

Opinion-based Task Allocation Strategy for Mobile Sensor Networks

Ziqiao Zhang, Wencen Wu, and Fumin Zhang

Abstract—In this paper, we propose an opinion-based task allocation strategy for mobile sensor networks to conduct simultaneous multi-source tracking while doing measurement collection and information estimation. Mobile sensors form opinions of the field based on the information obtained by sensors, and opinion dynamics describe how sensors communicate with each other and generate decisions on task assignments for multi-source tracking. Triggering conditions of running opinion dynamics for task allocation are also provided, determining when it is necessary to divide sensors into groups. Simulation results with a 12-sensor network tracking two different sources validate the proposed opinion-based task allocation strategy.

I. INTRODUCTION

Mobile sensor networks have been widely applied to collect information for severe natural processes such as hurricanes, ocean eddies, and forest fires for their high efficiency and reliability [1]–[4]. In real-world applications, communication among sensors is often limited due to low bandwidth, power limitations, high deployment expenses, and communication losses [5], [6]. This explains why distributed communication and forming different groups are more welcome in practice with robustness and scalability.

During the process of information collection, mobile sensors need to complete tasks such as source tracking or field mapping, collaboratively. Most of the time, the field of interest is relatively large compared to the size of the sensor network and sensors need to be divided into groups to move towards different areas to conduct different tasks simultaneously [7]. This requires task allocation among these sensors so that all tasks can be completed. The mobile sensors considered in this paper can collect field measurements, exchange information with neighbors, and estimate field information. We consider the scenario when a large number of sensors can be deployed in the field at a low cost, and aim to solve the task allocation problem for mobile sensor networks without considering an explicit utility cost.

Task allocation problem has been studied to assign a finite number of agents to complete a finite number of tasks efficiently. Task allocation algorithms can be classified into several categories: market-based, optimization-based, game theory-based, learning-based, and hybrid approaches, where

the first two are the most commonly used [8], [9]. In existing literature [10]–[12], the problem of task allocation or swarm grouping is usually addressed by solving an optimization problem of the utility cost. Opinion dynamics has also been applied for task allocation between two options through decentralized switching transformations where the grouping portion is determined by the designed switching [13]. Our goal is to solve the task allocation problem using another type of opinion dynamics, where opinions are modeled as unit-length vectors.

Individual opinions are usually composed of different perspectives and can be modeled by vectors. During the process of opinion exchange, it is more convenient and efficient to share normalized opinions such that each individual follows the same standard [14]. Our previous works in opinion dynamics [15]–[17] focus on studying the opinion evolution process on the unit sphere for any dimensional spaces. As opinion states evolve on the sphere while exchanging information with neighbors, rich convergence results of consensus and various dissensus with stability guarantees can be obtained by applying different types of interaction control. Such convergence behaviors can divide individuals into different groups or clusters in a natural way, which maximizes the differences among different groups as well as the similarities within the same group. This also avoids solving an optimization problem on utility, and makes task allocation among individuals simple to conduct.

In this paper, we propose using opinion dynamics to assign tasks in mobile sensor networks for source tracking. The major novelty in this proposed work is that no explicit utility information is needed and no corresponding optimization problem needs to be solved for task assignments. Here we only need to run the dynamics on a faster time scale to obtain the convergence results for sensor grouping. Different from the consensus-based auction algorithm where maximum consensus has been implemented for convergence of a list of winning bids [11], the consensus or dissensus algorithms will be applied for grouping directly in this work. The grouping results are determined by the initial conditions and the type of opinion dynamics rather than predetermined groups.

Our major contribution is the application of opinion dynamics in task allocation problems for simultaneous multi-source tracking using mobile sensor networks, especially the initialization of opinion states using sensor information and the triggering conditions for running opinion dynamics. The initialization builds the relationships between the *concrete* sensor information and the *abstract* opinions on the field of interest. The triggering conditions allow abstract opinions to develop. The proposed strategy enables the mobile sensors

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to detect multiple sources and form different groups after information exchange with each other, offering more flexibility to mobile sensors in complicated scenarios.

The problem of opinion-based task allocation is formulated in Section II. Information-based opinion-developing design is provided in Section III. Section IV introduces the triggering conditions for running the opinion dynamics. Section V presents the proposed opinion-based task allocation strategy for simultaneous multi-source tracking. Simulation results are illustrated in Section VI. Conclusions and future work follow in Section VII.

II. PROBLEM FORMULATION

In this section, we formulate the task allocation problem of mobile sensor networks in a spatial-varying field in d -dimensional space, where $d \in \mathbb{Z}_+$ and $d \geq 2$.

We assume that the field can be described by $z(\mathbf{r}, t) \in \mathbb{R}$ in a spatial domain $\Omega \subseteq \mathbb{R}^d$, where $\mathbf{r} \in \Omega$ represents location, $t \in \mathbb{R}_+$ represents time, z is the field value with spatial-varying property.

Suppose a group of N mobile sensors are deployed in the field described by $z(\mathbf{r}, t)$, taking discrete field measurements z with time interval Δt . There are multiple unknown sources in this field for the robots to explore, and the total number of sources is known as N_s . Denote the location of sensor i at the k th time step t_k as $\mathbf{r}_i^k$ and the noisy measurement taken by this sensor at t_k as $p(\mathbf{r}_i^k, k)$, written as

$$p(\mathbf{r}_i^k, k) = z(\mathbf{r}_i^k, k) + n_i, \quad (1)$$

where $n_i \in \mathbb{R}$ is an independent and identically distributed Gaussian noise. Utilizing the cooperative filtering strategy in [18]–[20], the mobile sensors can estimate the information of field value and corresponding gradient, denoted as $\hat{z}(\mathbf{r}_i^k, k)$ and $\nabla \hat{z}(\mathbf{r}_i^k, k)$, respectively. We denote $p(\mathbf{r}_i^k, k), \hat{z}(\mathbf{r}_i^k, k), \nabla \hat{z}(\mathbf{r}_i^k, k)$ as $p_i^k, \hat{z}_i^k, \nabla \hat{z}_i^k$ for simplicity. Denote the information obtained by sensor i at t_k as $\mathbf{I}_i^k$ where

$$\mathbf{I}_i^k = [p_i^k, \hat{z}_i^k, (\mathbf{r}_i^k)^\top, (\nabla \hat{z}_i^k)^\top]^\top \in \mathbb{R}^{2d+2}. \quad (2)$$

While mobile sensors are moving in the field of interest and collecting discrete measurements, multiple tasks are detected which requires the sensors to be split into different groups by solving the task allocation problem. We focus on one specific task allocation problem in which sensors need to be assigned to an unknown source for source-seeking.

Based on the estimated and collected field information, each sensor forms its own opinion $\mathbf{o}_i$ and communicates with its neighbors. After sufficient communication, they form several groups or clusters based on opinion convergence. In this paper, we model the opinions as unit-length vectors and apply opinion dynamics on the unit sphere. The relationship between estimated information and opinion state of sensor i at time t_k can be written as

$$\mathbf{o}_i(\tau_0) = \mathbf{f}(\tilde{\mathbf{I}}_i^k) \quad (3)$$

where $\mathbf{o}_i \in \mathbb{S}^{D-1}$ is the opinion state, $\tilde{\mathbf{I}}_i^k$ is the preprocessed information state based on $\mathbf{I}_i^k$ and $\mathbf{f}(\cdot) : \mathbb{R}^{2d+2} \mapsto \mathbb{S}^{D-1}$ is the relationship function to be designed.

Since it is difficult to say whether two opinion vectors are similar or different directly, we consider utilizing opinion dynamics to obtain a convergence result from which an assignment result can be easily obtained. We run the opinion dynamics in a faster time scale τ to avoid waiting for opinion convergence results and expedite task assignment. Under the control input $\mathbf{u}_i$, the opinion dynamics on the sphere can be written as

$$\frac{d}{d\tau} \mathbf{o}_i(\tau) = (\mathbf{I} - \mathbf{o}_i(\tau) \mathbf{o}_i^\top(\tau)) \mathbf{u}_i(\tau), \tau \geq \tau_0, \quad (4)$$

where $\mathbf{I} \in \mathbb{R}^{D \times D}$ is the identity matrix. The projection matrix $(\mathbf{I} - \mathbf{o}_i(\tau) \mathbf{o}_i^\top(\tau))$ projects $\mathbf{u}_i(\tau)$ onto the tangent space of $\mathbf{o}_i(\tau)$, which guarantees the unit-length property of $\mathbf{o}_i$.

Driven by opinion dynamics, the opinion states will converge to various stable equilibria and form different clusters based on the initial opinions and interactions with neighbors. Suppose there are N_s different opinion states $\mathbf{O}_1, \dots, \mathbf{O}_{N_s}$ at equilibria. Then the opinion states form N_s groups $G_1, \dots, G_{N_s}$ such that

$$\lim_{\tau \rightarrow \infty} \mathbf{o}_i(\tau) = \mathbf{O}_g, \forall i \in G_g, \quad (5)$$

where $g = 1, \dots, N_s$. The N_s sets $G_1, \dots, G_{N_s}$ provide the task assignment for each sensor and N_s is the number of sources being detected.

Unlike the measurement collection and information estimation process, mobile sensors are not expected to run opinion dynamics all the time. Instead, they only need to run these when task allocation is required. In this way, the initiation of opinion dynamics is event-triggered.

Problem Statement

In this paper, we propose an opinion-based task allocation strategy for mobile sensor networks. The first goal is to design the opinion-developing function $\mathbf{f}(\cdot)$ in (3) that models how sensors forge field opinions based on collected and estimated information. The second goal is to determine the triggering conditions to run opinion dynamics for task allocation. The last goal is to define proper opinion dynamics to achieve desired task assignments among mobile sensors.

III. INFORMATION-BASED OPINION-DEVELOPING DESIGN

In this section, we will present the design of the opinion-developing function based on the information collected and estimated by mobile sensors.

Each mobile sensor collects discrete measurements and generates estimated information while moving in the field of interest. After mobile sensors have been deployed in the field for a period of time, the estimated information becomes accurate and sensors start to form opinions about the field of interest using the obtained information. Compared with conventional mobile sensors, the ones modeled in this paper are equipped with opinion-developing capabilities.

Before defining the opinion-developing function, we need to preprocess the information states. Instead of using information states directly, we extract the average information from each information state and define the preprocessed information state as follows

$$\tilde{\mathbf{I}}_i^k \triangleq \mathbf{I}_i^k - \bar{\mathbf{I}}^k, \forall i = 1, \dots, N, \quad (6)$$

where $\bar{\mathbf{I}}^k = \frac{1}{N} \sum_{i=1}^N \mathbf{I}_i^k$. The preprocessed information states satisfy the zero-summation property

$$\sum_{i=1}^N \tilde{\mathbf{I}}_i^k = \mathbf{0}. \quad (7)$$

The opinion-developing function $\mathbf{f}(\cdot)$ for sensor i in (3) can be further written as

$$\mathbf{f}(\tilde{\mathbf{I}}_i^k) = \frac{\mathbf{C}\mathbf{I}_i^k}{\|\mathbf{C}\mathbf{I}_i^k\|_2}, \quad (8)$$

where $\mathbf{C} \in \mathbb{R}^{D \times (2d+2)}$ is the constant transition matrix modeling how information is transformed into opinions.

Transition Matrix Design

The design of matrix $\mathbf{C}$ in (8) decides how mobile sensors form opinions based on the information $\mathbf{I}_i^k$ defined in (2), which includes position, measurement, and estimation. The transition matrix $\mathbf{C}$ has the following structure

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0}^{2 \times d} & \mathbf{0}^{2 \times d} \\ \mathbf{0}^{d \times 2} & \mathbf{C}_2 & \mathbf{0}^{d \times d} \\ \mathbf{0}^{d \times 2} & \mathbf{0}^{d \times d} & \mathbf{C}_3 \end{bmatrix}, \quad (9)$$

where $\mathbf{C}_1 \in \mathbb{R}^{2 \times 2}$ models how scalar information of $p, \hat{z}$ is utilized, $\mathbf{C}_2 \in \mathbb{R}^{d \times d}$ represents the inclusion of position information, and $\mathbf{C}_3 \in \mathbb{R}^{d \times d}$ describes the fusion of gradient information. When the elements of $\mathbf{C}_2$ are relatively large compared with $\mathbf{C}_1, \mathbf{C}_3$, this setup models the scenario when opinions are position-oriented. When the elements of $\mathbf{C}_3$ are relatively large, the opinions are developed mostly based on the spatial variation of the field. In this case, the dimension of opinion states is $D = 2 + 2d$ and d is the field dimension.

IV. TRIGGERING CONDITIONS FOR OPINION DYNAMICS

In this section, we will provide the derivation of triggering conditions for running opinion dynamics among mobile sensors for task allocation.

Task allocation is needed when multiple tasks are detected and are required to be conducted by different sensors simultaneously. Our goal is to characterize the detection of multiple tasks based on the information collected and estimated by mobile sensors.

Before providing the triggering conditions for running opinion dynamics, we introduce some essential assumptions.

Assumption IV.1 *The field of interest is relatively large compared with the area covered by the whole mobile sensor network, and the distance between any two sources is larger than the range of area covered by the sensor network.*

This assumption models the scenario when mobile sensors are moving in a large field for exploration and exploitation,

and this also avoids the case when two sources are in the area covered by mobile sensors at the same time.

Assumption IV.2 *The total number of mobile sensors is large enough that there will be plenty of sensors tracking different sources.*

Assumption IV.3 *Each mobile sensor is capable of conducting at most one task at one time, and is allowed to have no task assignment.*

The above two assumptions are reasonable according to the low cost of deploying mobile sensors from both group and individual perspectives. These assumptions enable us to remove the constraints on the total number of sensors and the number of tasks conducted by sensors, which are usually considered in the task allocation problem setup.

Next, we will provide detailed derivations of the triggering conditions, which are determined by the detection of multiple sources. The detection of multiple sources is based on the information differences between neighboring sensors, e.g. distance between sensors, gradient estimation, field value, etc. Among all the information, gradient estimation reflects the field variation and plays a key role in the detection.

Fig.1 presents two mobile sensors i, j in the field at time t_k as well as the estimated gradients $\nabla \hat{z}_i^k, \nabla \hat{z}_j^k$. When we plot the corresponding vector extensions, we can obtain the intersection point of the extended vectors, denoted as $\mathbf{s}_{ij}$. Define the following three angles

$$\begin{aligned} \theta_{ij}^1 &= \arccos\left(\frac{\langle \nabla \hat{z}_i^k, \nabla \hat{z}_j^k \rangle}{\|\nabla \hat{z}_i^k\|_2 \cdot \|\nabla \hat{z}_j^k\|_2}\right) \in (0, \pi], \\ \theta_{ij}^2 &= \arccos\left(\frac{\langle \nabla \hat{z}_i^k, \mathbf{r}_j^k - \mathbf{r}_i^k \rangle}{\|\nabla \hat{z}_i^k\|_2 \cdot \|\mathbf{r}_j^k - \mathbf{r}_i^k\|_2}\right) \in (0, \pi], \\ \theta_{ij}^3 &= \arccos\left(\frac{\langle \mathbf{r}_i^k - \mathbf{r}_j^k, \nabla \hat{z}_j^k \rangle}{\|\mathbf{r}_i^k - \mathbf{r}_j^k\|_2 \cdot \|\nabla \hat{z}_j^k\|_2}\right) \in (0, \pi], \end{aligned}$$

as shown in Fig. 1.

We provide two cases regarding the relative position of the two estimated gradients:

- **Case 1** The extended vectors and vector $\mathbf{r}_i^k - \mathbf{r}_j^k$ form a closed area, and gradient estimations $\nabla \hat{z}_i^k, \nabla \hat{z}_j^k$ are part of the edges, as shown in Fig.1(a). The vectors $\theta_{ij}^1, \theta_{ij}^2, \theta_{ij}^3$ satisfy the following equation

$$\Theta_{ij} \triangleq \theta_{ij}^1 + \theta_{ij}^2 + \theta_{ij}^3 = \pi, \quad (10)$$

which is based on the angle sum property of a triangle.

- **Case 2** The extended vectors and vector $\mathbf{r}_i^k - \mathbf{r}_j^k$ form a closed area, but gradient estimations $\nabla \hat{z}_i^k, \nabla \hat{z}_j^k$ are not part of the edges, as shown in Fig.1(b)(c). The vectors $\theta_{ij}^1, \theta_{ij}^2, \theta_{ij}^3$ satisfy

$$\begin{aligned} \Theta_{ij} &\triangleq \theta_{ij}^1 + \theta_{ij}^2 + \theta_{ij}^3 \\ &= \pi + 2 \min\{\theta_{ij}^1, \theta_{ij}^2, \theta_{ij}^3\} > \pi, \end{aligned} \quad (11)$$

where two of three angles are exterior angles of a close triangle while the last one is an interior angle of the triangle.

According to **Assumption IV.1**, **Case 1** indicates that the directions of estimated gradients of sensors i, j are pointing towards the same source, while **Case 2** indicates that the directions of estimated gradients of sensors i, j are pointing towards different sources.

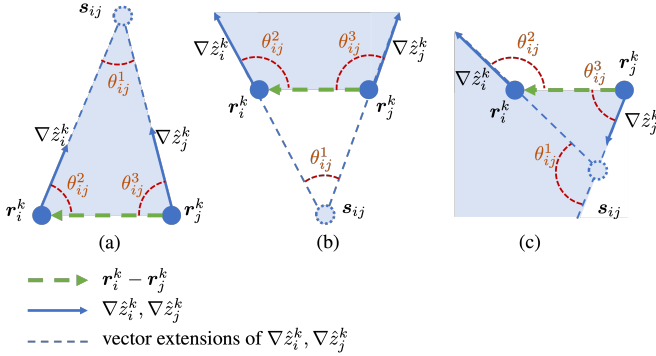


Fig. 1. Illustration of tracking the same source (a) and different sources (b)(c) among two sensors i, j , where the blue solid circles represent the sensor locations and the dashed ones represent the intersections between extended vectors.

When **Case 2** happens, different sources are detected by mobile sensors, which implies the necessity of group division among sensors.

Define an indicator function $1_\pi(x)$ as

$$1_\pi(x) = \begin{cases} 1, & \text{if } x > \pi, \\ 0, & \text{if } x \leq \pi. \end{cases} \quad (12)$$

The triggering condition for running opinion dynamics in mobile sensor networks can be characterized as

$$\sum_{i,j=1}^N 1_\pi(\Theta_{ij}) > 0, \quad (13)$$

which means that at least one $\Theta_{ij} = 1$ and at least two sensors are tracking different sources.

Then to obtain the number of sources N_s , we need to find a loop that visits the boundary agents of the sensor network that form the largest convex area covering the whole sensor network. Denote the edge set induced by the loop as $\mathcal{E}$, and the number of sources can be calculated as

$$N_s = \sum_{(i,j) \in \mathcal{E}} 1_\pi(\Theta_{ij}). \quad (14)$$

V. OPINION-BASED TASK ALLOCATION STRATEGY FOR MOBILE SENSOR NETWORKS

In this section, we will provide the opinion-based task allocation strategy for mobile sensor networks in a spatial-varying field, after the triggering conditions are satisfied.

With different control inputs u_i , the opinion states will perform various convergence behaviors driven by the dynamics in (4). The choice of control input is based on the number of detected sources, obtained by Algorithm 1. The number of detected sources N_s determines the number of different opinion states at equilibrium. In [17], we have developed a

generalized Kuramoto model for opinion dynamics on the sphere, where the control input can be written as

$$u_i^{N_s} = \sum_{j \in \mathcal{N}_i} \frac{1}{k} g'_{N_s}(\mathbf{o}_i^\top \mathbf{o}_j) \mathbf{o}_j, \quad (15)$$

where $g'_{N_s}(x) \triangleq \frac{dg_{N_s}(x)}{dx}$, and $g_{N_s}(x)$ is the first kind Chebyshev polynomial [21] with degree N_s defined by

$$\begin{aligned} g_{N_s}(x) &= \cos(N_s \arccos x) \\ &= \sum_{l=0}^{\lfloor \frac{N_s}{2} \rfloor} (-1)^l \binom{N_s}{2l} (1-x^2)^l x^{N_s-2l}, x \in [-1, 1]. \end{aligned} \quad (16)$$

Under the Kuramoto-type control input $u_i^{N_s}$, the opinion states will converge to N_s different groups with stability guarantee. Such convergence result naturally divides sensors into different groups and provides a solution to minimize the opinion difference within each group. Sensors within the same group will be assigned to the same source-seeking task. The choice of source follows the nearest-source strategy and avoids repetition as well as contradiction.

Algorithm 1 Multi-Source Detection

Initialize the number of sources $N_s = 1$;

- 1: **while** $i, j = 1, \dots, N$ **do**
 - 2: Calculate Θ_{ij} ;
 - 3: **end while**
 - 4: **if** $\sum_{i,j=1}^N 1_\pi(\Theta_{ij}) > 0$ **then**
 - 5: Find a loop $\mathcal{E}$ that forms the largest convex area covering the whole sensor network;
 - 6: $N_s = \sum_{(i,j) \in \mathcal{E}} 1_\pi(\Theta_{ij})$;
 - 7: **else** $N_s = 1$;
 - 8: **end if**
-

Remark V.1 Algorithm 1 also provides one scenario of $N_s = 1$, which can merge different groups back into one larger group when grouping assignments are no longer valid. This makes the proposed algorithm work efficiently not only for dividing sensors into smaller groups but also for reforming a large sensor network. This adds flexibility properties to the task allocation of mobile sensor networks. Regrouping of larger groups will be in the future work of this paper.

According to Algorithm 2, once the triggering conditions are satisfied, the opinion states will be initialized and opinion dynamics will be run in a faster time scale τ to assign mobile sensors into groups to track different sources based on the convergence results.

Assumption V.2 Mobile sensors with the same task assignment, share all-to-all communication within the group.

We utilize a gradient-based strategy for the velocity control of the mobile sensor network for multi-source seeking

$$\begin{aligned} \mathbf{r}_i^{k+1} &= \mathbf{r}_i^k + \mathbf{v}_i^k, \\ \mathbf{v}_i^k &= a \nabla \hat{z}_i^k + b \mathbf{f}_i^k, \end{aligned} \quad (17)$$

Algorithm 2 Opinion-based Task Allocation Strategy

For each mobile sensor $i, i = 1, \dots, N$:

Initialize the estimated information, $\{\hat{z}_i^0, \nabla \hat{z}_i^0\}_{i=1, \dots, N}$;

Initialize the field measurements, $\{p_i^1\}_{i=1, \dots, N}$;

Initialize the locations of mobile sensors, $\{r_i^1\}_{i=1, \dots, N}$;

Initialize time step $k = 1$;

- 1: **while** true **do**
 - 2: Receive measurements $\{p_j^k\}_{j \in N_i}$ from neighboring mobile sensors and generate estimated field value and gradient information $\hat{z}_i^k, \nabla \hat{z}_i^k$;
 - 3: Communicate the information state I_i^k to the neighboring sensors $j \in \mathcal{N}_i$, and check the triggering conditions for opinion dynamics;
 - 4: **if** Triggering condition (13) is satisfied **then**
 - 5: run opinion dynamics in (4) with control input in (15), and form new sensor groups and task assignments;
 - 6: **else**
 - 7: keep previous groups and task assignments;
 - 8: **end if**
 - 9: Move to the next mobile sensor locations, r_i^{k+1} ;
 - 10: $k = k + 1$;
 - 11: **end while**
-

where a, b are tuning parameters. The formation terms f_i^k are defined as

$$f_i^k = \sum_{i' \neq i} \frac{\|r_{i'}^k - r_i^k\|_2 - d_{ii'}^k}{\|r_{i'}^k - r_i^k\|_2} (r_{i'}^k - r_i^k), \quad (18)$$

where $d_{ii'}^k$ is a desired separation distance between sensors i and i' .

VI. SIMULATION RESULTS

In this section, we provide simulation results using the proposed dynamic group strategy for mobile sensor networks in multi-source seeking.

We consider a 2-dimensional scenario where the field is modeled by the following equation in $\mathbb{R}^2$

$$z(r, t) = a \sin(\omega_1 r_1) \sin(\omega_2 r_2), \quad (19)$$

where $r = [r_1, r_2]^T \in \mathbb{R}^2$, and $a, \omega_1, \omega_2 \in \mathbb{R}$ are tuning parameters. In this simulation setup, the parameters are chosen as $a = 100, \omega_1 = \frac{1}{40}, \omega_2 = \frac{1}{50}$, and the mobile sensors are deployed with region $r_1 \in [-100, 100], r_2 \in [-100, 100]$ for 1500 time steps with time interval $\Delta t = 1s$.

The size of the mobile sensors is relatively small compared with the field of interest, and these sensors are modeled as points moving in the field. A group of 12 mobile sensors are initially released within the region $\Omega_0 = \{r = [r_1, r_2]^T \in \mathbb{R}^2 | r_1 \in [-2, 13], r_2 \in [-11, 1]\}$ of the field collecting measurements of the field and generating estimation information (field value and gradient), as shown in Fig. 2. Meanwhile, mobile sensors are driven by the gradient-based velocity control law (17) using the estimated gradient information.

The initialization process takes 50 time steps, and the estimated information converges to the true one even with a

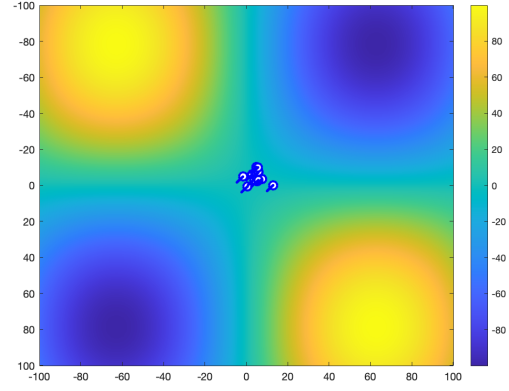


Fig. 2. The initialization process of a 12-sensor network in a spatial-varying field described by (19) within first 50-time steps, where the blue lines and blue circles represent the trajectories and the current locations of mobile sensors, respectively.

bad initial guess of the information, as shown in Fig. 3 where the estimated information is represented by small circles and the true value is denoted by smooth lines.

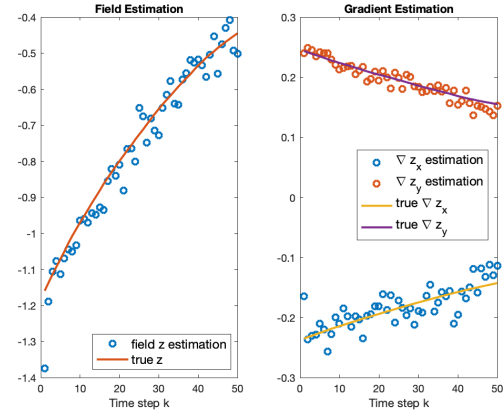


Fig. 3. Estimated information generated by mobile sensor network during the initialization process of first 50 time steps. The left figure represents the field estimation and the right one represents the gradient estimation, where the estimated information is represented by small circles and the true value is denoted by smooth lines.

After the initialization process, two sources are detected, indicating that the mobile sensors need to be divided into two separate groups, $N_s = 2$. In the opinion-developing function (3), we choose matrices $C_1 = I_{2 \times 2}, C_2 = 0.1I_{2 \times 2}, C_3 = 100I_{2 \times 2}$ to form the big transition matrix C with the structure in (9), where $I_{2 \times 2} \in \mathbb{R}^{2 \times 2}$ is the identity matrix. The developed opinion states are unit-length vectors in $\mathbb{R}^6$.

Once the opinions are developed, mobile sensors are required to run opinion dynamics to be divided into $N_s = 2$ groups. Fig. 4 shows the evolution of the inner products $\{\langle o_1, o_i \rangle\}_{i=1}^{12}$ towards $\{\pm 1\}$, which shows the transition of opinion states from non-equilibrium to bipartite dissensus equilibrium while evolving on the unit circle. The convergence result separates the mobile sensors into

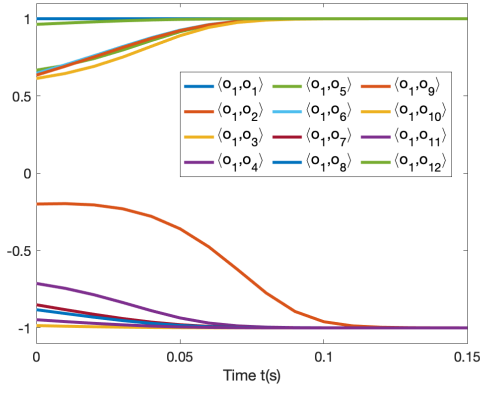


Fig. 4. The evolution of the inner products $\{\langle \mathbf{o}_1, \mathbf{o}_i \rangle\}_{i=1}^{12}$ towards $\{\pm 1\}$.

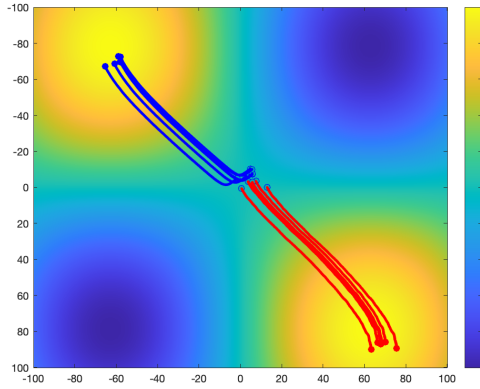


Fig. 5. Mobile sensors move in two different groups towards two different sources, represented by blue and red colors respectively.

2 different groups of $G_1 = \{1, 5, 6, 9, 10, 12\}$ and $G_2 = \{2, 3, 4, 7, 8, 11\}$. It takes less than $0.15s$ for the mobile sensors to achieve convergence results, which is less than the time interval $\Delta t = 1s$. This demonstrates the fast convergence speed and high efficiency of applying opinion dynamics for grouping in mobile sensor networks.

After forming different groups, mobile sensors take another 1450 time steps to track 2 different sources. Sensors in each group move in similar directions and move towards the same source. Additionally, sensors from different groups move in different directions (close to opposite directions in Fig.5), and are able to conduct different source-tracking tasks separately at the same time. Both groups gradually move to the source locations.

VII. CONCLUSIONS AND FUTURE WORK

In this paper, we propose an opinion-based task allocation strategy for mobile sensor networks in a spatial-varying field. This method enables the application of a large number of mobile sensors for simultaneous multi-source tracking with dynamic grouping, and also avoids solving optimization problems on utility costs. In our future work, we will generalize this strategy for distributed mobile sensor networks

which are more welcome in real-world scenarios.

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