2024 American Control Conference (ACC)
July 8-12, 2024. Toronto, Canada

Decentralized Optimal Merging Control for Mixed Traffic with Vehicle Inference

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Abstract— **This paper addresses the optimal control of vehicles arriving from two curved roads at a merging point where the objective is to jointly minimize the travel time, energy consumption, and passenger discomfort. Unlike prior work where traffic consists entirely of Connected and Automated Vehicles (CAVs), we consider optimal controls for CAVs in mixed traffic including Human-Driven Vehicles (HDVs) behaving according to some car-following model that includes random actions. The control applied to CAVs is based on partial information about the presence and states of HDVs which is inferred from local observations available to the CAVs. The passing order of HDVs at the merging point is determined and assisted by CAVs using a proposed Minimum-Effort Merging Contract (MEMC) that uses Control Barrier Functions (CBFs) to guarantee safety. A coordinator is used to manage both the CAV information and inferred HDV information such that the problem can still be solved in a decentralized way. Our approach first determines an analytically tractable unconstrained optimal solution. We then use the joint Optimal Control and Barrier Function (OCBF) method to obtain a controller which optimally tracks such a solution while also guaranteeing all safety and control constraints, including a safe merging contract between CAVs and HDVs. Simulation examples are included to compare the performance under different CAV penetration rates.**

I. INTRODUCTION

Traffic management at merging points (usually, highway on-ramps) is one of the most challenging problems within a transportation system in terms of safety, congestion, and energy consumption, in addition to being a source of stress for many drivers [1], [2], [3]. Advancements in next-generation transportation system technologies and the emergence of CAVs have the potential to drastically improve a transportation network's performance by better assisting drivers in making decisions, ultimately reducing energy consumption, air pollution, congestion and accidents.

A number of decentralized merging control mechanisms have been proposed [4], [5], [6], [7], [8]. In this case, all computation is performed on-board each vehicle and shared only with a small number of other vehicles which are affected by it. Optimal control problem formulations are used in some of these approaches, while Model Predictive Control (MPC) techniques are employed as an alternative, primarily to account for additional constraints and to compensate for disturbances by re-evaluating optimal actions [9], [10]. An alternative to MPC is provided in [11] by the use of Control Barrier Functions (CBFs).

Most works for traffic merging control mentioned above are based on the assumption that all traffic consists of CAVs. Clearly, in the foreseeable future, CAVs will coexist with Human-Driven Vehicles (HDVs), and such a mixed traffic setting presents a new challenge for optimizing system performance. Most existing works assume some models for HDVs [12] [13] [14]; however, this can be unrealistic as human driving behaviors are usually unknown and hard to accurately predict. On-board CAV sensors can be used to detect HDVs [15], but this is not possible when some HDVs are hidden by other vehicles making them hard to detect. Reinforcement/machine learning can be used for mixed traffic without the HDV model information [16] [17], but there are no safety guarantees. Moreover, the works mentioned above ignore the *curvature of the roads* which is often the case, especially in highway interchanges. Such curvature induces additional nonlinear safety constraints making it important to take into account centrifugal comfort and lateral rollover avoidance. A joint framework for travel time, energy consumption, and comfort in optimal merging control has been studied in [18], but it is based on the assumption of an all-CAV setting.

In this paper, we consider the traffic merging optimal control problem in a *mixed traffic* setting, in which case the driving policies of all HDVs are random and unknown. In order to account for "aggressive" HDVs, we infer the existence of HDVs and their states based on whatever information is available to CAVs, assuming that a rear-end safety constraint is satisfied among all vehicles. We show that in this setting we can still solve the problem in a decentralized way, as in [18], for all CAVs. We also propose a Minimum-Effort Merging Contract (MEMC) for the HDV passing order at the merging point using CBFs. This contract is enforced by CAVs so that the merging of HDVs is assisted by CAVs. Our simulation studies show that, with HDV inference, CAVs can guarantee safety in a mixed traffic setting, while HDVs tend to get close to each other when crossing the merging point. A simulation study of an actual curved road merging problem that arises in a Boston area segment of the Massachusetts Turnpike is included. Our results show that significant improvements in both CAV and HDV performance metrics can be achieved with the proposed framework, under different CAV penetration rates, compared to a baseline consisting exclusively of HDVs.

This work was supported in part by NSF under grants ECCS-1931600, DMS-1664644, CNS-1645681, CNS-2149511, and by ARPA-E under grant DE-AR0001282 (the statements made herein are solely the responsibility of the authors).

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II. PROBLEM FORMULATION

The merging problem arises when traffic must be joined from two different roads, usually associated with a main road and a merging road as shown in Fig. 1. We consider the case where the traffic consists of both CAVs and HDVs randomly arriving at the two curved roads joined at the Merging Point (MP) M where a lateral collision may occur. We assume all CAVs and HDVs are equipped with on-board sensors to detect local vehicle information (however, this cannot be communicated by HDVs). The segment from the origin O or O' to the merging point M has a length L for both roads and radii $r_{main} > 0, r_{merg} > 0$ for the main and merging lanes, respectively, and is called the Control Zone (CZ). A coordinator is associated with the MP whose function is to maintain a First-In-First-Out (FIFO) queue of CAVs based on their arrival time at the CZ and enable real-time communication with the CAVs in the CZ including the last one leaving it. The coordinator is also employed in managing the inferred information regarding all HDVs. Thus, there are some HDVs whose presence is directly detected, as well as some "inferred" HDVs.

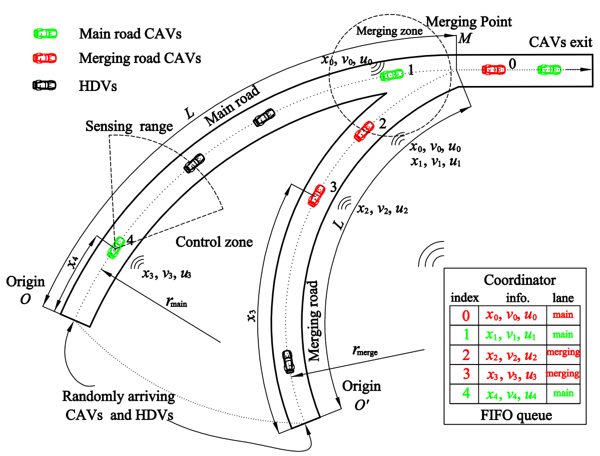

Fig. 1. Mixed traffic merging problem for roads with curvature without vehicle inference. Some HDVs are hard to be detected by CAVs. However, as all vehicles approach the MP, they can detect each other.

Let $S(t)$ be the set of FIFO-ordered (acccording to arrival times at O or O') indices of all CAVs located in the CZ at time t along with the CAV (whose index is 0 as shown in Fig.1) that has just left the CZ. Let $N(t)$ be the cardinality of $S(t)$. Thus, if a CAV arrives at O or O' at time t, it is assigned the index $N(t)$. All CAV indices in $S(t)$ decrease by one when a CAV passes over the MP and the vehicle whose index is -1 is dropped.

The vehicle dynamics for each CAV $i \in S(t)$ along the lane to which it belongs take the form

$$\dot{x}_i(t) = v_i(t), \qquad \dot{v}_i(t) = u_i(t), \tag{1}$$

where $x_i(t)$ denotes the distance to the origin O (O') along the main (merging) lane if the vehicle i is located in the main (merging) lane, $v_i(t)$ denotes the velocity, and $u_i(t)$ denotes the control input (acceleration). $\boldsymbol{x}_i = (x_i, v_i)$. We consider three objectives for each CAV subject to four constraints, as detailed next.

Objective 1 (Minimizing travel time): Let t_i^0 and t_i^m denote the time that CAV $i \in S(t)$ arrives at the origin O or O' and the merging point M, respectively. We wish to minimize the travel time $t_i^m - t_i^0$ for CAV i.

Objective 2 (Minimizing energy consumption): We also wish to minimize energy consumption for each CAV $i \in S(t)$:

$$\min_{u_i(t)} \int_{t_i^0}^{t_i^m} C_i(u_i(t))dt, \tag{2}$$

where $C_i(\cdot)$ is a strictly increasing function of the norm of its argument, and it usually takes the quadratic form: $C_i(u_i(t)) = u_i^2(t)$.

Objective 3 (Maximizing centrifugal comfort): In order to minimize the centrifugal discomfort (or maximize the comfort) for each CAV, we wish to minimize the centrifugal acceleration

$$\min_{u_i(t)} \int_{t_i^0}^{t_i^m} \kappa(x_i(t))v_i^2(t)dt, \tag{3}$$

where $\kappa : \mathbb{R} \to \mathbb{R}^{\geq 0}$ is the curvature of the road at position x_i. The curvature usually has a sign, but we assume it is always positive. It is determined by $\frac{1}{r(x_i)}$, where $r : \mathbb{R} \to \mathbb{R}$ is the radius of the road at x_i.

Constraint 1 (Safety constraints): Let i_p denote the index of the vehicle which physically immediately precedes i in the CZ (if one is present). We require that the distance $z_{i,i_p}(t) := x_{i_p}(t) - x_i(t)$ be constrained by the speed $v_i(t)$ of vehicle $i \in S(t)$ so that

$$z_{i,i_p}(t) \geq \varphi v_i(t) + \delta, \quad \forall t \in [t_i^0, t_i^m], \tag{4}$$

where φ denotes the reaction time (as a rule, $\varphi = 1.8$ is used, e.g., [19]).

Constraint 2 (Safe merging): There should be enough safe space at the MP M for a vehicle (which eventually becomes CAV 1, as shown in Fig. 1) to cut in, i.e.,

$$z_{1,0}(t_1^m) \geq \varphi v_1(t_1^m) + \delta. \tag{5}$$

Constraint 3 (Vehicle limitations): There are constraints on the speed and acceleration for each CAV $i \in S(t)$:

$$\begin{aligned} v_{i,min} \leq v_i(t) \leq v_{i,max}, \quad \forall t \in [t_i^0, t_i^m], \\ u_{min} \leq u_i(t) \leq u_{max}, \quad \forall t \in [t_i^0, t_i^m], \end{aligned} \tag{6}$$

where $v_{i,max} > 0$ and $v_{i,min} \geq 0$ denote the maximum and minimum speed allowed in the CZ, while $u_{min} < 0$ and $u_{max} > 0$ denote the minimum and maximum control, respectively.

Constraint 4 (Lateral safety constraint): Finally, there is a constraint on the centrifugal acceleration to avoid lateral rollover for each CAV $i \in S(t)$:

$$\kappa(x_i(t))v_i^2(t) \leq \frac{w_i^h}{h_i} g, \quad \forall t \in [t_i^0, t_i^m], \tag{7}$$

where $w_i^h > 0$ denotes the half-width of the vehicle, $h_i > 0$ denotes the height of the center of gravity with respect to the ground, and g is the gravity constant. The above lateral safety constraint is obtained through the Zero Moment Point (ZMP) [20] method (assuming the road lateral slope is zero) that balances the CAV considering both gravity and inertia.

Problem Formulation. Our goal is to determine a control law to achieve objectives 1-3 subject to constraints 1-4 for each CAV $i \in S(t)$ governed by the dynamics (1). We can formulate a joint optimal control problem based on the three objectives and normalize each term, as shown in [18].

Note that this problem is formulated only for CAVs, since they are the only ones subject to control. Nonetheless, the performance of HDVs may also be improved with the partial penetration of CAVs (as illustrated in our simulation results). However, to ensure that the problem is well-posed, we need to make some assumptions regarding the behavior of HDVs, starting with the following.

Assumption 1: All HDVs always satisfy the rear-end safety constraint (4) with their preceding vehicle in the CZ under the same conditions as CAVs (i.e., φ, δ in (4) are the same for all vehicles).

The above assumption may affect the performance of CAVs if it is violated, but it will not affect safety guarantees as this assumption is only used to infer the number of HDVs between CAVs.

HDV types. Regarding the overall behavior of HDVs, there are three types one can consider. (i) *Smart HDVs without communication capabilities*, in which case the HDVs have on-board sensors to assist them in determining the passing order at the MP. (ii) *Communication-enabled HDVs*, so that they can read and follow signals from CAVs (such as who should go first when reaching the MP) through V2V or V2I/I2V links. (iii) *Non-cooperative HDVs*, in which case HDVs make their own decisions, and all CAVs have to ensure the safe merging constraint (5) with no HDV cooperation. We also assume that HDVs are operated by human drivers using some random policies which are unknown to CAVs.

III. Optimal Control Framework with Inference

If all the vehicles in the CZ are CAVs, then the problem can be locally solved by each CAV i (see [18]). However, if there are any HDVs in the CZ, we cannot solve the merging problem using the above framework as some information from vehicles which are HDVs (and correspond to i_p or $i-1$ above) may be missing. In order to address this, we proceed by using local information to infer all (global) vehicle information, as shown in the next section.

A. HDV Information Inference

In a mixed traffic merging problem, we must rely on local CAV information to infer the presence and states of all vehicles in the CZ. Although it is possible to have infrastructure at the MP capable of sensing vehicles, we will not rely on any such assumption, especially since the road curvatures would limit this anyway. For any CAV $i \in S(t)$, it will share its own information with the coordinator. However, as the CAV has on-board sensors, it can also detect the presence and estimate the state of its preceding vehicle (possibly a HDV, e.g., the one before CAV 4 in Fig. 1), as well as detect the presence and estimate the state of vehicles from the other road when they are both close to the MP (e.g., CAV 1 can detect vehicle 2, even if it were a HDV, in Fig. 1). Therefore, each CAV i can share its own information and local information (such as its i_p and information on some vehicles in the other road when CAV i is close to the MP) with the coordinator. In this way, the coordinator can include some HDV information in its queue, but not all since some HDVs (such as the preceding vehicle of HDV i_p if it too is a HDV) may be hidden by other vehicles or obstacles.

In order to figure out hidden HDVs that cannot be directly detected, we proceed by inferring HDV information based on the local CAV information present in the coordinator table. First, at time t, we can get existing known information from all CAVs in the CZ, and then update the ordered set $S(t)$. This set includes information from CAVs and their preceding vehicles which may be HDVs. Therefore, there may already be some HDVs entered in $S(t)$.

After obtaining the preliminary ordered set $S(t)$, we can conduct information inference consistent with Assumption 1. For any vehicle $i \in S(t)$, there may be some HDVs that cannot be detected ahead of it (e.g., the vehicle behind CAV 1 in Fig. 1). However, as long as these HDVs maintain some safe distance following Assumption 1, then we can estimate the speed of some HDV j between i and i_p by:

$$\hat{v}_j(t) = P_v(\boldsymbol{x}_i(t), \boldsymbol{x}_{i_p}(t)), \tag{8}$$

where $P_v : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ is some prediction function (such as from a machine learning model). A simple choice for $P_v(\cdot,\cdot)$ is the average function of $v_i(t)$ and $v_{i_p}(t)$. An example of this process in Fig. 1 arises with $i = 4$ and $i_p = 1$: the HDV ahead of i is directly detected, but not the HDV ahead of it. The index j applies to either HDV for which a speed estimate is obtained through (8). If there is no i_p for i, then we simply remove the dependence on i_p in (8).

Following Assumption 1, the number of possible HDVs $N_i(t)$ at time t between i and i_p can then be predicted by:

$$N_i(t) = \max\left(\text{floor}\left(\frac{z_{i,i_p}(t) - \varphi v_i(t) - \delta}{\varphi \hat{v}_j(t) + \delta}\right), 0\right) \tag{9}$$

where floor$(\cdot)$ takes the maximum integer that is smaller than or equal to its input.

After predicting the number of HDVs between i and i_p, we can also estimate the positions of these HDVs that satisfy the rear-end safety constraint (4) through some $P_x : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ using a similar approach as (8) based on the state information of i and i_p. Once this is done, we incorporate the inferred HDVs into $S(t)$, update all indices, and resequence all vehicles in $S(t)$. This results in an inferred queue table. As an example, the original queue table shown in Fig. 1 results in the updated table shown in Fig. 2. Due to the information inference, the queue table shown in Fig. 1 is updated at each time step t in addition to times when a CAV arriving or exiting the CZ event occurs (the only required update time in a full-CAV traffic setting). At each time step, we get an inferred queue table that allows us to derive optimal controls for all CAVs as detailed in the next few sections.

Inferred Table

index	info.	lane	type	source
0	x_0, v_0, u_0	main	CAV	Wireless
1	x_1, v_1, u_1	main	CAV	Wireless
2	x_2, v_2, u_2	merging	CAV	Wireless
3	$\hat{x}_3, \hat{v}_3, ?$	main	HDV	Inferred
4	x_4, v_4, u_4	merging	CAV	Wireless
5	x_5, v_5, u_5	main	HDV	Detected
6	x_6, v_6, u_6	main	CAV	Wireless
7	$\hat{x}_7, \hat{v}_7, ?$	merging	HDV	Inferred

FIFO queue

Fig. 2. Information inferred queue table. All HDVs have random control policies that are unknown. HDVs 3 and 7 may not actually exist. $\hat{x}_i, \hat{v}_i$ denote estimated HDV states. "Wireless" denotes wireless communication with the coordinator.

Merging Zone. It is important to point out that the uncertainty regarding the existence of inferred HDVs is eliminated as all vehicles approach the MP, since their relative physical distance becomes limited and on-board sensors can detect the existence of other vehicles, as long as at least one CAV is present. We define this as a *Merging Zone* (MZ), denoted by the dotted circle shown in Fig. 1. If there is no CAV present in the MZ, then HDVs are left to deal with safe merging constraints as in current practice.

B. Safe Merging Contract between HDVs and CAVs

When HDVs and CAVs approach the MZ (defined above), all vehicles must follow the same safe merging contract in order to guarantee collision avoidance. Within the MZ, on-board sensors can detect the existence of other vehicles, as well as their state values. We start with the case where sensors can accurately measure the states of other vehicles, and then discuss how to deal with measurement uncertainty.

1) Perfect Information case: There are three cases to consider: (i) The first two vehicles approaching the MP from the main and merging roads, respectively, are CAVs. In this case, there is no need to formulate another safe merging contract between them as these two CAVs will consider the safe merging constraint (5) upon arriving at the origins (as in our full CAV setting in [18]). Note that even if there are some HDVs between these two CAVs, their passing order can still be the same. (ii) The first two vehicles are HDVs. Then, we have no choice but to let the human drivers figure out the safe merging contract. (iii) One vehicle is a HDV and the other is a CAV. This is the only case of interest, considered in what follows. Let their indices be i, j, observing that whether i or j is a HDV makes no difference.

The "safe merging contract" could be any agreement between CAVs and HDVs, such as a priority-based contract (e.g., main road vehicle goes first), or a nearest-vehicle-to-MP contract (the one who is closer to the MP goes first). This is the same as any traffic rule agreed upon and honored by HDVs in today's transportation network (e.g., "vehicle from the right has priority in crossing an unsignalized intersection"). In this paper, we propose a Minimum-Effort Merging Contract (MEMC) that is based on $u_i^2(t)$ at time t (a similar formulation as the energy consumption in (2)). A MEMC considers the merging effort from the perspective of CBFs, as we will employ the joint optimal control and barrier function (OCBF) method to solve the problem online, as shown in the sequel.

Let t_s be the time when vehicles i and j can recognize each other upon entering the MZ. We define two CBFs b_{ij} and b_{ji} corresponding to whether j or i goes first, respectively, for the safe merging constraint (5):

$$\begin{aligned} b_{ij}(\boldsymbol{x}_i(t), \boldsymbol{x}_j(t)) &= z_{i,j}(t) - \varphi v_i(t) - \delta, \\ b_{ji}(\boldsymbol{x}_i(t), \boldsymbol{x}_j(t)) &= z_{j,i}(t) - \varphi v_j(t) - \delta, \end{aligned} \tag{10}$$

where $z_{i,j} = x_j - x_i$, $z_{j,i} = x_i - x_j$. Using the standard CBF approach [21], each constraint above of the general form $b(\boldsymbol{x}(t)) \geq 0$ is mapped onto a new constraint which is linear in the control input $u_i(t)$ and takes the general form

$$L_f b(\boldsymbol{x}(t)) + L_g b(\boldsymbol{x}(t)) u_i(t) + \gamma(b(\boldsymbol{x}(t))) \geq 0, \tag{11}$$

where L_f, L_g denote the Lie derivatives of $b(\boldsymbol{x}(t))$ along f and g respectively for any control-affine dynamic system $\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u}$ and $\gamma(\cdot)$ stands for any class-$\mathcal{K}$ function. It has been established that satisfaction of (11) implies the satisfaction of the original problem constraint $b(\boldsymbol{x}(t)) \geq 0$ because of the forward invariance property [21].

Applying (11) to (10) with the system dynamics (1) and a linear $\gamma(b(\boldsymbol{x}(t))) = kb(\boldsymbol{x}(t))$, the control $u_i(t)$ or $u_j(t)$ should satisfy the following constraints:

$$\begin{aligned} u_i(t) &\leq \psi_{ij}(\boldsymbol{x}_i(t), \boldsymbol{x}_j(t)), \\ u_j(t) &\leq \psi_{ji}(\boldsymbol{x}_i(t), \boldsymbol{x}_j(t)), \end{aligned} \tag{12}$$

where $\psi_{ij}(\cdot) = (1/\varphi)[v_j(t) - v_i(t) + kb_{ij}(\boldsymbol{x}_i(t), \boldsymbol{x}_j(t))]$, $\psi_{ji}(\cdot) = (1/\varphi)[v_i(t) - v_j(t) + kb_{ji}(\boldsymbol{x}_i(t), \boldsymbol{x}_j(t))]$, $k > 0$. The ψ_{ij} or ψ_{ji} functions can actually be viewed as "risk functions" for a merging collision, and they include all the necessary system state information. We will show how these may be used to determine the passing order between CAVs and HDVs in the sequel. Note that the right hand sides of the above equations are usually negative at time t_s as the following vehicle has to decelerate in order to satisfy the safe merging constraint which may be violated at t_s. We assume the CBF constraints (12) do not conflict with the control bound in (6); otherwise, we have to relax either (12) or (6) in order to guarantee safety. The CBFs (10) are very likely to be negative at t_s without vehicle inference as both i, j would be aggressive towards the MP. However, with vehicle inference, a CAV will always consider making space before reaching the MP with inferred vehicles (even though they may not exist). Thus, we adopt a more conservative guarantee for the safe merging constraint (5), which demonstrates the importance of vehicle inference.

Proposition 1: The MEMC at t_s is to let vehicle $k(t_s)$ go first at the MP, where

$$k(t_s) = \begin{cases} j, \text{ if } \psi_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j)|_{t=t_s} > \psi_{ji}(\boldsymbol{x}_i, \boldsymbol{x}_j)|_{t=t_s} \\ i, \text{ if } \psi_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j)|_{t=t_s} < \psi_{ji}(\boldsymbol{x}_i, \boldsymbol{x}_j)|_{t=t_s} \end{cases}, \tag{13}$$

In the corner case when $\psi_{ij}(\boldsymbol{x}_i(t), \boldsymbol{x}_j(t))|_{t=t_s} = \psi_{ji}(\boldsymbol{x}_i(t), \boldsymbol{x}_j(t))|_{t=t_s}$, the minimum control effort is the same whether i or j goes first at the MP. In this case, we can apply any other merging contract, such as the aforementioned priority-based contract.

In order to apply (13), it is important to consider which vehicle should evaluate the MEMC in Proposition 1. This depends on the type of HDV involved, recalling the three HDV types presented at the end of Section II. A type-(i) HDV has sensing capability so that both CAVs and HDVs can evaluate the MEMC in Proposition 1, and make a mutually consistent decision. A type-(ii) HDV can receive and follow signals from the CAV regarding who should go first at the MP. Thus, only the CAV can evaluate the MEMC in Proposition 1 and inform the HDV it shares the MZ with. A type-(iii) HDV ignores CAVs and will cross the MP by making its own decision. Thus, the CAV is the only one evaluating the MEMC in Proposition 1 and must also make a determination regarding the decision associated with HDVs.

While the first two cases are readily implementable, in case (iii) above the CAVs are also responsible for the potentially reckless decisions of HDVs (which can actually relieve the stress of human drivers). In order to consider the decision of HDVs, we propose the following approach.

Recurrent safe merging contract: The CAV (say i) makes an initial evaluation of the MEMC in Proposition 1 at t_s and makes a decision. If $k(t_s) = j$, then the CAV can simply let the HDV j cross the MP before it and this decision would remain unchanged as long as the HDV does not decide to decelerate. Thus, in general, the CAV will re-evaluate the condition (12) after some time $\Delta t > 0$. In either case, (the CAV or the HDV goes first at the MP), if $b_{ij}(\boldsymbol{x}_i(t_s+\Delta t), \boldsymbol{x}_j(t_s+\Delta t)) < b_{ij}(\boldsymbol{x}_i(t_s), \boldsymbol{x}_j(t_s))$ or $b_{ji}(\boldsymbol{x}_i(t_s+\Delta t), \boldsymbol{x}_j(t_s+\Delta t)) < b_{ji}(\boldsymbol{x}_i(t_s), \boldsymbol{x}_j(t_s))$, then the CAV reverses the merging order (between itself and the HDV) determined at t_s; otherwise, the CAV keeps its decision unchanged. As a HDV may change its mind at any time after t_s, the merging contract should evolve based on the real-time state and history trajectory of the HDV. If the order recursively changes, the CAV could yield to the HDV for safety, although this comes at the cost of performance. This is a problem that remains to be further studied.

The case of imperfect information can be considered similarly, and this is left for future work. Finally, we can solve the problem using the joint optimal control and barrier function (OCBF) method proposed in [11].

IV. SIMULATION EXAMPLES

We have used the Vissim microscopic multi-model traffic flow simulation tool as a baseline to compare with the optimal control approach we have developed. The car following model in Vissim simulates human psycho-physiological driving behavior. We have chosen a merging configuration that arises in highway I-90 (the Massachusetts Turnpike) in the Boston, USA, area, as shown in Fig. 3. All CAVs start to communicate with a coordinator (at the MP) in the resequencing/connection zone as shown in the figure.

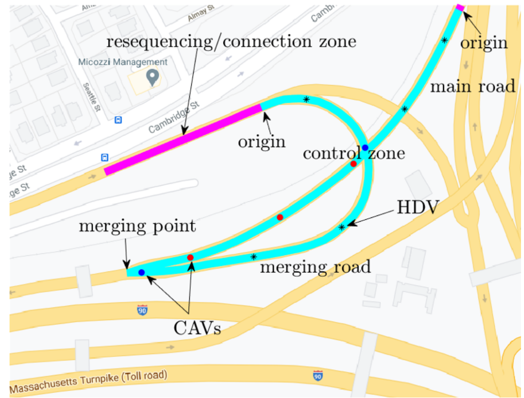

Fig. 3. A merging scenario of the I-90 Masschussets Turnpike, USA.

The parameters of the map are as follows: $L = 200m$, $\hat{\kappa} = \frac{1}{200}$ in the main lane, and $\hat{\kappa} = \frac{1}{50}$ in the merging lane. $v_{i,max} = 20m/s, \Delta = 0.1s, \varphi = 1.8s, v_{min} = 0m/s, u_{max} = -u_{min} = 0.4g, g = 9.81m/s^2, \alpha_1 = 0.3, \alpha_2 = 0.1$. The simulation under optimal control is conducted in MATLAB with the same arrival process input and initial conditions as in Vissim. Vehicles enter the CZ under a Poisson arrival process with initial speed in the range $6.5 - 12.5m/s$ at the origins, and whether a new arriving vehicle is a HDV or a CAV is randomly determined in accordance with the given CAV penetration rate. The MATLAB computation for the proposed framework is very efficient, i.e., less than $0.01s$ for each QP of the OCBF controller (Intel(R) Core(TM) i7-8700 CPU @ 3.2GHz×2).

The simulation results under two HDV driving policies compared to that in Vissim are summarized in Tables I-II. The traffic arriving rates in the main and merging lanes are 500 vehicles/h and 800 vehicles/h, respectively. We assume the HDVs can follow the merging signals from CAVs determined by the proposed MEMC in Prop. 1 with perfect information (i.e., type (ii) HDVs, and type (i) HDVs are similar in terms of the safe merging contract. We leave type (iii) for on-going work). The HDVs maintain a safe distance with their preceding vehicles with a random desired speed that uniformly takes value between $10m/s$ and $20m/s$.

When two HDVs compete at the MP and follow the FIFO rule, we set $\delta = 0m$ as vehicles will not stop at the MP. When a CAV competes with a HDV, the proposed MEMC is used. CAVs use a sensing range of 30m. The overall objective of CAVs improves about 50% with the proposed vehicle inference approach compared with Vissim, as shown in Table I. The travel time and energy consumption of CAVs are improved as the CAV penetration rate becomes larger. The energy consumption is a little worse than that of HDVs in Vissim. This is due to the random driving policies of HDVs. In this case, all CAVs can maintain a safe distance at the MP due to the vehicle inference, while the HDVs might stay very close to each other at the MP (see videos[1]). This demonstrates the advantage of vehicle inference at the cost

[1] https://sites.google.com/view/xiaowei2021/cav?authuser=0

TABLE I

COMPARISONS OF DIFFERENT CAV PENETRATION RATES; HDVS USE FIFO WITH RESPECT TO EACH OTHER

CAV fraction	Time(s) ↓	Comfort ↓	$\frac{1}{2}u_i^2(t)$ ↓	Obj. ↓
1.0 (prior*)	13.3404	44.2197	7.0785	68.3482
1.0	13.4248	43.8288	8.9633	70.4897
0.9	13.6568	43.5499	9.9823	72.2417
0.7	13.6411	42.8777	10.9780	73.1853
0.5	14.0733	42.5459	12.9866	76.5549
0.3	14.5235	44.8731	12.2254	77.2189
0.1	14.7556	32.6932	11.7607	77.4383
VISSIM	32.7797	22.4928	8.7537	140.1650

*With prior information that all vehicles are CAVs such that no inference is needed

Note: only counting for CAVs performance

TABLE II

COMPARISONS (CAV) OF DIFFERENT CAV PENETRATION RATES; HDVS ON MAIN ROAD HAVE PRIORITY OVER HDVS ON MERGING ROAD

CAV fraction	Time(s) ↓	Comfort ↓	$\frac{1}{2}u_i^2(t)$ ↓	Obj. ↓
1.0 (prior*)	13.3404	44.2197	7.0785	68.3482
1.0	13.0505	44.8097	17.3266	77.6417
0.9	13.8918	43.4173	20.5457	83.7673
0.7	15.6416	40.7137	25.7666	95.2678
0.5	16.4640	40.4495	28.8703	101.2967
0.3	16.7129	39.5317	29.4822	102.8022
0.1	17.1452	38.6328	30.8907	105.7633
VISSIM	32.7797	22.4928	8.7537	140.1650

*With prior information that all vehicles are CAVs such that no inference is needed

Note: only counting for CAVs performance

of some performance loss (shown by the rows "1.0 (prior)" and "1.0" in Table I).

When two HDVs compete at the MP and follow the priority rule, the one from the merging road yields to the HDV from the main road (assume the HDV observation distance is 50m) and it may stop at the MP. Thus, we set $\delta = 2m$. Once again, when a CAV competes with a HDV, the proposed MEMC is used. The performance of CAVs is worse than the one under the FIFO rule, as shown in Table II. Since randomness (hence congestion) is much higher under the priority rule than under the FIFO rule, we average the results of 20 simulations. When the CAV penetration rate is 0.1, the objective of CAVs is close to that of Vissim. The energy consumption is worse than the one in Vissim; this is due to the randomness of HDVs. Nonetheless, the throughput (travel time) of HDVs in our setting is significantly improved (about 50%) compared to the one in Vissim.

V. CONCLUSIONS

We have derived a vehicle inference approach for the mixed traffic merging problem that jointly minimizes the travel time and energy consumption, as well as the centrifugal discomfort of each CAV and guarantees that a speed-dependent safety constraint and a lateral rollover avoidance constraint are always satisfied. Ongoing research is extending the proposed framework to the "non-cooperative" HDV case through an interactive way and exploring the feasibility guarantee of the proposed framework under control bounds.

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