

Learning & Teaching

Mathematics Teacher: Learning and Teaching PK-12, is NCTM's newest journal that reflects the current practices of mathematics education, as well as maintains a knowledge base of practice and policy in looking at the future of the field. Content is aimed at preschool to 12th grade with peer-reviewed and invited articles. *MTLT* is published monthly.

ARTICLE TITLE:

AUTHOR NAMES:

DIGITAL OBJECT IDENTIFIER:

VOLUME:

ISSUE NUMBER:

Mission Statement

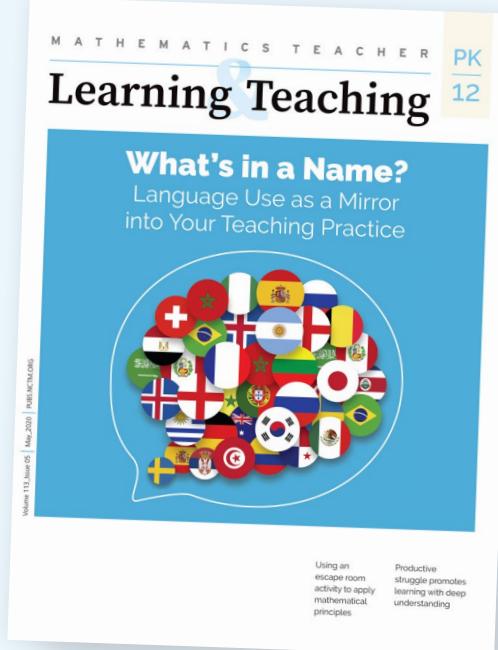
The National Council of Teachers of Mathematics advocates for high-quality mathematics teaching and learning for each and every student.

Approved by the NCTM Board of Directors on July 15, 2017.

CONTACT: mtlt@nctm.org



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS





Access digital content and takeaways at
nctm.org/mtlt11801g6.



Rate and Review

You rate movies, you rate restaurants, and you can also rate mathematics solutions!

Maria Nielsen Stewart, Noah Brown, Amber Candela, Samuel Otten, and Zandra de Araujo

We have been developing what we are calling instructional nudges as part of a larger research project (Otten et al., 2022). These instructional nudges are designed to be small but powerful changes to teachers' existing practices. Some instructional nudges focus on modifying tasks used in classrooms. We are sharing one with you that we call Rate and Review.

Think of the last time you were deciding what you wanted to eat, what movie you wanted to enjoy, or where to book your next oil change appointment. When you were making your decision, did you consider turning to reviews to help you decide? In an age of

unlimited access to information, people rely on reviews to help them sort through their many options for restaurants, movies, services, etc. We wondered, could students also rate mathematical solutions?

In today's mathematics classrooms, it is crucial to elicit students' thinking and spark conversations. This is useful for the teacher in assessing student understanding and beneficial for students in providing a space to demonstrate their knowledge. Although adding "explain" or "justify" to the end of a task is one way to encourage students' explanations, over time, this prompt gets repetitive and loses its punch. We wanted

to create a context where students had to Rate and Review mathematics strategies.

RATE AND REVIEW

When considering which solutions that students might rate, we were first drawn to worked examples. The benefit of a worked example is that it can provide students an opportunity to make sense of someone else's strategy while also showing students unique strategies for solving traditional problems (Star et al., 2015). Unique strategies are those that help students build connections across mathematics content without applying standard formulas or the most efficient strategies. As a research team, we developed the Rate and Review instructional nudge (Figure 1), which asks students to rate the solution strategy of a worked mathematics example on a scale of zero to five stars. Then, students write a review of the solution strategy to justify their rating. This task can be used for homework, independent work, a formative assessment, or for other instructional purposes. We believe the task is most effective when the teacher is able to explore student responses in some format (i.e., collecting student work or small group discussion). The benefit of Rate and Review is that it can be used with most mathematics content, and while the worked example

may change, the corresponding questions (rate, review) can stay the same. Using worked examples in Rate and Review encourages students to attend to the solution process, make connections between steps, and engage with mathematical ideas and relationships. We

Figure 1 Example of a Rate and Review Problem

Linear Equations Homework

1. Kai solved the following linear equation. Review Kai's solution strategy.
 $x - 1 = 5x + 3x - 8$

$$\begin{aligned} x - 1 &= 5x + 3x - 8 \\ -1 &= 5x + 2x - 8 \\ -1 &= 7x - 8 \\ -7x - 1 &= -8 \\ -7x &= -7 \\ x &= 1 \end{aligned}$$

The solution is $x = 1$

a. How many stars would you give Kai's solution?



b. Write a review of Kai's solution. Why did you give him that many stars?

Maria Nielsen Stewart, she/her, mntkd@umsystem.edu, is a doctoral candidate in mathematics education at the University of Missouri in Columbia, MO. She studies beginning mathematics teachers' experiences as they enter the profession and supports teacher learning.

Noah Brown, he/him, nobrown@wls4kids.org, is a seventh- and eighth-grade mathematics teacher at Washington Junior High in Toledo, OH, along with an adjunct instructor at Bowling Green State University in Bowling Green, OH. He seeks to provide instruction and assessment that support his students and connect their in- and out-of-school experiences to mathematics learning. He believes all students deserve opportunities to see themselves as "do-ers of mathematics."

Amber G. Candela, candelaa@umsl.edu, is an associate professor of mathematics education at the University of Missouri-St. Louis in St. Louis, MO. She enjoys engaging in professional development with teachers and supporting each and every student's access to high-quality mathematics.

Samuel Otten, he/him, ottensa@missouri.edu, is a professor of mathematics education and department chair at the University of Missouri in Columbia, MO. He studies students' mathematical reasoning and teachers' professional development, and he hosts the Math Ed Podcast.

Zandra de Araujo she/her, zdearaujo@coe.ufl.edu, is the math principal at the University of Florida's Lastinger Center for Learning. She also writes a blog, www.mathematicallyeducated.com, and co-creates videos for teachers at www.twominuteteachersguide.com.

doi:10.5951/MTLT.2024.0054

hypothesize that the more students engage with Rate and Review problems, their ability to articulate their justification becomes stronger.

When you first implement Rate and Review, have a conversation with students about the usefulness of reviews they read online, such as Google or Yelp. While reviews online can be overwhelmingly negative, with little justification for a rating, the goal of Rate and Review is for students to strengthen their justification skills by placing an emphasis on the strategy that was used, not only the answer given. By taking time before implementing Rate and Review to talk about how to be a helpful reviewer, you set students up to be more appreciative of the reasoning (Gutiérrez 2017) that has been done in the worked example. After discussing reviewing, give Rate and Review to your students and let them work. Then, you can have students turn in their Rate and Review, talk in small groups about their reviews, or have a whole-class discussion about the various ratings students gave the

strategy. If you choose to have students talk in small groups, we suggest listening to these conversations to get a sense of how students interpret the strategy. All of these options allow you to assess students' mathematical understanding, and when students are given the opportunity to share, you can assess their engagement level and mathematical communication. The illustration that follows shares the experience of one teacher as he implemented a Rate and Review in his classroom.

ILLUSTRATION OF RATE AND REVIEW

I, the second author, teach seventh- and eighth-grade mathematics in a suburban school in the Midwest. In a conversation with the first author, a member of the research team, she asked me to provide feedback on the Rate and Review instructional nudge. Upon hearing about it, I thought it would be a great addition to my classroom, so I immediately began thinking of ways to

Figure 2 Sample Rate and Review for Angle Relationships

Name: _____ Period: _____

Rate & Review

James found the missing angle below by doing the following steps.

Find $m\angle EBC$

Angle Relationship: Complementary

$$7x + 3x + 90 = 180$$

$$10x + 90 = 180$$

$$-90 \quad -90$$

$$\frac{10x}{10} = \frac{90}{10}$$

$$x = 9$$

$$3(9) = 27$$

$$\angle = 27^\circ$$

1. Give a rating to James' solution to the problem out of 5 stars. (shade the number of stars you want to use)

2. Why did you give the rating you did?

5 stars available for rating.

implement it the next day at school. Since then, I have done Rate and Review several times, modifying my implementation with each iteration.

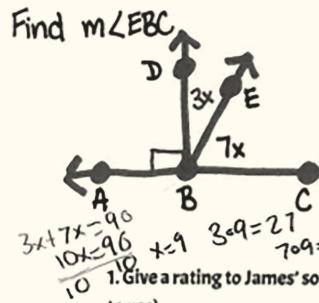
I was teaching a lesson about angle relationships that are formed by two parallel lines and a transversal, along with angles that would create a supplementary or complementary relationship. I gave students time to explore and practice these relationships in class for two days before the Rate and Review, in which I gave them a worked example. In this implementation of Rate and Review (Figure 2), I wanted to assess students' abilities to identify angle relationships and then use those relationships to construct an equation that would allow them to solve for a missing angle.

I began by creating a problem that intentionally stated the most frequently noticed angle relationship (i.e., complementary) but showed an equation for what might be considered the “least efficient” relationship (i.e., supplementary). Students might view supplementary as less efficient because of the “extra step” that comes from adding another angle and summing to 180° , rather than just the two angles shown in the diagram, which would sum to 90° . To take this task one step further, I also substituted the value I found for x into the wrong angle ($\angle DBE$) rather than the angle that the question asked for ($\angle EBC$). This was to ensure that students were encouraged to identify an angle within a complex relationship while

Figure 3 Asil's Approach to This Rate and Review Problem

Rate & Review

James found the missing angle below by doing the following steps.



Angle Relationship: Complementary

$$\begin{aligned}
 7x + 3x + 90 &= 180 \\
 10x + 90 &= 180 \\
 -90 &\quad -90 \\
 \frac{10x}{10} &= \frac{90}{10} \\
 x &= 9 \quad \rightarrow 3(9) = 27 \\
 4 &= 27^\circ
 \end{aligned}$$

1. Give a rating to James' solution to the problem out of 5 stars. (shade the number of stars you want to use)



2. Why did you give the rating you did?

He got the right answer, but he complicated the equation. The way he did it makes it look supplementary. Complementary = 90° not 180° . Also he found $m\angle DBE$ not $m\angle EBC$.

3. Would you recommend using James' strategy to a friend solving a similar problem? Why or why not?

No, because there's a much simpler way to do it, plus he didn't even get the correct angle.

also forcing them to think deeper about what they could use as reasoning in their review. In this Rate and Review, I asked students to look at the student work (Figure 2) and give a rating on a scale of zero to five stars of James' strategy, with evidence to support their ratings.

When I implemented my version of Rate and Review, students found many different approaches to the problem that aligned with their knowledge of the content. All names in this illustration are pseudonyms.

For example, Asil (Figure 3) may have thought that the worked example she was given had been overcomplicated a bit. She noticed in the example how the student believed it may have been a complementary relationship but set up the equation as if it were supplementary. Although that is what the picture might convince you of, there is enough evidence from the photo for us not to have a "need" for that 90° angle. Asil paid close attention to the directions where it asks for the $m\angle EBC$ but noticed that the student solved for

Figure 4 Jaheem's Approach to This Rate and Review Problem

Rate & Review

James found the missing angle below by doing the following steps.

Find $m\angle EBC$

$$\begin{aligned}
 & 7x + 3x + 40 = 180 \\
 & 10x + 40 = 180 \\
 & 10x = 140 \\
 & x = 14 \\
 & 7(14) = 98 \\
 & 3(14) = 42 \\
 & 98 + 42 = 140
 \end{aligned}$$

Angle Relationship: Complementary

1. Give a rating to James' solution to the problem out of 5 stars. (shade the number of stars you want to use)



2. Why did you give the rating you did?

I gave James 4 stars because he got all the math correct until the final part. He put in the angle for $D\angle E$, when he was supposed to put in $E\angle B$.

3. Would you recommend using James' strategy to a friend solving a similar problem? Why or why not?

Yes, because he is correct with the way he did it, but he just made a simple mistake in the last part.

the other angle, $\angle DBE$, making the overall answer incorrect. In the image in the worked example in Figure 3, you can notice Asil demonstrating mastery of identifying angle relationships along with solving equations by looking at her own mathematics work below the angles.

Jaheem (Figure 4) took a similar approach to Rate and Review but with some notable differences. For example, in his approach, Jaheem highlighted the angle that was being asked for and labeled the stars with what he believed to be appropriate descriptors for how to earn them. Jaheem either did not notice that the student in this worked example stated it as a

complementary relationship or agreed with it, but more importantly, Jaheem did the mathematics on his own as if he was checking to see if the answer was correct. This is both important and useful information for me as the teacher because I can directly observe Jaheem's mastery of the mathematics procedure, such as solving an equation formed from the diagram. However, I also know that it is important to check with this student, either through informal conversation or other formative assessment, to determine his conceptual understanding of angle relationships because of his lack of comment on the stated complementary relationship.

Figure 5 Cali's Approach to This Rate and Review Problem

Rate & Review

James found the missing angle below by doing the following steps.

Find $m\angle EBC$

Angle Relationship: Complementary

$$7x + 3x + 90 = 180$$

$$10x + 90 = 180$$

$$-90 \quad -90$$

$$10x = 90$$

$$\frac{10x}{10} = \frac{90}{10}$$

$$x = 9$$

$$3(9) = 27$$

$$4 = 27^\circ$$

1. Give a rating to James' solution to the problem out of 5 stars. (shade the number of stars you want to use)

2. Why did you give the rating you did?

Because he got the relationship wrong

3. Would you recommend using James' strategy to a friend solving a similar problem? Why or why not?

no Because the relationship is a big thing to get wrong

One of the best attributes of Rate and Review is the number of entry points to the task, as anyone can provide a rating. Additionally, there are no wrong answers as long as students provide sufficient justification. One student might give a rating based on the rigor of the mathematical argument presented, whereas another student might rate based on the clarity of the written work. Both can be valid and worth discussing as a classroom community.

If you notice in Cali's approach in Figure 5, she is able to enter and complete the task, giving her a sense of accomplishment and belief that she is a mathematician while also providing us, as her teachers, with some information about her understanding of the concept. We noticed that Cali recognized when angles form a complementary angle, but we might want to

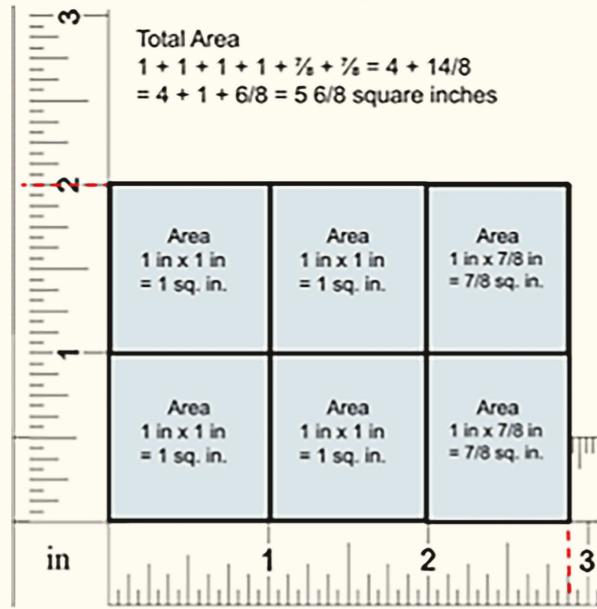
probe her understanding of the relationship by asking questions or allowing her to create a drawing based on a verbal description. Furthermore, we would like to know how she might set up an equation to solve the problem. Although each student receives the same solution strategy to rate at the beginning of the task, the work that the task might elicit from students individually could be vastly different. For this reason, Rate and Review can provide meaningful insight into the minds of students and allow the teacher to navigate conversations about multiple representations and multiple solution strategies.

While this task provided immediate insight into students' thinking that allowed for interventions, it also provided helpful information that could be used in the planning process for future lessons involving

Figure 6 Rate and Review With a Unique Strategy

RATE AND REVIEW

1. Salima solved this area problem in the following way.



First, I broke the rectangle up into smaller rectangles that I could find the area of. Then I added up all the small squares.

a. How many stars would you give this solution?



b. Write a review of this solution. Why did you give the strategy that many stars?

angle relationships in the long term. Reflecting on this lesson, in the future, I want to show students a variety of examples that could be worked using either a complementary or supplementary relationship. In the lessons before this, students had only engaged with a few examples of a relationship like this, but I failed to acknowledge how similar they could be. When I initially presented students with the Rate and Review task, they showed some confusion about how the fictional student had solved the problem in the task, but I witnessed the moment when they realized what the worked example was showing. After giving students some time to work individually on the task, I asked students to share their initial

reactions with table members. It was interesting to witness student discussion of the task as I collected their work, specifically the noticings they shared with one another about whether it should be solved using complementary or supplementary angles. Students discussed the validity of using one strategy over the other. The overall consensus was that “if [the student] wants to use a complementary relationship, then his equation should not have $a + 90$, and it should only equal 90.” Students also attended to this idea in another way, stating, “but based on the work that he provides, I think he just got the words confused, and he should have said they were supplementary angles.” The discussion provided a great way to gather evidence that

Figure 7 Rate and Review Problem With Two Worked Examples

Rate & Review

Noralie's car uses 20 gallons of gasoline to go 600 miles. Irshad uses the proportion $\frac{600}{20} = \frac{x}{4}$ to find the number of miles Noralie's car can travel with 4 gallons of gasoline. Jesse saw it differently, and says that you could use a unit rate to find the same information.

Irshad:

$$\frac{600}{20} = \frac{x}{4}$$

$$600 \cdot \frac{1}{5} = 120$$

$$x = 120 \text{ miles}$$

$$4 \div 20 = \frac{1}{5}$$

Jesse:

$$\text{Unit rate} = \frac{600 \text{ miles}}{20 \text{ gallons}}$$

$$600 \div 20 = 30$$

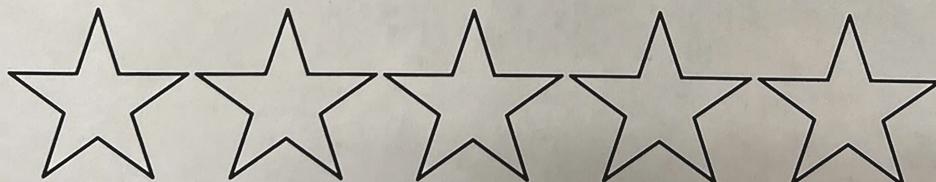
$$\text{Unit rate} = 30 \text{ mi/gal}$$

$$30 \text{ mi/gal} \times 4 \text{ gal} = 120$$

$$120 \text{ miles}$$

1. Who do you agree with?

2. Give a rating to the solution you agreed with out of 5 stars. (shade the number of stars you want to use)



students understood how both relationships provided a valid solution, but they may prefer one over the other based on their own understanding of the concepts. The process created time and a space for students to draw connections between the two relationships, allowing them to choose the strategy that works best for them both conceptually and mathematically.

CREATING A RATE AND REVIEW PROBLEM

The Rate and Review instructional nudge lends itself to many different mathematical standards and you can

adapt it and create scenarios that fit your grade level. When you create or select worked problems to Rate and Review, there are many options for the type of strategy that you can choose to showcase.

To create a new Rate and Review, we typically begin by thinking about the concept students are currently learning and common errors related to that concept. Highlighting a common error allows you to assess if students can identify the error and correct it. In the illustration shared above, the second author was able to assess two key concepts: 1) students' understanding of angle relationships in the various ways they can be

Figure 8 Student Work Only Identifying One Error

Rate & Review

James found the missing angle below by doing the following steps.

Find $m\angle EBC$

Angle Relationship: Complementary

$$7x + 3x + 90 = 180$$

$$10x + 90 = 180$$

$$-90 \quad -90$$

$$\frac{10x}{10} = \frac{90}{10}$$

$$x = 9$$

$$3(9) = 27$$

$$4 = 27^\circ$$

1. Give a rating to James' solution to the problem out of 5 stars. (shade the number of stars you want to use)

★ ★ ★ ★ ★

2. Why did you give the rating you did?

Because I like the way he set it up but the angle relationship is wrong it's actually supplementary

3. Would you recommend using James' strategy to a friend solving a similar problem? Why or why not?

Yes I would recommend using his strategy just not the angle measurement.

formed and 2) whether students still held the frequent misconceptions they considered previously about these types of problems.

Rate and Review can use a worked example that highlights a unique strategy, like the area problem in Figure 6, to discern if students connect the new strategy with something they are more familiar with. Additionally, you can present students with a comparison of two valid strategies, as in Figure 7, to assess if students are able to recognize value in both strategies while also identifying their similarities. Alternatively, you could provide students with a worked example that displays an inefficient strategy, like the solving equations example in Figure 1. When you use an inefficient strategy, it is helpful if the strategy still arrives at the correct answer because the goal is to get students to see the value of all strategies. Using alternate strategies can expose students to new strategies, which helps them to build their procedural fluency as well as their conceptual understanding. Ultimately, a student's view of a strategy will closely align with what makes sense to them and how well they may utilize the strategy.

If teachers store their Rate and Review problems in Google Drive, the "make a copy" feature lends itself easily to creating or modifying the problems in such a way that it almost becomes a template. The third question might change depending on the type of problem being used, but ultimately you may find yourself changing the names, printing the "template," and then hand-writing the "student solution" at the top, as seen with the gas problem in Figure 7.

RATE AND REVIEW: CAUTIONS

Although Rate and Review can provide meaningful data and insight into a student's understanding of both concept and procedure, there is a cautionary tale that comes with it. Designing, implementing, and using the data from a Rate and Review takes some practice. From our experiences of implementing Rate and Review problems, we found it very difficult to figure out the appropriate number of errors to include in the worked example. Having too many errors or trying too hard to hide the error in preparing your worked example can confuse students and make it too difficult for students to see any errors. In Figure 8, a student felt that because the mathematics was right, everything else must be as well. The only error they caught was that the relationship was named incorrectly

(i.e., complementary vs. supplementary), when a complementary relationship could still be justified as a valid strategy. When we create a space for students to think deeply about how to justify their reasoning, we need to be sure they know *how* to investigate a solution and what it looks like to justify before moving on to more complicated tasks that might have more errors in the worked example.

Another caution is to avoid creating problems that are too open, such as putting a strategy in the worked example that is not clear enough for the students to make sense of. When students read the worked example, they should be able to identify the strategy and not have to fill in any gaps. By giving a Rate and Review that is too open or has too many errors, the process can become hard for students to navigate without feeling overwhelmed. Additionally, students who give one or zero stars to correct (but unfamiliar) strategies and then give little to no justification for their rating may think there is only one correct way of solving those types of problems. Because the goal of Rate and Review is to see value in a variety of strategies, it is important to have a conversation with those students to further reveal their thought processes and clarify where appropriate.

The last piece of advice we offer, and probably the most important, is to avoid using your current students' work for Rate and Review solutions, as it may cause embarrassment or anxiety. Even though the worked example in the Rate and Review problem could be an anonymous student's work, your students will still be able to recognize their own work. If their classmates give their worked examples one star, it will be embarrassing. For this reason, it is best to create your own worked examples or find them elsewhere to include as your Rate and Review solutions.

LEVEL UP

After trying Rate and Review a few times with one specific example, as described above, there are some additional ways you could extend Rate and Review problems with your students. One way to level up a Rate and Review is by adding an additional question, or replacing the last question with, "What could this student have done to earn more stars?" Adding this question challenges students to not only identify any error or inefficiency in the solution but also find ways to amend the error. Even correct solutions can often be improved. Another way to level up is by having multiple

students compare their reviews. This allows students to observe a variety of justifications for different ratings. Additionally, this approach encourages student discussion about mathematical ideas.

If you really want to change up your Rate and Review, you could give students partially solved problems and tell them what rating this finished solution is intended to receive—students fill in the rest and try to hit the targeted rating. In this level up, students are challenged to solve the problem themselves but in unique, inefficient, or incorrect ways. Solving problems in these ways requires a deep understanding of the concept as well as knowledge of many alternative solution strategies. The opportunity to engage in this type of problem helps foster student flexibility in solution strategies as well as knowledge of common misconceptions.

While we have presented a few options for ways to level up Rate and Review problems, there are surely more possibilities. As the teacher, you have complete

freedom to level up the task based on what works best in your classroom and your own students' needs. However, we encourage you to try Rate and Review in its original form a few times before introducing the more complicated level ups.

CONCLUSION

The goal of Rate and Review is to support students in understanding that there are alternative solution strategies to what they might be used to using, provide students the opportunity to become more comfortable creating mathematical arguments, and develop a deeper understanding of a concept. Rate and Review provides formative assessment for teachers by supporting their reflections on students' understanding of current content and how students are making sense of common errors. For more examples of Rate and Review, please check out the supplementary material (link online). [—](#)

REFERENCES

Gutiérrez, R. (2017). Living mathematx: Towards a vision for the future. In E. Galindo & J. Newton (Eds.), *Proceedings of the 39th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (pp. 2–26). Hoosier Association of Mathematics Teacher Educators.

Otten, S., de Araujo, Z., Candela, A. G., Vahle, C., Stewart, M. N., Wonsavage, F. P., & Baah, F. (2022). Incremental change as an alternative to ambitious professional development. In A. E. Lischka, E. B. Dyer, R. S. Jones, J. N. Lovett, J. Strayer, & S. Drown (Eds.), *Proceedings of the forty-fourth annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (pp. 1445–1450). Middle Tennessee State University.

Star, J. A., Foegen, A., Larson, M. R., McCallum, W. G., Porath, J., Zbiek, R. M., Caronongan, P., Furgeson, J., Keating, B., & Lyskawa, J. (2015). *Teaching strategies for improving algebra knowledge in middle and high school students. Educator's practice guide*. Institute of Education Sciences

Linear Equations Homework

1. Kai solved the following linear equation. Review Kai's solution strategy.

$$x - 1 = 5x + 3x - 8$$

$$\begin{aligned}x - 1 &= 5x + 3x - 8 \\-1 &= 5x + 2x - 8 \\-1 &= 7x - 8 \\-7x - 1 &= -8 \\-7x &= -7 \\x &= 1\end{aligned}$$

The solution is $x = 1$

a. How many stars would you give Kai's solution?

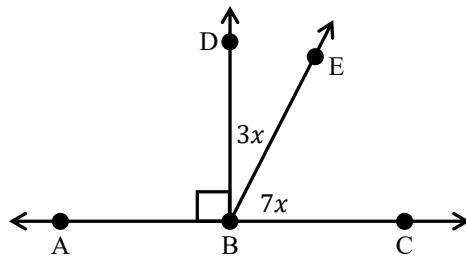


b. Write a review of Kai's solution. Why did you give him that many stars?

c. Would you recommend Kai's strategy for this type of problem to others? Why or why not?

Rate & Review

James found the $m\angle EBC$ below by doing the following steps.



Angle Relationship: Complementary

$$\begin{aligned}
 7x + 3x + 90 &= 180 \\
 10x + 90 &= 180 \\
 -90 & \quad -90 \\
 \frac{10x}{10} &= \frac{90}{10} \\
 x &= 9 \\
 3(9) &= 27 \\
 \text{so, } m\angle EBC &= 27^\circ
 \end{aligned}$$

a. Give a rating of James' solution to the problem out of 5 stars (shade the number of stars you want to use).



b. Why did you give the rating you did?

c. What could James do to improve his rating or solution in the future?