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Global Stabilization of Nash Equilibrium for Mixed Traffic

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Abstract— **We consider a traffic network in which the traffic is a mix of regular and connected-autonomous vehicles. We presume the headways for vehicles in each link in the network are distinct for the regular and autonomous vehicles, and the autonomous vehicle headways differ depending on type of vehicle being followed. We analyze the network in the context of a population game, with each population corresponding to an origin-destination pair and vehicle type. We assume the evolutionary dynamics of each population distribution are governed by an Impartial Pairwise Comparison (IPC) Protocol. For the regular vehicles, we presume the payoff mechanism is the negative of the travel time. For the autonomous vehicles, we presume the payoff mechanism is an algorithm that is controlled centrally, using feedback about the current state of the system. For this scenario, we propose a dynamic payoff control algorithm for the autonomous vehicles that guarantees global convergence to Nash equilibrium. Additionally, the algorithm assures that in steady-state, the regular and autonomous vehicles for each origin-destination pair equilibrate to the same optimum routes.**

I. Introduction

Connected-autonomous vehicles (CAVs) are expected to improve safety and mobility in traffic flow. Safety benefits may be realized by reducing crashes attributed to human error [1], while connected vehicle platooning has been shown to potentially double throughput in urban roads [2]. Mobility benefits are achieved by the ability of connected vehicles to drive with a shorter headway than non-connected vehicles [3]. This paper is concerned with analyzing the network stability of the mixed traffic environment. While the greatest societal benefits will be realized in a network with near full CAV market penetration, there will be a transition phase where regular vehicles (RVs) and CAVs coexist. Interactions between RVs and CAVs have been shown to cause unstable traffic flow behaviors [4], [5], thus motivating the study of a mixed autonomy traffic network.

Population games are a useful framework for modeling the dynamic interactions between many noncooperative agents that choose strategies to maximize their individual utility. Transportation networks are commonly modeled as a congestion game [6], [7], [8], which is a specific type of population game. Congestion games have been shown to be potential [9] and contractive [10], [11], which are two key properties for studying the existence and stability of equilibria. In this context, agents are drivers traveling across a roadway network, and agent populations are defined by a particular origin-destination (OD) pair and vehicle type (regular or autonomous), with each population having its own mass. Agents belonging to a population choose a strategy from a common set of paths, and they revise their paths at each time step with the goal of minimizing their individual travel time. The revision process for agent strategies is governed by an evolutionary dynamics model (EDM), where we consider EDMs belonging to the class of so-called impartial pairwise comparison (IPC) dynamics. The payoff for each population is defined as the negative of the travel time, and the system states are defined by the fraction of drivers adopting a particular strategy. The state in which no driver can unilaterally improve their travel time by switching paths is known as a Wardrop equilibrium [12], which is a special case of Nash equilibrium.

When considering a single vehicle type, assuming a continuously differentiable and monotone payoff function leads to the existence of a potential function which can be used to prove the stability of the system equilibria [13], [14], [15], [16]. However, no such potential function is believed to exist for the mixed traffic setting, as the payoff function is typically not monotone in the total traffic volume. Existing analysis has modeled mixed traffic networks as a weighted congestion game [17], [18], [19], [20], where the link travel time is an increasing function of the sum of the regular vehicle flow and the discounted autonomous vehicle flow (representing reduced headway). This approach was used in [21] to bound the price of anarchy for selfish routing, while [22], [23] consider passivity techniques for network control.

Reference [24] builds off the work introduced in [25] and continued in [26], [27] to present a passivity-based approach for proving global asymptotic stability of the so-called mean closed-loop model. This feedback interconnection consists of the revision process described by the EDM and the payoff mechanism described by the payoff dynamics model (PDM). The analysis derives stability conditions for the closed loop by leveraging the concept of delta-passivity, which describes stability conditions in terms of the derivative of the system input and the derivative of the system output. By modeling the mixed traffic environment as a weighted congestion game, global convergence to Nash equilibrium is assured.

This paper extends the approach developed in [24] by considering the mixed traffic model presented in [28]. In this model, the vehicle headway is dependent on the type of vehicle being followed (CAV following CAV, CAV following RV, RV following either RV or CAV). The link travel time is a strictly increasing function in one vehicle type flow when the other flow is held constant, but the travel time is not strictly increasing as a function of total flow. The methods in [24] cannot be used to assure global stability of Nash equilibrium, due to asymmetry of the payoff Jacobian. We

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propose a control algorithm for the CAV payoff vector, which guarantees global stability to the Nash equilibrium set. We then modify this control algorithm so that it also ensures equity of route choice between RV and CAV populations.

The paper is outlined as follows. Section II introduces the modeling notation for the EDM and the PDM and describes key assumptions used in the model. It reviews the notion of delta-passivity and its use to derive stability conditions for the case with homogenous traffic and a static payoff mapping. Section III introduces the mixed traffic network and summarizes the stability analysis using delta-dissipativity for the case with uniform headways for autonomous and regular vehicles. Section IV presents a model for mixed traffic in which vehicle headways are dependent on the type of vehicle being followed and shows that the techniques from prior analysis cannot be used to describe the stability of the system. A dynamic payoff control algorithm is presented that guarantees global convergence to Nash equilibrium. Section V provides some brief conclusions.

II. Preliminaries

This section introduces modeling conventions, which closely mirror those in [24].

A. Population Modeling

We assume ρ populations, each corresponding to a distinct combination of origin-destination pair and vehicle type. Each population is characterized as a vector $x^r(t) \in \mathbb{X}^r \subset \mathbb{R}^{n^r}_{\geqslant 0}$. Each component $x^r_i(t)$ represents the mass of population r that chooses the i^{th} available strategy (i.e., route). We then have

$$\mathbb{X}^r \triangleq \left\{ \xi \in \mathbb{R}^{n^r}_{\geqslant 0} : \sum_{i=1}^{n^r} \xi_i = m^r \right\} \tag{1}$$

where m^r is the total mass of population $r \in \{1, \dots, \rho\}$. We concatenate all populations, to obtain the total state vector

$$x(t) \triangleq \mathrm{vec}\{x^1(t), \dots, x^\rho(t)\}, \quad \mathbb{X} \triangleq \mathbb{X}^1 \times \cdots \times \mathbb{X}^\rho \tag{2}$$

We denote as $\mathbb{T}^r$, the associated tangent space of $\mathbb{X}$, i.e.,

$$\mathbb{T}^r \triangleq \left\{ z \in \mathbb{R}^{n^r} : \sum_{i=1}^{n^r} z_i = 0 \right\} \tag{3}$$

with the concatenated tangent space as $\mathbb{T} = \mathbb{T}^1 \times \cdots \times \mathbb{T}^\rho$.

B. Payoff Mechanism

For population $r \in \{1, \dots, \rho\}$ the payoff vector $p^r(t) \in \mathbb{R}^{n^r}$ represents the incentive rewarded for each strategy, i.e., $p^r_i(t)$ is the incentive for choosing strategy $i \in \{1, .., n^r\}$ at time t. The total payoff vector is just the concatenation for all populations, i.e.,

$$p(t) = \mathrm{vec}\{p^1(t), \dots, p^\rho(t)\} \tag{4}$$

A static payoff mechanism $F : \mathbb{X} \to \mathbb{R}^n$ is a feedback relationship that determines $p(t)$ from $x(t)$, i.e.,

$$p(t) = F\left(x(t)\right) \tag{5}$$

where we assume $F(\cdot)$ is continuously differentiable.

Definition 1: The *Nash equilibrium* associated with a memoryless payoff mechanism F is characterized by the set of all $\bar{x} \in \mathbb{X}$ with the property that

$$v^T F(\bar{x}) \leqslant \bar{x}^T F(\bar{x}), \quad \forall v \in \mathbb{X} \tag{6}$$

We denote the set of all Nash equilibria associated with F as $\mathbb{N}_F$. This set can be shown to be nonempty and closed.

Remark 1: Where it causes no ambiguity, we henceforth suppress time-dependency of x and p, to ease the notation.

C. Evolutionary Dynamic Model

We presume that the strategy vectors for populations $r \in \{1, \dots, \rho\}$ evolve according to EDMs of the form

$$\dot{x}^r = \nu^r(x^r, p^r) \tag{7}$$

where we assume each ν^r is a Lipschitz continuous function. We denote ν as the concatenation of the EDMs for each population, i.e.,

$$\nu(x, p) = \mathrm{vec}\{\nu^1(x^1, p^1), \dots, \nu^\rho(x^\rho, p^\rho)\}. \tag{8}$$

We assume $\nu(x, p) \in \mathbb{T}$ for all $x \in \mathbb{X}$ and $p \in \mathbb{R}^n$.

Definition 2: An EDM ν is said to exhibit *Nash stationarity* if the following equivalence holds for all $p \in \mathbb{R}^n$:

$$\nu(x, p) = 0 \quad \Leftrightarrow \quad v^T p \leqslant x^T p, \forall v \in \mathbb{X} \tag{9}$$

We consider EDMs that adhere to the IPC protocol. This may be expressed for each population $r \in \{1, \dots, \rho\}$ and each strategy $i \in \{1, \dots, n^r\}$ as

$$\nu^r_i(x^r, p^r) = \sum_{j=1}^{n^r} \left[x^r_j \phi^r_i(p^r_i - p^r_j) - x^r_i \phi^r_j(p^r_j - p^r_i) \right] \tag{10}$$

where each ϕ^r_i is Lipschitz continuous, and such that $\phi^r_i(\tilde{p}) > 0$ for all $\tilde{p} > 0$ and $\phi^r_i(\tilde{p}) = 0$ otherwise.

Theorem 1 ([7]): The IPC protocol exhibits Nash stationarity.

D. δ-Passivity

Definition 3: EDM ν is called *δ-passive* if there exist continuously-differentiable storage function $S = \mathbb{X} \times \mathbb{R}^n \to \mathbb{R}_{\geqslant 0}$ and dissipation function $\sigma : \mathbb{X} \times \mathbb{R}^n \to \mathbb{R}_{\geqslant 0}$ satisfying

$$\frac{\partial S(x,p)}{\partial x} \nu(x,p) + \frac{\partial S(x,p)}{\partial p} u \leqslant -\sigma(x,p) + u^T \nu(x,p) \tag{11}$$

for all $x \in \mathbb{X}$, $p \in \mathbb{R}^n$, and $u \in \mathbb{R}^n$, where

$$\sigma(x,p) = 0 \Leftrightarrow S(x,p) = 0 \Leftrightarrow \nu(x,p) = 0 \tag{12}$$

Theorem 2 ([26]): Let ν be an IPC EDM. Then it is δ-passive, with

$$S(x,p) = \sum_{r=1}^{\rho} \sum_{i=1}^{n^r} x^r_i \psi^r_i(p^r) \tag{13}$$

$$\sigma(x,p) = -\sum_{r=1}^{\rho} \sum_{i=1}^{n^r} \nu^r_i(x^r, p^r) \psi^r_i(p^r) \tag{14}$$

where

$$\psi_i^r(p^r) = \sum_{j=1}^{n^r} \int_0^{p_j^r - p_i^r} \phi_j^r(\tilde{p}) d\tilde{p}. \quad (15)$$

Theorem 3 ([26]): Suppose that EDM ν exhibits Nash stationarity, and is δ-passive. Suppose further that static payoff mechanism F satisfies

$$\zeta^T \left[W(x) + W^T(x)\right] \zeta \leqslant 0, \quad \forall \zeta \in \mathbb{T}, x \in \mathbb{X} \quad (16)$$

where $W(x) \triangleq \frac{\partial}{\partial x} F(x)$. Then $\mathbb{N}_F$ is a globally asymptotically stable set.

E. Application to Homogeneous Traffic

Let R be the routing matrix for a transportation network, i.e., let $R_{ij} = 1$ if strategy x_i contains link $j \in \{1, \dots, L\}$, and is 0 otherwise. The link flow for link j is

$$z = R^T x. \quad (17)$$

If each vehicle has the same headway (irrespective of whether it is a RV or CAV) then the travel time on link j can be assumed to simply be a function of the total link flow, i.e., $\Phi_j(z_j)$, where the function Φ_j is increasing and continuously-differentiable on $z_j \in \mathbb{R}_{\geqslant 0}$. Let the payoff p be the negative of the travel time for each strategy, i.e.,

$$p = F(x) = -R \operatorname{vec}\{\Phi_1(z_1), \dots, \Phi_L(z_L)\}. \quad (18)$$

Then we have that

$$\frac{\partial F(x)}{\partial x} = -R \operatorname{diag}\{\Phi_1'(z_1), \dots, \Phi_L'(z_L)\} R^T \quad (19)$$

where $\Phi_j'(z_j) = \frac{d}{dz_j}\Phi_j(z_j) > 0$ for all $z_j \geqslant 0$. Clearly, this matrix is symmetric and negative-semidefinite. As such, if ν is δ-passive and satisfies Nash-stationarity, it follows from Theorem 3 that $\mathbb{N}_F$ is a globally asymptotically-stable set.

III. The Case with Uniform Headways for Autonomous and Regular Vehicles

The results in this section are merely summarized from [24], and no novelty is claimed. Without loss of generality, we assume that the state vector x is partitioned as

$$x = \operatorname{vec}\{x^A, x^R\} \quad (20)$$

where x^A and x^R are the population vectors corresponding to each strategy, for the autonomous and regular vehicles, respectively. Respective state vectors for the each population (i.e., OD pair) are denoted x^{Ar} and x^{Rr}, for $r \in \{1, \dots, \varrho\}$ with $\varrho \triangleq n/2$. For $i \in \{1, \dots, n^r\}$ we assume components x_i^{Ar} and x_i^{Rr} correspond to the same strategy (i.e., route) for the two populations. As such, R can be partitioned as

$$R = \begin{bmatrix} \bar{R} \\ \bar{R} \end{bmatrix}. \quad (21)$$

The link flows for the CAVs and RVs, respectively, are

$$z^A = \bar{R}^T x^A, \quad z^R = \bar{R}^T x^R. \quad (22)$$

The travel time associated with a given link $j \in \{1, \dots, L\}$ is in general an independent function of both z^A and z^R, i.e.,

$$T_j = \Phi_j(z_j^A, z_j^R) \quad (23)$$

where $\Phi_j : \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{>0}$ is a monotonically increasing function of either argument with the other argument fixed. Formulating the payoff as the negative of the travel time for each strategy, we have that

$$p = \begin{bmatrix} p^A \\ p^R \end{bmatrix} = -\begin{bmatrix} \bar{R} \\ \bar{R} \end{bmatrix} \operatorname{vec}\{\Phi_1(z_1^A, z_1^R), \dots, \Phi_L(z_L^A, z_L^R)\}. \quad (24)$$

In this section, we assume the special case in which Φ can be reduced to a function of the form

$$\Phi_j(z_j^A, z_j^R) = \tilde{\Phi}_j(\mu z_j^A + z_j^R) \quad (25)$$

where $\tilde{\Phi}_j : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{>0}$ is an increasing function. In the above expression μ is the ratio of the headway of CAVs, to that of RVs. It is assumed that this ratio is uniform for all links, and the same irrespective of which type of vehicle the autonomous vehicle follows. In this case, the Jacobian matrix for the static payoff mechanism is asymmetric, as

$$W(x^A, x^R) = -\begin{bmatrix} \bar{R} \\ \bar{R} \end{bmatrix} \operatorname{diag}\{\tilde{\Phi}_1'(z_1), \dots, \tilde{\Phi}_L'(z_L)\} \begin{bmatrix} \mu\bar{R} \\ \bar{R} \end{bmatrix}^T. \quad (26)$$

It is straight-forward to verify that the Hermitian component of the Jacobian is negative-semidefinite only if $\mu = 1$. Therefore, Theorem 3 cannot be used to prove global stability. This issue is resolved via the introduction of δ-dissipativity.

In Definition 3 for δ-passivity, the term

$$s(x, u) \triangleq u^T \nu(x, p) \quad (27)$$

is called the *supply rate*. In the generalization to δ-dissipativity, the supply rate can be any function of the form

$$s(x, u) = \begin{bmatrix} \nu(x, p) \\ u \end{bmatrix}^T \Pi \begin{bmatrix} \nu(x, p) \\ u \end{bmatrix} \quad (28)$$

where $\Pi = \Pi^T$ can be any matrix. Then, in Theorem 3, we modify the condition for stability to accommodate generalized supply rates, resulting in the alteration of (16) to

$$\zeta^T \begin{bmatrix} I \\ \frac{\partial F(x)}{\partial x} \end{bmatrix}^T \Pi \begin{bmatrix} I \\ \frac{\partial F(x)}{\partial x} \end{bmatrix} \zeta \leqslant 0, \quad \forall \zeta \in \mathbb{T}, x \in \mathbb{X} \quad (29)$$

It is straightforward to verify that Theorem 3 still holds with the above generalization, which recovers the δ-passivity with

$$\Pi = \frac{1}{2}\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}. \quad (30)$$

Now, consider the alteration of S in (13) and σ in (14) such that all summand terms corresponding to CAV populations are multiplied by μ. Then it follows that the above storage function renders the system δ-dissipative with parameter

$$\Pi = \frac{1}{2}\begin{bmatrix} 0 & 0 & \mu I & 0 \\ 0 & 0 & 0 & I \\ \mu I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix}. \quad (31)$$

Meanwhile, we have that

$$\begin{bmatrix} I \\ \frac{\partial F(x)}{\partial x} \end{bmatrix}^T \Pi \begin{bmatrix} I \\ \frac{\partial F(x)}{\partial x} \end{bmatrix} = -\begin{bmatrix} \mu\bar{R} \\ \bar{R} \end{bmatrix} \operatorname{diag}\{\tilde{\Phi}_1'(z_1), \ldots, \tilde{\Phi}_L'(z_L)\} \begin{bmatrix} \mu\bar{R} \\ \bar{R} \end{bmatrix}^T \quad (32)$$

which is negative-semidefinite. As such, we conclude that the above model for mixed traffic exhibits global asymptotic stability to Nash equilibrium.

IV. Case with Non-Uniform Headways

It is interesting to ask what happens if we try to apply δ-dissipativity techniques to more realistic models for mixed traffic, in which $\Phi_j(\cdot,\cdot)$ cannot be reduced to a single-variate function $\tilde{\Phi}$ as in (25). In particular, it has been shown [28] that a more realistic model of the link travel time is

$$T_j = \tilde{\Phi}_j(z_j) \quad (33)$$

where z_j is related to z_j^A and z_j^R via

$$z_j = h_j^R z_j^R + h_j^C \frac{(z_j^A)^2}{z_j^A + z_j^R} + h_j^A \frac{z_j^A z_j^R}{z_j^A + z_j^R} \quad (34)$$

where $\{h_j^R, h_j^C, h_j^A\} \subset \mathbb{R}_{>0}$ with $h_j^A \geqslant h_j^R \geqslant h_j^C$. These constants are the critical time headways, respectively, for CAVs following RVs, RVs, and CAVs following each other. Assuming the static payoff mechanism (24), it is a straightforward calculation to show that the Jacobian is

$$W(x^A, x^R) = -\begin{bmatrix} \bar{R}\Delta^A(z^A,z^R)\bar{R}^T & \bar{R}\Delta^R(z^A,z^R)\bar{R}^T \\ \bar{R}\Delta^A(z^A,z^R)\bar{R}^T & \bar{R}\Delta^R(z^A,z^R)\bar{R}^T \end{bmatrix} \quad (35)$$

where

$$\Delta^A(z^A, z^R) \triangleq \operatorname{diag}\Big\{\tilde{\Phi}_1'(z_1)\left[h_1^C + (h_1^A - h_1^C)(1-\pi_1)^2\right], \ldots, \tilde{\Phi}_L'(z_L)\left[h_L^C + (h_L^A - h_L^C)(1-\pi_L)^2\right]\Big\}, \quad (36)$$

$$\Delta^R(z^A, z^R) \triangleq \operatorname{diag}\Big\{\tilde{\Phi}_1'(z_1)\left[h_1^R + (h_1^A - h_1^C)\pi_1^2\right], \ldots, \tilde{\Phi}_L'(z_L)\left[h_L^R + (h_L^A - h_L^C)\pi_L^2\right]\Big\}, \quad (37)$$

and

$$\pi_j \triangleq \frac{z_j^A}{z_j^A + z_j^R} \quad (38)$$

is the market share of autonomous vehicles on link j. (Note that the $\Delta^A > 0$ and $\Delta^R > 0$, due to the inequality assumptions made for h_j^A, h_j^R, and h_j^C.) Although each of the four blocks of $W(x^A, x^R)$ is Hermitian and negative-semidefinite, the Hermitian component of the matrix itself is always sign-indefinite unless $\Delta^A(z^A,z^R) = \Delta^R(z^A,z^R)$. Therefore, we conclude that condition (16) in Theorem 3 does not apply. Furthermore, even for the generalized notion of δ-dissipativity, Nash stability cannot be assured.

A. Nash Stabilization

Global asymptotic stability to Nash equilibrium can be assured, by changing the payoff mechanism for the CAV populations. We presume that the RVs retain the same static payoff function introduced in (24), which we denote

$$p^R = F^R(x). \quad (39)$$

For the CAVs, rather than a static payoff, we presume a first-order dynamic mechanism of the form

$$\dot{p}^A = G^A(x, p^A) \quad (40)$$

We assume $G^A : \mathbb{X} \times \mathbb{R}^{n/2}$ is such that the augmented system

$$\begin{bmatrix} \dot{x} \\ \dot{p}^A \end{bmatrix} = \begin{bmatrix} \nu\left(x, \begin{bmatrix} p^A \\ F^R(x) \end{bmatrix}\right) \\ G^A(x, p^A) \end{bmatrix} \quad (41)$$

has a solution $(x(t), p^A(t))$ for all initial conditions $(x(0), p^A(0)) \in \mathbb{X} \times \mathbb{R}^{n/2}$, and all $t \in [0, \infty)$.

Definition 4: The *Extended Nash equilibrium* associated with the pair of payoff mechanisms (G^A, F^R) is characterized by all ordered pairs $(\bar{x}, \bar{p}^A) \in \mathbb{X} \times \mathbb{R}^{n/2}$ for which

$$v^T \begin{bmatrix} \bar{p}^A \\ F^R(\bar{x}) \end{bmatrix} \leqslant \bar{x}^T \begin{bmatrix} \bar{p}^A \\ F^R(\bar{x}) \end{bmatrix}, \quad \forall v \in \mathbb{X} \quad (42)$$

We denote the extended Nash equilibrium set associated with G^A and F^R as $\mathbb{E}_{G^A,F^R}$.

Theorem 4: Suppose that EDM ν exhibits Nash stationarity, and is δ-dissipative with supply rate parameter Π. Let (G^A, F^R) be a pair of payoff mechanisms as described above, and satisfying

$$\Xi^T(x, p^A)\ \Pi\ \Xi(x, p^A) \leqslant 0, \quad \forall x \in \mathbb{X}, p^A \in \mathbb{R}^{n/2} \quad (43)$$

where

$$\Xi(x, p^A) \triangleq \begin{bmatrix} \nu\left(x, \begin{bmatrix} p^A \\ F^R(x) \end{bmatrix}\right) \\ G^A(x, p^A) \\ \frac{\partial F^R(x)}{\partial x}\nu\left(x, \begin{bmatrix} p^A \\ F^R(x) \end{bmatrix}\right) \end{bmatrix}. \quad (44)$$

Then $\mathbb{E}_{G^A,F^R}$ is a globally asymptotically stable set, with respect to differential equation (41).

On its own, Theorem 4 does not imply that either $x(t)$ or $p^A(t)$ approaches a well-defined limit as $t \to \infty$. It does guarantee that $\dot{x}(t) = \nu(x(t), p(t)) \to 0$ as $t \to \infty$, but it may be possible for this to be true and $\lim_{t\to\infty} x(t)$ to be undefined. We now establish a particular G^A which ensures convergence to $\mathbb{E}_{G^A,F^R}$.

Theorem 5: Let ν be a valid IPC model as in (10). Assume $p^R = F^R(x)$ where

$$F^R(x) = -\bar{R}\operatorname{vec}\left\{\Phi_1(z_1^A, z_1^R), \ldots, \Phi_L(z_L^A, z_L^R)\right\} \quad (45)$$

where with z^A and z^R as defined in (22) and Φ_i as in (23). Assume that $G^A(x, p^A, p^R)$ is

$$G^A(x, p^A) = -\bar{R}\Delta^C(z^A,z^R)\bar{R}^T\nu^A(x^A,p^A) - \bar{R}\Delta^A(z^A,z^R)\bar{R}^T\nu^R(x^R,p^R) \quad (46)$$

where

$$\Delta^C \triangleq \left[\Delta^A(z^A, z^R)\right]^2 \left[\Delta^R(z^A, z^R)\right]^{-1} \tag{47}$$

and Δ^A and Δ^R are defined in (36) and (37), respectively. Then $\mathbb{E}_{G^A,F^R}$ is a globally asymptotically stable set, with respect to differential equation (41).

Remark 2: It is possible to evaluate $p^A(t)$ in several different ways, rather than via equation (46). Consider that we may write

$$\dot{p}^A = -\bar{R}\Delta^A(z^A, z^R)\left[\Delta^R(z^A, z^R)\right]^{-1}\dot{T} \tag{48}$$

where we recall that T is the vector of travel times for each link in the network. Via (36) and (37) we have, equivalently,

$$\dot{p}^A = -\bar{R}\Theta\dot{T} \tag{49}$$

where

$$\Theta = \operatorname{diag}\left\{\frac{h_1^C + (h_1^A - h_1^C)(1-\pi_1)^2}{h_1^R + (h_1^A - h_1^C)\pi_1^2}, \ldots, \frac{h_L^C + (h_L^A - h_L^C)(1-\pi_L)^2}{h_L^R + (h_L^A - h_L^C)\pi_L^2}\right\} \tag{50}$$

where we recall that π_j is the market share of autonomous vehicles on link $j \in \{1, \ldots, L\}$. We can further manipulate this to avoid having to take a derivative of the travel time vector T, as the state space

$$\dot{\xi}(t) = \bar{R}\dot{\Theta}(t)T(t) \tag{51}$$

$$p^A(t) = \xi(t) - \bar{R}\Theta(t)T(t) \tag{52}$$

Note that the above implementation only requires real-time knowledge of the link travel times, the link market shares, and the rates of change of the link market shares.

B. Nash Stabilization with Payoff Equity

The payoff mechanisms in Theorem 5 stabilize the system to an extended Nash equilibrium set. However, in order to do this, it was necessary to redefine the payoff for the CAV populations. The payoff for the regular vehicles, p^R, is generated by a static payoff function, F^R, that has tangible physical meaning. Each component p_i^{Rr} is the negative of the travel time associated with a vehicle in population r taking route i. Meanwhile, as determined by the G^A given by Theorem 5, it is not clear what the meaning of p_i^{Ar}. In this section, we modify G^A such that in the limit as $t \to \infty$, the routes of maximum payoff are the same for both the RV and CAV populations, for each OD pair.

For autonomous vehicle population $r \in \{1, \ldots, \varrho\}$, let $\Sigma^r\left(p^{Ar}(t)\right)$ be a permutation matrix such that

$$\tilde{p}^{Ar}(t) \triangleq \Sigma^r\left(p^{Ar}(t)\right)p^{Ar}(t) \tag{53}$$

is in descending order, i.e., $\tilde{p}_1^{Ar}(t) \geqslant \tilde{p}_2^{Ar}(t) \geqslant \ldots \geqslant \tilde{p}_{n^r}^{Ar}(t)$. Next, for $i \in \{1, \ldots, n^r\}$, define

$$\tilde{q}^{Ar}(t) \triangleq \Sigma^r\left(p^{Ar}(t)\right)\nu^{Ar}\left(x^{Ar}(t), p^{Ar}(t)\right) \tag{54}$$

As such, q^{Ar} is just a rearrangement of $\dot{x}^{Ar}(t)$, in order of descending payoff. Next, define

$$\tilde{d}^{Ar}(t) \triangleq \sum_{j=1}^{i}\tilde{q}_j^{Ar}(t). \tag{55}$$

Then it follows that $d_i^{Ar}(t) \geqslant 0$ for all $i \in \{1, \ldots, n^r\}$, with $d_{n^r}^{Ar}(t) = 0$. To see this, consider, first, that

$$\tilde{q}^{Ar} = \begin{bmatrix} \tilde{\mu}_{12} & \tilde{\mu}_{13} & \cdots & \tilde{\mu}_{1n^r} \\ -\tilde{\sigma}_2 & \tilde{\mu}_{23} & \cdots & \tilde{\mu}_{2n^r} \\ 0 & -\tilde{\sigma}_3 & \cdots & \tilde{\mu}_{3n^r} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\tilde{\sigma}_{n^r} \end{bmatrix} \begin{bmatrix} \tilde{x}_2^{Ar} \\ \vdots \\ \tilde{x}_{n^r}^{Ar} \end{bmatrix} \tag{56}$$

where

$$\tilde{\mu}_{ij} \triangleq \tilde{\phi}_i^{Ar}(\tilde{p}_i - \tilde{p}_j), \qquad \tilde{\sigma}_j \triangleq \sum_{i=1}^{j-1}\tilde{\mu}_{ij} \tag{57}$$

and where $\{\tilde{\phi}_i^{Ar} : i = 1, \ldots, n^r\}$ is the permuted set of ϕ_i^{Ar} functions. The values of $\tilde{d}_i^{Ar}$ are obtained by adding the first i rows of the above expression. The first $i-1$ columns of these rows add to zero (for $i > 1$), and the remainder contain positive terms. Because $\tilde{x}_i^{Ar} \geqslant 0$, we conclude that $\tilde{d}_i^{Ar} \geqslant 0$. That $\tilde{d}_{n^r}^{Ar} = 0$ is verified by noting that each column of the above matrix has zero sum.

Let $\tilde{p}^{Rr}(t)$ be the values of $p^{Rr}(t)$ permuted according to the descending order of $p^{Ar}(t)$, i.e.,

$$\tilde{p}^{Rr}(t) = \Sigma^r\left(p^{Ar}(t)\right)p^{Rr}(t). \tag{58}$$

Now, the key idea here is to observe that if $\tilde{p}^{Rr}(t)$ is in descending order, then this implies that the RV and CAV populations for OD pair r have the same route(s) of maximum payoff. If this is the case as $t \to \infty$, then because the IPC EDM equilibrates at the routes of maximum payoff, both populations will converge to the same routes.

Definition 5: We say that $p^R(t)$ has *common priority* with $p^A(t)$ if $\tilde{p}^{Rr}(t)$ is in descending order for all $r \in \{1, \ldots, \varrho\}$.

Definition 6: Let $\beta(t)$ be a vector with elements defined for all $r \in \{1, \ldots, \varrho\}$ and all $i \in \{1, \ldots, n^r - 1\}$ as

$$\beta_i^r(t) = \min\left\{\tilde{p}_i^{Rr}(t) - \tilde{p}_{i+1}^{Rr}(t), 0\right\} \tag{59}$$

where the minimum is taken element-wise. We say that p^R *converges to common priority* with p^A if for any $\theta \in \mathbb{R}_{>0}$, it is the case that

$$\lim_{t\to\infty}\int_t^{t+\theta}\beta_i^r(\tau)d\tau = 0 \tag{60}$$

for all $r \in \{1, \ldots, \varrho\}$ and all $i \in \{1, \ldots, n^r - 1\}$.

The theorem below entails the modification of G^A from Theorem 5 such that for each $r \in \{1, \ldots, \varrho\}$, $\tilde{p}^{Rr}$ converges to common priority with p^{Ar}. To present the result concisely, we introduce the following matrices for $r \in \{1, \ldots, \varrho\}$:

$$U^r = \begin{bmatrix} 1 & \cdots & 1 \\ & \ddots & \vdots \\ O & & 1 \end{bmatrix} \in \mathbb{R}^{n^r \times n^r}, \quad E^r = \begin{bmatrix} I_{n^r-1} \\ 0_{1\times n^r-1}. \end{bmatrix} \tag{61}$$

Also, we define

$$\tilde{p}^R \triangleq \text{vec}\left\{\tilde{p}^{R1}, \ldots, \tilde{p}^{R\varrho}\right\} \quad (62)$$

$$\Sigma\left(p^A(t)\right) \triangleq \text{blockdiag}\left\{\Sigma^1\left(p^{A1}(t)\right), \ldots, \Sigma^\varrho\left(p^{A\varrho}(t)\right)\right\} \quad (63)$$

$$U \triangleq \text{blockdiag}\left\{U^1, \ldots, U^\varrho\right\} \quad (64)$$

$$E \triangleq \text{blockdiag}\left\{E^1, \ldots, E^\varrho\right\}. \quad (65)$$

Theorem 6: In Theorem 5, assume G^A is modified to

$$\begin{aligned} G^A(x, p^A) = &- \bar{R}\Delta^C(z^A, z^R)\bar{R}^T\nu^A(x^A, p^A) \\ &- \bar{R}\Delta^A(z^A, z^R)\bar{R}^T\nu^R(x^R, p^R) \\ &+ \Sigma\left(p^A\right)^T UAE\beta \end{aligned} \quad (66)$$

where $A = \text{blockdiag}\{a_1 I_{n^1}, \ldots, a_\varrho I_{n^\varrho}\}$, $\{a_1, \ldots, a_\varrho\}$ are positive constants, and β is defined as in (59). Then $\mathbb{E}_{G^A, F^R}$ is a globally asymptotically stable set, with respect to differential equation (41). Furthermore, p^R converges to common priority with p^A.

Remark 3: Note that G^A, as defined in (66) is discontinuous in p^A, due to the presence of the permutation matrix $\Sigma\left(p^A(t)\right)$, which changes discontinuously whenever the sorting order of $p^A(t)$ changes. At times when $\Sigma\left(p^A(t)\right)$ is non-unique (i.e., at times when $p_i^{Ar}(t) = p_j^{Ar}(t)$ for some $r \in \{1, \ldots, \varrho\}$ and some $i, j \in \{1, \ldots, n^r\}$ with $i \neq j$), we treat $G^A(x(t), p^A(t))$ as a set. Specifically, we treat it as the convex hull of all the permutation matrices that sort each $p^{Ar}(t)$ in non-ascending order, for $r \in \{1, \ldots, \varrho\}$. Equation (41) then becomes a differential inclusion at these times.

V. Conclusions

The objective of this paper has been to formulate a feedback control law for the CAV populations in a transportation network, such that global convergence to Nash equilibrium is achieved. In particular, we have focused on the case in which the static payoff mechanism, F, that generates the Nash equilibrium set is the negative of the travel time. Our objective has been accomplished through the formulation of a secondary, dynamic payoff mechanism, which governs route choice for the CAVs. This payoff mechanism takes the form of (40). Our main result is Theorem 6, which formulates this payoff mechanism such then global asymptotic stability to the Nash equilibrium set generated by F^R is assured.

Appendix

A. Proof of Theorem 4

If ν is δ-dissipative then it has storage function $S(x, p)$, dissipation function $\sigma(x, p)$, and supply rate parametrized by $\Pi = \Pi^T$. Define $V(x, p^A)$ as

$$V(x, p^A) \triangleq S\left(x, \begin{bmatrix} p^A \\ F^R(x) \end{bmatrix}\right) \quad (67)$$

Then

$$\dot{V} = \frac{\partial S(x,p)}{\partial x}\nu(x,p)\Big|_{p^R = F^R(x)} + \frac{\partial S(x,p)}{\partial p}\begin{bmatrix} G^A(x, p^A, p^R) \\ \frac{\partial F^R(x)}{\partial x}\nu(x,p) \end{bmatrix}\Bigg|_{p^R = F^R(x)}. \quad (68)$$

δ-dissipativity implies that the above is bounded by

$$\dot{V} \leqslant -\sigma(x,p)|_{p^R = F^R(x)} + \Xi^T(x, p^A)\ \Pi\ \Xi(x, p^A) \quad (69)$$

But $\sigma(x, p) \geqslant 0$ with the equality holding only if $\nu(x, p) = 0$. We conclude that if (43) holds then $\dot{V}(x(t), p^A(t)) < 0$ for all $x(t) \in \mathbb{X}$ and $p^A(t) \in \mathbb{R}^{n/2}$ except where $\nu(x(t), p(t)) = 0$. Let the set $\mathbb{F}$ be defined as

$$\mathbb{F} = \left\{(x, p^A) \in \mathbb{X} \times \mathbb{R}^{n/2} : \nu\left(x, \begin{bmatrix} p^A \\ F^R(x) \end{bmatrix}\right) = 0\right\} \quad (70)$$

Recall Definition 4 for extended Nash equilibrium, and Definition 2 for Nash stationarity, to conclude that $\mathbb{F} = \mathbb{E}_{G^A, F^R}$. Then because $V(x(t), p^A(t)) \geqslant 0$ and

$$(x, p^A) \in \mathbb{E}_{G^A, F^R} \Leftrightarrow V\left(x(t), p^A(t)\right) = 0 \quad (71)$$

we conclude that $V(x(t), p^A(t)) \to 0$ as $t \to \infty$, and $(x(t), p^A(t)) \to \mathbb{E}_{G^A, F^R}$. This implies that V is a global Lyapunov function with attracting set $\mathbb{E}_{G^A, F^R}$.

B. Proof of Theorem 5

It is known from Theorem 2 that ν is δ-passive with storage function (13) and dissipation function (14). It is therefore δ-dissipative with supply rate parameter Π as in (30). We need only verify that F^R and G^A, as defined, satisfy (43). Substitution of (45) and (46) into (43) results in

$$-\nu^T(x,p)\begin{bmatrix} \bar{R}\Delta^C(z^A, z^R)\bar{R}^T & \bar{R}\Delta^A(z^A, z^R)\bar{R}^T \\ \bar{R}\Delta^A(z^A, z^R)\bar{R}^T & \bar{R}\Delta^R(z^A, z^R)\bar{R}^T \end{bmatrix}\nu(x,p) \leqslant 0 \quad (72)$$

which is equivalent to

$$-\left[\Omega(z^A, z^R)\nu(x,p)\right]^T\left[\Omega(z^A, z^R)\nu(x,p)\right] \leqslant 0 \quad (73)$$

where

$$\Omega(z^A, z^R) \triangleq \begin{bmatrix} \bar{R}\Delta^A(z^A, z^R)\left[\Delta^R(z^A, z^R)\right]^{-1/2} \\ \bar{R}\left[\Delta^R(z^A, z^R)\right]^{1/2} \end{bmatrix}^T. \quad (74)$$

The inequality is therefore satisfied for all $x \in \mathbb{X}$ and $p \in \mathbb{R}^n$, thus concluding the proof.

C. Proof of Theorem 6

To show that $\bar{E}_{G^A, F^R}$ is a globally asymptotically stable set, it is sufficient to show that (43) is still satisfied after the addition of the extra term added to G^A in (66), beyond the terms in (46). Let $\Xi_0(x, p^A)$ be the value of $\Xi(x, p^A)$ without this additional term. Then we have that

$$\Xi(x, p^A) = \Xi_0(x, p^A) + \begin{bmatrix} 0_{n\times 1} \\ \Sigma(p^A)^T UAE\beta \\ 0_{n/2\times 1} \end{bmatrix} \quad (75)$$

It is straightforward to show that

$$\begin{aligned} \Xi^T(x, p^A)\ \Pi\ \Xi(x, p^A) = &\ \Xi_0^T(x, p^A)\ \Pi\ \Xi_0(x, p^A) \\ &+ \left[\nu^A(x^A, p^A)\right]^T\Sigma\left(p^A\right)^T UAE\beta \end{aligned} \quad (76)$$

It is known from Theorem 5 that the first term on the right-hand side is negative-semidefinite. We now show that the second term is as well. Using (54) and (55), this term is equivalently

$$(\tilde{q}^A)^T UAE\beta = (\tilde{d}^A)^T AE\beta \quad (77)$$

$$= \sum_{r=1}^{\varrho} \sum_{i=1}^{n^r-1} a_r \tilde{d}_i^{Ar} \beta_i^r \quad (78)$$

First, we evaluate this expression assuming $\Sigma\left(p^A(t)\right)$ is not at a time of transition, such that its value (and therefore those of $\tilde{d}_i^{Ar}$, $\tilde{p}_i^{Rr}$ and $\tilde{p}_{i+1}^{Rr}$) are unique. In this case, because it is known that $\tilde{d}_i^{Ar} \geqslant 0$, it follows that the summand in the above expression is nonpositive. Now, consider the case in which expression is evaluated at a time t at which $\Sigma^r\left(p^{Ar}(t)\right)$ jumps from one value to another, for some $r \in \{1, \ldots, \varrho\}$. This implies that there are sets $\mathbb{I}_k^{Ar}(t)$, for $k \in \{1, \ldots, K^r\}$ with $1 \leqslant K^r < n^r$, and a corresponding set of scalar values γ_k^r, such that

$$i, j \in \mathbb{I}_k^{Ar}(t) \quad \Leftrightarrow \quad p_i^{Ar}(t) = p_j^{Ar}(t) = \gamma_k^r. \quad (79)$$

(In other words, the values of γ_k^r constitute the set of unique values in the vector p^{Ar}.) With reference (56), let the configuration shown correspond to one of the possible orderings of the indices. Then relative to this ordering, the sets $\mathbb{I}_k^{Ar}(t)$ must each be consecutive. As such, any other admissible ordering of the indices in $\mathbb{I}_k^{Ar}$ would entail the switching of one or more consecutive rows corresponding to $p_i^{Ar} = \gamma_k^r$. But for consecutive rows with $\tilde{p}_i^{Ar} = \tilde{p}_j^{Ar} = \gamma_k^r$, it follows that $\tilde{\mu}_{ij} = 0$. Consequently, the value of $\tilde{d}_i^{Ar}(t)$ as in (55) is nonnegative for all $i \in \mathbb{I}_k^{Ar}$ irrespective of the reordering of the rows corresponding to this index set. It therefore follows that (78) is negative for all the values $\Sigma\left(p^A(t)\right)$ can take at time t, as well as over the entire convex hull of these values. We conclude that (43) holds, proving that $\mathbb{E}_{G^A,F^R}$ is a globally asymptotically stable set.

It remains to be shown that p^R converges to common priority with p^A. Let $W(t)$ be defined as

$$W(t) = \sum_{r=1}^{\varrho} \sum_{i=1}^{n^r-1} \left(\tilde{p}_i^{Ar}(t) - \tilde{p}_{i+1}^{Ar}(t)\right). \quad (80)$$

and consider that with differential equation (41) imposed, $\dot{W}(t)$ is

$$\dot{W}(t) = \sum_{r=1}^{\varrho} \sum_{i=1}^{n^r-1} a_r \beta_i^r(t) - \Upsilon^T(t)\nu(x(t), p(t)) \quad (81)$$

where

$$\Upsilon^T(t) = \omega(t)\Omega(z^A(t), z^R(t)) \quad (82)$$

with Ω as in (74),

$$\omega(t) \triangleq \hat{e}^T E^T U^{-1} \Sigma\left(p^A(t)\right) \bar{R} \Delta^A(z^A(t), z^R(t)) \times \left[\Delta^R(z^A(t), z^R(t))\right]^{-1/2} \quad (83)$$

and where $\hat{e}$ is a vector in which every component is 1. (As before, at times when $\Sigma\left(p^A(t)\right)$ is discontinuous we treat $\dot{W}(t)$ as a set. This set is comprised of $\dot{W}(t)$ evaluated over the convex hull of all possible values of $\Sigma\left(p^A(t)\right)$). Note that the components of $\omega(t)$ are uniformly bounded in t, and are all nonnegative. There is consequently a constant matrix $\bar{\omega} \geqslant 0$ such that

$$\omega(t)\Omega(z^A(t), z^R(t))\nu(x(t), p(t)) \geqslant -\bar{\omega}\left|\Omega(z^A(t), z^R(t))\nu(x(t), p(t))\right|. \quad (84)$$

We therefore have that

$$\dot{W}(t) \leqslant \sum_{r=1}^{\varrho} \sum_{i=1}^{n^r-1} a_r \beta_i^r(t) + \bar{\omega}\left|\Omega(z^A(t), z^R(t))\nu(x(t), p(t))\right| \quad (85)$$

and it follows that for any $t \geqslant 0$, and any $\theta > 0$,

$$W(t+\theta) \leqslant W(t) + \sum_{r=1}^{\varrho} \sum_{i=1}^{n^r-1} a_r \int_t^{t+\theta} \beta_i^r(\tau) d\tau + \bar{\omega} \int_t^{t+\theta} \left|\Omega(z^A(\tau), z^R(\tau))\nu(x(\tau), p(\tau))\right| d\tau \quad (86)$$

Now, consider that

$$\int_t^{t+\theta} \dot{x}^T(\tau)\dot{p}(\tau) d\tau = \int_t^{t+\theta} \begin{bmatrix} \nu^A(x^A(\tau), p^A(\tau)) \\ \nu^R(x^R(\tau), p^R(\tau)) \end{bmatrix}^T \begin{bmatrix} G^A(x(\tau), p^A(\tau)) \\ \frac{\partial F^R(x)}{\partial x}(\tau) \end{bmatrix} d\tau \quad (87)$$

$$= -\int_t^{t+\theta} \|\Omega(z^A(\tau), z^R(\tau))\nu(x(\tau), p(\tau))\|_2^2 d\tau + \sum_{r=1}^{\varrho} \sum_{i=1}^{n^r-1} \int_t^{t+\theta} \tilde{d}_i^{Ar}(\tau)\beta_i^r(\tau) d\tau \quad (88)$$

$$\geqslant S(x(t+\theta), p(t+\theta)) - S(x(t), p(t)) + \int_t^{t+\theta} \sigma(x(\tau), p(\tau)) d\tau \quad (89)$$

$$\geqslant -S(x(t), p(t)) + \int_t^{t+\theta} \sigma(x(\tau), p(\tau)) d\tau \quad (90)$$

Rearranging, we have that

$$S(x(t), p(t)) \geqslant \int_t^{t+\theta} \sigma(x(\tau), p(\tau)) d\tau - \sum_{r=1}^{\varrho} \sum_{i=1}^{n^r-1} \int_t^{t+\theta} a_r \tilde{d}_i^{Ar}(\tau)\beta_i^r(\tau) d\tau + \int_t^{t+\theta} \|\Omega(z^A(\tau), z^R(\tau))\nu(x(\tau), p(\tau))\|_2^2 d\tau \quad (91)$$

Each of the three terms on the right-hand side is positive, and therefore each is in the range $[0, S(x(t_0), p(t_0))]$. We therefore conclude that

$$\int_t^{t+\theta} \|\Omega(z^A(\tau), z^R(\tau))\nu(x(\tau), p(\tau))\|_2^2 d\tau \leqslant S(x(t), p(t)) \quad (92)$$

But via the Cauchy-Schwarz inequality,

$$\int_t^{t+\theta} \|\Omega(z^A(\tau), z^R(\tau))\nu(x(\tau), p(\tau))\|_1 d\tau$$
$$\leqslant \sqrt{L\theta}\left(\int_t^{t+\theta} \|\Omega(z^A(\tau), z^R(\tau))\nu(x(\tau), p(\tau))\|_2^2 d\tau\right)^{1/2} \tag{93}$$
$$\leqslant \sqrt{L\theta}\sqrt{S(x(t), p(t))} \tag{94}$$

Consequently, (86) implies that

$$W(t+\theta) \leqslant W(t) + \sum_{r=1}^{\varrho}\sum_{i=1}^{n^r-1}\int_t^{t+\theta}\beta_i^r(\tau)d\tau + \left(\max_i \bar{\omega}_i\right)\sqrt{L\theta}\sqrt{S(x(t), p(t))} \tag{95}$$

Let $\mathbb{Q}_\theta$ be the limit set for the quntity $[W(t+\theta) - W(t)]$ as $t \to \infty$. In other words, for every $Q \in \mathbb{Q}_\theta$, there is an increasing sequence of times $\{t_1, t_2, \ldots\}$ with $t_k \to \infty$ as $k \to \infty$, such that

$$\lim_{k\to\infty} [W(t_k+\theta) - W(t_k)] = Q. \tag{96}$$

Because $S(x(t), p(t)) \to 0$ as $t \to \infty$, we have that for all $Q \in \mathbb{Q}_\theta$,

$$Q \leqslant \lim_{t_0\to\infty}\sum_{r=1}^{\varrho}\sum_{i=1}^{n^r-1}\int_{t_k}^{t_k+\theta}\beta_i^r(\tau)d\tau \tag{97}$$

But $\beta_i^r(\tau) \leqslant 0$ so we conclude that $Q \leqslant 0$ for all $Q \in \mathbb{Q}_\theta$. Furthermore, the fact that $W(t) \geqslant 0$ for all t then implies that $Q \geqslant 0$, because otherwise (96) implies that $W(t_k)$ decreases without bound as $k \to \infty$. As such, we conclude that $\mathbb{Q}_\theta = \{0\}$. Taking the limit of (95) as $t \to \infty$ therefore gives that

$$0 \leqslant \lim_{t\to\infty}\sum_{r=1}^{\varrho}\sum_{i=1}^{n^r-1}\int_t^{t+\theta}\beta_i^r(\tau)d\tau \tag{98}$$

Because the integrand is negative-semidefinite, we conclude that the equality holds tightly, i.e.,

$$\lim_{t\to\infty}\sum_{r=1}^{\varrho}\sum_{i=1}^{n^r-1}\int_t^{t+\theta}\beta_i^r(\tau)d\tau = 0. \tag{99}$$

From Definition 6, we conclude that p^R converges to common priority with p^A.

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