Bispectrum Unbiasing for Dilation-Invariant Multi-Reference Alignment

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***Abstract*—Motivated by modern data applications such as cryo-electron microscopy, the goal of classic multi-reference alignment (MRA) is to recover an unknown signal $f : \mathbb{R} \to \mathbb{R}$ from many observations that have been randomly translated and corrupted by additive noise. We consider a generalization of classic MRA where signals are also corrupted by a random scale change, i.e. dilation. We propose a novel data-driven unbiasing procedure which can recover an unbiased estimator of the bispectrum of the unknown signal, given knowledge of the dilation distribution. Lastly, we invert the recovered bispectrum to achieve full signal recovery, and validate our methodology on a set of synthetic signals.**

***Index Terms*—Dilations, fourier invariants, multi-reference alignment.**

I. Introduction to MRA

In classic multi-reference alignment (MRA), one seeks to

(iii) $\{\varepsilon_j(x)\}_{j=1}^M$ are independent white noise processes on $[-\frac{1}{2}, \frac{1}{2}]$ with variance σ^2.

Methods for solving Model 1 can be grouped into two categories. The first approach is synchronization methods [3], [4], [5], [6], [12], [17], [18], [46], [55], [58], which try to recover each translation factor $\{t_j\}_{j=1}^M$, align the signals using the recovered translation factor, and average the aligned signals to get a smoother estimate for the ground truth signal. However, synchronization methods can be problematic when the signal-to-noise ratio (SNR) is small. Although synchronization is tenable at small noise levels, at high noise levels, the peaks are not recognizable and synchronization fails. The second approach involves estimating the signal directly, without estimation of the translation factors, using ideas such as the method of moments [28], [33], [53]. The method of invariants [3], [8], [19], [30], [31] is an important subclass of such methods in which one