Imaging of localized whispering-gallery-modes in a cylindrical fiber

Nitzan Shani[1],Fan Cheng[1],Lev Deych[2],and Tal Carmon[1]

[1]Department of Electric Engineering, Tel Aviv University, Tel Aviv, 6997801, Israel. [2]Department of Physics, Queens College, Flushing, NY, USA

Modes of a dielectric cylindrical waveguide are characterized by three numbers: azimuthal number m, radial number, s, and a continuous propagation constant β.[1] The azimuthal number determines the field dependence of the azimuthal angle φ of the cylindrical coordinate system with the polar axis along the axis of the cylinder in the form of $\exp(im\varphi)$. The radial number characterizes the behavior of the field in the radial direction and in an ideal cylinder is given by a cylindrical Bessel function $J_m(\kappa_{m,s}r)$. Discrete values of the parameter κ_s characterizing the radial dependence of the field is determined by Maxwell boundary conditions. The propagation constant β describes the propagation of the mode along the axis of the fiber (Z-axis of the cylindrical coordinate system). The frequency of the modes ω is related to the parameters β and $\kappa_{m,s}$ via a standard relation

$$\frac{\omega}{c} = \sqrt{\beta^2 + \kappa_{m,s}^2}, \tag{1}$$

where c is the vacuum speed of light. The value of $\beta = 0$ corresponds to the cutoff value of the frequency for a given mode $\omega_{m,s}^c = \kappa_{m,s}$such that for frequencies below $\omega_{m,s}^c$ no propagating modes exist.[1] Exactly at cut-off frequency the modes of the fiber with large values of the azimuthal number $m \gg 1$ can be described as purely whispering-gallery modes (WGM) formed by light traveling along the circumference of the fiber and confined due to total internal reflection.

In the vicinity of the cut-off frequency, when propagating constant β is small, the group velocity of the fiber mode also becomes very small. This makes the modes of the fiber extremely sensitive to the smallest variations of the fiber's radius.[2] If these variations are smooth enough one can apply the boundary conditions locally and neglect the dependence of the radial function $J_m(\kappa_{m,s}r)$ on coordinate z, which appears due to z-dependence of $\kappa_{m,s}$,and present the equation governing the propagation of the mode in the z direction as

$$\frac{d^2\Psi}{dz^2} + \sqrt{\frac{\omega^2}{c^2} - \kappa_{m,s}^2(z)}\Psi = 0 \tag{2}$$

In the vicinity of the cut-off frequency, this equation as was shown in Ref.[2] can be recast in the form of the Schrödinger equation describing motion in a potential. The form of the potential depends on the profile of the fiber's radius and can result in localization of the fiber modes in one or two directions.[3–6] If the radius decreases in both directions from some central location, the effective potential will take the form of a potential well resulting in formation of the analogue of discrete energy levels, or in optical terms, in standing waves of the whispering gallery modes. Experimentally such effect will manifest itself in the form of the interference fringes[2]and can be used, for instance for extremely accurate measurement of the variations in the fiber radius.[6]

In this work we (for the first time to the best of our knowledge) image the interference fringes arising in a cylindrical fiber due to variations of its radius and observe their evolution as we change the excitation wavelength. The sample fiber with nominal diameter $d = 125\,\mu m$ and refractive index $n_f = 1.466$ is placed in water, where fluorescent dye molecules are dissolved. The whispering gallery modes are excited by a tapered fiber positioned perpendicular to the sample fiber. We are scanning wavelengths between 776 and 781 nm with a speed of $0.16\,nm/sec$ while observing the resonances in the transmission spectra recorded from the tapered fiber (see Fig. A camera is used to collect fluorescence of the die molecules excited by light escaping from the sample fiber. The progression of the fringe patterns corresponding to different standing wave resonances is shown in the first three images in Fig.2. A rough estimate based on a simplified formula for the resonance wavelength λ_m of the WGMs in the fiber $\lambda_m = \pi n_f d/m$, indicates that we excite WGM characterized by azimuthal numbers m in the

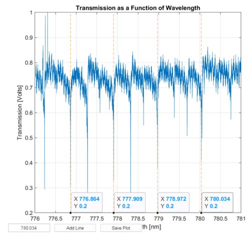

Figure 1. Transmission spectra collected from the tapered fiber showing the presence of WGM resonances of the sample fiber.

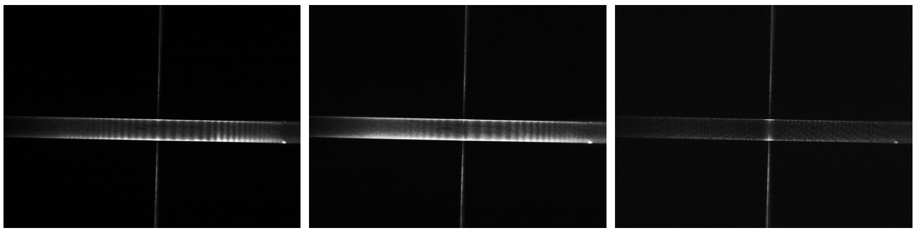

Figure 2. Evolution of fringe patterns with changing wavelength of the excitation.The last figure shows an unusual rhomboidal fringe pattern.

vicinity of $m = 740$ and that the separation between the adjacent resonances is about $1\,nm$. We can conclude, therefore, that the evolution of the fringes observed in our experiment represents transitions between different standing wave resonances originating from WGMs localized between two turning points created by variations of the diameter of the sample. We also observed an unusual rhomboidal fringe pattern, origins of which are still under investigation (Fig. 3).

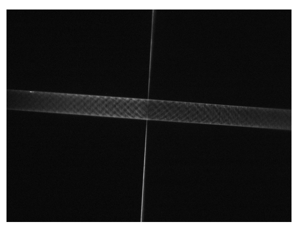

Figure 3. A rhomboidal fringe pattern observed in a sample fiber

To conclude, we observe WGM resonances in a cylindrical fiber and image interference fringe patterns arising due to localization of the fiber modes in the axial direction due to small variations of the fiber's diameter.

REFERENCES

1. E. Snitzer, "Cylindrical dielectric waveguide modes," *Journal of the Optical Society of America* **51**, p. 491, may 1961.
2. M. Sumetsky and J. M. Fini, "Surface nanoscale axial photonics," *Optics Express* **19**, p. 26470, dec 2011.
3. M. Sumetsky, "Whispering-gallery-bottle microcavities: the three-dimensional etalon," *Optics Letters* **29**, p. 8, Jan. 2004.
4. M. Sumetsky, "Localization of light on a cone: theoretical evidence and experimental demonstration for an optical fiber," *Optics Letters* **36**, p. 145, jan 2011.
5. M. Sumetsky, "Theory of SNAP devices: basic equations and comparison with the experiment," *Optics Express* **20**, p. 22537, sep 2012.
6. M. Scheucher, K. Kassem, A. Rauschenbeutel, P. Schneeweiss, and J. Volz, "Slow-light-enhanced optical imaging of microfiber radius variations with subangstrom precision," *Physical Review Applied* **14**, p. 064052, dec 2020.