

Magneto-photoluminescence fine structures of a two-dimensional electron gas in the extreme quantum limit

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Keywords: GaAs quantum well, photoluminescence fine structures, charged exciton, extreme quantum limit

We study magneto-photoluminescence (PL) spectra of a two-dimensional electron system in an ultrahigh-mobility GaAs/AlGaAs single quantum well at a very low temperature ~ 50 mK. In the extreme quantum limit up to 7.0 T (corresponding to Landau level filling factor $\nu \sim 0.29$), we observe an unexpectedly large number of sharp PL peaks. These fine structures are split from fundamental optical transitions assigned to negatively charged excitons (trions) and neutral excitons. They are found to exhibit unconventional circular polarization characteristics. The emergence of circularly polarized PL fine structures cannot be described by non-interacting band structures, indicating significant effects of many-body interactions.

1. Introduction

The two-dimensional electron gas (2DEG) confined in semiconductor quantum wells displays pronounced interaction effects under applied magnetic fields. In the extreme quantum limit where all electrons populate the lowest Landau level (LL), the interplay between localized and extended quantum Hall states give rise to a rich set of correlation-modified transport and optical phenomena [1]. Magneto-photoluminescence (PL) probes the response of 2DEGs to the creation of photoexcited holes in the valence bands. While many-body effects on PL are expected to be suppressed for an ideal two-dimensional system under magnetic fields [2], the realistic 2DEGs often break electron-hole symmetry in the extreme quantum limit and generate PL that is sensitively dependent on band structures and interacting electronic ground states [3–8].

Spectroscopic studies found evidence of localization of quantum Hall fluids formed in 2DEGs [9,10]. As the system enters the extreme quantum limit, a crossover of optical transitions from free electron-hole recombinations to negatively charged excitons (trions) was observed at $\nu < 2$ [6,9–13]. The trion active in PL measurements consists of two electrons bound to a photoexcited hole. Depending on the spin alignment of two electrons, trions can be either singlet or triplet. Singlet trions usually appear as the lowest energy PL features under applied magnetic fields. For $\nu < 1$,

higher energy PL transitions emerge sequentially and they are assigned to triplet trions, neutral excitons, or many-body exciton transitions [6,9,11–16]. Three different trion states were predicted theoretically [17,18] for a dilute 2DEG exposed to high magnetic fields. They include a singlet state and a bright triplet state, both of which are optically active, as well as an optically inactive dark triplet state. The energy of the bright triplet trions was found between those of singlet trions and neutral excitons. Above a critical magnetic field, the dark triplet trion instead of the singlet trion was suggested to be the ground state due to its larger binding energy relative to neutral excitons [17]. However, since the dark triplet trion cannot decay radiatively, its appearance in PL depends sensitively on sample parameters and measurement conditions [16,19–23]. The behaviors of these PL transitions remain the topic of ongoing research.

In this work, we study circularly polarized magneto-PL of a low-density ultrahigh mobility 2DEG in a 75 nm-wide single quantum well (SQW) around 50 mK. The magnetic field is applied with a small tilt angle. Under total magnetic fields below 2.0 T ($\nu > 1$), we observe the emergence of PL transitions attributed to the lowest energy singlet trion, the bright and dark triplet trions and the neutral exciton. Unexpectedly, these transitions further split into at least eleven new PL lines at higher magnetic fields up to 7.0 T ($\nu \sim 0.29$). The large number of well-separated emission lines mainly develop from the singlet trion states and neutral exciton states, which are much more than previous observations. On the other hand, the PL lines including those of triplet trions exhibit unconventional circular polarization features. While the bright triplet trion still possesses two circularly polarized transitions, their energies are found to cross each other with increasing magnetic fields. Moreover, the dark triplet trion is mainly detected under right-circular polarization (σ^+). The observed PL complexes with their novel magnetic field and polarization dependences go beyond previous understanding of charged and neutral excitons. Transitions between non-interacting GaAs band structures cannot fully explain our PL data. It suggests that many-body interactions in the extreme quantum limit contribute to the observed PL fine structures.

2. Methods

Magneto-PL studies were performed in an ultraclean 2DEG confined in a 75 nm-wide symmetrically modulation-doped GaAs/AlGaAs SQW. The GaAs layer is flanked by 300 nm $\text{Al}_{0.12}\text{Ga}_{0.88}\text{As}$ followed by 340 nm $\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$ barriers. The electron density of the wafer is $n = 4.2 \times 10^{10} \text{ cm}^{-2}$ with the ultrahigh mobility reaching $12 \times 10^6 \text{ cm}^2/\text{Vs}$ at 300 mK after illumination. According to previous studies [24,25], the electron density for such a SQW is too low to form a bilayer-like charge distribution. The transport measurement also revealed fractional quantum hall states in a characteristic sequence of a single 2DEG and a compressible $\nu = 1/2$ state corresponding to the low-density region in the phase diagram [24]. As shown in Fig. 1, the sample was mounted on a cold finger at the bore of a 14 T superconducting magnet inside the dilution refrigerator with windows for direct optical access. PL was measured in a backscattered geometry with a tilt angle $\theta = 25^\circ$ and the applied magnetic field B contributed a perpendicular component $B_\perp = B \cos\theta$ to the 2DEG. The sample was excited by a Ti-sapphire laser after passing through a polarizer. The linearly polarized incident photon energy was tuned to $\omega_L = 1589 \text{ meV}$ in PL measurements, higher than the band gap of the GaAs SQW. The power density was kept below $2 \mu\text{W}/\text{cm}^2$ to maintain the extremely low temperature around 50 mK. The emitted photons were selectively detected based on left- and right-circular polarization (σ^- and σ^+), dispersed by a triple-grating spectrometer, and

collected by a charge-coupled device detector.

Using the same experimental setup, we measured the depolarized inelastic light scattering spectra at $B = 6.15$ T as shown in the inset of Figure 1. The spin-wave excitation around $\nu \sim 1/3$ (estimated from the carrier density) emits a very sharp peak with the full width at half maximum ~ 0.03 meV. It sits accurately at the Zeeman energy $E_Z = |g_e|\mu_B B$, where μ_B is Bohr magneton and $g_e = -0.44$ is the conduction band Landé g-factor of GaAs [26]. This sharp spin-wave excitation detected by the high-resolution spectrometer is consistent with the ultrahigh mobility of the 2DEG.

3. Results and discussion

Figure 2 presents the magneto-PL spectra at selected fields in σ^+ (blue dashed lines) and σ^- (red solid lines) circular polarizations. At zero applied magnetic field, we observe a single prominent PL peak consisting of overlapping σ^+ and σ^- components, indicating a spin-degenerate ground state. It weakly depends on magnetic fields for $\nu > 2$. Entering the extreme quantum limit ($\nu < 2$ in the lowest LL), the dominant PL peak undergoes a smooth transformation into the transitions with quasi-quadratic energy shifts under increasing magnetic fields that are characteristics of trion states [9–12]. The lowest energy transitions evolving from the dominant PL peak are conventionally assigned to singlet trion states x_s^- . Above 1.2 T, higher energy transitions in both σ^+ and σ^- polarizations emerge, and their intensities are enhanced with increasing magnetic fields. They are assigned to the triplet trion states x_t^- as will be discussed later. Above 3.0 T, another pair of transitions with σ^+ and σ^- polarizations is observed on the higher energy side of x_t^- and attributed to the neutral exciton states x^0 . The assignments of singlet trion, triplet trion and neutral exciton transitions are consistent with previous studies [6,12,16,19], following the ascending energy sequence in which optical transitions emerge with increasing magnetic fields.

PL spectra under high magnetic fields > 3.8 T ($\nu < 0.53$) are shown in Fig. 2(b). Unexpectedly, several new PL transitions are found to develop from x_s^- and x^0 . Under each circular polarization, the dominant PL peak x_s^- becomes broader and develops a low-energy shoulder before additional splitting occurs at higher fields. At 3.6 T, the singlet trion transition x_s^- splits into two PL lines x_{s1}^- and x_{s4}^- in σ^- polarization and the other two PL lines x_{s2}^- and x_{s3}^- in σ^+ polarization. On the other hand, under higher magnetic fields x^0 evolves into x_1^0 (in σ^-) and x_2^0 (in σ^+) with different energies. The x_2^0 transition further develops two higher energy transitions x_3^0 and x_4^0 in σ^+ . Up to 7.0 T ($\nu \sim 0.29$), an additional weak σ^- peak x_5^0 appears as the highest energy transition of neutral excitons, as shown in Fig. 2(b). These PL fine structures are quite sharp with full width at half maximum of ~ 0.25 meV, corresponding to a lifetime larger than 2.63 ps, suggesting that they are fundamental optical transitions from a low-disorder electron system.

Triplet trions do not display unexpected splittings but possess unconventional circular polarization characteristics. The σ^+ component of triplet trion x_t^- has a strong dominant peak x_{tb2}^- with a low-energy shoulder x_{td}^- at low fields, as shown in Fig. 2(a). The shoulder x_{td}^- develops into an individual peak at higher fields with increasing binding energy relative to the neutral exciton x_2^0 . x_{td}^- is also the only transition that tends to cross with the singlet trion x_{s4}^- . The energy difference between x_{td}^- and x_{s4}^- decreases quickly from 0.58 meV to 0.17 meV as the

magnetic field increases from 3.0 T to 7.0 T ($0.29 < \nu < 0.67$). Therefore, we tentatively assign x_{td}^- to the dark triplet trion which has similar field dependence behaviors compared to previous studies [16–18,20–23]. On the other hand, the triplet trion x_t^- has only one single peak x_{tb1}^- in σ^- polarization. Its binding energy $E(x_1^0) - E(x_{tb1}^-)$ relative to the neutral exciton x_1^0 saturates as the magnetic field increases, reaching a stable value of ~ 0.31 meV above 4.0 T. The x_{tb1}^- and x_{tb2}^- are close in energies between the singlet trion and neutral exciton transitions so we assign them to two circularly-polarized components of bright triplet x_{tb}^- [12,14].

To discuss the magnetic field dependence and the unconventional polarizations of magneto-PL spectra, we summarize in Fig. 3 the energies of dominant circularly polarized PL transitions. Here blue lines (σ^+) and red lines (σ^-) with various makers label singlet trions, triplet trions and neutral excitons, which are defined continuously from the high field limit. The dashed lines in the bottom left of Fig. 3 track single-particle-like LL transitions close to the zero field for $\nu > 2$. As the filling factor decreases, the lowest energy PL transition sweeps across these LL transitions that are linearly dependent on applied fields. After LL transitions are suppressed below the filling factor $\nu \sim 2$, a dispersed σ^+ PL peak linearly shifts to higher energies and evolves into the triplet state x_{tb2}^- . The observed PL evolution in σ^+ polarization indicates a continuous lifting of degeneracy between singlet and triplet states.

As shown in Fig. 3, at low fields the bright triplet x_{tb2}^- in σ^+ has lower energy than x_{tb1}^- in σ^- , but above 5.0 T its energy crosses and exceeds that of x_{tb1}^- . Note that before this crossing occurs, the circular polarization of the bright triplet trion exhibits an unconventional order, different from previous results where the lower energy transition was usually detected in σ^- [13,14,16,19]. The unexpected field dependence of the energies of x_{tb1}^- and x_{tb2}^- indicates distinct magnetic responses of the states contributing to bright triplet transitions. Moreover, the dark triplet trion x_{td}^- evolves from the shoulder of bright triplet x_{tb2}^- to an individual peak in σ^+ up to the highest magnetic field. It is surprising that the triplet trion is mainly detected in the σ^+ circular polarization instead of the easier one σ^- due to a higher electron population in the lowest spin-up LL (inset of Fig. 3). We mention that dominant circular polarization of the lowest-energy singlet trions and neutral excitons transitions are σ^- polarization as expected.

Figure 3 illustrates the unconventional circular polarization of PL fine structures. For the singlet trions, both the highest and lowest energy PL lines x_{s1}^- and x_{s4}^- are σ^- polarized while the rest of PL lines with intermediate energies are σ^+ polarized. According to the selection rule, only two singlet states are allowed between spin $S_{hh} = \pm 3/2$ heavy holes and $S_e = \pm 1/2$ electrons (a lower energy one in σ^- polarization and its counterpart in σ^+ polarization) as plotted in the inset of Fig. 3. Previous experiment results are consistent with this simplified transition picture [12–14]. We thus assign the lowest two states x_{s1}^- and x_{s2}^- to conventional singlet trion transitions. At high fields, the saturating binding energy (of the additional electron) of the singlet trion is given by $E(x_1^0) - E(x_{s1}^-) \approx 1.27$ meV. It is smaller than that in a 20 nm SQW [16] as the Coulomb energy becomes softer in a wider SQW. However, x_{s3}^- and x_{s4}^- carrying a reversed order of circular polarizations go beyond the conventional interpretations of singlet trions. Similarly, for the neutral excitons, x_1^0 and x_5^0 are σ^- polarized while the rest of them are σ^+ polarized. Only the lowest two transitions x_1^0 and x_2^0 may be incorporated into conventional transitions between spin $\pm 3/2$ heavy holes and

175 $\pm 1/2$ electrons.

176
177 In the following, we discuss several factors that may contribute to the observed PL fine structures
178 and unconventionally polarized trion complex. Inspired by the unconventional polarization of the
179 singlet trions (x_{s1}^- to x_{s4}^-), we examine two possible transitions associated with light-hole valence
180 bands (with spin $S_{lh} = \pm 1/2$). These two transitions are at higher energies than those involving
181 heavy-hole bands. Following the selection rule, they exhibit a reversed order of circular
182 polarizations, as shown in the inset of Fig. 3. This picture seems to give a good explanation of the
183 PL fine structures of singlet trions and neutral excitons under high magnetic fields. However, the
184 observed convergence of these splitting lines at the zero field is inconsistent with light-hole band
185 diagrams in the SQW, as the SQW potential is expected to lift the degeneracy of light-hole and
186 heavy-hole bands and induces a finite energy gap between them [12,21]. Therefore, the single-
187 particle energy band diagrams are hard to explain our data completely.

188
189 Moreover, the 2DEG optical transitions are anticipated to be significantly affected by many-body
190 interactions as entering the extreme quantum limit with a partially populated LL. Many-body optical
191 transitions were suggested to coexist with emissions from localized charge puddles in a wide range
192 of filling factors [12]. We find that the primary PL peak evolves continuously from the PL of Fermi
193 liquids at zero-field and enhances in a range of small magnetic fields (Fig. 2(a)). It later gets weaker
194 at $\nu < 1$ and transforms into singlet trions. The higher energy PL peak at low fields has a similar
195 evolution which first enhances and then weakens into triplet trions with increasing magnetic fields,
196 as shown in Fig. 2. Although their smooth evolution may rule out effects from fractional quantum
197 hall quasiparticle excitations [3,6,7], the nontrivial field dependence in their energy splitting and
198 intensities indicates an interplay between many-body interactions and few-body binding energies.
199 A complete understanding of the origin of the PL fine structures and their polarization dependence
200 calls for a detailed theoretical framework and is out of the scope of this paper.

201 202 **4. Conclusion**

203 PL fine structures with unconventional circular polarization dependence were observed in an
204 ultrahigh mobility GaAs SQW under applied magnetic fields in the extreme quantum limit. The
205 large number of field-induced splittings in singlet trions and neutral excitons exhibits marked
206 discrepancies from previous results. Furthermore, the circular polarization characteristics of PL
207 peaks go beyond the framework of non-interacting band structures. Strong modifications from
208 many-body interactions may be involved. Our results suggest the ultrahigh mobility SQW as a new
209 platform to study the behaviors of charged and neutral excitonic transitions under high magnetic
210 fields. The observed PL fine structures may couple to collective excitations and offer key insights
211 into the correlated ground states in the fractional quantum hall regime.

212 213 **Acknowledgements**

214 The work at Nanjing University was supported by the National Natural Science Foundation of China
215 (Grant No. 12074177), Program for Innovative Talents and Entrepreneur in Jiangsu and a Faculty
216 Initiative Fund of Nanjing University. The work at Columbia University was supported by the
217 National Science Foundation, Division of Materials Research (Award DMR-2103965). The
218 Molecular Beam Epitaxy (MBE) growth and transport characterization at Princeton was supported

by the Gordon and Betty Moore Foundation through the EPiQS initiative grant GBMF9615 and by the National Science Foundation MRSEC grant DMR 1420541.

Competing interests

The authors declare no competing interests.

Data availability

Data will be made available on request.

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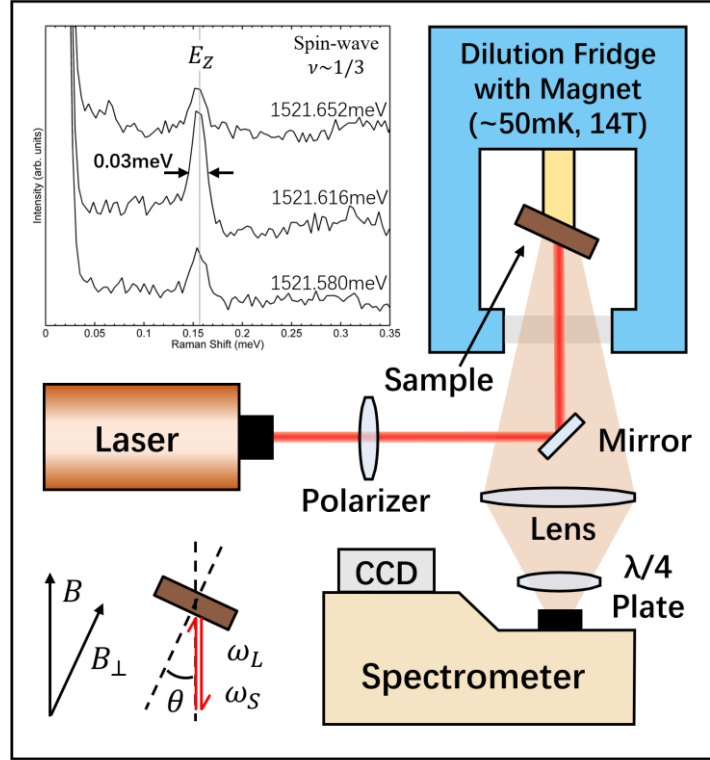


Fig. 1. (color online) Experimental setup of optical spectroscopy of the sample in a dilution fridge with bottom optical access. A quarter wave plate is inserted in front of the spectrometer to detect circularly polarized photons in PL. Inset: sharp spin wave (SW) excitations around filling factor $\nu \sim 1/3$ measured by resonant inelastic light scattering.

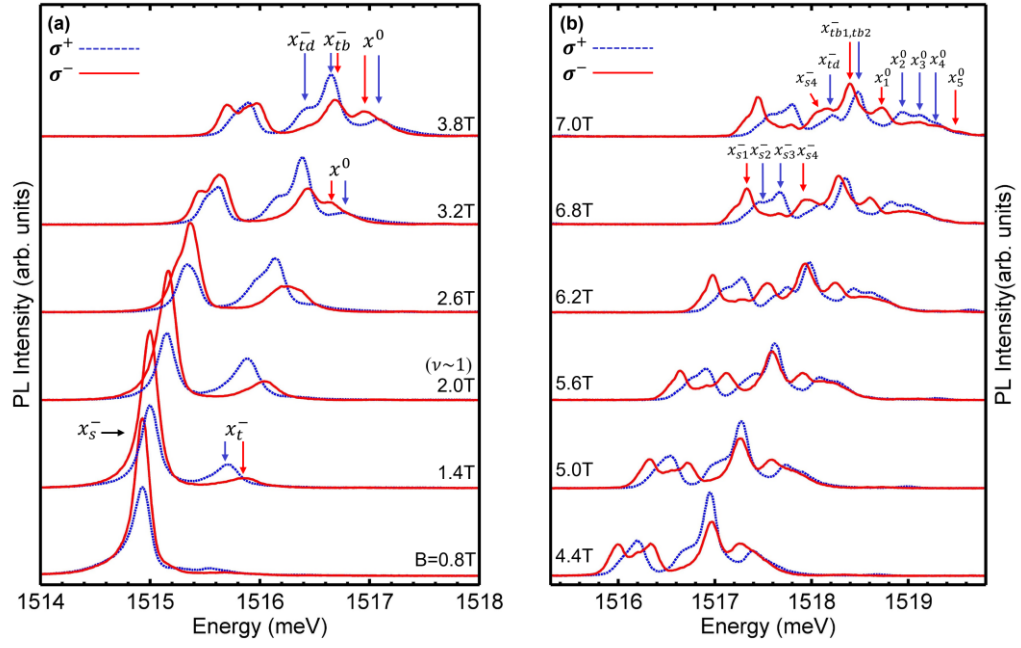


Fig. 2. (color online) (a)(b) Circularly polarized PL spectra at selected total magnetic fields B . Red solid and blue dashed PL lines plot from σ^- and σ^+ polarized photons, respectively. PL features are found to split with increasing magnetic fields.

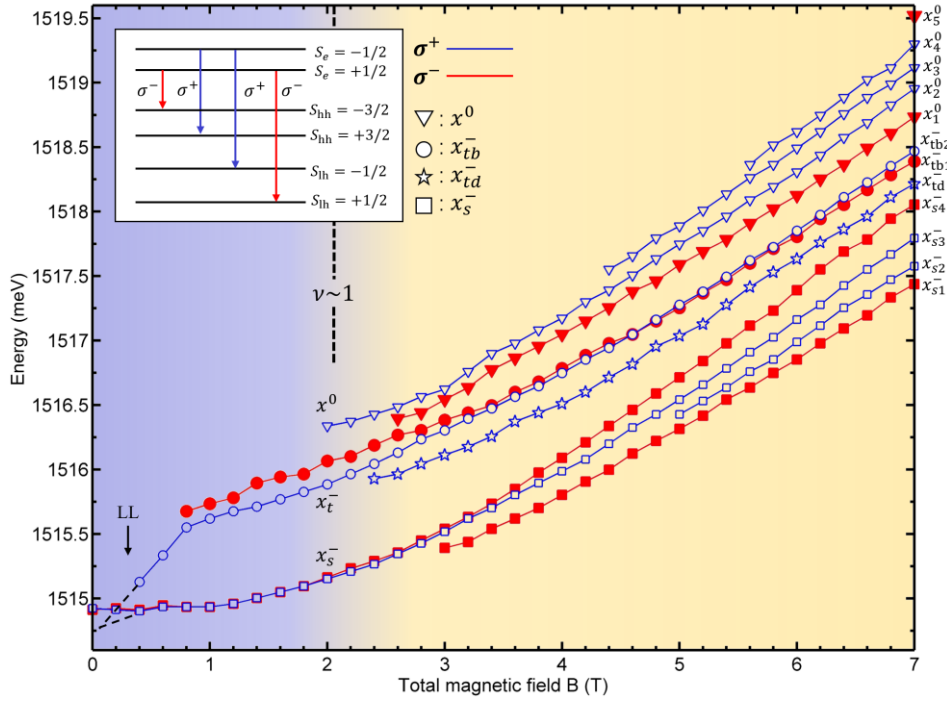


Fig. 3. (color online) Energies of various circularly polarized PL transitions as a function of total magnetic field. Red closed and blue open markers label left- and right-circularly polarized PL peak energies from singlet trions (squares), triplet trions (stars and circles) and excitons (triangles), respectively. LL transitions (dash lines) in the violet region at filling factor $\nu > 2$ smoothly evolve into PL fine structures in the $\nu < 1$ region where unexpected splittings occur. Inset: transitions allowed by the selection rule in the single-particle band diagram between conduction bands ($S_e = \pm 1/2$) and heavy-hole ($S_{hh} = \pm 3/2$) or light-hole ($S_{lh} = \pm 1/2$) bands.