

Grid-Forming Inverter Optimal Allocation Framework for Voltage Stability Improvement of a Rural Weak Grid

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Abstract—Highly penetrated renewable energy to weak rural grids results in voltage instability and higher power loss due to the backflow of power to the load center and higher R/X ratio. Incorporating traditional methods (i.e., capacitor banks) or connecting grid-forming inverters can help to achieve voltage stability. This paper proposes a grid-forming inverter optimal allocation framework to enhance voltage stability and profile during islanded operations. This framework can also enable optimal capacitor bank (CB) placement in grid-connected settings. A particle swarm optimization (PSO) algorithm is employed to allocate the grid-forming inverters or CB optimally. The radial bus distribution network is also analyzed using the current summation method. A simulation case study adopts a modified IEEE-33 radial weak bus distribution network. The optimal GFM allotment improves the weakest bus voltage profile by 8.57% and reduces the total active power loss by 86.7%, maintaining the same load condition as the base case.

Index Terms—grid-forming inverter, optimal allocation, renewable energy, rural weak grid, voltage stability

I. INTRODUCTION

The accelerating adoption of renewable energy on the grid via inverter-based resources (IBRs) introduces challenges such as voltage stability, low inertia, and intermittent generation. In particular, rural grids are experiencing a substantial increase in the total share of this growth due to the considerable number of remote solar or wind farms [1]. Moreover, rural grid usually has fewer substations, a lower short circuit ratio (SCR), and a higher R/X ratio, making it more challenging to adopt IBRs. Due to the non-inertial IBRs and radial bus architecture, frequency and voltage regulation become poor, and line loss becomes dominant.

A rural grid faces an elevated risk of natural calamities such as wildfire [2] and storms [3] that can isolate the affected lines until conditions improve to ensure the broader power system stability. In such scenarios, the absence of electricity supplied by operational lines can lead to power outages for downstream customers. Grid-following inverters (GFLs) are the commonly employed topology for grid connection. However, they exhibit low inertia and are prone to instability when linked to a weak grid [5]. In contrast, Grid-forming inverter-based resources

offer the potential to minimize downtime during emergencies by forming isolated rural micro-grid [4]. Usually, grid-forming inverters (GFMs) based IBRs with energy storage devices (ESS) can facilitate voltage profile and stability improvement during islanded operation. Optimizing the size and placement of GFMs can improve stability without requiring an increased number of GFMs.

Many conventional methods are adopted to overcome the distribution network's voltage regulation and stability issues, e.g., capacitor banks (CBs), FACTS devices (STATCOM, SSSCs, SVCs), and synchronous condensers [6]. FACTS devices offer faster response, better controllability, and dynamic adjustments among these techniques but are costlier. However, optimal placement of the capacitor bank can still help improve the voltage stability and profile and minimize the installation cost. Extensive research is being conducted to address compensation devices' optimal size and allocation in distribution networks [7]. Ref [8] shows different methods proposed to optimally allocate the distributed generations and shunt capacitor banks using various methods, including genetic algorithm, particle swarm algorithm, and analytical with power flow approaches. However, these studies need to consider the GFMs-based IBRs technique, which can provide at least 0.5 p.u. reactive power output of its base rating [9].

Rural grids suffer from dispersed loads with higher IBRs, low inertial response, and extreme weather, which present novel challenges and research opportunities in GFM allocation to improve resiliency and efficiency. This paper focuses on the optimal allocation of GFM-based IBRs to enhance voltage stability and reduce line loss in the weak rural grid during islanding. Considering the reactive power support capability and synchronous generator characteristics of GFMs, they are regarded as a means to enhance voltage stability. The detailed modeling approach of GFMs is explained in Section II. A particle swarm multi-objectives optimization algorithm minimizes voltage deviation and avoids voltage instability at each IEEE-33 radial distribution network bus. At each iteration, the radial distribution network is solved using the current summation power flow method until the optimal location and size of the GFMs and CB are acquired, which is discussed in section III. Simulation results and comparative studies between GFMs and

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CB-based voltage improvement method is covered in Section IV.

II. GRID-FORMING INVERTER MODEL

GFM enable renewable energy grid integration while maintaining control over voltage and frequency to meet active and reactive power requirements. An overview including different subsystems of the renewable energy generation to the grid integration is depicted in Fig. 1. The generation-side converter

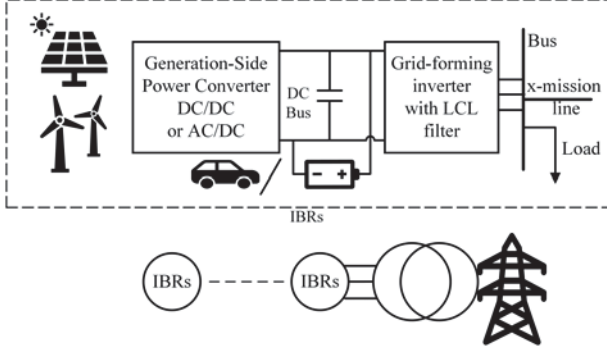


Fig. 1. Inverter-based renewable energy system (IBRs) and its connection to the bulk power grid.

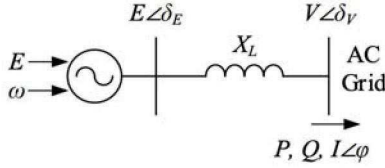


Fig. 2. Grid-forming inverter basic model. [9]

helps to achieve maximum power extraction, maintain DC bus voltage, and supply DC loads or store surplus power in the battery. The GFM with an LCL filter can supply power into the grid depending on the load demand. GFMs behave as a controllable voltage source behind a coupling reactance (X_L), as shown in Fig. 2. Internal voltage E and angular frequency ω can be controlled using a vector controller to achieve P-F and Q-V droop functionality. Details of modeling and control of the GFMs are not explained in this article; readers are referred to [9] for detailed information.

The following section explores the optimal allocation framework using PSO and distribution power flow analysis. Power flow analysis spans a longer timescale than the GFMs switching transients, assuming that GFMs operate ideally within this timescale.

III. OPTIMIZATION FRAMEWORK MODELING

The optimization framework is modeled in MATLAB, and details of the modeling approach are explained in the following subsections.

A. Modified Distribution Network

Since this study focuses on the rural distribution architecture, as an example, a modified IEEE-33 radial bus distribution network is considered, as shown in Fig. 3. The bus architecture is modified by removing branch connections between buses (21-8, 9-15, 12-22, 18-33, 25-29) to imitate a weak rural network. Moreover, shunt CB from the bus node to neutral is added to meet the reactive power demand and improve the voltage profile of the farthest bus from the synchronous generator. The choice of the bus node for adding CB is determined through the proposed optimization framework. Due to the intermittent nature of renewable energy generation,

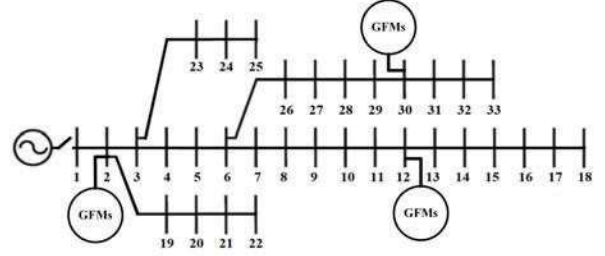


Fig. 3. Modified IEEE-33 bus during islanding with GFMs connected.

the voltage rise and fall during higher and lower generations cause voltage stability issues in the rural grid. Moreover, rural distribution networks could become more vulnerable during natural disasters and may experience isolation from the bulk power system. Fig. 3 shows the isolated operation of the rural grid with multiple GFMs placed in different buses. We assume that total GFMs active and reactive power capacity equals the total power demand of the isolated community. This grid-forming inverter-connected bus acts as a PV bus, and one of them is assigned as a slack bus to perform the power flow analysis at every iteration of the proposed optimization process. All of these modifications are encountered in the modified grid modeling framework.

B. Current Summation AC Power Flow Method

Current, voltage, and power summation AC power flow methods are widely used for radial distribution networks. Among these, the current summation AC method (CSM) is employed to solve the power flow for the proposed framework. The CSM can solve the power flow even in an unbalanced distribution system, making it suitable for a modified radial distribution network. Details of implementing CSM used in this article can be found in [10].

C. Capacitor Bank Model

A circuit breaker (CB) is included to manage reactive power. The diagram in Fig. 4 illustrates the single line setup with the CB at bus $n+1$. The specific distribution line's reactive power is detailed in [7]

$$Q_{n+1} = Q_n - Q_{Ln+1} - X_n \left(\frac{P_n^2 + jQ_n^2}{|V_n|^2} \right) + Q_{c,n+1} \quad (1)$$

where Q_c represents the shunt CB reactive power rating.

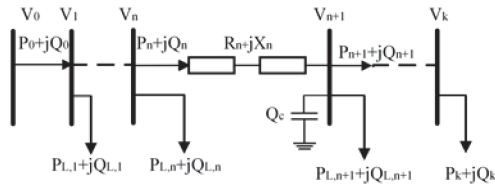


Fig. 4. Radial distribution system with a capacitor connected at $n+1$ bus.

D. System Constraints

The proposed optimization framework with the connected GFM model, discussed in the preceding Section, is subject to equality and inequality constraints associated with the electrical distribution network [7]. In this study, the following constraints are taken into consideration.

- Equality constraints- The active and reactive power flow constraints are considered equality constraints as

$$\sum_{i=1}^{n_g} P_{GFM_s} = \sum_{i=1}^n P_L(i) + \sum_{j=1}^{n_b} P_{loss}(j) \quad (2)$$

$$\sum_{i=1}^{n_g} Q_{GFM_s} = \sum_{i=1}^n Q_L(i) + \sum_{j=1}^{n_b} Q_{loss}(j) \quad (3)$$

where P_{GFM_s} and Q_{GFM_s} are active and reactive power supplied from the GFM-based IBRs, respectively. P_L and Q_L are the active and reactive loads, respectively. n_g , n , and n_b are the number of GFM-based IBRs, bus number, and branch number, respectively.

- Inequality Constraints- These inequality constraints include bus voltage and thermal limit constraints. Bus voltage constraints:

$$v_{min} \leq v_i \leq v_{max}, i = 1, 2, \dots, n \quad (4)$$

Thermal limit constraints: The current passing through branches and GFMs must stay within acceptable limits

$$I_i \leq I_{max,i}, i = 1, 2, \dots, n_b \quad (5)$$

$$I_{G(j)} \leq I_{G,max}, j = 1, 2, \dots, n_g \quad (6)$$

where I_i is the branch current of i^{th} branch, and $I_{G(j)}$ supplied current by GFM connected at j bus.

- For the CB-based voltage profile improvement approach, we need to consider total injected reactive power must be equal to or less than the total reactive load as

$$Q_{c,total} \leq \sum_{i=1}^n Q_L(i) \quad (7)$$

E. Particle Swarm Optimization Algorithm

PSO, a population-based iterative optimization algorithm, uses fish schooling and bird flocking as examples of swarm behavior to direct particle movement while searching for globally optimum solutions [12]. PSO is selected for this study due to its more straightforward implementation process and generic nature, irrespective of models. This method can

effectively choose the optimal CB or GFM placement and total power loss minimization. The bounded variables are bus numbers and sizing (active and reactive power limit) for each subsystem (CBs or GFMs). PSO parameters used in this study are shown in Table I.

TABLE I
PSO PARAMETERS

Parameters	Value	
	GFM	CB
Number of variables	9	6
Maximum stall iterations	20	20
Number of particles	90	60

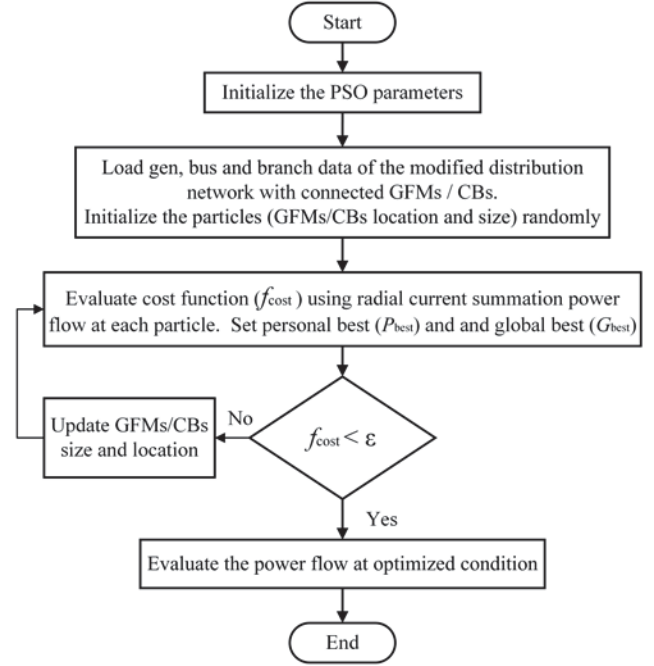


Fig. 5. Particle swarm optimization method for optimal allocation of GFMs/CBs to improve voltage stability.

The PSO flow diagram focused on the voltage stability improvement method is illustrated in Fig. 5. The cost function considers the total fast voltage stability index (FVSI) and voltage profile variation. FVSI can be expressed, considering Fig. 2, as

$$FVSI = \frac{4Z^2Q}{E^2X_L} \quad (8)$$

where Z is the line impedance, $FVSI < 1$ indicates stable, and $FVSI \geq 1$ refers to unstable conditions.

$$f_{cost} = \zeta_1 \sum_{i=1}^n FVSI(i) + \zeta_2 \sum_{i=1}^n \sqrt{(v_{ref} - v_i)^2} \quad (9)$$

where f_{cost} , ζ , v_{ref} , and n are the cost function, weighting factor, reference voltage (1 p.u.), and total bus number. Since FVSI only considers sending bus voltage E , voltage profile variation is also considered to improve the receiving end bus voltage profile. Moreover, all system constraints outlined in

the preceding subsection are incorporated into the optimization framework by assigning a high-cost value if these constraints are violated.

IV. SIMULATION CASE STUDY

A simulation case study is performed considering the following two different operating modes.

A. Grid-Connected Condition with Optimal Allocation of CBs

The modified IEEE-33 bus network without CBs/GFM connection is a base case scenario. The voltage profile of the base case is calculated by employing current summation AC power flow, as shown in Fig. 6, which illustrates that severe voltage fall occurs at the farthest buses (18 and 33) from the synchronous generator. After adding capacitors at optimal bus

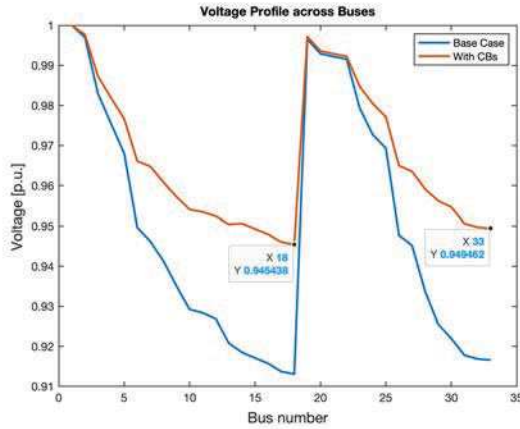


Fig. 6. Voltage profile improvement of a rural distribution network through optimal shunt capacitor bank placement.

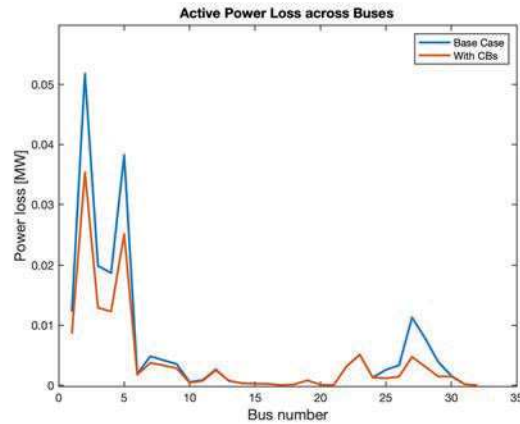


Fig. 7. Power loss reduction of a rural distribution network through optimal shunt capacitor bank placement.

locations 7, 14, and 30 with 500, 338.5, and 438.7 kVAR found from the optimization framework, both bus 18 and 33 voltage profiles improve by 3.81%. The injection of reactive power in the network improves the power factor and reduces total active power loss by 32.47%, as shown in Fig. 7. Additionally, FVSI shows a lower magnitude than the base case, as shown in Fig. 8, referred to as a more stable network.

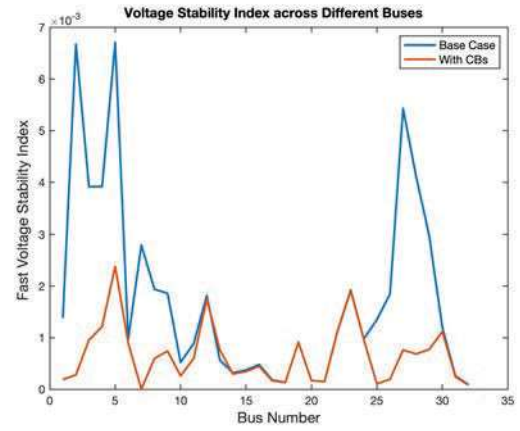


Fig. 8. Voltage stability improvement through optimal shunt capacitor bank placement.

B. Islanding Operation with Optimal Allocation of GFMs

The whole rural distribution network is considered disconnected from the bulk power grid during this operation mode. Local grid-following inverter-based IBRs still produce power and meet the local power demand. However, in the absence of a synchronous generator, three GFM units are considered to form the microgrid, as shown in Fig. 3. Table II shows the optimal GFMs bus positions and sizes, attained by iteratively minimizing the cost function, illustrated in Fig. 9. Fig. 10

TABLE II
OPTIMAL GFMS LOCATION AND SIZE

Serial no.	Bus number	Active Power [MW]	Reactive Power [MVAR]
GFM-1	2	1.8	1.054
GFM-2	12	1.018	0.986
GFM-3	30	1.785	1.085

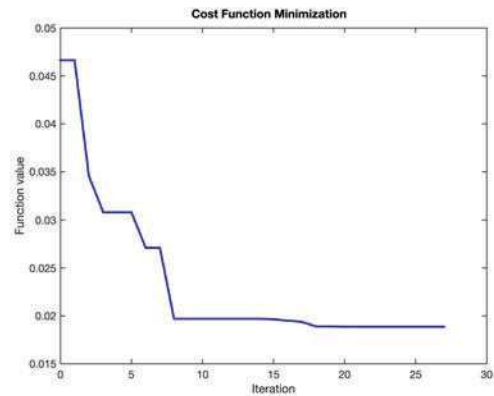


Fig. 9. Cost function minimization process using PSO algorithm.

shows the voltage profile improvement by 8.12% and 8.57% at bus 18 and 33, respectively. Both active power loss and fast voltage stability index reduction are evident, as shown in Fig. 11 and Fig. 12, respectively. The total active power

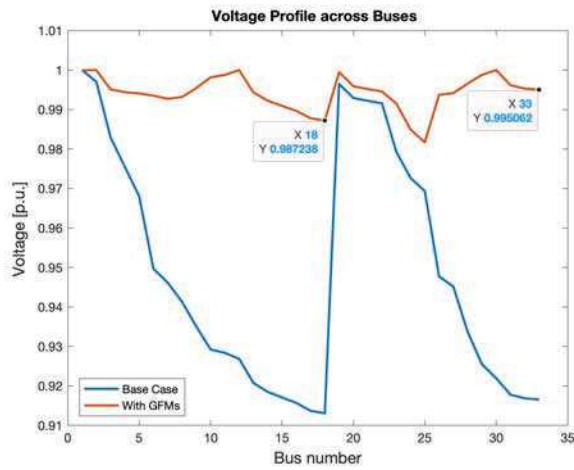


Fig. 10. Voltage profile improvement of a rural distribution network through optimal GFMs placement.

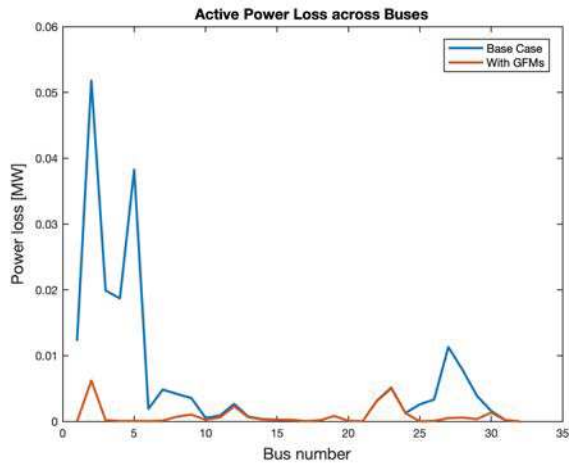


Fig. 11. Power loss reduction of a rural distribution network through optimal GFMs placement.

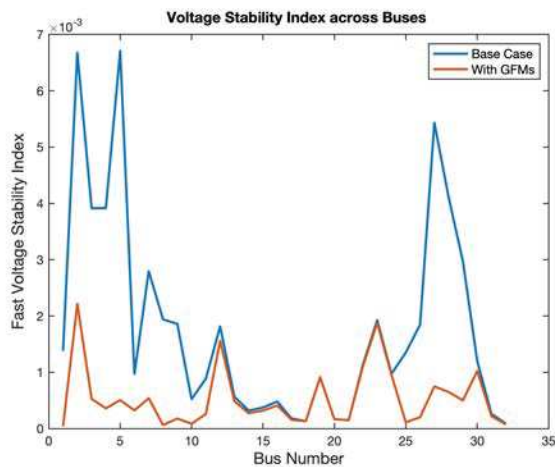


Fig. 12. Fast voltage stability index of a rural distribution network through optimal GFMs placement.

loss (202.7 kW for base case and 26.8 kW for GFMs case) and total FVSI (0.058 for Base case and 0.017 for GFMs case) across all the buses is reduced by 86.7% and 70.5% due to the decentralized GFMs deployment, which reduces the distance between sources and load centers.

V. CONCLUSION

The PSO-based optimal allocation framework of GFMs and CBs for voltage stability improvement in a weak rural grid with system constraints is tested in a modified IEEE-33 radial bus system. Simulation results show that CB placement improves the voltage profile of the most affected bus by 3.81%. During islanding operation, GFMs can improve voltage profile by 8.57%, reduce total FVSI index by 70.5%, and reduce total active power loss by 86.7%. With the decentralized GFMs' optimal deployment and their optimal size can form an efficient, robust, and resilient rural microgrid. This paper excludes the GFMs' dynamic performance; hence, comparing the dynamic performance between optimally and randomly allocated GFMs will be worth investigating.

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