Exploring the Limits of Differential Privacy

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Abstract—Differential Privacy (DP) is a powerful technology, but not well-suited to protecting corporate proprietary information while computing aggregate industry-wide statistics. We elucidate this scenario with an example of cybersecurity management data, and consider an alternative approach that relies on a pragmatic assessment of harm to add noise to the data.

Index Terms—Public Policy Issues, Data Sharing, Privacy

I. Introduction

Differential Privacy (DP) is widely recognized as a useful and powerful privacy-enhancing technology, but there are contexts in which it is currently not well suited because the underlying data models or needs of data users do not match existing DP approaches. This paper explores using DP in one such context: protecting corporate proprietary information while computing aggregate industry-wide query results.

Some firms are willing to support research by providing sensitive and potentially damaging confidential data, making the assumption that their corporate interests will be protected if their data are aggregated with data from other firms prior to release. Dwork and Roth's Fundamental Law of Information Recovery [1] states that this is a fallacy, and recommends using differential privacy to protect confidential information.

This paper's contribution includes a brief overview of DP's goals, a worked example that shows why DP poorly addresses some reputational harms that firms might suffer from some kinds of aggregate statistics, and a discussion of why changing the specifics of the questions being asked of the confidential data—the query set—may make it easier to both apply DP and reason about possible harms. For our example, we use synthetic cybersecurity management data.

Finally, we consider an alternative approach for performing privacy-preserving data analysis that is inspired by DP's intuition and epistemological principles, but where the amount of noise added is derived based on a pragmatic assessment of harm.

II. Background

There are distinct and complex challenges when trying to protect proprietary information at the firm level. First, firms have fundamentally different concerns regarding the disclosure of their proprietary information than individuals have regarding the disclosure of their personal data, as the kinds of harms that firms can suffer are fundamentally different from the harms experienced by individuals. Second, the sample size (the number of firms providing data) is often small, which is a challenge for DP. The challenge is further exacerbated by the significant differences between individual firms: in general there are considerably more differences in measurable characteristics between firms than between people.

There are also practical challenges in using DP. First, DP is formally described in terms of a mathematical abstraction called privacy loss, which maps to some relative potential improvement in capability that an attacker may enjoy as a result of a data release, but does not map well to absolute increases in harm. While the ultimate goal of DP is to prevent harm to the data subjects, the potential for actual harm must take into account the context of the query.

Our goal here is to avoid the traditional mathematical framing of DP as much as possible, and instead focus on the utility (and limitations) of DP in an intuitive way based on visualizations of simulated results.

There are a number of ways to think about and use DP, but the simple version we use in this paper is to imagine that there is a database with a number of records, each with a fixed number of fields. Each record represents confidential data from a different entity. The goal is to produce a useful statistical release while providing some degree of protection for the confidential data.

DP provides protection by adding a degree of noise to the result of each query against the confidential records. The noise makes it difficult to reconstruct one or more of the true, confidential values of any record or combination of records. Equivalently, DP limits the ability of an attacker (which we will call the data hacker) to ascertain if data from a particular entity are or are not included in the confidential dataset.

In general, DP mechanisms fall into three broad modes of operation:

- 1) Local mode. Noise is added to every element of every record of the entire database, after which the entire noisy database can be used for any number of statistical operations without further privacy loss. Alternatively, the entire noisy database can be publicly released. The local mode requires comparatively high levels of noise to achieve significant privacy protection, which limits the usefulness of this approach.
- 2) Trusted curator mode. A trusted data curator collects confidential data, computes statistics, and adds noise to each result. Multiple queries that address the same records increase the overall privacy loss that data subjects experience.
- 3) Trusted curator with synthetic data. The trusted curator performs queries on the data to produce a noisy statistical model, which it then uses to generate synthetic data. This data can be used or published without additional privacy loss. The challenge with this approach is creating synthetic data that have sufficient fidelity

and accuracy. In practice, this is an open research problem.

In this paper we explore the use of DP solely in the trusted curator mode, avoiding the high levels of noise required by the local mode and the immaturity of methods to generate synthetic data.

The goal of DP is to assure that an analysis of a database containing an individual's confidential data should not differ by more than a small amount from an analysis of a similar database that does not contain the individual's data.

DP uses a parameter ϵ to quantify what we mean by "a small amount." If $\epsilon = 0$, there should be no difference, which means that queries on the database can have no relationship to the data stored in the database. If $\epsilon = \infty$, then any difference is acceptable. In practice, $\epsilon = \infty$ allows a query to precisely release any value in the database, or all of them.

We are interested in the range $0 < \epsilon < \infty$, where there is a trade-off between the accuracy of the output and the amount of privacy loss incurred. The higher the accuracy, the more privacy loss.

If the intended use of the data requires more accuracy, or alternatively if the data does not require so much protection, then less noise can be used, and there is more privacy loss and more accurate statistics.

Here is the formal statement of the protection provided by any scheme that is consistent with the traditional DP framework. For a query function M that returns a noisy answer to a query of a database:

- for all subsets of the database d_1 and d_2 that differ by one record:
- for all subsets S of the range(M):

$$\frac{P(M(d_1) \in S}{P(M(d_2) \in S} < e^{\epsilon} \tag{1}$$

That is, the ratio of the probability that M() operating on d_1 produces some answer S to the probability that the same function M() operating on the database d_2 produces the same answer S must approach 1 as ϵ approaches 0, which

means that they produce the same answer with high probability for different databases at this limit, and there is little privacy loss. But when ϵ grows larger, the probabilities that $M(d_1)$ and $M(d_2)$ produce the same result become much lower, so the results of the two queries can be distinguished, implying increased privacy loss.

III. A simple example of harms from querying firm-level data

Consider the following simple example: 100 firms using a well-known and widely used system or application each complete a survey reporting the fraction of systems they are running that have been upgraded to the latest security patch. Each of these reports consists of a single number between 0.0 and 1.0 and is stored in database D that is operated by the trusted curator. Our goal is to get a sense, industrywide, of whether firms are keeping their systems up-to-date with respect to security patches.

Once the trusted curator receives the reports, the curator computes one or more queries on the confidential data and publishes the result to the public.

One obvious query of public interest would be the average of the values. In practice, we might wish to weigh each sample based on the size of the firm, but for this simple example, we assume a query that solely computes the mean of the values returned from each firm.

Our first question is whether releasing the mean of these values (with no noise added) can cause harm to the firms that provided the inputs. What if the mean is 0.5? This might imply that the contributing firms have all upgraded half of their systems to the latest patch level. Alternatively it might be that precisely half of the firms reported patching all of their systems, and half reported patching none.

A. The Data Hacker

In the specific case above, unless we assume that the data hacker knows the statistic for 99 of the firms and is attempting to learn the data for the firm that remains, it is unlikely that revealing that the average is 0.5 will harm any one firm. (We return to this assumption in Section IV-C.)

However, this harmless situation may not hold with other averages. What if the computed mean is 0.0? Then it would have to be true (from the math) that each of the firms returned the value 0.0 as their firm's response to the query. No firm has upgraded any of their systems. The release of the average would cause reputational harm to all of the contributing firms. Note that if the average had been 1.0, the firms might be very happy to reveal that result—it would show that they all did great. Actual harm (or the potential for actual harm) depends on both the result and the context of the query, not the underlying math.

In this article we term this harm the binomial pathology, making an intentional analogy to the binomial theorem, in that there are many ways to take 50 balls out of an urn with 100 balls in it (without replacement), but only one way to take out 0 or all 100. As the returned value of the query gets closer to the minimum or maximum of the possible range for the mean, there are fewer and fewer combinations of data values that can yield the result. So the potential for harm is data dependent.

The binomial pathology can arise in other contexts. Consider a census block where the average age is 45. There could be much younger and much older people contributing to that average, so we learn little about them as individuals. But if the average is 85, it is a good guess that most of the people in that block are old. The data hacker, seeing that the average age is 85, can only guess about a given individual in the census block, but a guess can be good enough to cause harm. Also, the guess can become arbitrarily more accurate with additional public data—for example, learning that a couple living on the block married just before the husband was drafted to serve in the Korean War.

B. Privacy loss vs. harm

While the ultimate goal of DP is to prevent harm to the data subject, the protection provided by DP is defined not by the possible harm of a data release, but on the maximum amount of privacy loss that could result.

It is the relative privacy loss that is data independent: the absolute protection very much depends upon the global data context. A given amount of privacy loss will be more damaging in the hands of a data hacker who has substantial knowledge about the world in which the data subjects reside.

DP's protection is defined by the degree that the potential harm caused by the result of a query is independent of whether the individual's data were considered when evaluating the query. The classic DP example is a query that tries to establish a link between smoking and cancer. If that linkage is accepted, a smoker might see their health or life insurance rates go up. The smoker is harmed by the result of the query, but not because of privacy loss: it made no difference whether the smoker's confidential data were in the database or not. So the difference between the harm suffered whether or not the smoker's data were considered is zero, which is why the approach is called differential privacy. The smokers in the data were harmed, but so were the smokers not in the data.

In our security example above, if we publish that the average patch rate is 0.0, the firms are individually harmed, but so is the broader community: it will be guilt by association.

This kind of harm may not be acceptable in the case of corporate confidentiality. If we seek voluntary release of data (as opposed to data release that is compelled by regulation or law), the fear of this kind of harm may cause firms to refuse to release data. For example, corporations may fear that making confidential data available to produce industry-wide statistics may help create a body of evidence that will be used to regulate the industry.

This is a harm that DP is not designed to mitigate, because this is a harm outside of DP's definition of privacy loss. But recognizing this limitation, can adding noise to a result contribute to the mitigation of this sort of harm?

IV. Adding noise, in the DP way

DP protects privacy by adding noise to the result of each query, creating uncertainty for a data hacker attempting to learn the contents of the confidential database.

There are many approaches for adding noise that are consistent with DP; here we use the Laplace Mechanism, which adds noise drawn from a Laplace distribution with zero mean to the result of each query. The zero mean assures that the noise added to the true answer is equally likely to be positive or negative, so that there is no implicit bias added to the query results. The magnitude of the noise added (the width of the Laplace distribution) is determined by two factors: ϵ (discussed above), and a factor called sensitivity. While ϵ gets all the attention in discussions of DP, the concept of sensitivity is equally significant, as the amount of noise added is a function of both.

A. Sensitivity

DP sensitivity $(\Delta(f(D) - f(D')))$ is the maximum amount that a query result (in this case, mean) can change if the data associated with the unit of protection—typically a single database record—is changed or removed. (Here we ignore the subtle difference between removing a record and changing it.)

The sensitivity of a query is not based on the actual values in the current database, but on the theoretical maximal impact that a single record change could cause for the universe of all possible databases. In fact, it is an error to use the contents of a particular database to compute query sensitivity: it must be inferred from the range of values that might be in the database. In our example, with 100 samples between 0 and 1, the maximal impact than a single firm could have would be the situation where $(x_1...x_{99}) = 0$ and $x_{100} = 1$. In this case, the mean will either be $\frac{x_{100}}{100}$, or .01. or else 0, if x_{100} is changed to 0. So the global sensitivity S is .01.

There is much more that could be said about query sensitivity, but now that we have a value, we can explore how the Laplace distribution will be scaled.

B. The Laplace noise function

The zero-mean Laplace distribution is defined as follows: for a possible value x of noise to be added to the true value, the probability of adding that noise value is

$$P = \frac{1}{2b} exp(-abs(\frac{x}{b}))$$

where b is defined as $\frac{S}{\epsilon}$, and S is the sensitivity. Most papers that introduce differential privacy include a plot of the Laplace distribution; ours appears below, in figure 1. The Y-axis height of the red line indicates the probably that a single value drawn from this Laplace distribution will result in the value indicated on the X-axis. For this distribution with a mean of 0.5 and a scale of 0.01, the most probable value is 0.5, and 95% of the values will be between 0.47 and 0.53. These values correspond to using the Laplace Mechanism with an $\epsilon = 1.0$ and sensitivity $\Delta f = 0.01$ to add noise to a value of 0.5.

Note that while it looks like 0.50 is the most probable value, a value that is close to 0.5 is vanishly improbable: fewer than 10% of the values are between 4.999 and 0.501

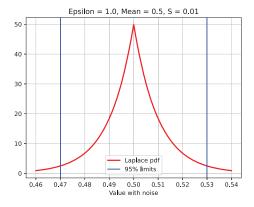


Fig. 1. Noisy result of the computation of the mean

For this paper we would like to derive the required level of noise (or other harm mitigation) from an assessment of the real potential for harm, rather than an abstract notion of privacy loss based on ϵ . In practice, it is difficult to asses the potential for harm. Instead, many data analysts take the reverse approach: they review the amount of noise that a given level of ϵ will add to the statistic and decide if the result is still useful. If not, the amount of noise is decreased; if so, the amount of noise is increased. Eventually the analyst finds the maximum amount of noise that can be added while still allowing the statistic to be fit-foruse. This emphasis on utility may dominate an assessment of harm.

In our example, in which the query is designed to reveal the fraction of systems patched to the latest release, the privacy protected value will be between 0.47 and 0.53 with high probability, and between 0.45 and 0.55 with almost absolute certainity. Many people consider $\epsilon=1.0$ to be a high amount of privacy protection, and in this case it still produces a useful answer! But this is not always the case.

C. The worst-case assumption

DP's definition causes it to make a worstcase assumption about the prior knowledge of the data hacker, short of knowing the actual value that DP is trying to protect.. That is, it assesses an upper-bound of the potential loss of privacy that might result from a data release, independent of what the data hacker's prior knowledge or computational capabilities. So consider the case of a data hacker that happens to know the actual answer for 99 of the firms, and wants to learn the answer for the final firm. If no noise is added to the answer, then the hacker can easily reverse the computation of the mean and derive the answer for that firm. Thus, the data curator decides to protect the result with DP.

Assume that the mean of the known 99 values was 0.5. If the remaining (unknown) value is 0.0, the true mean of all the values will be 0.495. If

the remaining value is 1.0, the true mean would be 0.505. (Note that we have just recomputed the Global Sensitivity in this case—the difference is 0.01.) In these two extreme cases, what would the Laplace distributions be for the noisy answer with an epsilon of 1?

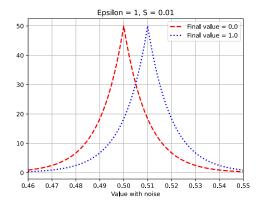


Fig. 2. Range of noisy results depending on actual value of the remaining data item.

The data hacker (as before) faces a 95% certainty that the returned value is ± 0.03 from the actual answer. The hacker does not know from which distribution (anywhere between the lowest and highest pictured in Figure 2) the returned value came. All the hacker sees is a single number. If that number happened to be .5, the value is equally likely to have come from the lowest and the highest alternative, so the hacker has learned nothing. However, if the answer were (for example) .48, it is much more likely that this result was from a distribution at the lower range of the options. In other words, the hacker cannot guess the true value of the final value, knowing the other 99 values, but may be able (for some noisy results) to guess that the remaining value is "lowish" or "highish." Whether this degree of guess is harmful is not a question of math, but must be answered from the actual context.

One question we might ask is whether we need to address the actual worst case. A hacker that knows all but 2 of the values would learn

essentially nothing from the range of possible noisy answers. If we allow ourselves to relax the worst case assumption in assessing the potential for actual harm, we may get a more realistic assessment of what a data hacker can actually learn, but the mathematical foundation of DP are of no help to us.

V. Addressing the binomial pathology

The previous illustrations showed the distribution of added noise if the actual mean was 0.5. What if the true mean was really 0.0—that is, what if all of the firms had patched none of their systems? Figure 3 shows the resulting distribution of noisy answers.

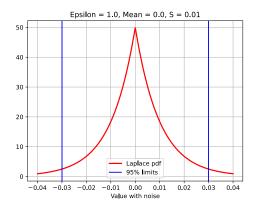


Fig. 3. Range of noisy results if the true mean of the 100 values is 0.0.

With this degree of noise, the hacker can reasonably infer that the actual mean (that is, the fraction of systems that have been patched to the current release) has a 95% probably of being less than 0.03. The hacker cannot know that the actual value was 0.0, but knowing that it is highly unlikely that the value was above 0.03 may be enough to cause harm. Would this degree of uncertainty allow any single firm that had contributed data to plausibly claim that while the overall number was really low, they had actually done a good job? Probably not.

Note that the returned protected value may be less than zero or greater than 1. That is, the trusted curator might declare that -10% of all companies are fully patched. In this case, all parties (both legitimate data analysts and the data hacker) would understand that this cannot be a true value, and that it was either be the result of a small number (such as 0) being treated with a small negative value from the Laplace distribution, or else (with lower probability) a larger value (such as .5, or even 1.0) being treated with an even more negative value from the Laplace distribution.

To minimize the public's confusion, the organization producing the protected statistics might resolve to only report values ≥ 0 and ≤ 1.0 This robs downstream data users (and hackers) of some information, but it does leave the reporting agency less open to ridicule. (This exact problem faced the US Census Bureau in its use of differential privacy for the 2020 Census; it resolved the problem by publishing two sets of statistics: one set having only non-negative integer counts, and a second set, the so-called noisy measurements file, containing negative and fractional numbers.)

How about an organization that has not contributed to the dataset? If the names of the firms are themselves confidential, an organization could claim that their data was not included in the computation, but there is no way to prove this. DP cannot help this organization. Even if the organization can establish through some kind of audit that its data truly was not included, that organization will still likely suffer reputational harm because it will be tarred with the same brush as the poorly performing organizations that did participate.

These sorts of harms are not the harms that DP is designed to prevent. This is like the case of smoking and cancer. Some reputational harm may attach to firms of the sort surveyed, whether or not they were in the sample.

VI. Small samples exacerbate DP privacy loss

What if there were only 10 firms in the database, rather than 100? In that case, the global sensitivity S would be 10 times greater,

and if ϵ were still 1, the distribution of added noise would look like Figure 4.

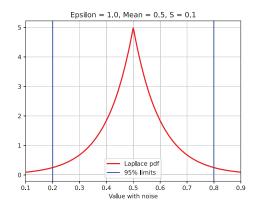


Fig. 4. Range of noisy results if the true mean of the 10 values is 0.5.

This amount of noise, given a global sensitivity of .1, significantly decreases the utility of any published results.

How can we improve the utility of the result? One obvious answer is to increase ϵ . If we increase ϵ to 10, the plot will be exactly the same as Figure 1. This should not be a surprise: if we increase ϵ by 10 and increase the Global Sensitivity by 10, the two changes cancel out. What is different is that the data hacker can now form a far more accurate hypothesis regarding the underlying confidential data (see Figure 5).

Because the global sensitivity has changed by a factor of 10, the curves representing the Laplace distribution for the minimum and maximum value of the one sample that the worstcase hacker does not know have moved 10 times further apart. In this case, while the hacker cannot guess an exact number for that remaining hidden value, as Figure 5 shows, the hacker can make a very good guess.

The high quality of the guess is consistent with DP's definition of privacy loss in equation 1: in the context of a single query protected using the Laplace mechanism, there is little protection with $\epsilon = 10$. It specifies that the ratio of the two

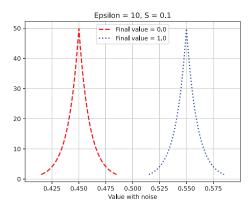


Fig. 5. Range of noisy results depending on actual value of the final data item, with 10 samples.

probabilities (with d_1 differing from d_2 by one record) must differ by no more than e^{ϵ} , which in this case is 22,026. This is a mathematically well-formed way of saying that in the worst case the privacy loss is substantial.

This does not mean that DP's approach to privacy protection by adding noise is a useless exercise with small samples. What it means is that we must step away from the strict, worse-case assumption of DP and accept a more pragmatic assessment of the capabilities and motivations of the data hacker. We can use the same method to add noise, so the result is nominally consistent with DP, but to assess harm we cannot rely on the concept of privacy loss. We should also consider other advantage of DP that legacy data protection mechanisms lack, which we have not discussed in this paper.

VII. Making the situation better

We have identified two risks that DP was not designed to mitigate (and does so poorly): small data set sizes and the binomial pathology. Regarding the first risk, as the sample size (the number of firms) shrinks, we must rely on a pragmatic, not worst-case, analysis of the capabilities and intentions of the hacker to assess potential harm. But what can we do about the binomial pathology?

A. Sampling the data records

One approach that is used to try to protect individual entries in a database is to sample the database and compute a noisy result across a subset of the sample (here, the firms). In this case, a firm can try to claim that the result may not apply to them because they may not even have been in the sample. In this simplistic example, would sampling address the privacy concerns of the firms?

Sadly, probably not. Here, the sample size works against that claim. If there were 100 firms, and the trusted agent that computes and returns the noisy answer declares that it has used the data from only 90 of them, what then? Statisticians, when looking at the behavior of firms, can often justify the assumption that the variables that define their behavior are i.i.d. In this case, using a sample of 90 to predict the behavior of a population of 100 is a highly robust statistical assumption. The more firms that are in the data, the stronger the justification for a statistical conclusion that a subsample is a robust predictor of the population. Sampling cannot help us here.

B. Change the query

Another way out of this dilemma is not to compute the mean, but to devise an alternative query that provides sufficient information for the needs of the data analyst but avoids pitfalls such as the binomial pathology. These alternative queries may also benefit from added noise, but with careful design can avoid dangerous results from low-probability data values. Indeed, this experience is common for those attempting to adopt a statistical analysis to incorporate differential privacy: frequently it is necessary not simply to add noise to the statistics that are reported, but to change the statistics that we choose to report.

One kind of query might be some form of quantile. For example, the query might be: "what fraction of the firms have patched more than 50% of their systems." This is essentially a histogram with two bins, and a count for each.

Of course, none of the firms might have patched more than 50% of their systems, so 100 would be in the lower bin, and none of them in the upper bin. Would this outcome represent a harm to an individual firm? None of the firms have patched more than 50%—that fact is known about each individual firm. But the firms might find this degree of reputational loss to be tolerable, since all the others are in the same boat. And any single firm could argue that they had done 49% of their systems.

With DP we protect histograms by adding noise to the counts for each bin. If there are 100 firms with fewer than 50% of their systems patched and 0 firms with more than 50% of their systems patched, the DP computation might add noise of +2 to the first number and -1 to the second number, with the result of 102 firms in the firm bin and -1 firms in the second.

Small counts is still a problem, however. If there is a bin with 100 firms in it, adding or subtracting one or two as we add noise does not greatly change the utility. For a bin with one firm in it, adding enough noise that the answer might be two or zero or even negative one is potentially a huge loss in the precision (and utility) of that small bin size. If we created more bins, the expected number of firms in each bin would be lower, so the degree of uncertainty in the results would go up. If there were only 10 firms in the data, and we split them up into more than a very few bins, the added noise would render the results less useful.

Alternatively, we could ask the median of the percent of patched systems across all the firms. If the median is 0.0, then at least half the firms have done nothing, but we know nothing about the other half. Again, we have to assess this query through the lens of the potential harm to the individual firms, and whether the use of DP could further mitigate these potential harms.

C. Add noise based on the actual data

If it is necessary to use a query (such as mean) that has a low-probability data disclosure pathology, we could consider abandoning

the logic of DP and adding noise (or more noise) only when the actual data triggers the pathology. To do so steps completely outside the philosophy of DP, because the fact that additional noise has been added to a particular result (which has to be disclosed) itself reveals important facts about the data. In our example with 100 firms, the trusted agent could add additional noise as the true value of the result approaches 0.0. This would prevent a data hacker from making a precise guess, but would still make obvious that the number was unfortunately low. Such ad hoc systems are difficult to analyze and are brittle if there exists external data that can be used to undo their protection mechanisms: it was the analysis of such schemes and the dissatisfaction with them that led to the development of DP.

Once the trusted curator has committed to releasing the query of a mean, there are no easy ways to mask the pathological outcomes. Refusing to return a result itself reveals something about the data. Adding lots of noise based on the data reveals something about the data, which is why DP requires that the trusted curator must make the decision about how much noise to add before looking at the confidential data. This is an example of the "Fienberg Problem." [2]

It is critical to disclose the amount of noise that a trusted curator has added to a result, both to inform the legitimate data analyst and as well a possible data hacker. A frustrating harm can occur when an hacker does not understand how the added noise has limited the validity of his conclusion, and publishes an unjustified conclusion, causing reputational harm that could have been prevented had the hacker been aware of the added noise. Showing some form of error bars on an answer may be a way to make the point forcefully. However, the error bars must not be presented in a way that reveals anything further about the actual data.

VIII. Conclusions

The astute reader may have observed that we could have written this paper from a different

starting point, with a title such as: "The Hidden Perils of Computing a Mean," and gotten much of the way through the development without even mentioning DP. We chose this course through the material both to introduce the basic ideas of DP, and to point out that there are queries on specific kinds of datasets that must lead to either poor privacy or poor utility outcomes even for systems that implement DP.

While most papers that discuss DP focus on ϵ , perhaps the most important element of a strategy for industry cooperation is to develop types of queries that provide sufficient utility but are free of low-probability disclosure of firm-specific information. Query design is a critical part of effective use of DP that is often not discussed in introductory texts on DP.

One of the problems that DP faces is to help practitioners map from a value of ϵ to a practical assessment of harm. The approach we have taken here (exploiting the simplicity of our example) is to start with an assessment of harm, and tune ϵ based on that assessment.

Adding noise to the result of a query can be an important method to reduce potential harm, even if the amount of noise added is not framed as a form of DP, but in that case there is no mapping from the noise to the formal DP specification. Pragmatic assessment of potential harm is a space that is fraught with failure.

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