Facilitating Spectrum Sharing With Passive Satellite Incumbents

Jonathan Chamberlain, *Member, IEEE*, David Starobinski, *Senior Member, IEEE*, and Joel T. Johnson, *Fellow, IEEE*

***Abstract*— Space-Air-Ground Integrated Networks will facilitate seamless user experiences across a variety of 6G applications. The deployment of these networks will necessitate new approaches to spectrum allocation. Spectrum access by passive microwave sensors for earth-based and space-based scientific applications represents a spectrum use application having unique attributes that motivate consideration of spectrum sharing between these "incumbents" and commercial users to ensure the most efficient utilization of available frequencies across applications. Toward this end, we propose an economic framework where incumbents have priority use, with a primary and secondary commercial tier underneath. For commercial users, the option to join the primary tier is based on a model of short term post-paid leases of spectrum, while the secondary tier is available to join at no cost. Using a joint game-theoretic and queuing-theoretic model, we find that for practical parameters the revenue maximizing equilibrium is: 1) stable in the Evolutionary Stable Strategy sense; 2) associated with the maximum priority upgrade fee customers are willing to pay; 3) associated with an equilibrium where all customers wish to join the priority class; and 4) socially optimal. We validate our findings leveraging trace data from satellite radiometers operating in the vicinity of Boston, Massachusetts.**

***Index Terms*— Spectrum management and engineering, integrated communications, scheduling of communication, network economics, game theory.**

I. INTRODUCTION

THE vision for the future of wireless communication includes support for a range of applications, such as precision agriculture, automatic traffic monitoring, connectivity to remote regions, and smart healthcare, with impacts to effectively every industry imaginable [1]. The service classes required to support these applications range from terabit per second transmission rates, to combining high-fidelity voice transmissions with ultra-low power consumption, to low-bit rate transmissions with ultra-low delays [2]. To ensure seamless user experiences in these dynamic environments while providing ubiquitous coverage, future 6G networks are expected to leverage a Space-, Air-, Ground- Integrated Network (SAGIN) paradigm: a complex resource stack with components operating on distinct frequency bands and requiring coordination for efficient utilization [3], [4], [5], [6]. SAGINs must balance access requirements among users, including any existing incumbents operating in frequency ranges targeted for integration.

In particular, scientific applications such as the remote sensing Earth Exploration Satellite Service (EESS) for environmental monitoring operating in the high band (>6 GHz) of the wireless spectrum represent an important current and future spectrum use category with unique attributes with respect to innovative spectrum coexistence frameworks. These EESS radiometers are passive observers operating on multiple frequencies spread throughout the spectrum. As seen in Figure 1, the frequency allocation is based upon the resonance properties of the phenomena being observed. For example, observations near 23 GHz support measurement of atmospheric water vapor given the water vapor molecule's absorption line near this frequency. Further, the observations made by these radiometers are particularly susceptible to interference from anthropogenic radio transmissions.

Thus, within the SAGIN context, this necessitates a situation where frequencies are intermittently available to continue supporting the important environmental information acquired by passive microwave observations [7], either at frequencies currently used for such applications or in other portions of the spectrum. As seen in the example orbital track of a Global Precipitation Measurement (GPM) radiometer [8] in Figure 2, low Earth orbiting (LEO) satellites observe a given location on Earth only during the brief interval in which the sensor's typically high gain antenna pattern is directed toward that location. Taken collectively, this results in a situation where there are frequent interruptions, but of durations of at most seconds at a time [9].

As a result, while space-based microwave radiometers have a theoretical negative outcome on commercial users due to regular interruptions, in practice this is mitigated by a collective low occupancy rate. For example, the tracks shown in the vicinity of Boston, MA result in no access requests from the GPM sensor (which observes simultaneously at frequencies near 10.6, 18.7, 23, 37, 89, 166, and 183 GHz) 99% of the time [10]. While this access level is applicable only for one satellite, and deployment of space-based Earth observing microwave radiometers is expected to grow in the future, it is nevertheless reasonable to assume that access

Received 6 March 2024; revised 28 June 2024; accepted 5 August 2024. Date of publication 16 September 2024; date of current version 22 November 2024. This work was supported in part by the U.S. National Science Foundation under Grant CNS-1908087, Grant AST-2229103, and Grant AST-2229104. *(Corresponding author: Jonathan Chamberlain.)*

Jonathan Chamberlain and David Starobinski are with the Department of Electrical and Computer Engineering, Boston University, Boston, MA 02215 USA (e-mail: jdchambo@bu.edu; staro@bu.edu).

Joel T. Johnson is with the Department of Electrical and Computer Engineering and the ElectroScience Laboratory, The Ohio State University, Columbus, OH 43210 USA (e-mail: johnson.1374@osu.edu).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/JSAC.2024.3459034.

Digital Object Identifier 10.1109/JSAC.2024.3459034

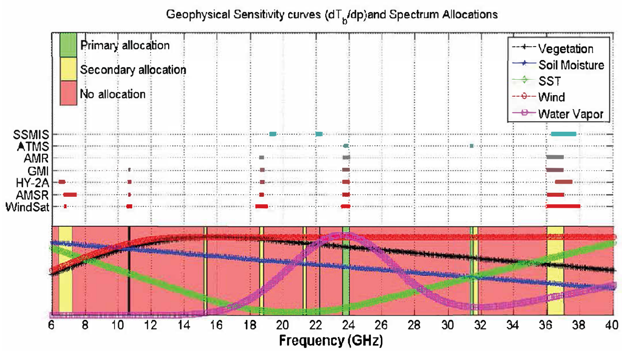


Fig. 1. Above, the observational frequencies for a selection of Earth Exploration Satellite Service passive remote sensing radiometers. Below, sensitivity curves for geophysical properties of interest being measured, as compared to the radiometer frequency allocations, both primary and secondary.

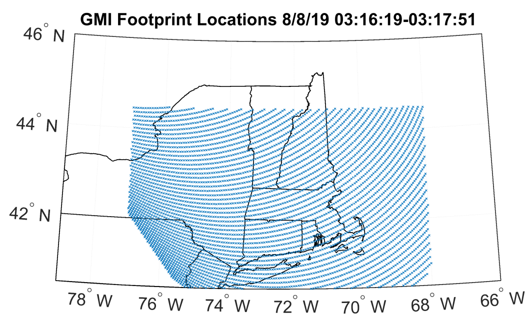


Fig. 2. Footprint locations of the GPM radiometer over an example approximate 90 second period, in a sample region encompassing portions of the Northeastern United States.

requirements will remain moderate when examined versus time well into the future. This drives an overarching question of how to accommodate commercial use given the presence of space-based EESS measurements. Using EESS-passive systems as a representative example, we envision an innovative spectrum access paradigm whereby such measurements are designated as "prioritized incumbents" similar to those considered in other spectrum sharing designs.

Specifically, we envision a three-tier spectrum sharing scenario akin to the Citizens Broadband Radio Service (CBRS) [11], [12], where *incumbents* are pre-assigned the highest priority. Commercial users wanting to utilize spectrum have the option to either pay a fee for access to a *primary commercial tier* (i.e., the second highest priority tier), or forgo the fee and access under a *secondary commercial tier* (i.e., the third, lowest priority). The fee charged to primary commercial users is on a short-term *pay-as-you-go* basis, much like in-flight WiFi access passes offered by airlines today [13], [14]. For commercial users, the decision of which tier to utilize is driven by whether the better Quality of Service (QoS) performance for priority access outweighs the fee, as secondary commercial users experience higher delay and more frequent preemption.

The incumbents' absolute requirement that commercial users vacate the spectrum when a satellite covers the area further complicates this decision, because customers cannot avoid preemption altogether, even if purchasing priority. While the passive nature of the radiometers complicates coordination efforts, it has been demonstrated that spectrum sharing from a technical perspective is feasible with EESS-passive satellites [9]. However, as discussed in Section II, to our knowledge the economic impacts of the passive incumbents' presence on the customer decision-making process within such a multi-tier setup have not been previously studied. This motivates the following questions given this framework:

i) How do Quality of Service factors such as delay and preemption impact customers' tier utilization decision?
ii) How does strategic (selfish) customer behavior impact a provider's revenue guarantees?
iii) What cost does a provider need to set to induce specific customer behavior to maximize revenues?
iv) What impact do the passive incumbent arrival patterns and the service provider's behavior have on the overall social welfare?

Our contributions are as follows:

1) We formalize the customer tier decision and provider price decision within a *queuing game* framework [15], which captures both the game-theoretic and queuing-theoretic aspects of the system. Our model explicitly accounts for preemption costs and incumbent traffic load.
2) We determine that there exist several possible equilibrium regions, including some with mixed equilibria which are stable in the *Evolutionary Stable Strategy* (ESS) sense, and others featuring multiple unstable mixed equilibria.
3) We prove that despite the possibility of regions with unstable equilibria, the maximal provider revenue for fixed parameters is always associated with stable equilibria.
4) We show that the customers' uncoordinated purchasing decision results in a theoretical lack of an upper bound on the negative externalities they impose on one another (i.e. their *Price of Anarchy* [16]). Nevertheless, we argue that in practice, this theoretical worst-case does not prevail with a revenue maximizing provider.
5) We validate our delay model and assert our equilibrium claims by leveraging trace data from satellites on the 23 GHz band and prior studies on practical parameters for commercial users on shared spectrum bands [17], [18], [19].

The remainder of the paper is organized as follows. In Section II we provide an overview of related works. In Section III we detail our system model and the associated game. In Section IV we analyze the resulting possible equilibria states, which we leverage in Section V to evaluate how this impacts the provider's behavior. In Section VI we consider how the resulting provider decision impacts the social state and whether this imposes negative externalities on the provider. We present numerical results in Section VII and provide concluding remarks in VIII.

II. Related Work

Reuse of frequencies to avoid interference is integral in SAGIN design to facilitate seamless communication via components on separate layers. Yet, while spectrum sharing is implicitly assumed to be a part of 6G design in general (e.g., designing cellular towers with both communication and sensing in mind [20]), frequency reuse is an identified fundamental challenge in enabling SAGIN frameworks to be stood up at all [4]. Multiple mechanisms for spectrum sharing have been proposed [21]; among these is the three-tier CBRS [11], [12], which has been a topic of interest for research in light of ongoing license allocations and the resulting deployments. The CBRS consists of three tiers of users: extant highest-priority incumbents, priority license holders, and general licensed by rule users. While currently applied to the 3.5 GHz mid-band, CBRS has been cited as a paradigm which may be expanded to other areas of spectrum, such as those containing sub-bands allocated for passive science and space exploration use [22]. CBRS is also cited as an inspiration for demonstrating the technical feasibility of sharing with EESS satellites, including proposing a similar spectrum access scheme to coordinate spectrum utilization [9]. Subsequent technical work has demonstrated the ability to establish real time geofencing through decentralized coordination [23]; however these works only consider the feasibility of sharing from a technical standpoint, and not the economic aspects as we do here.

To our knowledge there have been no previous studies of passive incumbents' impacts on the economics of multi-tier systems. In general, existing literature on multi-tier sharing does not directly account for incumbent behavior in lower tier decisions. Examples include the provider decision of which tier to operate [24]; the impact of small-cell resource allocation on deployment decisions by multiple providers [25]; the problem of sub-leasing priority access to other providers [26]; and minimizing the impact of free-riders within a shared spectrum scenario [27].

The use of queuing games to analyze priority purchasing decisions in the face of potential delays has been explored in several prior works [28], [29], [30]. In particular, models with Poisson distributed arrivals and general service distribution in particular are commonly used in modeling cognitive radio as noted in prior surveys [31]. The question of a customer priority purchasing decision under general service distribution has been previously considered under a two-tier scenario [32], [33].

A preliminary version of this work considered an alternate incumbent characterization, specifically those of active incumbents under a CBRS setting (i.e. Naval radars) [34]. Here, we consider passive satellite incumbents, who therefore have differing spectrum access characteristics. In particular, [34] assumes in its analysis that CBRS incumbents can potentially occupy 5% of spectrum, a rate much greater than is currently indicated by the EESS-passive satellites considered here. In addition, our paper addresses the social welfare resulting from the provider's pricing actions, as well as validation of our model with traces and equilibrium convergence simulation, all of which were not considered in [34].

Additional work considers a two-tier system, with a single customer class and the passive satellite incumbents [10]. The customers in that system faced a decision of whether to pay a fee to join the system, or not join at all (i.e. *balk*). Here, we have an additional secondary customer tier; customers have the choice between joining the primary commercial tier for a fee, or the secondary tier for free, and therefore customers do not balk. As a result, the equilibria analysis is much more involved, with a key result here being that a *Follow the Crowd* [15] equilibrium is a more likely outcome. In the two-tier case, *Avoid the Crowd* [15] is more likely. This contributes to the result that the Price of Anarchy here has no fixed upper bound; whereas in the two-tier case, the PoA is unity everywhere.

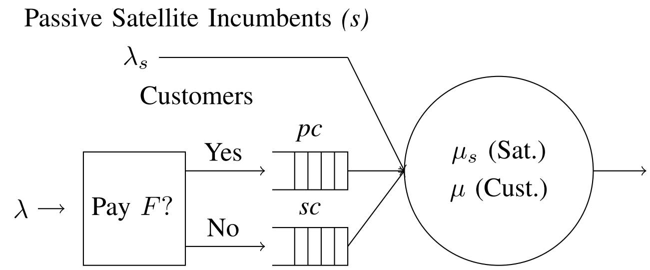


Fig. 3. The multi-tier spectrum access model expressed as a queuing model. Spectrum access by incumbent passive satellites force customers to vacate. Conversely, customers choose between paying a fee F to access the Primary (pc) commercial tier queue, or forgo payment and default to Secondary (sc) tier queue, which are effectively the 2nd and 3rd tiers due to the satellites causing service interruptions.

III. Economic Model and Problem Statement

In this section, we present our economic model for multi-tier spectrum access including the assumptions underpinning the formal definition of the priority purchasing game the commercial user agents (hereafter, *customers*) engage in. The parameters and variables of interest for the model and subsequent purchasing game are defined in Table I.

A. Model of Spectrum Usage

We begin by introducing a statistical model accounting for spectrum utilization by incumbents and customers. We assume that, given a fixed geographic region, customers operate within a specific frequency band. Access to this frequency band is intermittent due to the presence of satellite based radiometers in LEO which cause interruptions when passing overhead performing remote sensing measurements. The assumption of customers operating on specific frequency bands is consistent with existing licensing structures, such as CBRS Priority Access Licenses [11], and is consistent with other assumptions regarding intermittent spectrum access [23]. We consider this reasonable in any event due to the low utilization rate of available spectrum capacity by the EESS satellites, as shown by the trace data in Section VII. We additionally assume that a service provider processes requests for spectrum utilization. As part of this process, the provider offers priority upgrade to the customers:

- If the customer agrees to pay an upgrade fee F to the provider, they are granted access to a *primary commercial tier*, with preemptive priority.
- If the customer forgoes paying the fee, they are relegated to the *secondary commercial* tier.

As noted in Section I, this business model parallels that of limited-term spectrum purchases.

To describe customer behavior, we leverage queuing-theoretic frameworks. Specifically, the customers form an unobservable queue, following a First Come First Serve

TABLE I

DEFINITION OF PARAMETERS AND VARIABLES RELATED TO THE MULTI-TIER QUEUING GAME MODEL. THE FIRST SIX ARE INPUT PARAMETERS WHICH ARE TRAFFIC PARAMETERS AND COSTS BASED ON THE CUSTOMERS' OWN VALUATION OF SERVICE; THE SEVENTH IS A SUBSTITUTION FOR REPEATING EXPRESSIONS WHICH IS A COMBINATION OF THESE PARAMETERS. THE NEXT THREE ARE DERIVED FROM THE ANALYSIS. THE FEE F (AND BY EXTENSION THE AVERAGE REVENUE R) IS THE PARAMETER WHICH THE PROVIDER SEEKS TO OPTIMIZE. THE SOCIAL WELFARE S PROVIDES A MEANS OF MEASURING THE EXTENT TO WHICH THE PROVIDER OPTIMAL FEE DIFFERS FROM THE SOCIALLY OPTIMAL SYSTEM STATE

Parameter	Definition
λ, λ_s	Arrival rate of customers and passive satellites, respectively.
μ, μ_s	Service rate of customers and passive satellites, respectively (equal to 1 over mean service length).
ρ, ρ_s	Traffic load of customers and passive satellites, respectively (equal to arrival rate over service rate).
K, K_s	Service variance parameter, where the second moment of service is K/μ^2, K_s/μ_s^2 for customers and passive satellites, respectively.
C_d	The customers' self-imposed cost of system delay, per time unit.
C_p	The customers' self cost of preemption by a higher class user or server breakdown, per preemption.
α, β, γ	Substitutions for the repeating expressions $1-\rho_s$, $1-(\rho_s+\rho\phi)$, and $1-(\rho_s+\rho)$, respectively.
D_{pc}, D_{sc}	Random variable representing the total system delay for primary and secondary customers, respectively.
P_{pc}, P_{sc}	Random variable representing the number of service preemptions for primary and secondary customers, respectively.
ϕ	Fraction of customers who are priority customers.
F	Fee to become a priority customer.
R	Average provider revenue per time unit.
S	The social welfare (net utility) of all user agents in the system.

(FCFS) service discipline with priorities, which we visualize in Figure 3. While Processor Sharing models exist which capture situations where multiple individuals of different priorities are served simultaneously [35], [36], CBRS-type models do not allow for simultaneous service of users in different tiers on the same channel in the same location [11]. Thus, we leverage FCFS to capture a sense of the traffic while respecting the CBRS restriction. Note that as the customers themselves are service providers and not individual users, an arrival can be considered as a flow consisting of a sequence of packets utilizing the spectrum representing multiple individuals.

The customer arrivals to the queue follow a Poisson process with rate λ, an assumption supported by measurement studies such as those by Willkomm et al. [17] analyzing spectrum usage in cellular networks. Service times are *generally* distributed service with mean $1/\mu$ for some $\mu > 0$ and a second moment K/μ^2 for some $K \geq 1$; the same measurement studies which justify Poisson arrivals do not justify the common assumption in the literature of exponential service, which is why we adopt a general service model here. Knowledge of the mean and the second moment defines a specific service distribution; in the next subsections we show that the priority joining decision depends in part on whether the variance in service times as determined by the second moment is greater than that of exponential service. This yields the parameter $K \geq 1$ defined in terms of the second moment; of note, $K = 1$ corresponds to a deterministic distribution and $K = 2$ corresponds to an exponential one. Finally, we describe the *traffic load* as the ratio of arrivals to departures, expressed as $\rho = \lambda/\mu$. Per the standard assumptions on an unobservable queue, the choice to join the queue, or upgrade, or not is based on statistical knowledge [29]. Specifically, the arrival process is continuous and the number of customers is not fixed *a priori* at any given moment in time. Thus, we reason primarily in terms of this traffic load ρ as it can be viewed synonymously with spectrum utilization rate.

As for the incumbents, as orbital radiometers they represent *passive agents*. As the satellites pass overhead, a blackout is imposed for the duration to ensure the EESS radiometer access to the frequencies required for measurements. Under our model, we assume the provider has the ability to coordinate such a blackout via a geo-fencing model [23] due to the passive remote sensing nature of the radiometers, which do not have a means to directly communicate their presence. These blackout periods create intermittent frequency availability for the customers. While the overpass times of specific satellites can be predicted deterministically, these times can be widely distributed and more complex when multiple satellites are considered. Therefore, we conclude that the satellites impose a stochastic *service interruption* process from the standpoint of the customers. Interruptions are triggered by the satellite arrivals, which follow a Poisson process with rate λ_s. The blackout periods during which radiometers are conducting remote sensing in the region also follow a general distribution, with mean length $1/\mu_s$ for some $\mu_s > 0$, and second moment K_s/μ_s^2, for some $K_s \geq 1$. The resulting incumbent traffic load is $\rho_s = \lambda_s/\mu_s$. As with the customers, incumbent arrivals form a continuous process, thus the number of interruptions is not fixed *a priori*.

In terms of the relative priority of the user classes, the incumbents' requirements clearly necessitate they have absolute priority over all customers. The nature of the blackout periods results from the assumption of preemptive priority. The provider's priority upgrade offer also grants primary customers preemptive priority over secondary customers. As procedures exist to hold lower priority transmissions until they can be resumed [37], we can impose the assumption that customers who are preempted by a higher priority user reenter service from the point of interruption. This corresponds to a *preemptive-resume* queuing model [38, p. 67]. In addition, we also employ other standard assumptions from the queuing literature: specifically that users' service distributions are independent and identically distributed (i.i.d.) and that the system is stable; that is, new users do not arrive faster than the current users can be serviced [28], [29]. We further assume that while the incumbent and customer streams have differing statistical parameters, users within each stream are homogeneous. Regarding the stability condition, we specifically assume that the relation $\rho + \rho_s < 1$ holds. With the queuing framework established, we are now able to analyze the game theoretic decision of which tier customers join for given values of the queuing parameters.

B. Priority Purchasing Game

In establishing the game, we only have two (groups) of players: the provider, and the customers. Incumbents, while present and impacting the decision making processes of the other agents, are passive agents not engaging in a meaningful decision to utilize spectrum. Each (set of) agent(s) has(/have) their own set of actions available to them. For the provider, who operates spectrum access and priority upgrade mechanisms, there is only one meaningful decision: the level at which to set the priority upgrade fee F. The provider has no restriction on the level at which to set the fee beyond an implicit assumption that $F > 0$. In a priority purchasing environment, it can be assumed the provider has no incentive to provide a subsidy to customers to induce them to choose to join the primary tier. As a result, the primary focus will be on the customers' reaction to this decision. As we consider an unobservable queue, decisions of which tier to join are made on the basis of the expected values of the metrics of interest. As customers are assumed to be homogeneous, their benefit from service is identical regardless of class. Therefore, customers will choose the class which minimizes their costs. Specifically a rational customer will be concerned with the following:

1) The fee F to upgrade to the primary tier (if paid);
2) The self-imposed costs of time spent waiting for service completion;
3) The self-imposed costs of preemption by other users.

Additionally, due to the nature of the continuous stream of customers a) once a customer identifies a need to utilize the frequencies on offer there is no reason not to join the queue; and b) the statistics do not change, thus providing no appreciable benefit to attempt to jockey for position and indeed, dropping out of the queue and rejoining provides negative benefit by extending the expected time before service is completed. Thus, we assume that customers do not balk or renege from the queue as no incentive exists to do so. Therefore, as the game is based on the customer decision to upgrade or not, the possible system states are some fraction $\phi \in [0, 1]$ of customers opting for priority over general status. This ϕ also represents the customers' collective strategy.

Our analysis concerns itself with the factors influencing a particular fraction ϕ to upgrade. Specifically, we address:

i) How the customers reach a particular collective strategy ϕ for the upgrade decision?
ii) What is the cost the provider needs to set to induce a particular strategy ϕ which maximizes its revenue?
iii) What are the impacts of passive satellite incumbent arrival patterns and the cost of preemption on these decision making processes?

We are interested in establishing the states ϕ which result in *(Nash) equilibrium* states under steady-state conditions. This occurs when either one class always has a lower cost to join than the other, or when customers are indifferent between their options. This indifference occurs when the costs to join each class are equal. Let C_d denote the self-imposed costs of the system delay per time unit; $\mathbb{E}[D_{pc}]$ and $\mathbb{E}[D_{sc}]$ the expected costs of delay as defined in Subsection III-C below; C_p the self-imposed cost of preemption, per each preemption; and $\mathbb{E}[P_{pc}]$ and $\mathbb{E}[P_{sc}]$ the expected number of service interruptions due to preemption for each class. Then in terms of our defined quantities, customers are indifferent between their options when the costs of joining each class are equal:

$$C_d E[D_{pc}] + C_p E[P_{pc}] + F = C_d E[D_{sc}] + C_p E[P_{sc}]. \quad (1)$$

C. Analysis of System Delays and Preemptions

To engage in an equilibrium analysis we must first define expressions for the per-class system delay and expected number of preemptions. We begin with the system delay, defined as the total time spent in the system from initial entry to the queue to completion of service. Let D_{pc} and D_{sc} be, respectively, the random variables governing system delay for priority and secondary customers. We approximate our system as a three class $M/G/1$ preemptive-resume with priorities [38]. We justify this view due to the fact that a second satellite arrival can overlap with an-in-progress sweep due to differences in orbits, overlapping protection zones, and/or sweeps covering distinct partial segments of the same sector within a similar time frame. Our trace data in Section VII justifies this view as a reasonable framing due to the relative infrequency of satellite arrivals compared to customer arrivals. Taking the incumbents as the highest class, and letting $\alpha = 1 - \rho_s$, $\beta = 1 - (\rho_s + \rho\phi)$, and $\gamma = 1 - (\rho_s + \rho)$ be substitutions for repeating expressions we employ for space saving constraints, we derive the system delay for the remaining classes by applying the formula for preemptive-resume priority queuing systems [38]:

$$\mathbb{E}[D_{pc}] = \frac{1}{\mu\alpha} + \frac{K_s\rho_s/\mu_s + K\rho\phi/\mu}{2\alpha\beta};$$
$$\mathbb{E}[D_{sc}] = \frac{1}{\mu\beta} + \frac{K_s\rho_s/\mu_s + K\rho/\mu}{2\beta\gamma}. \quad (2)$$

For the preemptions, the expected number of preemptions by higher class customers is a function of the arrival rate of the higher class customers and the service rate of the current customer. For each class this is expressed as

$$\mathbb{E}[P_{pc}] = \lambda_s/\mu, \quad \mathbb{E}[P_{sc}] = (\lambda_s + \lambda\phi)/\mu. \quad (3)$$

IV. Equilibria Analysis

We now have the means to establish equilibria conditions for fixed parameters, and address the questions raised in Subsection III-B. As the provider has no restrictions on how they set F, while customers react to the value of F and the queue statistics, providers may use their knowledge of said statistics and corresponding candidate equilibria to optimize their revenues. As a result, we derive a function $\mathcal{F}(\phi)$ relating equilibrium candidate strategies ϕ to the corresponding priority upgrade fees F. To define $\mathcal{F}(\phi)$ we solve Equation (1) for F and apply Equations (2) and (3) for the expected system delays and preemptions. Additionally, WLOG we fix $C_d = 1$ and normalize costs with respect to the system delay, equating time with money. The result is

$$\mathcal{F}(\phi) = \frac{\rho\Big(K_s\mu\rho_s + 2\mu_s\phi\gamma + K\mu_s(\alpha - \phi\gamma)\Big)}{2\mu\mu_s\alpha\beta\gamma} + C_p\rho\phi. \quad (4)$$

As a result of this expression, we consider two issues: a) how many and what types of equilibria candidates exist for a given F; and b) whether these equilibria candidates are stable given a dynamic customer environment. We immediately observe from the expression that only primary customer preemption of secondary customers explicitly impacts the upgrade incentive - while satellite incumbent arrivals lengthen queuing periods and thus implicitly impact incentives through the delay costs. The fact that customers are preempted regardless of priority due to satellite arrivals results in those costs of preemption cancelling out when determining which class to join. In considering the existence and stability of equilibria, this factor leads to non-intuitive results, especially compared to a simpler join-or-balk model with breakdowns [10].

A. Equilibrium Existence

We begin by determining the existence and classification of equilibrium states. As the customers' action state is a binary decision of which priority class to join, there are three possible equilibrium types:

1) *All upgrade*: $\phi = 1$, i.e. customers are all primary;
2) *None upgrade*: $\phi = 0$, i.e. customers are all secondary;
3) *Some upgrade*: $\phi \in (0, 1)$ of customers are primary, the rest are secondary. Such values of ϕ are found through Equation (1) for fixed queuing parameters.

Based on the relationship between F and $\mathcal{F}(\phi)$, we make the following high level assertion, whose proof is contained in Appendix A:

Lemma 1: Given $\mathcal{F}(\phi)$ as defined as in Equation (4) for fixed queuing parameters, and define $F_0 = \mathcal{F}(0)$ and $F_1 = \mathcal{F}(1)$. The following equilibria types are possible based on the value of F relative to the values of $\mathcal{F}(\phi)$:

1) If $\min \mathcal{F}(\phi) < F < \max \mathcal{F}(\phi)$, at least one *some upgrade* equilibrium is possible.
2) If $F < F_1$, a *all upgrade* equilibrium is possible.
3) If $F < \min \mathcal{F}(\phi)$, there is a unique *all upgrade* state.
4) If $F > F_0$, a *none upgrade* equilibrium is possible.
5) If $F > \max \mathcal{F}(\phi)$, there is a unique a *none upgrade* state.

Leveraging Lemma 1, we derive exact conditions for the regions in which specific combinations of equilibria types exist given fixed parameters. We accomplish this through an analysis of $\mathcal{F}(\phi)$ as defined in Equation (4). We begin by defining threshold preemption costs $\mathsf{C}_\mathsf{p}^{(\mathsf{L})}$,$\mathsf{C}_\mathsf{p}^{(\mathsf{M})}$, $\mathsf{C}_\mathsf{p}^{(\mathsf{H})}$ and a threshold traffic load ρ^T. These thresholds determine boundaries between regions where different combinations of equilibria co-exist:

$$\mathsf{C}_\mathsf{p}^{(\mathsf{L})} = \frac{\mu_s\alpha(K(\gamma - \rho) - 2\gamma) - K_s\mu\rho\rho_s}{2\mu\mu_s\alpha^3\gamma}, \quad (5)$$

$$\mathsf{C}_\mathsf{p}^{(\mathsf{M})} = \frac{\mu_s\alpha(K(\gamma - \rho) - 2\gamma) - K_s\mu\rho\rho_s}{2\mu\mu_s\alpha^2\gamma^2}, \quad (6)$$

$$\mathsf{C}_\mathsf{p}^{(\mathsf{H})} = \frac{\mu_s\alpha(K(\gamma - \rho) - 2\gamma) - K_s\mu\rho\rho_s}{2\mu\mu_s\alpha\gamma^3}, \quad (7)$$

$$\rho^T = \frac{(K-2)\mu_s\alpha^2}{2(K-1)\mu_s\alpha + K_s\mu\rho_s}. \quad (8)$$

Theorem 1: Given $\mathcal{F}(\phi)$ as defined in Equation (4); $\mathsf{C}_\mathsf{p}^{(\mathsf{L})}$, $\mathsf{C}_\mathsf{p}^{(\mathsf{M})}$, and $\mathsf{C}_\mathsf{p}^{(\mathsf{H})}$ defined in Equations (5), (6), and (7); and ρ^T as defined in Equation (8); there are five possible equilibrium regimes which can occur, conditioned on the value of F set by the provider and the queuing parameters for each class:

(I) If $F < \min \mathcal{F}(\phi)$, *all upgrade* is the sole equilibrium.
(II) If $F > \max \mathcal{F}(\phi)$, *none upgrade* is the sole equilibrium.
(III) If $K > 2$, $\rho < \rho^T$ AND
  a) $\min \mathcal{F}(\phi) < F < \max \mathcal{F}(\phi)$ and $C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{L})}$, OR
  b) $\min \mathcal{F}(\phi) < F < F_0$ and $\mathsf{C}_\mathsf{p}^{(\mathsf{L})} < C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{M})}$,
  a *some upgrade* is the sole equilibrium.
(IV) If $K > 2$, $\rho < \rho^T$, AND
  a) $F_0 < F < \max \mathcal{F}(\phi)$ and $\mathsf{C}_\mathsf{p}^{(\mathsf{L})} < C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{M})}$, OR
  b) $F_1 < F < \max \mathcal{F}(\phi)$ and $\mathsf{C}_\mathsf{p}^{(\mathsf{M})} < C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{H})}$,
  there are three equilibria: two *some upgrade* and one *none upgrade*
(V) Otherwise, there are three possible equilibria states: an *all upgrade*, a *none upgrade*, and a single *some upgrade*.

A formal proof is contained in Appendix B. Notably, the monotone decreasing regime (III) corresponds to that of *Avoid the Crowd* customer behavior [15], where an increasing fraction of customers upgrading service decreases the incentive for newly arriving customers to follow suit. Conversely, customer behavior in the monotone increasing region (V) corresponds to that of *Follow the Crowd* where an increasing fraction of customers opting to upgrade service increases the incentive for newly arriving customers to do the same [15]. In fact, we find from a practical standpoint that all of the following must be true for a scenario other than *Follow the Crowd* behavior to occur: customer traffic load must be sufficiently light, the variance in customer traffic must be sufficiently high (i.e. greater than exponential), and customers must be sufficiently tolerant of preemption. We see an example of this in Figure 4 for an arbitrary customer distribution, and a satellite incumbent distribution derived from our trace data in Section VII. Measurement studies of commercial cognitive radios suggest that service distributions are likely to have variances less than that of the exponential distribution [17], [18], [19]. Thus, while *Avoid the Crowd* and multiple mixed equilibrium behaviors are theoretical possibilities that must be accounted for, *Follow the Crowd* behavior (regime (V)) is most likely to prevail for practical parameters.

B. Equilibrium Stability

In this section, we investigate an enhanced equilibrium definition to account for the dynamic nature of the system, given the existence of regions where there exist non-unique best response strategies to given fees F. We adopt the notion of stability according to the Evolutionary Stable Strategy (ESS) definition to capture behavior over time as customers enter the system, assuming steady-state queuing parameters [39]:

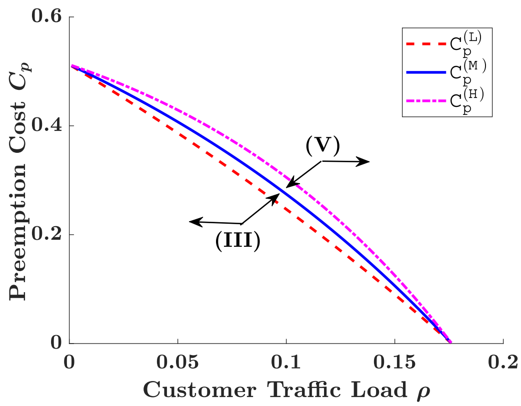


Fig. 4. Plot of the boundaries between equilibrium regimes, assuming $\min \mathcal{F}(\phi) < F < \max \mathcal{F}(\phi)$ for passive satellite incumbents with parameters $\mu_s = 0.02$, $K_s = 3.22$, $\rho_s = 0.01$ and an arbitrary customer distribution with $\mu = 1$ and $K = 3$, showing how the boundaries are impacted by changes to the customer traffic load ρ and cost of preemption load C_p. While type (III), type (IV) (co-exists with the other types between $C_p^{(L)}$ and $C_p^{(H)}$) and type (V) regimes are all possible, ρ and C_p must be sufficiently low for customer behavior other than *Follow the Crowd* (type (V)) to emerge.

Definition 1: An equilibrium strategy x is stable in the *Evolutionary Stable Strategy (ESS)* sense if no alternate equilibrium strategy x' is a better response against itself than x.

Even if equilibrium strategy x is the initial strategy chosen, if some subset of players choose the alternate best response strategy x' in subsequent rounds, if x is not ESS but x' is, then the latter will eventually become the equilibrium state adopted by the population as a whole [15, p.5]. While the concept of ESS equilibria originated with evolutionary biology, the concept has been widely studied in mathematics and economics to consider dynamic systems more generally [40]. With Definition 1 in mind, we assert the following, which we prove in Appendix C:

Theorem 2: Pure equilibria, i.e. *all upgrade* or *none upgrade* are always ESS stable. *Some upgrade* equilibria are ESS stable if and only if it is the sole equilibrium state.

Thus, the provider can always be assured that if an *all upgrade* equilibrium leads to maximum revenues, the revenues will be guaranteed due to equilibrium stability. If however maximum revenues are associated with a *some upgrade equilibrium*, the situation becomes more uncertain. Thus, part of the next section is determining whether we can assert that revenue streams are guaranteed in a meaningful sense, provided the queue is in a steady state.

V. Revenue Analysis

With equilibria classification and stability criteria established for a given upgrade fee F, we next determine how the provider leverages their knowledge of $\mathcal{F}(\phi)$ and position for revenue (and therefore profit) maximization. However, as the number of customers is not fixed, we reason in terms of expected revenues per time-unit. Thus, if the upgrade fee is set to $F = \mathcal{F}(\phi)$ for known queue parameters and some fixed ϕ, then the revenue function $\mathcal{R}(\phi)$ can be defined as the product of F times the number of primary customers arriving to the system, or $\lambda\phi\mathcal{F}(\phi)$:

$$\mathcal{R}(\phi) = C_p\rho\lambda\phi^2 + \frac{\rho^2\phi\Big(K_s\mu\rho_s + 2\mu_s\phi\gamma + K\mu_s(\alpha - \phi\gamma)\Big)}{2\mu_s\alpha\beta\gamma}. \tag{9}$$

Because of this relation between $\mathcal{R}(\phi)$ and $\mathcal{F}(\phi)$, for fixed parameters we can assert a) the fee leading to maximal revenues, and b) whether the associated equilibrium ϕ^* is ESS. If ϕ^* is not ESS, the possibility exists that customers will divert to an alternative equilibrium strategy; specifically, whenever multiple candidate equilibria strategies are possible for a given F, the *none upgrade* strategy is a potential outcome. We claim in practice, however, this is not a concern:

Theorem 3: For any valid and fixed user traffic statistics, there exists a stable Nash Equilibrium resulting in revenue arbitrarily close to the maximum.

As noted in Subsection IV-A, for practical parameters *Follow the Crowd* behavior is the most likely behavior. However, these results show that even in the event that customer behavior matches corner cases where other equilibrium regimes are possible, providers have guarantees of their revenue streams as the resulting equilibrium is stable. The stability guarantees assume that providers have knowledge of the steady state traffic. Should the provider still be learning the traffic statistics, there be measurement error, or there is a change in the customer traffic, the results may not align with this guarantee. This has been identified as a target for future research.

VI. Social Welfare and Price of Anarchy

As the customers are reacting to a monopolistic provider in this setting, the provider's action plays an outsized role in determining the game outcome. As such, we are motivated to consider the *Social Welfare*, or net utility, of all user agents in the system. The goal is to determine the extent to which the users impose negative externalities on each other for participation in the market, and in particular the extent that the provider imposes such externalities on the customers through their pricing decision. This in turn determines whether regulatory intervention is necessary to set conditions on which a provider is given the right to manage spectrum access services in this shared setting. To measure the extent of these externalities, we leverage a measure of noncooperation of the customers, known as the *Price of Anarchy (PoA)* [16]. As *PoA* is defined in terms of Social Welfare, we consider the latter first.

A. Social Welfare

In considering the Social Welfare, we note the following:

- The queue is of a work conserving nature, therefore the service order of customers does not matter and the total cost to service customers is independent of service order.
- The provider's reward is the fee F received from the customers who opt to upgrade. This however is a cost from the customer perspective. Therefore, this is a payment transfer which cancels out.

- Customers are homogeneous and see identical reward from service regardless of class, per the assumption that cost minimization is the only factor in the upgrade decision.

Taken together, this leaves the expected costs of system delay and preemption as the only relevant costs or benefits to consider. Thus, the Social Welfare is really a *Social Cost*, and the resulting function $\mathcal{S}(\phi)$ is the weighted average of the costs in each class through application of Equations (2) and (3):

$$\begin{aligned}\mathcal{S}(\phi) &= \phi\left(\frac{1}{\mu\alpha}+\frac{K_s\rho_s/\mu_s+K\rho\phi/\mu}{2\alpha\beta}+\frac{C_p\lambda_s}{\mu}\right)\\ &\quad+(1-\phi)\left(\frac{1}{\mu\beta}+\frac{K_s\rho_s/\mu_s+K\rho/\mu}{2\beta\gamma}+\frac{C_p(\lambda_s+\lambda\phi)}{\mu}\right).\end{aligned} \tag{10}$$

where the terms in the first set of parentheses are the costs of delay and preemption of the primary class $\mathbb{E}[D_{pc}]$ and $C_p\mathbb{E}[P_{pc}]$ weighted by the fraction ϕ of primary customers, and the terms in the second and third sets are the costs of delay and preemption of the secondary class $\mathbb{E}[D_{sc}]$ and $C_p\mathbb{E}[I_{pc}]$ weighted by the remaining fraction $1-\phi$ of secondary customers. The final step before addressing the Price of Anarchy is determining the states which lead to the optimal and worst case social costs, as these essentially define the *PoA* limits. To do so, we require the definitions of the following special states, where where j is the imaginary square root of -1:

$$\phi^* = \left[2C_p\mu\rho(\rho\alpha+4\alpha^2)-(K-2)\rho-\sqrt[3]{\omega_1+\omega_2} - \frac{\rho^2(K-2+2C_p\mu\alpha(\alpha+\gamma))^2}{\sqrt[3]{\omega_1+\omega_2}}\right]\Big/ 12C_p\mu\rho^2\alpha, \tag{11}$$

$$\phi^{**} = \left[\left(\frac{\rho^2(K-2+2C_p\mu\alpha(\alpha+\gamma))^2}{\sqrt[3]{\omega_1+\omega_2}}+\sqrt[3]{\omega_1+\omega_2}\right)\times(1-j\sqrt{3})+2\rho(K-2-2C_p\mu(4\alpha+\rho)\alpha)\right]\Big/ 24C_p\mu\rho^2\alpha, \tag{12}$$

$$\begin{aligned}\omega_1 = 12\Bigg[3\Big(&C_p^2(K-2)\mu^2\rho^6\alpha^3\gamma-\big(8-K^3\\ &-6K^2\big(C_p\mu\alpha(\alpha+\gamma)-1\big)-12K\big(1+C_p\mu\alpha(C_p\mu(\rho^2\\ &+5\alpha(\rho-\alpha))\alpha-2(\alpha+\gamma))\big)-8C_p\mu\alpha(3(\alpha+\gamma)\\ &+C_p\mu\alpha(15\alpha(\alpha-\rho)+C_p\mu\alpha(\alpha+\gamma)^3-3\rho^2))\big)\Big)\Bigg]^{\frac{1}{2}},\end{aligned} \tag{13}$$

$$\begin{aligned}\omega_2 = \rho^3(&-8+K^3+6K^2(-1+C_p\mu\alpha(\alpha+\gamma))+12K(1\\ &+C_p\mu\alpha(C_p\mu(\rho^2+14\rho\alpha-14\alpha^2)\alpha-2(\alpha+\gamma)))\\ &-8C_p\mu\alpha(-3(\alpha+\gamma)+Cp\mu\alpha(3\rho^2+42\rho\alpha-42\alpha^2\\ &-C_p\mu\alpha(\alpha+\gamma)^3)))\end{aligned} \tag{14}$$

The appearance of complex conjugates is the result of the cubic root expression. However as we are restricted to the real subset $\phi\in[0,1]$, then the resulting values of ϕ^* and ϕ^{**} will be real valued for valid parameter ranges which satisfy the conditions for each being the best and/or worst case social states. We formalize our claim in the following Lemma:

Lemma 2: Let $\mathcal{S}(\phi)$ be as defined in Equation (10), and ϕ^* and ϕ^{**} be mixed states defined in Equations (11) and (12).

1) If $K\le 2$, or if $K>2$ and $C_p\ge\frac{K-2}{2\mu\alpha\gamma}$, $\phi\in\{0,1\}$ are optimal social states and $\phi=\phi^*$ is the worst case state;
2) If $K>2$, and $C_p\le\frac{K-2}{2\mu\alpha^2}$, $\phi=\phi^{**}$ is the optimal social state, and $\phi\in\{0,1\}$ are the worst case social states.
3) If $K>2$ and $\frac{K-2}{2\mu\alpha^2}<C_p<\frac{K-2}{2\mu\alpha\gamma}$, $\phi=\phi^{**}$ is the optimal social state, and $\phi=\phi^*$ is the worst case social state.

The proof is deferred to Appendix E. That the Social Cost is dominated by C_p is perhaps not surprising; the cost of preemption is related the sensitivity to being preempted. The larger the value of C_p, the lower the tolerance for preemption, and thus each preemption engenders a larger Social Cost, providing incentive for customers to join the same class to limit preemptions to those which are out of their control (i.e. preemptions by satellite incumbents). That K also plays a role in determining the optimal Social Cost is less obvious at first glance. Recall however that K is defined in terms of the second moment of service, and thus is related to the variance in service. In particular, whenever $K>2$, the variance in customer service times is greater than that of exponential service. By extension, the residual service time of the customer currently in service $K/(2\mu)$ is greater than the expected mean service time $1/\mu$ in this situation. Therefore, we find from a social stand point that preemption yields a net savings in terms of system delay and is desirable, if the variance in service is sufficiently high *and* the cost of preemption is sufficiently low.

B. Price of Anarchy

With the Social Cost now defined, we now address the question of how to quantify the impact of the provider's choice of F on the social outcome. We introduce the *Price of Anarchy (PoA)*: while there are multiple equivalent definitions, the relevant definition for our Social Cost function $\mathcal{S}(\phi)$ is as follows [16], where E is the set of candidate equilibrium states for the provider's choice of F:

$$PoA = \max_{\phi\in E}\mathcal{S}(\phi)\Big/\min_{\phi\in[0,1]}\mathcal{S}(\phi). \tag{15}$$

That is, PoA is expressed as a ratio of social costs: the worst case cost resulting from the candidate equilibria states for the given F, to the optimal state for the given queuing parameters. The optimal state need not necessarily itself be an equilibrium. We are motivated to determine whether a bound exists on the PoA, or whether a provider's choice of F could conceivably result in there being no bounds on the externalities imposed on the customers, a situation which would require regulatory intervention to avoid. We claim that, in general, no such bound on the PoA exists; to show this we require a related Lemma. The proofs of the following claims are deferred to Appendices F and G, respectively:

Lemma 3: As $C_p \to \infty$, the worst case social state $\phi^* \to \frac{1}{2}$, regardless of the values of other parameters.

Theorem 4: The Price of Anarchy is unbounded in general.

Thus, no upper bound on the PoA exists as $C_p \to \infty$. That is, as customers become infinitely intolerant of preemption, any preemption imposes an infinite social cost. As the socially optimal state in this scenario is one where all customers are in the same tier, any deviation from this state creates an infinitely intolerable situation as inter-customer tier preemptions are introduced. For fixed parameters, precise bounds can be computed per application of Lemma 2 and Equation (15), which give rise to more refined analyses.

However, we claim that in practice, the unbounded PoA is not a concern despite being a theoretical possibility:

Corollary 1: Consider a revenue maximizing provider. As $C_p \to \infty$, the socially optimal state prevails, and $PoA = 1$.

The proof is deferred to Appendix H. Thus, for an arbitrarily large C_p, the provider incentive will be aligned with the customers' actions, and for the unbounded PoA scenario to occur, the provider would have to be acting in an irrational manner by setting F to a level significantly lower than revenue maximizing, which goes against our assumptions on their behavior. The reasons for this alignment in incentive are ultimately tied to the cost of preemption C_p. As noted in the discussion in Subsection VI-A, a large value of C_p represents a low tolerance of preemption and therefore a greater willingness to pay a greater fee F to avoid preemptions.

As a result, the same factors that result in a potentially unbounded Price of Anarchy result in it not being a concern in practice for two reasons. The first being that the offending equilibrium state will not be stable over time due to the intolerance to preemption, which eventually leads to an evolution of the system where all customers opt to join the same tier, which is the socially optimal outcome. The second being that the equilibrium which leads to the worst case equilibrium is not the provider revenue maximizing state, and per Corollary 1 the provider will specifically induce the system into one where all customers join the primary tier.

C. Comparison to Two-Tier Model

We conclude this section by comparing these results against a two-tier system featuring passive incumbents and a single customer tier [10]. Under this model, the service provider offers the upgrade fee under the same short term lease provision as our three-tier model; however, the customer decision is instead one of whether to join the queue in exchange for the posted admission fee F, or balk and not receive service. Under the two-tier model the Price of Anarchy is 1 everywhere, as the socially optimal state and provider maximizing equilibrium state will coincide regardless of the values of the system parameters. By contrast, under the three-tier model here, we show this holds as the cost of preemption tends towards infinity, but not necessarily in general. Thus, from a social and regulatory standpoint there is less complexity in the two-tier vs the three-tier system as specific information is required in the latter to determine the extent to which a provider's actions deviates from the socially optimal state if at all.

TABLE II
EESS RADIOMETERS INCLUDED IN OUR TRACES AS REPRESENTATIVE INCUMBENT PASSIVE SATELLITE RADIOMETERS

Satellite	Space Agency	Sensor	Source
Aqua	NASA	AMSU-A	[42]
GCOM-W	JAXA	AMSR2	[43]
GPM Core Observatory	NASA	GMI	[44]
Metop-B	EUMETSAT	AMSU-A	[45]
Metop-C	EUMETSAT	AMSU-A	[45]
NOAA-15	NOAA	AMSU-A	[46]
NOAA-18	NOAA	AMSU-A	[47]
NOAA-19	NOAA	AMSU-A	[47]
JPSS-1 (NOAA 20)	NOAA	ATMS	[48]
JPSS-2 (NOAA 21)	NOAA	ATMS	[48]
Sentinel-3A	ESA	MWR	[49]
Sentinel-3B	ESA	MWR	[49]
Sentinel-6A	EUMETSTAT	AMR-C	[50]
SNPP	NOAA	ATMS	[47]

This does not imply however that the customers or provider are necessarily better off under the two-tier vs the three-tier. Such an analysis is relegated to future work, as the model in [10] utilizes a different model. Specifically, the radiometers in the two-tier model are modeled using an on-off model [41]. While similar in performance to a multi-class $M/G/1$ queue, the on-off nature of the queue results in an *effective* service rate which is not immediately directly comparable to the equivalent rate under the three-tier model.

VII. NUMERICAL RESULTS

In this section we present numerical simulations of our key equilibrium results. Specifically, validation of the delay model and the equilibrium stability claims in Theorems 2 and 3 *vis a vis* the guarantees to the provider's maximum revenues. In support of these validations, we leverage level 1 brightness temperature data traces of spectrum access by Earth Exploration Satellite Service radiometer sensors at or near 23.8 GHz. These measurements were identified for situations where observations occurred within 100km of 42.36 deg latitude, -70.06 longitude (i.e. Boston, MA) during the month of September 2023; this distance represents a buffer to protect radiometers from transmissions from both the main beam and the sidelobes. 783 unique passes were identified from accessible datasets cited in Table II, with mean time between arrivals of $3 \times 10^3 s$, mean access duration of $26.71s$, and a variance in service slightly greater than that of exponential. In terms of our parameters, this translates to $\lambda_s = 3 \times 10^{-4} s^{-1}$, $\mu_s = 0.04 s^{-1}$, $K_s = 2.11 s^{-2}$. To generate customer data, we apply a distribution derived from studies of commercial cognitive radios under high usage [19]. The commercial users have a mean service time of $6.47s$ ($\mu = 0.16s^{-1}$) distributed with a variance in-between deterministic and exponential ($K = 1.49s^{-2}$). Customers joining the system arrive according to a Poisson process with mean arrival rates falling within the range $\lambda \in (0.13, 0.15)s^{-1}$ (i.e. the mean inter-arrival times range between $6.67s$ and $7.69s$), resulting in traffic rates in a range $\rho \in (0.82, 0.94)$. The scenarios we consider thus represent heavy customer traffic but not a fully saturated system.

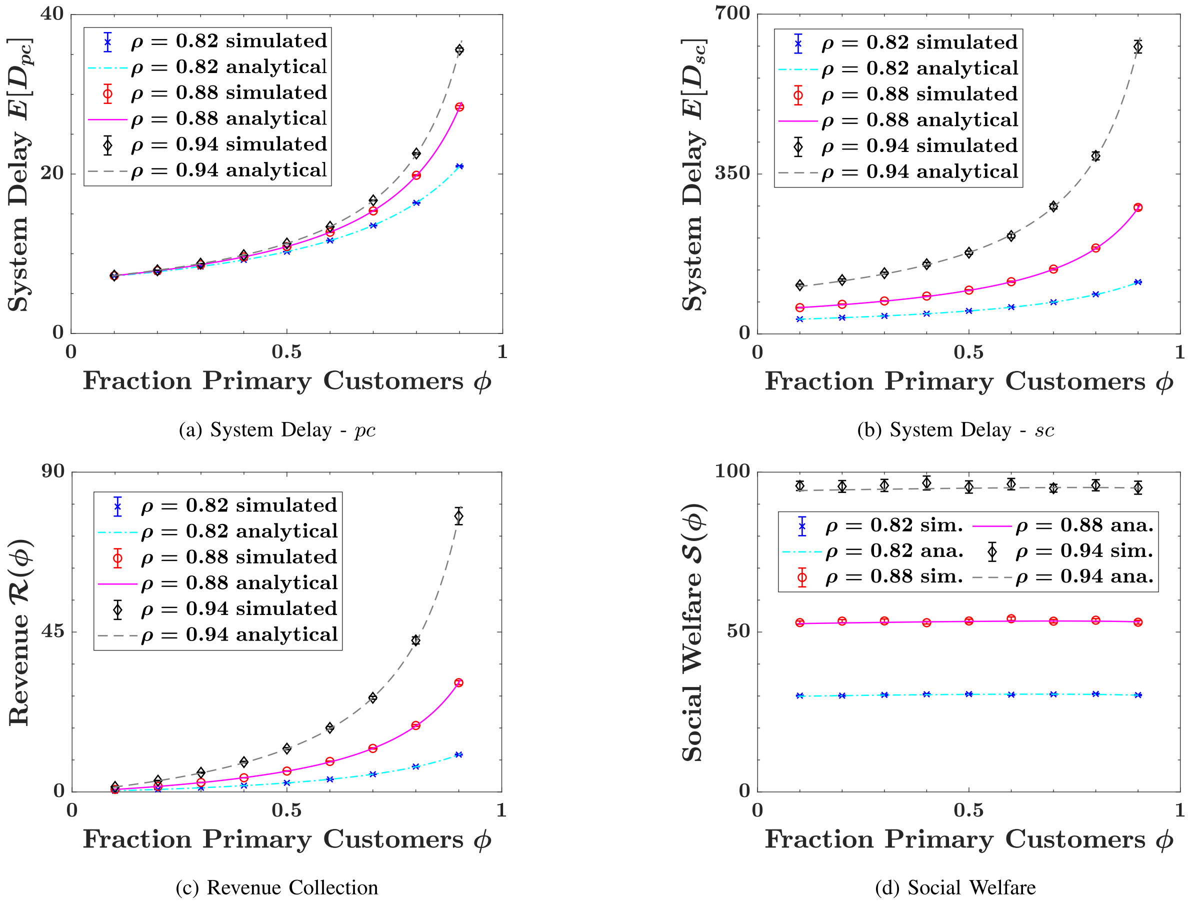


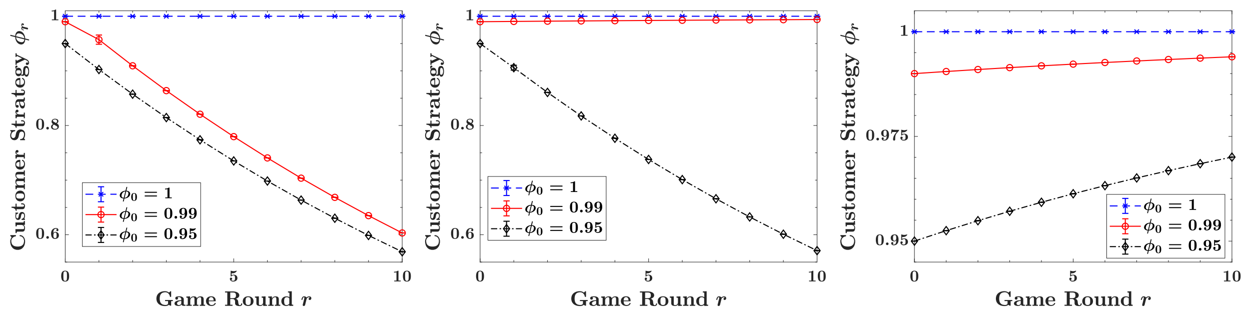
Fig. 5. Comparison of analytical results to simulated results for the system delay and corresponding revenue and social welfare based on radiometer traces sourced from datasets in Table II with parameters $\lambda_s = 3 \times 10^{-4}$, $\mu_s = 0.04$, $K_s = 2.11$, and simulated commercial users based on studies of commercial cognitive radios with parameters $\lambda \in (0.13, 0.15)$, $\mu = 0.16$, $K = 1.49$ [19]. Figures (a) and (b) show that the simulated values of the system delay for the primary and secondary classes respectively consistently fall within a 95% confidence interval of the analytical values of $\mathbb{E}[D_{pc}]$ and $\mathbb{E}[D_{sc}]$. Figures (c) and (d) respectively show that the corresponding revenue and social welfare also consistently fall within a 95% confidence interval of the analytical values given by $\mathcal{R}(\phi)$ and $\mathcal{S}(\phi)$ for a cost of preemption $C_p = 1$.

A. Validation of Delay Model

In Figure 5(a) and (b), we plot validations of the delay model for the primary and secondary customer classes, leveraging the trace data to represent incumbent arrivals. Running simulations generating customers according to our defined distribution, we let $\phi \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ of the customers be in the primary class and the remainder in the secondary, comparing the simulated system delays to the expectation defined in Equation 2. For each value of ϕ, simulation is repeated over 30 iterations with the mean of the delays returned by each simulation computed along with the corresponding confidence interval. We find that when comparing the simulated values to the expected delay, the simulated values fall within a 95% confidence interval of the analytical values of $\mathbb{E}[D_{pc}]$ and $\mathbb{E}[D_{sc}]$. This shows a good fit of our model to the data, and that delays are consistent with $M/G/1$. We also observe that, particularly for a nearly saturated system where $\rho = 0.94$, the secondary class delay rises to nearly 700 seconds due to repeated preemptions as the fraction of primary class customers increases. While this may be considered unreasonable, especially for delay sensitive applications, the result reinforces the notion of *Follow the Crowd* behavior being in effect, and the customer parameters are such that a regime (V) equilibrium is in effect. Thus in practice, a service provider would price such that all customers would join the primary class, which as observed in Figure 5(a), has comparatively reasonable delays due to the sole preemption source being the relatively less frequent passive incumbents.

B. Validation of Revenue and Social Welfare Models

In Figure 5(c) and (d), respectively, we plot validations of the revenue and social welfare corresponding to the same trace data and defined customer distribution. We let $C_p = 1$ to represent an arbitrarily high valuation on aversion to preemption, relative to the reward from service in this instance (i.e., the cost of preemption is equal to the reward from service). As before, we run 30 simulations for each value of $\phi \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$, collecting the relevant cost data and computing the mean of the simulated values and associated confidence intervals. To validate the revenue, we assume that F is set equal to the corresponding $\mathcal{F}(\phi)$ for our given parameters and the current ϕ, allowing us to test against the corresponding $\mathcal{R}(\phi)$ as derived in Equation (9) from $\mathcal{F}(\phi)$. We find that comparing mean revenues collected per second to the expected values from



(a) $Cp = 10$ (b) $Cp = 100$ (c) $Cp = 1000$

Fig. 6. Examples of Dynamic games taking place over multiple rounds, using the queuing parameters from Figure 5, and an upgrade fee of $F = 450$ arbitrary currency units. As this is slightly below $F_1 = 479.81$ when $C_p = 10$, when the initial strategy $\phi_0 = 1$, the customers converge on that strategy over time. However, even an initial strategy of $\phi_0 = 0.99$ has a corresponding $F(\phi) < 450$, causing customers to instead converge towards $\phi = 0$ over time as the upgrade fee is greater than the difference in wait times. If the customers' cost of preemption is greater the assumed $C_p = 10$ however, then the threshold between where customers will converge to $\phi = 1$ and $\phi = 0$ decreases; the initial strategy can thus start further away from $\phi_0 = 1$ and still ultimately converge to $\phi = 1$.

$\mathcal{R}(\phi)$, the simulated means fall within the 95% confidence intervals of the expected values. Similarly, comparing the simulated costs to compute the social welfare, we find that these costs generally fall within the confidence intervals of the expected value from the analytical $\mathcal{S}(\phi)$ as defined in Equation (10). Thus, we demonstrate through the simulations the validity of our model definitions. We conclude this section by demonstrating convergence of the equilibria under varying circumstances.

C. Equilibrium Convergence Simulation

We demonstrate equilibrium convergence in the sense of Theorem 2, using SimPy [51] to define a learning game (which by extension demonstrates Theorem 3). Leveraging the trace data, our defined customer distribution with $\lambda = 0.14$ (thus $\rho = 0.88$), and a cost of preemption C_p value of 10, the corresponding revenue maximizing fee is $F_1 = 479.81$ in arbitrary currency units. For the purposes of our simulation however, we assume the provider takes a conservative approach, to avoid the impact of measurement errors.

This cautious provider instead sets $F = 450$, associated with a mixed state $\phi = 0.993$. Per Theorems 1 and 2, given the values of our queuing parameters we have a type (V) equilibrium regime where customers *Follow The Crowd*: $\phi \in \{0, 1\}$ are ESS equilibrium, and $\phi = 0.993$ is not. To demonstrate the stability results, the customers play a dynamic learning game, which has the following high level structure:

1) Customers select some initial strategy ϕ_0.
2) Simulate over one month's worth of trace data.
3) During each iteration, system delay and preemption data is collected for customers of each tier.
4) After iteration ends, compare resulting costs for each tier.
5) If secondary tier has lower cost, reduce ϕ_k by α;
6) Else, if primary class has lower cost, increase ϕ_k by α.
7) Repeat Steps 2-6 for 30 iterations.
8) Compute mean of results from previous step as next ϕ_k, return to step 2 and start next round.

The rationale for repeating a round for multiple iterations and taking the mean result is to account for outliers which result in measurement errors. The results for a game over 10 rounds with $\alpha = 0.05$ are plotted in Figure 6(a). We consider three initial strategies for the customers: $\phi_0 \in \{1, 0.99, 0.95\}$. Choosing $\phi_0 = 1$ as the initial strategy results in the customers never deviating from the strategy. Therefore, $\phi = 1$ is indeed an equilibrium strategy as claimed. However, if the customers choose initial strategies of $\phi = 0.99$ or $\phi = 0.95$, the corresponding $\mathcal{F}(\phi) < 450$. Therefore, the upgrade fee is greater than the customers are willing to pay in this instance. Thus over time, the customers instead progressively opt out against upgrading, and had the game been allowed to continue the strategy would have eventually reached $\phi_k = 0$.

However, if instead the customers' valuation of C_p is greater than 10, but the provider prices F under the assumption $C_p = 10$, then customers' behavior changes. For instance, if $C_p = 100$, then $F = 450$ is associated with a mixed equilibrium state $\phi = 0.968$. Thus, starting with an initial strategy of $\phi_k = 0.99$ results in the primary customers' costs being less than the upgrade fee, and thus customers converge towards $\phi_k = 1$ over time instead; however the initial strategy $\phi_k = 0.95$ still converges to $\phi_k = 0$ over time. This is seen in Figure 6(b). If instead $C_p = 1000$, the associated mixed equilibrium state with a fee of $F = 450$ is $\phi = 0.421$, thus each of the ϕ_0 strategies we consider will ultimately converge to $\phi = 1$, as the upgrade fee will be less than the difference in the costs between the two classes. If it is the case however that $C_p = 1000$, the associated $F_1 = 1363.64$ in our arbitrary currency units, or 2.84 times the corresponding F_1 for a cost of preemption two orders of magnitude smaller. This underscores the need for the provider to ensure that they are pricing accurately to ensure revenue collection is maximized, as while all customers are incentivized to upgrade, they pay a lower fee than they would be otherwise willing to given their heightened sensitivity to preemptions.

VIII. Conclusion

In this work we established a queuing game based framework for spectrum sharing between passive satellite incumbent users and commercial users which themselves have access to multiple tiers: a preemptive priority primary tier accessible for a fee, or a secondary tier accessible under a general use policy. We find that there are multiple equilibrium regimes which may appear, including one in which there is a lone equilibrium state where only some fraction of users opt to join the primary tier, that is *Avoid the Crowd* in the sense of Hassan and Haviv [15]; one in which multiple mixed equilibria candidate states are possible, along with a state in which no customers upgrade; and one in which there is a single mixed equilibrium candidate state along side states where every customer upgrades and no customers upgrade (i.e. *Follow the Crowd*). We find that despite the differing possible equilibrium regime types, the equilibrium which maximizes revenue under steady state conditions is associated with a stable equilibrium, a result validated by simulations backed by real-world satellite traces.

In fact, our results show that for practical parameters, the profit maximizing equilibrium is also the socially optimal state. Thus, the specter of preemption is a key factor in determining whether a customer will pay for a premium tier if all other factors are equal. This result holds in the face of potentially large negative externalities the customers place on each other by preempting each other's service in the worst case scenario, as this outcome drives the customers to avoid splitting between the two tiers in the first place. This points to this mode of spectrum sharing having potential for maximum benefits for all. Future work in this area involves additional measurement studies to determine the precise amounts of white-space available for sharing, as well as identifying preferred frequencies, and determining how opportunities for individual clients to pool their spectrum access mitigates the negative impacts of individual intermittent access, as in the scenario proposed by Mu and Berry in [52]. In addition, further refinement of the assumptions underpinning the model is warranted, particularly determining the precise impacts of measurement error and long term shifts in incumbent utilization of relevant frequencies on the provider pricing action as opposed to assuming perfect knowledge of system statistics.

Appendix A
Proof of Lemma 1

Recall the definition of $\mathcal{F}(\phi)$ from Equation (4):

$$\mathcal{F}(\phi) = \frac{\rho\Big(K_s\mu\rho_s + 2\mu_s\phi\gamma + K\mu_s(\alpha - \phi\gamma)\Big)}{2\mu\mu_s\alpha\beta\gamma} + C_p\rho\phi.$$

Let the values of the queuing parameters be fixed but arbitrary. We claim $\mathcal{F}(\phi)$ must be continuous and finite valued for $\phi \in [0,1]$. As $\mathcal{F}(\phi)$ is composed of basic arithmetic functions, the only candidates for discontinuity occur through division by zero. By assumption μ, μ_s are both greater than 0. α, β, and γ are respectively substitutions for the expressions $1-\rho_{in}$, $1-(\rho_{in}+\rho\phi)$. and $1-(\rho_{in}+\rho)$. By consequence of the stability conditions and the restriction on ϕ, none of these can be equal to zero either.

Therefore, the function is continuous and finite valued. If then $F < \mathcal{F}(1)$, F is below the minimum fee that prompts any customer to opt against upgrading, and the *all-upgrade* equilibrium is possible. If $F < \min \mathcal{F}(\phi)$ it follows that the *all-upgrade* equilibrium is unique. Analogous arguments establish that that $F > \mathcal{F}(0)$ results in the *none-upgrade* equilibrium being possible, and $F > \max \mathcal{F}(\phi)$ results in it being unique.

If on the other hand F is between the minimum and maximum values of the function, the fact that $\mathcal{F}(\phi)$ is continuous results in there being at least one solution to $F = \mathcal{F}(\phi)$. By definition, any resulting solution will be an equilibrium state, and because F is strictly between the minimum and the maximum values of the function at least one mixed solution $\phi \in (0,1)$ must exist, therefore there is at least one *some-upgrade* equilibrium.

Appendix B
Proof of Theorem 1

The existence of regimes (I) and (II) follow directly from Lemma 1; therefore, assume $\min \mathcal{F}(\phi) < F < \max \mathcal{F}(\phi)$. As the continuous behavior of function $\mathcal{F}(\phi)$ on the domain $\phi \in [0,1]$ is also established in Lemma 1, we analyze the derivative $\mathcal{F}'(\phi)$ to determine the function behavior:

$$\mathcal{F}'(\phi) = \frac{\rho\Big(K_s\mu\rho_s\rho + \mu_s\alpha(2\gamma - K(\gamma-\rho))\Big)}{2\mu_s\mu\alpha\beta^2\gamma} + C_p\rho. \quad (16)$$

Evaluating the sign of the derivative, we determine that there are three possible behaviors for the function:

1) Monotone decreasing if $K > 2$, $\rho < \rho^T$, and $C_p \leq \mathsf{C}_\mathsf{p}^{(\mathsf{L})}$.
2) Unimodal with unique maximum if $K > 2$, $\lambda < \lambda^T$, and $\mathsf{C}_\mathsf{p}^{(\mathsf{L})} < C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{H})}$.
3) Monotone increasing otherwise.

Applying Lemma 1 to the monotone decreasing case, the result is $\min \mathcal{F}(\phi) = F_1$ and $\max \mathcal{F}(\phi) = F_0$, and there is exactly one solution to $F = \mathcal{F}(\phi)$ for a fixed but arbitrary F; therefore there is a single *some upgrade* equilibrium which will be possible under these conditions, and thus regime (III) is in place. We can make similar arguments to show that one equilibrium of each type must be possible when the monotone increasing case is in effect, satisfying regime (V). However, if $\mathcal{F}(\phi)$ is instead unimodal, it will transition from increasing to decreasing at some $\phi^{max} \in (0,1)$. As a result, there will be values of F for which there are multiple solutions to $F = \mathcal{F}(\phi)$, while the remaining region may be locally monotone increasing or locally monotone decreasing.

If $\min \mathcal{C}(\phi) = F_1$, then a locally monotone decreasing region will exist; this occurs when $\mathsf{C}_\mathsf{p}^{(\mathsf{L})} < C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{M})}$. Thus, regime (III) also occurs when $\mathsf{C}_\mathsf{p}^{(\mathsf{L})} < C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{M})}$ and $\min \mathcal{F}(\phi) < F < F_0$. Otherwise, when $\mathsf{C}_\mathsf{p}^{(\mathsf{L})} < C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{M})}$ and $F_0 < F < \max \mathcal{F}(\phi)$, the unimodal nature of the function results in two solutions existing to $F = \mathcal{F}(\phi)$. As $F > F_0$ also holds in this case, the *none upgrade* equilibrium is also a candidate equilibrium state as a consequence of Lemma 1, and thus we have an equilibrium regime (IV) state.

If on the other hand $\mathsf{C}_\mathsf{p}^{(\mathsf{M})} < C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{H})}$, then $\min \mathcal{C}(\phi) = F_0$, and we have a locally monotone increasing region. By extension to the previous arguments, $F_0 < F < F_1$ results in a regime (V) situation, while $F_1 < F < \max \mathcal{F}(\phi)$ results in a regime (IV) situation.

Appendix C
Proof of Theorem 2

If a given equilibrium is the sole equilibrium possible, then as no alternative strategy exists the equilibrium is ESS by default; therefore a lone *all upgrade* or *none upgrade* equilibrium is ESS stable, and this proves the if direction of the claim on *some upgrade* equilibrium. To show the only if direction for the *some upgrade* equilibrium, regardless of which regime is in effect $F > F_0$ follows and the fee is greater than that which any customer is willing to pay. Therefore, $\phi = 0$ will be a better response to itself than any potential mixed strategy and no mixed strategy can be ESS if $\phi = 0$ is a candidate equilibrium.

By extension, as the mixed strategy cannot be a better response to $\phi = 0$, to finish the claim on the pure equilibria, assume regime (V) is in effect; we claim the pure equilibria cannot be a better response to each other than to themselves. Assume that all customers are primary customers. As a result, a newly arriving customer opting to join the secondary class faces a greater wait time and preemption cost than what is saved by not paying the upgrade fee. If we assume the opposite situation is in effect and a newly arriving customer upgrades in the face of a queue where all others remained in the secondary class, then the upgrade fee is greater than the time saved by asserting a higher priority. As a result, if all previous customers have taken the same action, the best response is to follow suit.

Appendix D
Proof of Theorem 3

As $\mathcal{R}(\phi) = \lambda\phi\mathcal{F}(\phi)$, it follows that $\mathcal{R}(\phi)$ is continuous because $\mathcal{F}(\phi)$ was shown to be previously. As a result, we evaluate the derivative $\mathcal{R}'(\phi)$ to determine the conditions where $\mathcal{R}(\phi)$ is maximized:

$$\mathcal{R}'(\phi) = \frac{\rho^2}{2\mu_s\alpha\beta^2\gamma}\Big(K_s\mu\rho_s\alpha + 2\mu_s\phi\gamma(\alpha+\beta) + K\mu_s(\alpha^2 + \phi^2\rho\gamma - 2\phi\alpha\gamma)\Big) + 2C_p\rho^2\mu\phi. \tag{17}$$

As a result, we find that $\mathcal{R}(\phi)$ is either monotone increasing, or unimodal with a unique maximum at some $\phi^{opt} \in (0,1)$. For the latter to be the case, the following must be satisfied for any $K_s \geq 1$, $\mu_s > 0$, and $\mu > 0$:

$$\begin{aligned} &K > 4 \\ &\rho_s < \frac{\mu_s(K-4)}{K_s\mu + \mu_s(K-4)} \\ &\rho < \frac{3\alpha}{2} - \frac{1}{2}\sqrt{\frac{\alpha\big((5K-2)\mu_s\alpha + 4K_s\mu\rho_s\big)}{(K-2)\mu_s}} \\ &C_p \leq \frac{K\mu_s(\rho^2 - 3\rho\alpha + \alpha^2) - K_s\mu\alpha\rho_s - 2\mu_s\gamma(\alpha+\gamma)}{4\mu\mu_s\alpha\gamma^3} \end{aligned} \tag{18}$$

And the conditions on K, ρ_s, ρ, C_p in Equation (18) holding result in $K > 2$ $\rho < \rho^T$ and $C_p < \mathsf{C}_\mathsf{p}^{(\mathsf{L})}$ also holding. Therefore an *Avoid the Crowd* equilibrium regime (III) must be in effect if the revenue maximizing equilibrium is associated with a mixed equilibrium (and in fact $\mathcal{F}(\phi)$ is itself strictly monotone decreasing). Thus, if the conditions in Equation (18) hold, then by Theorems 1 and 2, the maximum revenue is associated with a lone, but stable, mixed equilibrium.

Otherwise, $\mathcal{R}(\phi)$ is monotone increasing. In this case, revenues are maximized when $F = F_1$. As this is a boundary case between equilibrium regimes, in practical terms we assume the cost is instead set to $F = F_1 - \epsilon$ for some arbitrarily small $\epsilon > 0$. Per Theorem 1, depending on the queuing parameters either there is a sole *all upgrade* equilibrium, in which case all customers are primary anyway, or there is a *some upgrade* ϕ^* associated with this F. ϕ^* is non-ESS, but as it is arbitrarily close to $\phi = 1$ by construction and per the proof of Theorem 2, the *all upgrade* equilibrium is a better response to itself than the *none upgrade*, we conclude that the provider is able to steer the customers into all joining the primary class through this pricing decision.

Appendix E
Proof of Lemma 2

We note that whenever all customers are in the same tier, the result is identical to a single class First Come, First Serve queue, therefore $\mathcal{S}(0) = \mathcal{S}(1)$ follows. We proceed in similar fashion to the analyses of $\mathcal{F}(\phi)$ and $\mathcal{R}(\phi)$ by computing the derivative $\mathcal{S}'(\phi)$. The approach is valid on the restricted domain $\phi \in [0,1]$ by analogous arguments:

$$\mathcal{S}'(\phi) = \frac{\rho\Big(2C_p\mu(1-2\phi)\alpha\beta^2 + (K-2)((\alpha+\beta)\phi - \alpha)\Big)}{2\mu\alpha\beta^2}. \tag{19}$$

If $K \leq 2$, the derivative transitions from positive to negative at ϕ^*, regardless of the value of C_p. Hence, $\mathcal{S}(\phi)$ has a maximum at $\phi = \phi^*$ and minimums at $\phi \in \{0,1\}$. As a result, the states $\phi \in \{0,1\}$ correspond to the optimal social states, and $\phi = \phi^*$ is the worst case state.

If however $K > 2$, we determine that the behavior of the function will also depend on the value of C_p relative to the other parameters. Specifically, when $C_p < \frac{K-2}{2\mu\alpha\gamma}$, the derivative transitions from negative to positive at ϕ^{**}. If however $C_p > \frac{K-2}{2\mu\alpha^2}$, the derivative also transitions from positive to negative at ϕ^* as in the $K \leq 2$ case. Combining these facts leads to the following:

- If $C_p \geq \frac{K-2}{2\mu\alpha\gamma}$, the $\mathcal{S}(\phi)$ is unimodal with a unique maximum at $\phi = \phi^*$; thus the function behaves identically to the case when $K \leq 2$ with $\phi \in \{0,1\}$ as socially optimal with $\phi = \phi^*$ as the worst case state.
- If on the other hand $C_p \leq \frac{K-2}{2\mu\alpha^2}$, the $\mathcal{S}(\phi)$ is unimodal with a unique minimum at $\phi = \phi^{**}$. Thus, $\phi = \phi^{**}$ is socially optimal, while $\phi \in \{0,1\}$ are the worst case states.
- When $\frac{K-2}{2\mu\alpha^2} < C_p < \frac{K-2}{2\mu\alpha\gamma}$, the $\mathcal{S}(\phi)$ has a local maximum at ϕ^* and a local minimum at ϕ^{**}. As $\mathcal{S}(0) = \mathcal{S}(1)$, and $\mathcal{S}(\phi)$ is continuous on $\phi \in [0,1]$, the local extrema are also the global extrema on the domain. Therefore, we conclude that for this case the optimal social social state is $\phi = \phi^{**}$ while the worst case state is $\phi = \phi^*$.

APPENDIX F
PROOF OF LEMMA 3

Consider $\mathcal{S}'(\phi)$ given in Equation (19). As $C_p \to \infty$, the limit of $\mathcal{S}'(\phi)$ equals $Sign(1-2\phi)\infty$. The sign of $1-2\phi$ is positive when $\phi \in [0, \frac{1}{2})$, negative when $\phi \in (\frac{1}{2}, 1]$, and the expression is 0 when $\phi = \frac{1}{2}$. Thus, as C_p tends to ∞, $\mathcal{S}(\phi)$ transitions from increasing to decreasing at $\phi = \frac{1}{2}$. Thus, for arbitrarily large C_p $\frac{1}{2}$ of customers opting for priority is the worst case social state and $\phi^* = \frac{1}{2}$.

APPENDIX G
PROOF OF THEOREM 4

Applying Lemma 2 for fixed arbitrary queuing parameters $\mu, \rho, K, \mu_s, \rho_s, K_s$, and letting the cost of preemption C_p increase without bound, we find ourselves in the first situation: $\phi \in \{0, 1\}$ are the best case social states, and ϕ^* as defined in Equation (11) is the worst case social state. Assume F is set such that ϕ^* is an equilibrium candidate, then applying the PoA definition from Equation (15), the PoA is bounded by the value $\mathcal{S}(\phi^*)/\mathcal{S}(1)$. Taking the limit as $C_p \to \infty$, the result is an undefined ∞/∞, thus we apply L'Hopital's rule to define the limit:

$$\lim_{C_p \to \infty} \frac{\mathcal{S}(\phi^*)}{\mathcal{S}(1)} = \lim_{C_p \to \infty} \frac{(1-\phi^*)\phi^*\rho + \rho_{in}}{(1-1)(1)\rho + \rho_{in}}. \tag{20}$$

Per Lemma 3, the limit of ϕ^* as $C_p \to \infty$ is $1/2$. Therefore, this limit reduces to the following:

$$1 + \frac{(1-\frac{1}{2})\frac{1}{2}\rho}{\rho_s} = 1 + \frac{\rho}{4\rho_s}. \tag{21}$$

And as $\rho_s \to 0$, representing a situation where incumbents are not present, the expression in Equation (21) increases without bound. Therefore, there is no upper limit on the PoA.

APPENDIX H
PROOF OF COROLLARY 1

Per application of Theorem 1, an arbitrarily large C_p results in an equilibrium regime (V) in place, where customers *Follow the Crowd*. Consequently, the revenue maximizing equilibrium must be one where $F = \mathcal{F}(1) - \epsilon$ for some arbitrarily small ϵ per Theorem 3, and the revenue maximizing equilibrium is $\phi = 1$. But per Lemma 2, this is the optimal social state for an arbitrarily large C_p regardless of the service distribution of the customers; $PoA = 1$ follows as a consequence.

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Jonathan Chamberlain (Member, IEEE) received the B.A. degree in mathematics and the M.S. degree in systems engineering from Boston University, Boston, MA, USA, in 2012 and 2019, respectively, where he is currently pursuing the Ph.D. degree with the Department of Electrical and Computer Engineering, advised by David Starobinski. He additionally was a Technical Services Representative with Epic Systems Corporation, Verona, WI, USA, from 2012 to 2013, and a Business Analyst with Hewlett Packard Enterprise, Madison, WI, USA, from 2014 to 2016. His thesis work involves conducting research into the economic, policy, and security aspects of the co-existence of user agents in wireless spectrum, mobile computing, and micro-service environments. His particular interests include advancing the fundamentals of game theory to provide a foundational structure for reasoning about user decisions, and ensuring protection of fundamental scientific activity while also allowing expansion of commercial use of resources. He was the lead author of a paper that received the Best Paper Runner-Up consideration in the Policy track at the IEEE International Symposium on Dynamic Spectrum Access Networks in 2024.

David Starobinski (Senior Member, IEEE) received the Ph.D. degree in electrical engineering from the Technion—Israel Institute of Technology, in 1999. He was a Visiting Post-Doctoral Researcher with the EECS Department, UC Berkeley, from 1999 to 2000, an Invited Professor with EPFL from 2007 to 2008, and a Faculty Fellow with the U.S. DoT Volpe National Transportation Systems Center from 2014 to 2019. He is currently a Professor of electrical and computer engineering, systems engineering, and computer science with Boston University. His research interests include cybersecurity, wireless networking, blockchain and cryptocurrency, and network economics. He received the CAREER Award from the U.S. National Science Foundation in 2002, the Career Principal Investigator (ECPI) Award from the U.S. Department of Energy in 2004, and the BU ECE Faculty Teaching Awards in 2010 and 2020. He co-authored papers that received the Best Paper Awards at the WiOpt Symposium in 2010, the IEEE Conference on Communications and Network Security (CNS) in 2016, and the IEEE International Conference on Blockchain and Cryptocurrency (ICBC) in 2020 and 2023. He served on the Editorial Boards of IEEE TRANSACTIONS ON INFORMATION FORENSICS AND SECURITY, IEEE/ACM TRANSACTIONS ON NETWORKING, and IEEE OPEN JOURNAL OF THE COMMUNICATIONS SOCIETY.

Joel T. Johnson (Fellow, IEEE) received the bachelor's degree in electrical engineering from the Georgia Institute of Technology, in 1991, and the S.M. and Ph.D. degrees from the Massachusetts Institute of Technology in 1993 and 1996, respectively. He is currently the Burn and Sue Lin Professor of the Department of Electrical and Computer Engineering and the ElectroScience Laboratory, The Ohio State University. His research interests include microwave remote sensing, propagation, and electromagnetic wave theory. He is a member of commissions B and F of the International Union of Radio Science (URSI), and a member of Tau Beta Pi, Eta Kappa Nu, and Phi Kappa Phi. He received the 1993 Best Paper Award from the IEEE Geoscience and Remote Sensing Society, was named an Office of Naval Research Young Investigator, the National Science Foundation Career Awardee, and PECASE Award recipient in 1997, and was recognized by the U. S. National Committee of URSI as a Booker Fellow in 2002. He serves as the Deputy Editor-in-Chief for IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING.