by the following equations.

$$\begin{aligned} \dot{x}_{i,m+1} &= x_{i+1,m+1} + f_{i,m+1}(\bar{x}_{i,m+1}), && (4)\\ &\quad i = 1, \ldots, n-1 \\ \dot{x}_{n,m+1} &= u_{m+1} + f_{n,m+1}(\bar{x}_{n,m+1}) && (5)\\ y_{m+1} &= x_{1,m+1} && (6) \end{aligned}$$

where u_{m+1} is a known time-varying function. The virtual system is labeled as $(m+1)$-th system.

The systems indexed by j for $1 \leq j \leq m$ are referred to as the follower systems, while the system indexed by $(m+1)$ is referred to as the leader system. In the context of m follower systems and one leader system, there is communication between systems. If we consider each system as a vertex and the indexes of the vertexes are the same as the labels of the systems, the communication between systems can be defined by a graph denoted as $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. Here, $\mathcal{V} = \{v_j\}_{j=1}^{m+1}$ represents the vertex set, and $\mathcal{E} = \{E_{kj}\}_{1 \leq k \neq j \leq m+1}$ denotes the edge set. It is assumed that the communication between systems is bidirectional. An edge E_{kj} means that the information of system j is available to system k and vice versa. For convenience, we assign a *tail* and a *head* for each edge. Without loss of generality, vertex k is considered as the head and vertex j is considered as the tail for edge E_{kj}. A *path* between vertex k and vertex j is a set of edges which connect vertex k and vertex j. A path forms a *simple cycle* if the path is closed. A graph is *connected* if for every pair of vertexes there is a path to connect them. A *tree* is defined as a connected graph without cycles. A tree is a *spanning tree* if it contains all the vertexes in the graph.

For a vertex j, the set of its neighbors, represented as $\mathcal{N}_j$, comprises all vertexes directly connected to vertex j through edges. For $(m+1)$ vertexes and K edges, the incidence matrix $D(\mathcal{G}) = [d_{kj}] \in R^{(m+1)\times K}$ can be defined to characterize the graph structure. For K edges, we label them by $1, 2, \ldots, K$. If the p-th edge is E_{kj}, $d_{kp} = 1$ and $d_{jp} = -1$. It is obvious that $D(\mathcal{G})$ is not unique and depends on the labels of the edges. The Laplacian matrix of the graph $\mathcal{G}$ is denoted as $L = D(\mathcal{G})D(\mathcal{G})^\top$. The edge Laplacian matrix is defined as $L_e = D(\mathcal{G})^\top D(\mathcal{G})$.

For system j, it is assumed that system k is one of its neighbors. We define the tracking error between system j and system k as

$$e_{jk} = y_j - y_k \tag{7}$$

for $1 \leq j \leq m$ and $k \in \mathcal{N}_j$. The transient and steady-state performance requirements on the tracking error e_{jk} can be defined by a PPF [25]. With the aid of a given PPF, the problem considered in this paper is as follows.

Control Problem: For the follower systems in (1)-(3) and the leader system in (4)-(6), the problem considered in this article is to design a distributed control law for system j using its neighbors' information such that

$$|e_{jk}(t)| < \rho(t) \tag{8}$$

$$\lim_{t\to T} |y_j(t) - y_{m+1}(t)| \leq \epsilon \tag{9}$$

$$\lim_{t\to\infty} (y_j(t) - y_{m+1}(t)) = 0 \tag{10}$$

for $1 \leq j \leq m$ and $i \in \mathcal{N}_j$, where ρ is a PPF, T and ϵ are prescribed time constant and the threshold of the tracking errors.

In order to solve our problem, the following assumptions are made on the communication graph.

Assumption 1: The communication graph $\mathcal{G}$ has a spanning tree with the node $m+1$ as the root node.

Assumption 2: The state of the leader system is bounded.

Assumption 1 indicates that the information of the leader can be shared among all follower systems. This assumption is crucial for controller design and is a requirement in all literature on leader-following control problems. Assumption 2 is reasonable, as it is practical for all state values of a system to be bounded.

III. Distributed Controller Design

A. Prescribed Fixed-time Performance Function

In order to meet the transient performance in (8) and the steady-state performance in (9), the prescribed performance function is chosen as

$$\rho(t) = \begin{cases} (\rho_0 - \rho_\infty)\exp\left(-\frac{c_3 T t}{T-t}\right) + \rho_\infty, & 0 \leq t < T \\ \rho_\infty, & t \geq T \end{cases}$$

where $|e_{ji}(0)| < \rho_0$ for $1 \leq j \leq m$ and $1 \leq i \leq m+1$, $\rho_\infty < \frac{\epsilon}{m}$, and $c_3 > 1$. The constant T is the prespecified maximum allowable convergence time for $\rho(t)$ converging from the given maximum initial error ρ_0 to the maximum allowable steady-state error ρ_∞, and c_3 denotes the prespecified minimum convergence rate. The PPF has the following properties [25, 26]: 1) $\rho(t)$ is bounded, i.e., $0 < \rho_\infty \leq \rho(t) \leq \rho_0$ and $\dot{\rho}(t) \leq 0$; and 2) $\lim_{t\to T} \rho(t) = \rho_\infty$ and $\rho(t) = \rho_\infty$ for any $t \geq T$.

With the aid of the PPF, the following tracking error is defined

$$v_{jk} = F(e_{jk}) = \ln\left(\frac{e_{jk} + \rho}{\rho - e_{jk}}\right) \tag{11}$$

where $F(e_{jk})$ is a natural logarithm function of e_{jk}. Then,

$$\dot{v}_{jk} = A_{jk}(\dot{e}_{jk} + \Lambda e_{jk}) \tag{12}$$

where

$$A_{jk} = \frac{1}{e_{jk} + \rho} + \frac{1}{\rho - e_{jk}}, \quad \Lambda = -\frac{\dot{\rho}}{\rho}.$$

For the PPF ρ, the following results have been proved in [26].

Lemma 1: For the PPF ρ,

1) $\rho(t)$ is monotonically decreasing and $\Lambda > 0$.
2) $A_{jk} > \frac{2}{\rho} \geq \frac{2}{\rho_0} > 0$.

Lemma 2: For the transformation (11), if v_{jk} is bounded for $1 \leq j \leq m$ and $k \in \mathcal{N}_j$, eqns. (8)-(9) are satisfied.

B. Controller Design

Under Assumption 1, the graph $\mathcal{G}$ has a spanning tree. The edge set $\mathcal{E}$ can be written as $\mathcal{E}_t \cup \mathcal{E}_c$ where $\mathcal{E}_t$ includes the edges of the spanning tree and $\mathcal{E}_c$ includes the edges which are not in the spanning tree. Based on the decomposition of the edges, the graph $\mathcal{G}$ can be decomposed as $\mathcal{G} = \mathcal{G}_t \cup \mathcal{G}_c$. Since there are $m+1$ vertexes, the number of edges in $\mathcal{E}_t$ is m. We label the edges in $\mathcal{E}_t$ first and then label the edges in $\mathcal{E}_c$. The corresponding incidence matrix $D(\mathcal{G})$ can be written as

$$D = [D_t, D_c] \tag{13}$$

where $D_t \in R^{(m+1)\times m}$ and $D_c \in R^{(m+1)\times(K-m)}$ represent, respectively, the incidence matrices corresponding to the spanning tree edges and other edges. It is shown that the columns of D_c are linearly depend on the columns of D_t [27, 28] and

$$D_t Z = D_c$$

where

$$Z = (D_t^\top D_t)^{-1} D_t^\top D_c.$$

The incidence matrix D can also be written as the following block matrix

$$D = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} D_{1t} & D_{1c} \\ D_{2t} & D_{2c} \end{bmatrix} \tag{14}$$

where $D_1 \in R^{m\times K}$, $D_2 \in R^{1\times K}$, $D_{1t} \in R^{m\times m}$, $D_{1c} \in R^{m\times(K-m)}$, $D_{2t} \in R^{1\times m}$, and $D_{2c} \in R^{1\times(K-m)}$. It can be verified that

$$D_{1t}Z = D_{1c}, \quad D_{2t}Z = D_{2c}, \quad Z = D_{1t}^{-1} D_{1c}.$$

Let

$$\eta = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_K \end{bmatrix} = D^\top \begin{bmatrix} y \\ y_{m+1} \end{bmatrix} = \begin{bmatrix} \vdots \\ e_{jk} \\ \vdots \end{bmatrix} \tag{15}$$

where $y = [y_1, \ldots, y_m]^\top$. Noting the definition of the incidence matrix, we have

$$\begin{aligned} \eta &= D^\top \begin{bmatrix} y \\ y_{m+1} \end{bmatrix} - D^\top \mathbf{1} y_{m+1} \\ &= [D_{1t}, D_{1c}]^\top \tilde{y} = \begin{bmatrix} D_{1t}^\top \tilde{y} \\ Z^\top D_{1t}^\top \tilde{y} \end{bmatrix} \\ &= \begin{bmatrix} I_{m\times m} \\ Z^\top \end{bmatrix} D_{1t}^\top \tilde{y} \end{aligned} \tag{16}$$

where $\mathbf{1}$ is a vector whose elements are one and $\tilde{y} = y - \mathbf{1} y_{m+1}$.

Let

$$z = \begin{bmatrix} z_1 \\ \vdots \\ z_K \end{bmatrix} = F(\eta) = \begin{bmatrix} F(\eta_1) \\ \vdots \\ F(\eta_L) \end{bmatrix} = \begin{bmatrix} \vdots \\ v_{jk} \\ \vdots \end{bmatrix} \tag{17}$$

with the aid of (12), we have

$$\dot{z} \;=\; A(\dot{\eta} + \Lambda\eta) = A D_1^\top (\dot{\tilde{x}}_{1*} + \Lambda \tilde{x}_{1*}) \tag{18}$$

where $A = \text{diag}(A_{ji})$ is a diagonal positive definite matrix and $\tilde{x}_{1*} = [\tilde{x}_{11}, \ldots, \tilde{x}_{1m}]^\top = [x_{11} - x_{1,m+1}, \ldots, x_{1m} - x_{1,m+1}]^\top$.

Lemma 3: Under Assumption 1, if z is bounded, and

$$\lim_{t\to\infty} z^\top A D_1^\top D_1 A z = 0 \tag{19}$$

then $\lim_{t\to\infty} \eta = 0$ and eqns. (8)-(9) are satisfied.

Proof: Let $\eta_I = [\eta_1, \ldots, \eta_m]^\top$ and $\eta_{II} = [\eta_{m+1}, \ldots, \eta_K]^\top$. By eqn. (16), we have

$$\eta_{II} = Z^\top \eta_I. \tag{20}$$

Let $z_I = [z_1, \ldots, z_m]^\top$ and $z_{II} = [z_{m+1}, \ldots, z_K]^\top$. By the mean value theorem and the property of the function $F(\cdot)$ in Lemma 1,

$$\begin{aligned} z_I &= F(\mathbf{0}) + G\eta_I = G\eta_I \\ z_{II} &= F(\mathbf{0}) + H\eta_{II} = HZ^\top \eta_I \end{aligned}$$

where $G = \text{diag}([G_1, \ldots, G_m])$ and $H = \text{diag}([H_1, \ldots, H_m])$ are diagonal matrices with $G_i \geq \frac{2}{\rho_0} > 0$ and $H_i \geq \frac{2}{\rho_0} > 0$. Eqn. (19) means that $\lim_{t\to\infty} D_1 A z = 0$. Noting that

$$\begin{aligned} D_1 A z &= [D_{1t}, D_{1c}] A \begin{bmatrix} G \\ HZ^\top \end{bmatrix} \eta_I \\ &= D_{1t}[I, Z] A \;\; \text{diag}(G, H) \begin{bmatrix} I \\ Z^\top \end{bmatrix} \eta_I \\ &= D_{1t}(A_I G + Z A_{II} H Z^\top)\eta_I = D_{1t} \Pi \eta_I \end{aligned}$$

where $A = \text{diag}(A_I, A_{II})$, $A_I \in R^{m\times m}$, and $A_{II} \in R^{(K-m)\times(K-m)}$, $\Pi = (A_I G + Z A_{II} H Z^\top)$ is a positive definite matrix. So, eqn. (19) means that $\lim_{t\to\infty} \eta_I = 0$. By (20), $\lim_{t\to\infty} \eta = 0$.

By Lemma 2, the boundedness of z means that eqns. (8)-(9) are satisfied. □

Lemma 4: Under Assumption 1, if $\lim_{t\to\infty} \eta = 0$, and $|e_{jk}(t)| \leq \rho(t)$ for $1 \leq j \leq m$ and $k \in \mathcal{N}_j$, then eqn. (10) is satisfied.

Proof:

$$\lim_{t\to\infty} L\begin{bmatrix} y \\ y_{m+1}\end{bmatrix} = \lim_{t\to\infty} DD^\top \begin{bmatrix} y \\ y_{m+1}\end{bmatrix} = \lim_{t\to\infty} D\eta = 0.$$

If the graph $\mathcal{G}$ has a spanning tree, the elements of y and y_{m+1} reach consensus based on the property of the Laplacian matrix L. So, $y - \mathbf{1}y_{m+1}$ converges to zero. □

Next, we design distributed controllers with the aid of the universal approximation theorem of continuous function and the backstepping technique.

In the dynamics (1)-(2), there is uncertainty $f_{ij}(\bar{x}_{ij})$. With the aid of the uniform approximation theorem of functions [29, 30], in the compact set $\mathcal{X}$ for selected basis vectors $\psi_{ij}(\bar{x}_{ij})$ there exists an ideal constant weight vector θ_{ij} such that

$$f_{ij}(\bar{x}_{ij}) = \psi_{ij}^\top(\bar{x}_{ij})\theta_{ij} + \epsilon_{ij} \tag{21}$$

where ϵ_{ij} is the approximation error and is bounded by unknown constants δ_{ij} and $\bar{\delta}$ (i.e. $|\epsilon_{ij}| \le \delta_{ij} \le \bar{\delta}$). Then (1)-(2) can be written as

$$\dot{x}_{ij} = x_{i+1,j} + \psi_{ij}^\top(\bar{x}_{ij})\theta_{ij} + \epsilon_{ij}, \quad i = 1, \ldots, n-1 \tag{22}$$

$$\dot{x}_{nj} = u_j + \psi_{nj}^\top(\bar{x}_{nj})\theta_{nj} + \epsilon_{nj}. \tag{23}$$

Substitute (22) into (18) for $i = 1$, we have

$$\dot{z} = AD_1^\top(x_{2*} + \psi_{1*}^\top\theta_{1*} + \Lambda x_{1*} - \Lambda x_{1,m+1}\mathbf{1} - \dot{x}_{1,m+1}\mathbf{1} + \epsilon_{1*}) \tag{24}$$

where $x_{1*} = [x_{11}, \ldots, x_{1m}]^\top$, $x_{2*} = [x_{21}, \ldots, x_{2m}]^\top$, $\psi_{1*} = \mathrm{diag}([\psi_{11}, \ldots, \psi_{1m}])$, and $\epsilon_{1*} = [\epsilon_{11}, \ldots, \epsilon_{1m}]^\top$.

Step 1: Assume that x_{2j} is a virtual control input. The virtual controller is chosen as

$$\alpha_{1j} = -\lambda_1 \sum_{l\in\mathcal{N}_j} A_{jl}z_{jl} - \Lambda x_{1j} - \psi_{1j}^\top\hat{\theta}_{1j} - \frac{s_{1j}\sum_{l\in\mathcal{N}_j} A_{jl}z_{jl}}{\sqrt{\left(\sum_{l\in\mathcal{N}_j} A_{jl}z_{jl}\right)^2 + h(t)}} \tag{25}$$

$$= -\lambda_1(D_1Az)_j - \Lambda x_{1j} - \psi_{1j}^\top\hat{\theta}_{1j} - \frac{s_{1j}(D_1Az)_j}{\sqrt{((D_1Az)_j)^2 + h(t)}} \tag{26}$$

where $\lambda_1 > 0$, $\hat{\theta}_{1j}$ is an estimate of θ_{1j} and will be chosen later, s_{1j} is the magnitude of a robust term and will be chosen later, $(\cdot)_j$ denotes the j-th element of its argument vector, and $h(t)(> 0)$ satisfies

$$\int_0^\infty \sqrt{h(t)}dt < \infty. \tag{27}$$

There are different choices of h. For example, $h(t) = e^{-t}$, $h(t) = \frac{1}{(t+1)^4}$, etc.

In order to find the update laws for $\hat{\theta}_{1j}$ and s_{1j}, we choose a Lyapunov function

$$V_1 = \frac{1}{2}z^\top z + \frac{1}{2}\sum_{j=1}^m \tilde{\theta}_{1j}^\top\Gamma_{1j}^{-1}\tilde{\theta}_{1j} + \frac{1}{2}\sum_{j=1}^m \gamma_{1j}^{-1}\tilde{s}_{1j}^2 \tag{28}$$

where Γ_{1j} is a positive definite constant matrix, γ_{1j} is a positive constant, and

$$\tilde{\theta}_{1j} = \theta_{1j} - \hat{\theta}_{1j} \tag{29}$$

$$\tilde{s}_{1j} = s_{1,m+1} - s_{1j} \tag{30}$$

where

$$s_{1,m+1} = \max_{1\le j\le m}\max_{t\in[0,\infty)} |\dot{x}_{1,m+1} + \Lambda x_{1,m+1} - \epsilon_{1j}|$$

is a positive constant. The derivative of V_1 is

$$\begin{aligned}
\dot{V}_1 &= z^\top AD_1^\top(x_{2*} - \alpha_{1*}) - \lambda_1 z^\top AD_1^\top D_1Az \\
&\quad + z^\top AD_1^\top\psi_{1*}^\top\tilde{\theta}_{1*} - \sum_{j=1}^m \frac{s_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2 + h}} \\
&\quad + z^\top AD_1^\top(\epsilon_{1*} - \Lambda x_{1,m+1}\mathbf{1} - \dot{x}_{1,m+1}\mathbf{1}) \\
&\quad + \sum_{j=1}^m \tilde{\theta}_{1j}^\top\Gamma_{1j}^{-1}\dot{\tilde{\theta}}_{1j} + \sum_{j=1}^m \gamma_{1j}^{-1}\tilde{s}_{1,j}\dot{\tilde{s}}_{1j} \\
&= z^\top AD_1^\top(x_{2*} - \alpha_{1*}) - \lambda_1 z^\top AD_1^\top D_1Az \\
&\quad - \sum_{j=1}^m \frac{s_{1,m+1}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2 + h}} \\
&\quad + \sum_{j=1}^m (D_1^\top Az)_j(\epsilon_{1j} - \Lambda x_{1,m+1} - \dot{x}_{1,m+1}) \\
&\quad + \sum_{j=1}^m \tilde{\theta}_{1j}^\top(\Gamma_{1j}^{-1}\dot{\tilde{\theta}}_{1j} + \psi_{1j}(D_1Az)_j) \\
&\quad + \sum_{j=1}^m \tilde{s}_{1j}\left(\gamma_{1j}^{-1}\dot{\tilde{s}}_{1j} + \frac{((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2 + h}}\right)
\end{aligned}$$

$$
\begin{aligned}
&\leq z^\top A D_1^\top (x_{2*} - \alpha_{1*}) - \lambda_1 z^\top A D_1^\top D_1 A z \\
&\quad - \sum_{j=1}^{m} \frac{s_{1,m+1}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2 + h}} \\
&\quad + \sum_{j=1}^{m} s_{1,m+1}(D_1^\top Az)_j \\
&\quad + \sum_{j=1}^{m} \tilde{\theta}_{1j}^\top \Gamma_{1j}^{-1}(\dot{\tilde{\theta}}_{1j} + \Gamma_{1j}\psi_{1j}(D_1Az)_j) \\
&\quad + \sum_{j=1}^{m} \tilde{s}_{1j}\left(\gamma_{1j}^{-1}\dot{\tilde{s}}_{1j} + \frac{((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2 + h}}\right) \\
&\leq z^\top A D_1^\top (x_{2*} - \alpha_{1*}) - \lambda_1 z^\top A D_1^\top D_1 A z \\
&\quad + m s_{1,m+1}\sqrt{h(t)} + \sum_{j=1}^{m} \tilde{\theta}_{1,j}^\top \Gamma_{1j}^{-1}(\dot{\tilde{\theta}}_{1j} \\
&\quad + \Gamma_{1j}\psi_{1,j}(D_1Az)_j) \\
&\quad + \sum_{j=1}^{m} \tilde{s}_{1j}\gamma_{1j}^{-1}\left(\dot{\tilde{s}}_{1j} + \frac{\gamma_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2 + h}}\right)
\end{aligned}
$$

where $\alpha_{1*} = [\alpha_{11}, \ldots, \alpha_{1m}]^\top$ and we apply the inequality $\zeta - \frac{\zeta^2}{\sqrt{\zeta^2+h}} \leq \sqrt{h}$ for any scalar ζ.

If we choose the update laws

$$
\dot{\hat{\theta}}_{1j} = \Gamma_{1j}\psi_{1,j}(D_1Az)_j =: \tau_{1j}^{[1]} \tag{31}
$$

$$
\dot{s}_{1j} = \frac{\gamma_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2 + h(t)}}, \tag{32}
$$

then

$$
\begin{aligned}
\dot{V}_1 &\leq z^\top A D_1^\top (x_{2*} - \alpha_{1*}) - \lambda_1 z^\top A D_1^\top D_1 A z \\
&\quad + m\sqrt{h}.
\end{aligned} \tag{33}
$$

Step 2: Since x_{2j} is not the control input, it cannot be α_{1j}. Let

$$
\xi_{2j} = x_{2j} - \alpha_{1j}. \tag{34}
$$

Then

$$
\dot{\xi}_{2j} = x_{3j} + \psi_{2j}^\top \theta_{2j} + \epsilon_{2j} - \dot{\alpha}_{1j} \tag{35}
$$

$$
\begin{aligned}
&= x_{3j} + \psi_{2j}^\top \theta_{2j} + \epsilon_{2j} - \frac{\partial \alpha_{1j}}{\partial x_{1j}}(x_{2j} + \psi_{1j}^\top \theta_{1j}) \\
&\quad - \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{1j}}{\partial x_{1l}}(x_{2l} + \psi_{1l}^\top \theta_{1l}) - \frac{\partial \alpha_{1j}}{\partial x_{1j}}\epsilon_{1j} \\
&\quad - \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{1j}}{\partial x_{1l}}\epsilon_{1l} - \frac{\partial \alpha_{1j}}{\partial \hat{\theta}_{1j}}\dot{\hat{\theta}}_{1j} - \frac{\partial \alpha_{1j}}{\partial h}\dot{h} \\
&\quad - \frac{\partial \alpha_{1j}}{\partial s_{1j}}\dot{s}_{1j}
\end{aligned} \tag{36}
$$

It is assumed that x_{3j} is a virtual control input. The following virtual input is proposed as

$$
\begin{aligned}
\alpha_{2j} &= -\lambda_{2j}\xi_{2j} - \psi_{2j}^\top \hat{\theta}_{2j} - (D_1Az)_j \\
&\quad - \frac{s_{2j}\xi_{2j}\beta_{2j}^2}{\sqrt{\xi_{2j}^2\beta_{2j}^2 + h}} + \frac{\partial \alpha_{1j}}{\partial x_{1j}}(x_{2j} + \psi_{1j}^\top \hat{\theta}_{1j}) \\
&\quad + \frac{\partial \alpha_{1j}}{\partial \hat{\theta}_{1j}}\dot{\hat{\theta}}_{1j} + \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{1j}}{\partial x_{1l}}(x_{2l} + \psi_{1l}^\top \hat{\theta}_{1l}) \\
&\quad + \frac{\partial \alpha_{1j}}{\partial h}\dot{h} + \frac{\partial \alpha_{1j}}{\partial s_{1j}}\dot{s}_{1j}
\end{aligned} \tag{37}
$$

where

$$
\begin{aligned}
\beta_{2j} &= 1 + \sqrt{\left(\frac{\partial \alpha_{1j}}{\partial x_{1j}}\right)^2 + h} \\
&\quad + \sqrt{\left(\sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{1j}}{\partial x_{1l}}\right)^2 + h}
\end{aligned}
$$

$\lambda_2 > 0$, $\hat{\theta}_{2j}$ and s_{2j} are estimates of θ_{2j} and $\bar{\delta}_{2*} = \max_{1\leq j\leq m}\delta_{2j}$, respectively. In order to design the update laws for $\hat{\theta}_{1j}$, $\hat{\theta}_{2j}$, and s_{2j}, we choose a Lyapunov function

$$
\begin{aligned}
V_2 &= V_1 + \frac{1}{2}\sum_{j=1}^{m}\xi_{2j}^2 + \frac{1}{2}\sum_{j=1}^{m}\tilde{\theta}_{2j}^\top \Gamma_{2j}^{-1}\tilde{\theta}_{2j} \\
&\quad + \frac{1}{2}\sum_{j=1}^{m}\gamma_{2j}^{-1}\tilde{s}_{2j}^2
\end{aligned} \tag{38}
$$

where $\tilde{\theta}_{2j} = \theta_{2j} - \hat{\theta}_{2j}$, $\tilde{s}_{2j} = \bar{\delta}_{2*} - s_{2j}$, Γ_{2j} is a positive definite constant matrix, and γ_{2j} is a positive constant.

The derivative of V_2 is

$$
\begin{aligned}
\dot{V}_2 &\leq z^\top A D_1^\top (x_{2*} - \alpha_{1*}) - \lambda_1 z^\top A D_1^\top D_1 A z \\
&\quad + m s_{1,m+1}\sqrt{h(t)} + \sum_{j=1}^{m}\tilde{\theta}_{1,j}^\top \Gamma_{1j}^{-1}(\dot{\tilde{\theta}}_{1j} + \tau_{1j}^{[1]}) \\
&\quad + \sum_{j=1}^{m}\tilde{s}_{1j}\gamma_{1j}^{-1}\left(\dot{\tilde{s}}_{1j} + \frac{\gamma_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2 + h}}\right) \\
&\quad + \sum_{j=1}^{m}\xi_{2j}(x_{3j} - \alpha_{2j}) + \sum_{j=1}^{m}[-\lambda_{2j}\xi_{2j}^2 \\
&\quad - \xi_{2j}(D_1AZ)_j - \frac{s_{2j}\xi_{2j}^2\beta_{2j}^2}{\sqrt{\xi_{2j}^2\beta_{2j}^2 + h}}
\end{aligned}
$$

$$
\begin{aligned}
&+\xi_{2j}\psi_{2j}^\top\tilde{\theta}_{2j} + \xi_{2j}\epsilon_{2j} - \xi_{2j}\frac{\partial\alpha_{1j}}{\partial x_{1j}}\psi_{1j}^\top\tilde{\theta}_{1j} \\
&-\xi_{2j}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{1j}}{\partial x_{1l}}\psi_{1l}^\top\tilde{\theta}_{1l} - \xi_{2j}\frac{\partial\alpha_{1j}}{\partial x_{1j}}\epsilon_{1j} \\
&-\xi_{2j}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{1j}}{\partial x_{1l}}\epsilon_{1l}] \\
&+\sum_{j=1}^m\tilde{\theta}_{2j}^\top\Gamma_{2j}^{-1}\dot{\tilde{\theta}}_{2j} + \sum_{j=1}^m\gamma_{2j}^{-1}\tilde{s}_{2j}\dot{\tilde{s}}_{2j} \\
\leq\ & -\lambda_1 z^\top AD_1^\top D_1Az + ms_{1,m+1}\sqrt{h} \\
&+\sum_{j=1}^m\tilde{\theta}_{1j}^\top\Gamma_{1j}^{-1}(\dot{\tilde{\theta}}_{1j} + \tau_{1j}^{[1]} \\
&-\Gamma_{1j}\psi_{1j}\xi_{2j}\frac{\partial\alpha_{1j}}{\partial x_{1j}} - \Gamma_{1j}\psi_{1j}\frac{\partial\alpha_{1j}}{\partial x_{1j}}\sum_{l\in\mathcal{N}_j}\xi_{2l}) \\
&+\sum_{j=1}^m\tilde{s}_{1j}\gamma_{1j}^{-1}\left(\dot{\tilde{s}}_{1j} + \frac{\gamma_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2+h}}\right) \\
&+\sum_{j=1}^m\xi_{2j}(x_{3j}-\alpha_{2j}) + \sum_{j=1}^m[-\lambda_{2j}\xi_{2j}^2 \\
&-\frac{s_{2j}\xi_{2j}^2\beta_{2j}^2}{\sqrt{\xi_{2j}^2\beta_{2j}^2+h}} \\
&+|\xi_{2j}|\left(1+\sqrt{\left(\frac{\partial\alpha_{1j}}{\partial x_{1j}}\right)^2+h}\right. \\
&\left.+\sqrt{\left(\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{1l}}{\partial x_{1l}}\right)^2+h}\right)\bar{\delta}_{2*} \\
&+\sum_{j=1}^m\tilde{\theta}_{2j}^\top(\Gamma_{2j}^{-1}\dot{\tilde{\theta}}_{2j} + \psi_{2j}\xi_{2j}) \\
&+\sum_{j=1}^m\gamma_{2j}^{-1}\tilde{s}_{2j}\dot{\tilde{s}}_{2j} \\
\leq\ & -\lambda_1 z^\top AD_1^\top D_1Az + (s_{1,m+1}+\bar{\delta}_{2*})m\sqrt{h} \\
&+\sum_{j=1}^m\tilde{\theta}_{1j}^\top\Gamma_{1j}^{-1}(\dot{\tilde{\theta}}_{1j} + \tau_{1j}^{[1]} \\
&-\Gamma_{1j}\psi_{1j}\xi_{2j}\frac{\partial\alpha_{1j}}{\partial x_{1j}} - \Gamma_{1j}\psi_{1j}\frac{\partial\alpha_{1j}}{\partial x_{1j}}\sum_{l\in\mathcal{N}_j}\xi_{2l}) \\
&+\sum_{j=1}^m\tilde{s}_{1j}\gamma_{1j}^{-1}\left(\dot{\tilde{s}}_{1j} + \frac{\gamma_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2+h}}\right) \\
&+\sum_{j=1}^m\xi_{2j}(x_{3j}-\alpha_{2j}) - \sum_{j=1}^m\lambda_{2j}\xi_{2j}^2 \\
&+\sum_{j=1}^m\tilde{\theta}_{2j}^\top\Gamma_{2j}^{-1}(\dot{\tilde{\theta}}_{2j} + \Gamma_{2j}\psi_{2j}\xi_{2j}) \\
&+\sum_{j=1}^m\gamma_{2j}^{-1}\tilde{s}_{2j}\left(\dot{\tilde{s}}_{2j} + \frac{\gamma_{2j}\xi_{2j}^2\beta_{2j}^2}{\sqrt{\xi_{2j}^2\beta_{2j}^2+h}}\right)
\end{aligned}
$$

We choose the update law (32) for s_{1j} and the update laws for $\hat{\theta}_{1j}$, $\hat{\theta}_{2j}$, and s_{2j} as

$$
\begin{aligned}
\dot{\hat{\theta}}_{1j} &= \tau_{1j} - \Gamma_{1j}\psi_{1j}\xi_{2j}\frac{\partial\alpha_{1j}}{\partial x_{1j}} \\
&\quad -\Gamma_{1j}\psi_{1j}\frac{\partial\alpha_{1j}}{\partial x_{1j}}\sum_{l\in\mathcal{N}_j}\xi_{2l} =: \tau_{1j}^{[2]} \qquad (39)
\end{aligned}
$$

$$
\dot{\hat{\theta}}_{2j} = \Gamma_{2j}\psi_{2j}\xi_{2j} =: \tau_{2j}^{[2]} \qquad (40)
$$

$$
\dot{s}_{2j} = \frac{\gamma_{2j}\xi_{2j}^2\beta_{2j}^2}{\sqrt{\xi_{2j}^2\beta_{2j}^2+h}} \qquad (41)
$$

Then

$$
\begin{aligned}
\dot{V}_2 \leq\ & -\lambda_1 z^\top AD_1^\top D_1Az - \sum_{j=1}^m\lambda_{2j}\xi_{2j}^2 \\
&+\sum_{j=1}^m\xi_{2j}(x_{3j}-\alpha_{2j}) + (s_{1,m+1}+\bar{\delta}_{2*})m\sqrt{h}.
\end{aligned}
$$

Step i: Since x_{ij} is not a real control input, it cannot be $\alpha_{i-1,j}$. Let

$$
\xi_{ij} = x_{ij} - \alpha_{i-1,j} \qquad (42)
$$

Then

$$
\begin{aligned}
\dot{\xi}_{ij} &= x_{i+1,j} + \psi_{ij}^\top\theta_{ij} + \epsilon_{ij} - \dot{\alpha}_{i-1,j} \qquad (43) \\
&= x_{i+1,j} + \psi_{ij}^\top\theta_{ij} + \epsilon_{ij} \\
&\quad -\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial x_{kj}}(x_{k+1,j} + \psi_{kj}^\top\theta_{kj} + \epsilon_{kj}) \\
&\quad -\sum_{k=1}^{i-1}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{i-1,j}}{\partial x_{kl}}(x_{k+1,l} + \psi_{kl}^\top\theta_{kl} + \epsilon_{kl}) \\
&\quad -\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}_{kj}}\dot{\hat{\theta}}_{kj} - \sum_{k=1}^{i-1}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}_{kl}}\dot{\hat{\theta}}_{kl} \\
&\quad -\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial s_{kj}}\dot{s}_{kj}. \qquad (44)
\end{aligned}
$$

It is assumed that $x_{i+1,j}$ is a virtual control input. The following virtual control input is proposed.

$$\begin{aligned}\alpha_{ij} &= -\lambda_{ij}\xi_{ij} - \psi_{ij}^{\top}\hat{\theta}_{ij} + \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1,j}}{\partial x_{kj}}(x_{k+1,j} \\ &\quad +\psi_{kj}^{\top}\hat{\theta}_{kj}) + \sum_{k=1}^{i-1}\sum_{l\in\mathcal{N}_j}\frac{\partial \alpha_{i-1,j}}{\partial x_{kl}}(x_{k+1,l} \\ &\quad +\psi_{kl}^{\top}\hat{\theta}_{kl}) + \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1,j}}{\partial \hat{\theta}_{kj}}\dot{\hat{\theta}}_{kj} \\ &\quad + \sum_{k=1}^{i-1}\sum_{l\in\mathcal{N}_j}\frac{\partial \alpha_{i-1,j}}{\partial \hat{\theta}_{kl}}\dot{\hat{\theta}}_{kl} - \frac{s_{ij}\xi_{ij}\beta_{ij}^2}{\sqrt{\xi_{ij}^2\beta_{ij}^2+h}} \\ &\quad + \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1,j}}{\partial s_{kj}}\dot{s}_{kj} - \xi_{i-1,j} \end{aligned} \tag{45}$$

where

$$\begin{aligned}\beta_{ij} &= 1 + \sum_{k=1}^{i-1}\sqrt{\left(\frac{\partial \alpha_{i-1,j}}{\partial x_{kj}}\right)^2 + h} \\ &\quad + \sum_{k=1}^{i-1}\sum_{l\in\mathcal{N}_j}\sqrt{\left(\frac{\partial \alpha_{i-1,j}}{\partial x_{kl}}\right)^2 + h}\end{aligned}$$

$\lambda_{ij} > 0$, $\hat{\theta}_{ij}$ and s_{ij} are estimates of θ_{ij} and $\bar{\delta}_{i*} = \max_{1\le j\le m}\delta_{ij}$, respectively. In order to design the update laws for $\hat{\theta}_{ij}$ and s_{ij}, we choose a Lyapunov function

$$\begin{aligned}V_i &= V_{i-1} + \frac{1}{2}\sum_{j=1}^{m}\xi_{ij}^2 + \frac{1}{2}\sum_{j=1}^{m}\tilde{\theta}_{ij}^{\top}\Gamma_{ij}^{-1}\tilde{\theta}_{ij} \\ &\quad + \frac{1}{2}\sum_{j=1}^{m}\gamma_{ij}^{-1}\tilde{s}_{ij}^2\end{aligned} \tag{46}$$

where $\tilde{\theta}_{ij} = \theta_{ij} - \hat{\theta}_{ij}$, $\tilde{s}_{ij} = \bar{\delta}_{i*} - s_{ij}$, Γ_{ij} is a positive definite constant matrix, and γ_{ij} is a positive constant.

The derivative of V_i is

$$\begin{aligned}\dot{V}_i &\le -\lambda_1 z^{\top}AD_1^{\top}D_1Az + \left(s_{1,m+1} + \sum_{k=2}^{i-1}\bar{\delta}_{k*}\right)m\sqrt{h} \\ &\quad + \sum_{k=1}^{i-1}\sum_{j=1}^{m}\tilde{\theta}_{kj}^{\top}\Gamma_{kj}^{-1}(\dot{\tilde{\theta}}_{kj} + \tau_{kj}^{[i-1]}) \\ &\quad + \sum_{j=1}^{m}\tilde{s}_{1j}\gamma_{1j}^{-1}\left(\dot{\tilde{s}}_{1j} + \frac{\gamma_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2+h}}\right) \\ &\quad + \sum_{j=1}^{m}\xi_{ij}(x_{i+1,j} - \alpha_{ij}) - \sum_{k=2}^{i}\sum_{j=1}^{m}\lambda_{kj}\xi_{kj}^2 \\ &\quad + \sum_{k=2}^{i-1}\sum_{j=1}^{m}\gamma_{kj}^{-1}\tilde{s}_{kj}\left(\dot{\tilde{s}}_{kj} + \frac{\gamma_{kj}\xi_{kj}^2\beta_{kj}^2}{\sqrt{\xi_{kj}^2\beta_{kj}^2+h}}\right) \\ &\quad + \sum_{j=1}^{m}\xi_{ij}\psi_{ij}^{\top}\tilde{\theta}_{ij} + \sum_{j=1}^{m}\xi_{ij}\epsilon_{ij} \\ &\quad - \sum_{j=1}^{m}\xi_{ij}\sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1,j}}{\partial x_{kj}}(\psi_{kj}^{\top}\tilde{\theta}_{kj} + \epsilon_{kj}) \\ &\quad - \sum_{j=1}^{m}\xi_{ij}\sum_{k=1}^{i-1}\sum_{l\in\mathcal{N}_j}\frac{\partial \alpha_{i-1,j}}{\partial x_{kl}}(\psi_{kl}^{\top}\tilde{\theta}_{kl} + \epsilon_{kl}) \\ &\quad - \sum_{j=1}^{m}\frac{s_{ij}\xi_{ij}^2\beta_{ij}^2}{\sqrt{\xi_{ij}^2\beta_{ij}^2+h}} \\ &\quad + \sum_{j=1}^{m}\tilde{\theta}_{ij}^{\top}\Gamma_{ij}^{-1}\dot{\tilde{\theta}}_{ij} + \sum_{j=1}^{m}\gamma_{i,j}^{-1}\tilde{s}_{ij}\dot{\tilde{s}}_{ij}\end{aligned}$$

Noting that $|\epsilon_{ij}| \le \bar{\delta}_{i*}$, we have

$$\begin{aligned}\dot{V}_i &\le -\lambda_1 z^{\top}AD_1^{\top}D_1Az \\ &\quad + \left(s_{1,m+1} + \sum_{k=2}^{i-1}\bar{\delta}_{k*}\right)m\sqrt{h} \\ &\quad + \sum_{k=1}^{i-1}\sum_{j=1}^{m}\tilde{\theta}_{kj}^{\top}\Gamma_{kj}^{-1}(\dot{\tilde{\theta}}_{kj} + \tau_{kj}^{[i-1]} \\ &\quad -\Gamma_{kj}\psi_{kj}\xi_{ij}\frac{\partial \alpha_{i-1,j}}{\partial x_{kj}} - \Gamma_{kj}\psi_{kj}\frac{\partial \alpha_{i-1,j}}{\partial x_{kj}}\sum_{l\in\mathcal{N}_j}\xi_{il}) \\ &\quad + \sum_{j=1}^{m}\tilde{s}_{1j}\gamma_{1j}^{-1}\left(\dot{\tilde{s}}_{1j} + \frac{\gamma_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2+h}}\right) \\ &\quad + \sum_{j=1}^{m}\xi_{ij}(x_{i+1,j} - \alpha_{ij}) - \sum_{k=2}^{i}\sum_{j=1}^{m}\lambda_{kj}\xi_{kj}^2 \\ &\quad + \sum_{k=2}^{i-1}\sum_{j=1}^{m}\gamma_{kj}^{-1}\tilde{s}_{kj}\left(\dot{\tilde{s}}_{kj} + \frac{\gamma_{kj}\xi_{kj}^2\beta_{kj}^2}{\sqrt{\xi_{kj}^2\beta_{kj}^2+h}}\right) \\ &\quad + \sum_{j=1}^{m}|\xi_{ij}|\beta_{ij}\bar{\delta}_{i*} - \sum_{j=1}^{m}\frac{\bar{\delta}_{i*}\xi_{ij}^2\beta_{ij}^2}{\sqrt{\xi_{ij}^2\beta_{ij}^2+h}} \\ &\quad + \sum_{j=1}^{m}\frac{\tilde{s}_{ij}\xi_{ij}^2\beta_{ij}^2}{\sqrt{\xi_{ij}^2\beta_{ij}^2+h}} + \sum_{j=1}^{m}\tilde{\theta}_{ij}^{\top}\Gamma_{ij}^{-1}(\dot{\tilde{\theta}}_{ij}\end{aligned}$$

$$+\Gamma_{ij}\psi_{ij}\xi_{ij}) + \sum_{j=1}^{m} \gamma_{i,j}^{-1}\tilde{s}_{ij}\dot{\tilde{s}}_{ij}$$

Next, we apply

$$|\xi_{ij}|\beta_{ij} \leq \frac{\xi_{ij}^2\beta_{ij}^2}{\sqrt{\xi_{ij}^2\beta_{ij}^2 + h}} + \sqrt{h}$$

then

$$\begin{aligned}
\dot{V}_i \leq & -\lambda_1 z^\top A D_1^\top D_1 A z \\
& + \left(s_{1,m+1} + \sum_{k=2}^{i}\bar{\delta}_{k*}\right) m\sqrt{h} \\
& + \sum_{k=1}^{i-1}\sum_{j=1}^{m}\tilde{\theta}_{kj}^\top \Gamma_{kj}^{-1}(\dot{\tilde{\theta}}_{kj} + \tau_{kj}^{[i-1]} \\
& -\Gamma_{kj}\psi_{kj}\xi_{ij}\frac{\partial\alpha_{i-1,j}}{\partial x_{kj}} - \Gamma_{kj}\psi_{kj}\frac{\partial\alpha_{i-1,j}}{\partial x_{kj}}\sum_{l\in\mathcal{N}_j}\xi_{il}) \\
& + \sum_{j=1}^{m}\tilde{s}_{1j}\gamma_{1j}^{-1}\left(\dot{\tilde{s}}_{1j} + \frac{\gamma_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2 + h}}\right) \\
& + \sum_{j=1}^{m}\xi_{ij}(x_{i+1,j} - \alpha_{ij}) - \sum_{k=2}^{i}\sum_{j=1}^{m}\lambda_{kj}\xi_{kj}^2 \\
& + \sum_{k=2}^{i-1}\sum_{j=1}^{m}\gamma_{kj}^{-1}\tilde{s}_{kj}\left(\dot{\tilde{s}}_{kj} + \frac{\gamma_{kj}\xi_{kj}^2\beta_{kj}^2}{\sqrt{\xi_{kj}^2\beta_{kj}^2 + h}}\right) \\
& + \sum_{j=1}^{m}\tilde{\theta}_{ij}^\top\Gamma_{ij}^{-1}(\dot{\tilde{\theta}}_{ij} + \Gamma_{ij}\psi_{ij}\xi_{ij}) \\
& + \sum_{j=1}^{m}\gamma_{ij}^{-1}\tilde{s}_{ij}(\dot{\tilde{s}}_{ij} + \frac{\gamma_{ij}\xi_{ij}^2\beta_{ij}^2}{\sqrt{\xi_{ij}^2\beta_{ij}^2 + h}})
\end{aligned}$$

We choose the update laws for s_{kj} $(1 \leq k \leq i-1)$ the same as before and the update laws for $\hat{\theta}_{kj}$ $(1 \leq k \leq i)$ and s_{ij} as follows.

$$\dot{\hat{\theta}}_{kj} = \tau_{kj}^{[i-1]} - \Gamma_{kj}\psi_{kj}\xi_{ij}\frac{\partial\alpha_{i-1,j}}{\partial x_{kj}} \quad (47)$$

$$-\Gamma_{kj}\psi_{kj}\frac{\partial\alpha_{i-1,j}}{\partial x_{kj}}\sum_{l\in\mathcal{N}_j}\xi_{il} =: \tau_{kj}^{[i]} \quad (48)$$

$$1 \leq k \leq i-1,$$

$$\dot{\hat{\theta}}_{ij} = \Gamma_{ij}\psi_{ij}\xi_{ij} =: \tau_{ij}^{[i]} \quad (49)$$

$$\dot{s}_{ij} = \frac{\gamma_{ij}\xi_{ij}^2\beta_{ij}^2}{\sqrt{\xi_{ij}^2\beta_{ij}^2 + h}} \quad (50)$$

Then

$$\begin{aligned}
\dot{V}_i \leq & -\lambda_1 z^\top A D_1^\top D_1 A z - \sum_{k=2}^{i}\sum_{j=1}^{m}\lambda_{kj}\xi_{kj}^2 \\
& + \sum_{j=1}^{m}\xi_{ij}^\top(x_{i+1,j} - \alpha_{ij}) \\
& + \left(s_{1,m+1} + \sum_{k=2}^{i}\bar{\delta}_{k*}\right) m\sqrt{h}.
\end{aligned}$$

Step n: Since x_{nj} is not a real control input, it cannot be $\alpha_{n-1,j}$. Let

$$\xi_{nj} = x_{nj} - \alpha_{n-1,j} \quad (51)$$

Then

$$\begin{aligned}
\dot{\xi}_{nj} = & u_j + \psi_{nj}^\top\theta_{nj} + \epsilon_{nj} - \dot{\alpha}_{n-1,*} \\
= & u_j + \psi_{nj}^\top\theta_{nj} + \epsilon_{nj} \\
& - \sum_{k=1}^{n-1}\frac{\partial\alpha_{i-1,j}}{\partial x_{kj}}(x_{k+1,j} + \psi_{kj}^\top\theta_{kj} + \epsilon_{kj}) \\
& - \sum_{k=1}^{n-1}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{i-1,j}}{\partial x_{kl}}(x_{k+1,l} + \psi_{kl}^\top\theta_{kl} + \epsilon_{kl}) \\
& - \sum_{k=1}^{n-1}\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}_{kj}}\dot{\hat{\theta}}_{kj} - \sum_{k=1}^{n-1}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}_{kl}}\dot{\hat{\theta}}_{kl}
\end{aligned}$$

The control input u_j is proposed as

$$\begin{aligned}
\alpha_{nj} = & -\lambda_{nj}\xi_{nj} - \psi_{nj}^\top\hat{\theta}_{nj} \\
& + \sum_{k=1}^{n-1}\frac{\partial\alpha_{i-1,j}}{\partial x_{kj}}(x_{k+1,j} + \psi_{kj}^\top\hat{\theta}_{kj}) \\
& + \sum_{k=1}^{n-1}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{i-1,j}}{\partial x_{kl}}(x_{k+1,l} + \psi_{kl}^\top\hat{\theta}_{kl}) \\
& + \sum_{k=1}^{n-1}\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}_{kj}}\dot{\hat{\theta}}_{kj} + \sum_{k=1}^{n-1}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}_{kl}}\dot{\hat{\theta}}_{kl} \\
& - \frac{s_{nj}\xi_{nj}\beta_{nj}^2}{\sqrt{\xi_{nj}^2\beta_{nj}^2 + h}} - \xi_{n-1,j} \quad (52)
\end{aligned}$$

where

$$\begin{aligned}
\beta_{nj} = & 1 + \sum_{k=1}^{n-1}\sqrt{\left(\frac{\partial\alpha_{n-1,j}}{\partial x_{kj}}\right)^2 + h} \\
& + \sum_{k=1}^{n-1}\sum_{l\in\mathcal{N}_j}\sqrt{\left(\frac{\partial\alpha_{n-1,j}}{\partial x_{kl}}\right)^2 + h}
\end{aligned}$$

$\lambda_{nj}>0$, $\hat\theta_{nj}$ and s_{nj} are estimates of θ_{nj} and $\bar\delta_{n*}=\max_{1\le j\le m}\delta_{nj}$, respectively. In order to design the update laws for $\hat\theta_{nj}$ and s_{nj}, we choose a Lyapunov function

$$V_n = V_{n-1}+\frac{1}{2}\sum_{j=1}^{m}\xi_{nj}^2+\frac{1}{2}\sum_{j=1}^{m}\tilde\theta_{nj}^\top\Gamma_{nj}^{-1}\tilde\theta_{nj} +\frac{1}{2}\sum_{j=1}^{m}\gamma_{nj}^{-1}\tilde s_{nj}^2 \quad (53)$$

where $\tilde\theta_{nj}=\theta_{nj}-\hat\theta_{nj}$, $\tilde s_{nj}=\bar\delta_{n*}-s_{nj}$, Γ_{nj} is a positive definite constant matrix, and γ_{nj} is a positive constant.

The derivative of V_n is

$$\begin{aligned}\dot V_n \le\ & -\lambda_1 z^\top AD_1^\top D_1Az +\left(s_{1,m+1}+\sum_{k=2}^{n}\bar\delta_{k*}\right)m\sqrt{h} \\ &+\sum_{k=1}^{n-1}\sum_{j=1}^{m}\tilde\theta_{kj}^\top\Gamma_{kj}^{-1}(\dot{\tilde\theta}_{kj}+\tau_{kj}^{[n-1]} \\ &-\Gamma_{kj}\psi_{kj}\xi_{ij}\frac{\partial\alpha_{n-1,j}}{\partial x_{kj}}-\Gamma_{kj}\psi_{kj}\frac{\partial\alpha_{n-1,j}}{\partial x_{kj}}\sum_{l\in\mathcal{N}_j}\xi_{il}) \\ &+\sum_{j=1}^{m}\tilde s_{1j}\gamma_{1j}^{-1}\left(\dot{\tilde s}_{1j}+\frac{\gamma_{1j}((D_1Az)_j)^2}{\sqrt{((D_1Az)_j)^2+h}}\right) \\ &-\sum_{k=2}^{n}\sum_{j=1}^{m}\lambda_{kj}\xi_{kj}^2 \\ &+\sum_{k=2}^{n-1}\sum_{j=1}^{m}\gamma_{kj}^{-1}\tilde s_{kj}\left(\dot{\tilde s}_{kj}+\frac{\gamma_{kj}\xi_{kj}^2\beta_{kj}^2}{\sqrt{\xi_{kj}^2\beta_{kj}^2+h}}\right) \\ &+\sum_{j=1}^{m}\tilde\theta_{nj}^\top\Gamma_{nj}^{-1}(\dot{\tilde\theta}_{nj}+\Gamma_{nj}\psi_{nj}\xi_{nj}) \\ &+\sum_{j=1}^{m}\gamma_{nj}^{-1}\tilde s_{nj}(\dot{\tilde s}_{nj}+\frac{\gamma_{nj}\xi_{nj}^2\beta_{nj}^2}{\sqrt{\xi_{nj}^2\beta_{nj}^2+h}})\end{aligned}$$

We choose the update laws for s_{kj} ($1\le k\le n-1$) the same as before and the update laws for s_{nj} and $\hat\theta_{kj}$ ($1\le k\le n$) as follows.

$$\dot s_{nj} = \frac{\gamma_{nj}\xi_{nj}^2\beta_{nj}^2}{\sqrt{\xi_{nj}^2\beta_{nj}^2+h}} \quad (54)$$

$$\begin{aligned}\dot{\hat\theta}_{kj} =\ & \tau_{kj}^{[n-1]}-\Gamma_{kj}\psi_{kj}\xi_{ij}\frac{\partial\alpha_{n-1,j}}{\partial x_{kj}} \\ &-\Gamma_{kj}\psi_{kj}\frac{\partial\alpha_{n-1,j}}{\partial x_{kj}}\sum_{l\in\mathcal{N}_j}\xi_{il}) =: \tau_{kj}^{[n]} \quad (55)\\ &1\le k\le n-1\end{aligned}$$

$$\dot{\hat\theta}_{nj} = \Gamma_{nj}\psi_{nj}\xi_{nj} =: \tau_{nj}^{[n]} \quad (56)$$

then

$$V_n \le -\lambda_1 z^\top AD_1^\top D_1Az-\sum_{k=2}^{n}\sum_{j=1}^{m}\lambda_{kj}\xi_{kj}^2 +\left(s_{1,m+1}+\sum_{k=2}^{n}\bar\delta_{k*}\right)m\sqrt{h}.$$

With the aid of the above design procedure, the following theorem can be proved.

Theorem 1: For the systems in (1)-(2), under Assumptions 1-2 the distributed control law

$$u_j = \alpha_{nj} \quad (57)$$

with the update laws

$$\dot{\hat\theta}_{ij} = \tau_{ij}^{[n]} \quad (58)$$

$$\dot s_{ij} = \frac{\gamma_{ij}\xi_{ij}^2\beta_{ij}^2}{\sqrt{\xi_{ij}^2\beta_{ij}^2+h}}, 1\le i\le n, 1\le j\le m \quad (59)$$

ensure that (8)-(10) are satisfied, where the control parameters are defined in the above controller design procedure.

Proof: With the control law (57), we have

$$\begin{aligned}\dot V_n \le\ & -\lambda_1 z^\top AD_1^\top D_1Az-\sum_{k=2}^{n}\sum_{j=1}^{m}\lambda_{kj}\xi_{kj}^2 \\ &+\left(s_{1,m+1}+\sum_{k=2}^{n}\bar\delta_{k*}\right)m\sqrt{h} \quad (60)\\ \le\ & \left(s_{1,m+1}+\sum_{k=2}^{n}\bar\delta_{k*}\right)m\sqrt{h}. \quad (61)\end{aligned}$$

Integrating both sides of (61), we have

$$\begin{aligned}V_n(t) \le\ & V_n(0) +\left(s_{1,m+1}+\sum_{k=2}^{n}\bar\delta_{k*}\right)m\int_0^t\sqrt{h(\tau)}d\tau \\ <\ & \infty\end{aligned}$$

which means that $V(t)$ is bounded (i.e., $V\in L_\infty$). Therefore, z, ξ_{ij}, $\hat\theta_{ij}$, and s_{ij} are bounded for all i

and j. Integrating both sides of (60), we have

$$\lambda_1 \int_0^t z^\top A D_1^\top D_1 A z d\tau + \sum_{k=2}^{n}\sum_{j=1}^{m} \lambda_{kj} \int_0^t \xi_{kj}^2 d\tau \leq V_n(0) - V_n(t) + \left(s_{1,m+1} + \sum_{k=2}^{n} \bar{\delta}_{k*}\right) m \int_0^t \sqrt{h(\tau)} d\tau < \infty$$

which means that $D_1 Az$ and ξ_{ij} ($2 \leq i \leq n$, $1 \leq j \leq m$) are square-integrable. With the aid of Barbalat's lemma, $D_1 Az$ and ξ_{ij} ($2 \leq i \leq n$, $1 \leq j \leq m$) converge to zero. By Lemma 3, eqns. (8)-(9) are satisfied. Since the communication graph $\mathcal{G}$ is connected, eqn. (10) is satisfied by Lemma 4. □

Leveraging the properties of the prescribed performance function ρ, the proposed controllers facilitate the convergence of tracking errors between neighboring systems to a specified value within finite time. In the controller design, to simplify notation, the PPF ρ remains consistent across different systems. However, one can substitute ρ with ρ_j specifically for system j. Furthermore, the PPF ρ can be tailored to different functions, accommodating diverse transient and steady-state performance requirements. Within the controller design, the transformation (11) is represented by a natural logarithm function. However, alternative choices for ρ are viable.

To approximate unknown functions, one can choose the basis to be polynomial functions, sigmoid functions, logistic functions, or other functions. To effectively implement the proposed controllers, obtaining the partial derivatives of the virtual controllers is imperative. With the aid of the command filtered backstepping technique [31–33] , simplified distributed controllers can be proposed. Due to space limitations, it is omitted.

IV. Simulation

Consider three second-order systems in (1)-(3), where

$$\begin{aligned} f_{1j} &= x_{1j}^2 + \sin(2x_{1j}) \\ f_{2j} &= x_{2j}^2 + 3x_{1j}^2 + 2\sin 5x_{2j}. \end{aligned}$$

There is one second-order leader system in (4)-(5) with $f_{14} = \sin(2x_{14})$, $f_{24} = 5\cos 3x_{14} - 2x_{24}$, and $u_4 = 3\cos 2t$. The communication between systems is shown in Fig. 1. It is obvious that the communication graph $\mathcal{G}$ is connected. The incidence matrix D is

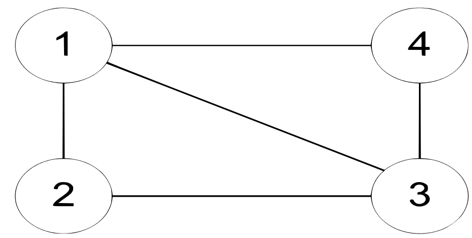

Fig. 1. The communication graph $\mathcal{G}$ between systems.

$$D = \left[\begin{array}{c} D_1 \\ \hline D_2 \end{array}\right] = \left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 \\ \hline 0 & 0 & -1 & -1 & 0 \end{array}\right].$$

The PPF is chosen as

$$\rho(t) = \begin{cases} 20\exp\left(\frac{-20t}{10-t}\right) + 0.1, & \text{if } 0 \leq t < 10 \\ 0.1, & \text{if } t \geq 10 \end{cases}$$

The control problem is to design distributed controllers such that $y_j - y_4$ converge to zero for $1 \leq j \leq 3$ and the performance (9) is satisfied.

In the controller design, we choose

$$\psi_{1j} = [1, x_{1j}, x_{1j}^2]^\top, \quad \psi_{2j} = [1, x_{1j}, x_{1j}^2, x_{2j}, x_{2j}^2]^\top$$

The boundedness of the approximation errors is evident. The distributed controllers proposed in Sections 3 and 4 effectively address the control problem.

The proposed controllers from Theorem 1 were implemented in a simulation with specified control parameters. Fig. 2 illustrates the convergence of the response $y_j - y_4$ for $1 \leq j \leq 3$ to zero. In Fig. 3, the responses of e_{12}, e_{14}, e_{23}, e_{34}, $-\rho$, and ρ are depicted, revealing that e_{12}, e_{14}, e_{23}, and e_{34} are bounded by $-\rho$ and ρ, thereby confirming the satisfaction of (9). Fig. 4 showcases the response of $\hat{\theta}_{1j}$ for $1 \leq j \leq 3$. Additionally, Fig. 5 illustrates the response of s_{1j} for $1 \leq j \leq 3$. Figure 6 presents the response of $\hat{\theta}_{2j}$ for $1 \leq j \leq 3$. Lastly, Fig. 7 demonstrates the response of s_{2j} for $1 \leq j \leq 3$. The simulation results affirm that θ_{ij} and s_{ij} are bounded for $1 \leq i \leq 2$ and $1 \leq j \leq 3$. These findings robustly substantiate the validity of the claim made in Theorem 1.

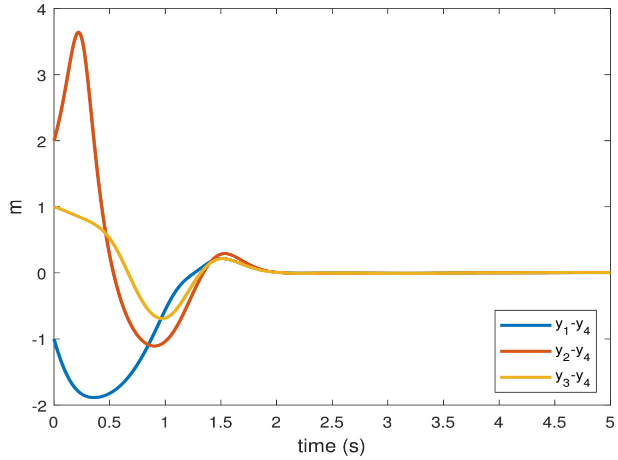

Fig. 2. The tracking error of $y_j - y_4$ for $1 \leq j \leq 4$.

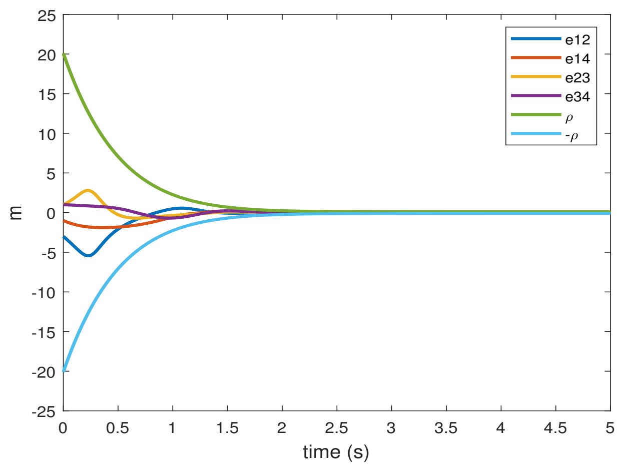

Fig. 3. The tracking errors e_{12}, e_{14}, e_{23}, and e_{34} and $-\rho$ and ρ.

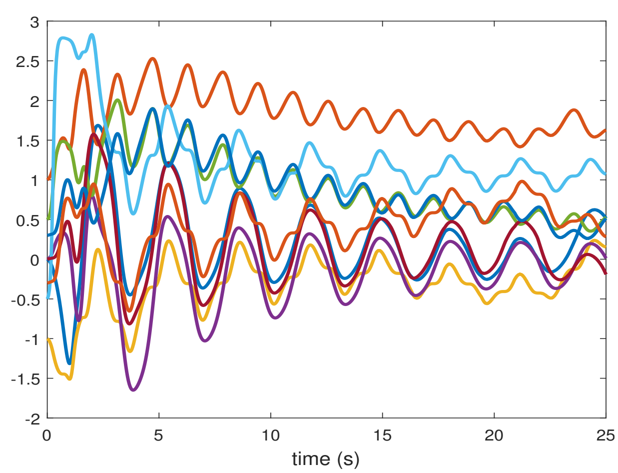

Fig. 4. The response of $\hat{\theta}_{1j}$ for $1 \leq j \leq 3$.

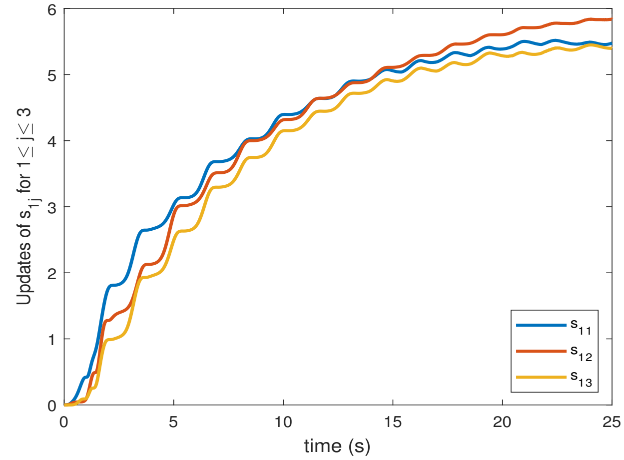

Fig. 5. The response of s_{1j} for $1 \leq j \leq 3$.

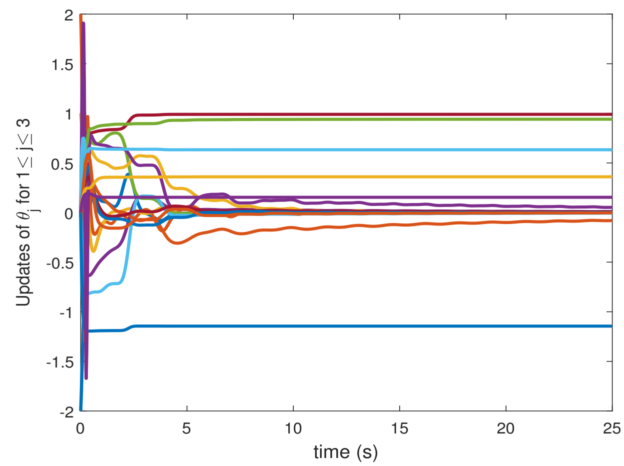

Fig. 6. The response of $\hat{\theta}_{2j}$ for $1 \leq j \leq 3$.

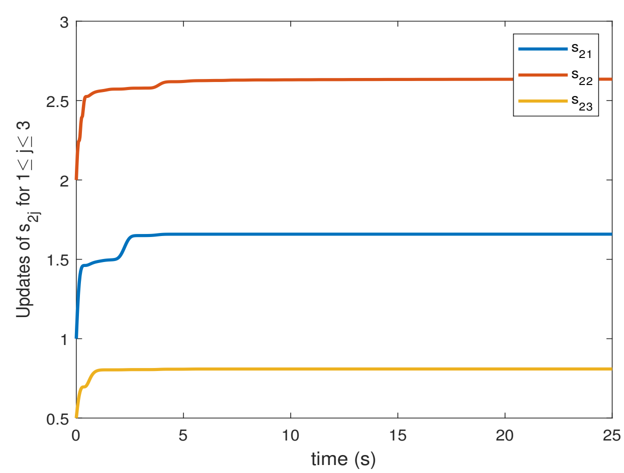

Fig. 7. The response of s_{2j} for $1 \leq j \leq 3$.

V. Conclusion

In this paper, the distributed tracking control of high-order uncertain nonlinear systems with prescribed performance requirements was studied. Distributed robust adaptive controllers were proposed to ensure that tracking errors converge to a small neighborhood around the origin within a specified finite time while satisfying prescribed performance criteria. The results presented in this paper provide a new approach to addressing the distributed control of uncertain nonlinear systems with performance requirements. Throughout our study, bidirectional communication between systems is assumed; however, these findings can potentially be extended to scenarios where the communication graph is directed.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Data availability statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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