Data-Centric Theories and Singular Value Decomposition (SVD) for Identification of Inverter-Based Resources

1st Shunjie Zhang
Electrical and Computer Engineering,
Lehigh University
Bethlehem, PA, U.S.
shz923@lehigh.edu

2nd Javad Khazaei*, Senior Member, IEEE
Electrical and Computer Engineering,
Lehigh University
Bethlehem, PA, U.S.
jak921@lehigh.edu

***Abstract*—This paper presents a pioneering data-centric approach for model identification of inverter-based resources (IBRs) in smart grids, which includes renewable energy systems and energy storage technologies. Unlike traditional methods that depend on fixed models and extensive system identification tools, our approach leverages emerging systems behavioral theories and combines them with singular value decomposition (SVD) to efficiently identify IBR models from data. The SVD's ability to reduce the dimension of collected data is instrumental in capturing critical dynamic features from minimal data inputs. By applying the principle of persistence of excitation and organizing input/output data into a Hankel matrix form, we derive a robust, model-free representation of IBR dynamics that requires significantly less data than conventional machine learning methods. The effectiveness of our approach is validated through comprehensive time-domain simulations, demonstrating its potential for model-free IBR control applications.**

***Index Terms*—Singular Value Decomposition (SVD),Inverter-based Resources, Willems' Fundamental Lemma.**

I. Introduction

Inverter-based resources (IBRs), such as renewable energy systems and storage units, are indispensable for achieving the global 2050 net-zero emissions goal. However, challenges arise due to the potential destabilization of the power grid caused by inadequate control mechanisms, which can result in significant power outages. The integrity and effectiveness of these IBR controllers hinge critically on the availability of precise and reliable dynamic models. Such models are essential to ensure both the robustness and stability of the overall control design.

Conventional control methods for IBRs are generally categorized into model-based and model-free strategies, each reliant on the presence or absence of an underlying model of the IBR system. Model-based control is further subdivided into linear strategies [1] and nonlinear approaches [2]. These model-based methods develop system models either from first principles or through data-driven techniques. The complexity involved in modeling diverse IBR configurations, the extensive nature of the grid, and the inability of traditional physics-based models to accommodate uncertainties have all driven the adoption of data-driven system identification tools.

A diverse array of methodologies has been introduced to facilitate the extraction of system dynamics directly from data. Notable among these are dynamic mode decomposition (DMD) [3], the Koopman operator [4], deep learning [5], [6], and sparse identification of nonlinear dynamics, also known as sparse regression [7], [8]. These techniques have been effectively applied within the realm of power systems management, ranging from delay-tolerant microgrid control [9]–[12], as demonstrated through the use of DMD [9], and deep learning for load identification [6], to the application of the Koopman operator for generator dynamic state estimation [12].

Despite the advancements in data-driven techniques for IBRs, these methods encounter several notable challenges that can impede their broader application. Primarily, the performance of these techniques is often restricted by the requirement for large datasets to achieve satisfactory results. This dependency on extensive data can limit the practicality and efficiency of the methods in real-world scenarios [13]. Moreover, the absence of an underlying physical model complicates the interpretability of the control strategies, posing challenges in comprehending the basis of the controller's decisions and troubleshooting control issues [13]. Additionally, these methods frequently struggle to generalize effectively to new, unseen conditions, which further restricts their utility in dynamic environments where operational conditions can vary unpredictably [8]. These traditional techniques often fail to capture the dynamically changing conditions inherent in modern power grids.

To address this challenge, this paper introduces a new approach that leverages a data-centric perspective alongside singular value decomposition (SVD) to identify IBR models from the minimum required amount of data. By integrating SVD with systems behavioral theories, the dimension of

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data can be further reduced and computational efficiency of proposed data-centric method can significantly be increased. Drawing inspiration from systems behavioral theory, as originally proposed in Willems' fundamental lemma [14], [15], our method is grounded in the principle of persistence of excitation. Several recent studies have explored the application of systems behavioral theories for data-centric modeling and control of dynamical systems so far [16]–[18]. This rigorous mathematical concept ensures that the model directly reflects the underlying data, adhering to the axiom "Model is Data." However, this approach requires constructing large-scale Hankel matrices of measured input/output data, which might introduce computational complexities especially for large-scale grid modeling purposes. Therefore, to address this gap, we propose a novel combination of SVD with systems behavioral theories to enable a low-rank factorization of Hankel matrix without losing important information in the measured trajectories. As a result, SVD plays a crucial role in the proposed method for enhancing the quality of data analysis. By breaking down data matrices into simpler orthogonal and diagonal matrices, SVD helps in isolating and eliminating noise and non-dominant data samples, while preserving the most significant features of the data set [19], [20]. This dual approach not only simplifies the modeling process but significantly increases the accuracy and efficiency of system identification and control strategies. Our simulation results in MATLAB validate the effectiveness of an SVD-based data-centric technique for model identification of IBRs. This analysis explores the robustness of the approach demonstrates its practical utility across different scenarios.

The remainder of the paper is structured as follows: Section II discusses the fundamental systems behavioral theories that underpin our methodology. Section III details the modeling of IBRs using our proposed approach. Section IV presents simulations and validation of our work, demonstrating the efficacy and potential of the techniques. Finally, Section V concludes the paper.

II. Fundamental Systems Behavioral Theories

In the context of data-centric modeling for IBRs, a foundational understanding of systems behavioral theories is crucial. This section delineates the essential theoretical framework required for constructing a data-centric representation of IBR dynamics, with a particular emphasis on Hankel matrices and principles derived from systems behavioral theories. Given a signal $x \in \mathbb{R}^n$, the aggregated data over the interval $[k, k+T]$ is represented as $\mathbf{x}[k, k+T] = \begin{bmatrix}\mathbf{x}(k) & \dots & \mathbf{x}(k+T)\end{bmatrix}^T$. The Hankel matrix for signal x, denoted $H(\mathbf{x})$, is defined to encapsulate the dynamics of the system over time. It is constructed as follows for indices k, t, and N,

$$H_{k,t,N}(\mathbf{x}) = \begin{bmatrix} \mathbf{x}(k) & \mathbf{x}(k+1) & \cdots & \mathbf{x}(k+N-1) \\ \mathbf{x}(k+1) & \mathbf{x}(k+2) & \cdots & \mathbf{x}(k+N) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}(k+t-1) & \mathbf{x}(k+t) & \cdots & \mathbf{x}(k+N+t-1) \end{bmatrix} \tag{1}$$

For cases where the Hankel matrix's order is one ($t = 1$), the matrix simplifies to:

$$H_{k,1,N}(\mathbf{x}) = \begin{bmatrix}\mathbf{x}(k) & \mathbf{x}(k+1) & \cdots & \mathbf{x}(k+N-1)\end{bmatrix} \tag{2}$$

The state-space representation involves the vector of state variables $\mathbf{x}$, control inputs $\mathbf{u}$, and outputs $\mathbf{y}$, given by

$$\dot{\mathbf{x}} = Ax + B\mathbf{u}, \quad \dot{\mathbf{y}} = C\mathbf{x} + D\mathbf{u} \tag{3}$$

For t samples of input-output data, these relationships can be represented using Hankel matrices [17]

$$\begin{bmatrix}\mathbf{y}_{[1,t]} \\ \mathbf{u}_{[1,t]}\end{bmatrix} = \begin{bmatrix} O_t & C_t \\ \mathbf{0}_{tm\times n} & \mathbf{I}_{tm}\end{bmatrix}\begin{bmatrix}\mathbf{x}_0 \\ \mathbf{u}_{[1,t]}\end{bmatrix} \tag{4}$$

Here, x_0 denotes the initial condition, and Ot and Ct are matrices derived from the system parameters C, A, B, and D as [17]

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{t-1}\end{bmatrix}, C = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{t-2}B & CA^{t-3}B & \cdots & D\end{bmatrix} \tag{5}$$

Consider a collection of input-output data over a fixed number of samples t, represented as $u^m_{[1,T]}$ and $y^m_{[1,T]}$, where the superscript m denotes these as measured data. These data samples can be effectively organized into Hankel matrices to analyze the system's dynamics. The input-output dynamics for this system are represented by the equation [17]

$$\begin{bmatrix} H_{1,T-t}(\mathbf{y}) \\ H_{1,T-t}(\mathbf{u})\end{bmatrix} = \begin{bmatrix} O_t & C_t \\ 0_{t\times n} & I_{t_m}\end{bmatrix}\begin{bmatrix} H_{1,T-t}(x) \\ H_{1,T-t}(u)\end{bmatrix} \tag{6}$$

In this context, $H_{1,T-t}(x)$ is the Hankel matrix for the state data x^m. This configuration captures the dynamics of the system by mapping the states and inputs to their outputs through the matrices O_t and C_t, with I_{t_m} being an identity matrix.

A. Willems' Fundamental Lemma

This lemma introduces a non-parametric approach for representing linear time-invariant systems through Hankel matrices, contingent on the persistence of excitation condition [14] [15].

1) Persistence of Excitation Lemma [14]: The lemma specifies that for a signal x in $\mathbb{R}^n$ which is persistently exciting of order t, the corresponding Hankel matrix for any T-long trajectory will exhibit full rank nt, i.e., [15]

$$\text{rank}(H_{1,T-t}(\mathbf{x})) = nt \tag{7}$$

2) Fundamental Lemma for Linear Systems: Building upon the initial lemma, the fundamental lemma for linear systems represented by (4) posits that if a T-long measurement of input u^m in $\mathbb{R}^m$ is persistently exciting of order $n + t$, then any t-long trajectory of system inputs and outputs can be effectively represented using a Hankel matrix of order t [15].

$$\begin{bmatrix}\mathbf{u}_{[k,k+t]} \\ \mathbf{y}_{[k,k+t]}\end{bmatrix} = \begin{bmatrix} H_{1,T-t}(\mathbf{u}^m) \\ H_{1,T-t}(\mathbf{y}^m)\end{bmatrix}\alpha \tag{8}$$

for some vector α that can be determined (see the proof in reference [17]).

B. Singular Value Decomposition

Singular value decomposition (SVD) is an essential matrix factorization technique in linear algebr. In the decomposition process, a matrix is expressed in terms of three other matrices, showcasing orthogonal and diagonal characteristics that are crucial for understanding the structure of the original matrix. The idea in this paper is to apply SVD to the Hankel matrix of data for increasing the computational efficiency of data-centric modeling of IBRs. The SVD [21] of a Hankel matrix Y is expressed as

$$Y = U\Sigma V^T \tag{9}$$

where U and V represent orthogonal matrices, and Σ is a diagonal matrix containing the singular values. The ordering of the singular values can be mathematically described by the following inequalities [19], [20]

$$\sum_{i=1}^{k} \sigma_i \geq c \cdot \sum_{i=1}^{k} \tau_i \quad \text{for } k = 1, 2, \ldots, n-1 \tag{10}$$

$$\sum_{i=1}^{n} \sigma_i = c \cdot \sum_{i=1}^{n} \tau_i \tag{11}$$

where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n$ are the ordered singular values of a matrix Y [21], and $\tau_1, \tau_2, \ldots, \tau_n$ are elements of another ordered set being compared, and c is the desired proportion of variance to retain (e.g., 0.95).

This factorization not only elucidates the structural attributes of Y but also facilitates various computations, including the derivation of a reduced-rank version of Y. Specifically, a reduced-rank approximation, denoted as Y_r, can be reconstructed by retaining only the first r largest singular values and corresponding singular vectors. This approximation is given by

$$Y_r = U_r \Sigma_r V_r^T \tag{12}$$

where U_r, Σ_r, and V_r are derived from the first r components of U, Σ, and V, respectively. The matrix Y_r thus represents the rank-r matrix that is closest to Y, offering a practical approach for dimensionality reduction and data compression.

III. Data-Centric Modeling of IBRs

Fig 1 illustrates the structure of the proposed framework for data-centric identifiaction an IBR, characterized by a voltage source converter (VSC) linked to the main electrical grid through a three-phase filter. This filter comprises inductance L_c, resistance r_c, and capacitance C_f, collectively known as grid impedance components. The main focus of this paper is on leveraging a data-centric modeling approach for elucidating the dynamics of IBRs and use SVD reconstruct a low-rank version of data-centric model to reduce data dimension and increase the computational efficiency of proposed data-centric modeling. For a deeper dive into control design methodologies pertinent to IBRs, interested readers may read additional detailed studies referenced in [22].

A. dq-frame Modeling of IBR

Phase-locked loop (PLL) is instrumental in achieving synchronization between the converter's voltage at the point of common coupling (PCC) and the grid frequency [22]. This synchronization ensures that the IBR operates harmoniously within the grid infrastructure.

Assuming a PLL exists, dynamics on the AC side of the IBR in the dq-frame are expressed as [23]

$$v_{cd} - v_{pd} + \omega_0 L_c i_{cq} = (L_c s + r_c) i_{cd} \tag{13}$$

$$v_{cq} - v_{pq} - \omega_0 L_c i_{cd} = (L_c s + r_c) i_{cq} \tag{14}$$

$$v_{pd} - v_{gd} + \omega_0 L_n i_{nq} = (L_n s + r_n) i_{nd} \tag{15}$$

$$v_{pq} - v_{gq} - \omega_0 L_n i_{nd} = (L_n s + r_n) i_{nq} \tag{16}$$

$$i_{cd} - i_{nd} + \omega_0 C_f v_{pq} = C_f s v_{pd} \tag{17}$$

$$i_{cq} - i_{nq} - \omega_0 C_f v_{pd} = C_f s v_{pq} \tag{18}$$

where s denotes the Laplace operator, ω_0 is the nominal frequency(377 rad/s) of the system, and $v_{cd}, v_{cq}, i_{cd}, i_{cq}$ are the dq-frame components of the converter output voltage and current, respectively. Additionally, the above equations can be represented in following state-space form as

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}, \quad \mathbf{y} = C\mathbf{x} \tag{19}$$

Where $\mathbf{x} = [i_{cd} \quad i_{cq} \quad i_{nd} \quad i_{nq} \quad v_{pd} \quad v_{pq}]^T$ is state vector, $\mathbf{u} = [v_{cd} \quad v_{cq} \quad v_{gd} \quad v_{gq}]^T$, and $\mathbf{y}$ is output vector. The system matrices A, B and C are found to be [23]

$$A = \begin{bmatrix} -\frac{r_c}{L_c} & \omega_0 & 0 & 0 & -\frac{1}{L_c} & 0 \\ -\omega_0 & -\frac{r_c}{L_c} & 0 & 0 & 0 & -\frac{1}{L_c} \\ 0 & 0 & -\frac{r_g}{L_g} & \omega_0 & \frac{1}{L_g} & 0 \\ 0 & 0 & -\omega_0 & -\frac{r_g}{L_g} & 0 & \frac{1}{L_g} \\ \frac{1}{C_f} & 0 & -\frac{1}{C_f} & 0 & 0 & \omega_0 \\ 0 & \frac{1}{C_f} & 0 & -\frac{1}{C_f} & -\omega_0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{L_c} & 0 & 0 & 0 \\ 0 & \frac{1}{L_c} & 0 & 0 \\ 0 & 0 & -\frac{1}{L_g} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_g} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = I \tag{20}$$

B. Data-Centric Modeling of IBRs

Assuming the system's inputs u_m and outputs x_m are collected, and assuming a full-state feedback $\mathbf{y} = \mathbf{x}$, the IBR dynamics can be described through persistence of excitation

$$\begin{bmatrix} \mathbf{u}_{[1,t]} \\ \mathbf{x}_0 \end{bmatrix} = \begin{bmatrix} H_{1,t,T-t}(\mathbf{u}^m) \\ H_{1,t,T-t}(\mathbf{x}^m) \end{bmatrix} \alpha, \tag{21}$$

where α is a vector that parameterizes the linear combination of past trajectory matrices to predict future system states.

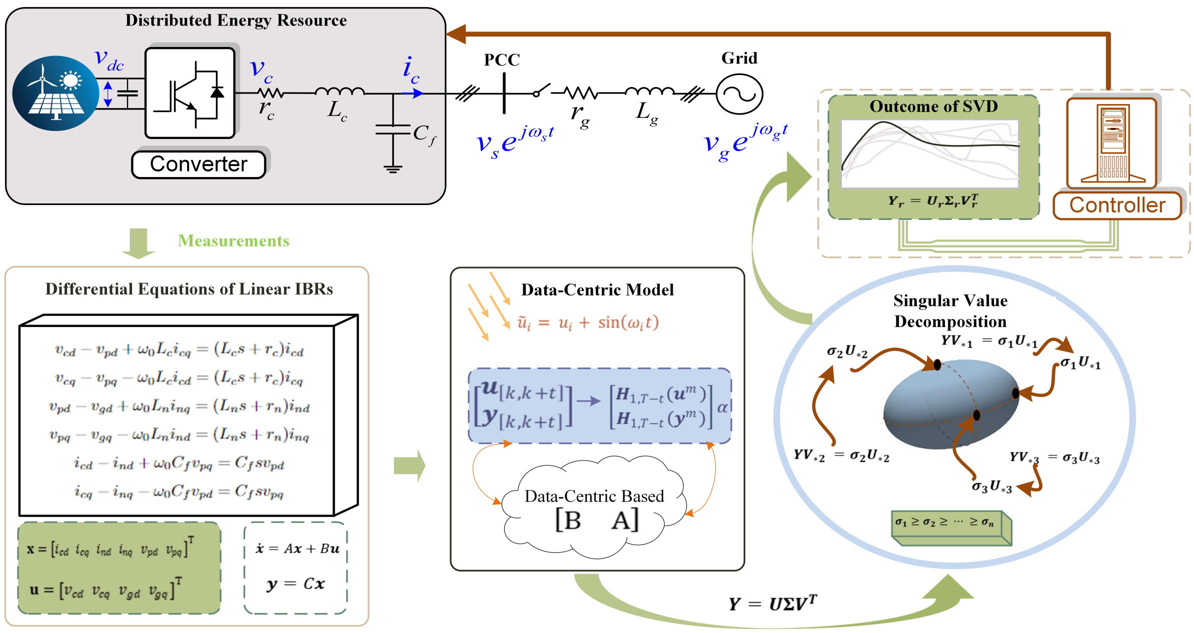

Fig. 1. Proposed data-centric modeling of a grid-connected IBR.

According to the Rouche-Capelli theorem, the system admits infinite solutions for α, which can be represented as [17]

$$\alpha = \begin{bmatrix} H_{1,t,T-t}(\mathbf{u}^m) \\ H_{1,t,T-t}(\mathbf{x}^m) \end{bmatrix}^{\dagger} \begin{bmatrix} \mathbf{u}_{[1,t]} \\ \mathbf{x}_0 \end{bmatrix} + (I - \begin{bmatrix} H_{1,t,T-t}(\mathbf{u}^m) \\ H_{1,t,T-t}(\mathbf{x}^m) \end{bmatrix}^{\dagger} \begin{bmatrix} H_{1,t,T-t}(\mathbf{u}^m) \\ H_{1,t,T-t}(\mathbf{x}^m) \end{bmatrix})\omega, \tag{22}$$

where ω is any vector in $\mathbb{R}^T$, highlighting the degrees of freedom in the model's predictions. The IBR dynamics in discrete form is then expressed as

$$\mathbf{x}(k+1) = H_{2,t,T-t+1}(\mathbf{x}^m)\alpha, \tag{23}$$

noting that $H_{2,t,T-t+1}(\mathbf{x}^m)$ is the Hankel matrix for advanced measurements or the derivative estimations given by

$$\dot{\mathbf{x}} \approx \frac{\mathbf{x}(i+1) - \mathbf{x}(i-1)}{2h}, \tag{24}$$

where h is the sampling interval. This differential approximation is essential for capturing dynamic transitions without direct derivative measurements. The data-centric model is then expressed as

$$\mathbf{x}(k+1) = H_{1,t,T-t+1}(\dot{\mathbf{x}})\alpha, \tag{25}$$

which indicates that the future state $x(k+1)$ can be predicted directly from past measurements encapsulated in the Hankel matrix, using the coefficient vector α calculated to best fit the data. This approach bypasses the need for conventional state-space models, providing a robust framework for modeling IBRs directly from input-output data. The validation of this modeling approach is achieved by demonstrating that

$$[B \quad A] = H_{1,t,T-t+1}(\dot{\mathbf{x}}) \begin{bmatrix} H_{1,t,T-t}(\mathbf{u}^m) \\ H_{1,t,T-t}(\mathbf{x}^m) \end{bmatrix}^{\dagger} \tag{26}$$

which shows that the system matrices A and B can be directly derived from the Hankel matrices of input and state data, see proof in [17].

C. Singular Value Decomposition of Data-Centric

Our approach involves the construction of Hankel matrices from T-sample time-series data, followed by the application of SVD to identify dominant singular values and vectors, which reveal significant modes and features of the data [19], [20]. The multidimensional time-series data from the IBR states, are horizontally concatenated into a single matrix X_n. A Hankel matrix, which is a structured matrix where each ascending skew-diagonal from left to right is constant, is then generated for each data series. This is performed by defining an initial segment length L, which effectively determines the matrix's ability to capture the temporal structure within the data

$$H_i = \begin{bmatrix} H_{1,L}(X_n^i) \\ H_{L,T}(X_n^i) \end{bmatrix} \tag{27}$$

here, $i = 1...6$, as there are 6 states in an IBR. These individual Hankel matrices $H1, H2, H3, H4, H5$, and $H6$ are subsequently stacked to form a larger matrix H, which consolidates the information from all data channels $H = [H_1 \quad H_2 \quad H_3 \quad H_4 \quad H_5 \quad H_6]^T$. The next step involves applying the SVD to the Hankel matrices H_i, which decomposes the matrices into three matrices U_i, S_i, and V_i as

$$H_i = U_i \Sigma_i V_i^T \tag{28}$$

From S_i, the diagonal elements are extracted and a threshold is set to identify significant singular values, which are then used to reconstruct a low-rank approximation of the Hankel matrix, i.e., threshold $= 0.95 \times \max(\text{diag}(S))$.

The determination of the rank k, which defines the number of significant singular values to retain, is pivotal for capturing the essential structure of the matrix while minimizing the complexity. Our operation identifies the indices of singular values that are greater than or equal to a pre-defined threshold. This threshold determines the largest of these indices, which specifies the cut-off rank k. This value k effectively limits the number of singular values and vectors considered in the reduced matrix, ensuring a balance between approximation accuracy and computational efficiency.

$$H_r = U_k \Sigma_k V_k^T \tag{29}$$

where U_k represents the matrix comprising the first k columns of U_i, which includes the left singular vectors associated with the largest k singular values, Σ_k is the $k \times k$ diagonal matrix containing the largest k singular values, and V_k^T is the transpose of the matrix consisting of the first k columns of V_i, incorporating the right singular vectors corresponding to these singular values.

This formulation ensures that H_r captures the most significant features of the original matrix H_i, providing an optimal balance between approximation accuracy and complexity reduction. The rank-k approximation effectively retains the principal structural and statistical properties of H_i, making it invaluable in various data processing and machine learning applications. The H_r is then converted back to a vector form to analyze or visualize the data effectively. Having the reduced-order Hankel matrix, a data-centric IBR model can then be obtained by

$$[B \quad A] = H_{r,[1,t,T-t+1]}(\dot{\mathbf{x}}) \begin{bmatrix} H_{r,[1,t,T-t]}(\mathbf{u}^m) \\ H_{r,[1,t,T-t]}(\mathbf{x}^m) \end{bmatrix}^{\dagger} \tag{30}$$

IV. CASE STUDIES

To validate the effectiveness of the proposed data-centric modeling and SVD-based IBR models, several case studies are carried out using time-domain simulation in MATLAB. The parameters of IBR were obtained from [23].

A. Case 1: Data-Centric IBR Model Identification

This case study aims to determine the minimum quantity of sample data required for successful identification of IBR models. Utilizing a simulation model of an IBR in an open-loop configuration [23], data were sampled over a duration with a granularity of 50 microseconds, resulting in a dataset comprising $T = 100$ samples. Initially, constant input vectors were applied to the system. However, it became apparent that such inputs did not sufficiently excite the system to meet the necessary rank conditions for effective model identification. Specifically, the condition $\text{rank}(H_{1,t,T-t}(x)) = nt = 6t$ for $t = 1$ was not satisfied, indicating the insufficiency of the data diversity with constant inputs [15]. To address this challenge, a small perturbation was introduced to all input signals by superimposing a sinusoidal waveform with amplitude less than 5% of the nominal input amplitude. This modification proved to be pivotal, as it enabled the achievement of the desired rank condition with merely 14 samples.

Following the enhancement of input excitation, a data-centric representation of the IBR model was developed and implemented in MATLAB as shown in Fig. 2. Results illustrate an accurate identification of IBR dynamics with minimum data and prove the efficacy of proposed data-centric methodologies to streamline the model identification process, ensuring accuracy while minimizing the requisite data volume.

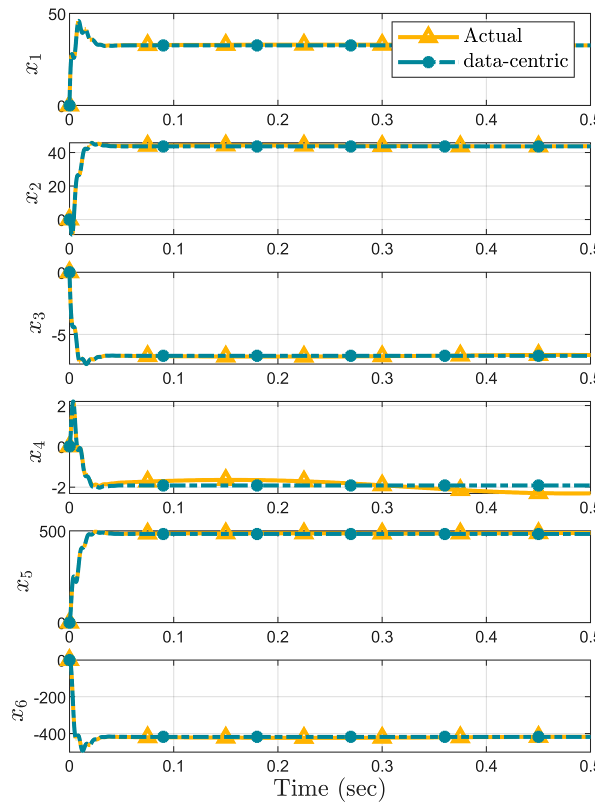

Fig. 2. Data-centric modeling of IBRs without SVD.

B. Case 2: SVD Reconstruction Data

This case study delves into the utilization of SVD to reconstruct a set of system responses, with a focus on transient reduction and accurate replication of the original data curves. Simulation results for this case are shown in Fig. 3. Analysis revealed that the reconstructed data is closely aligned with the original datasets, capturing the essential dynamics with high fidelity. This precise matching of the curves not only confirms the effectiveness of the SVD in reducing the dimension of Hankel matrix for improved computational efficiency, but also demonstrates the method's capacity to retain crucial information.

V. CONCLUSION

This paper proposed a computationally efficient approach for combining data-centric theories with singular value decomposition (SVD) for model-free identification of dynamics in

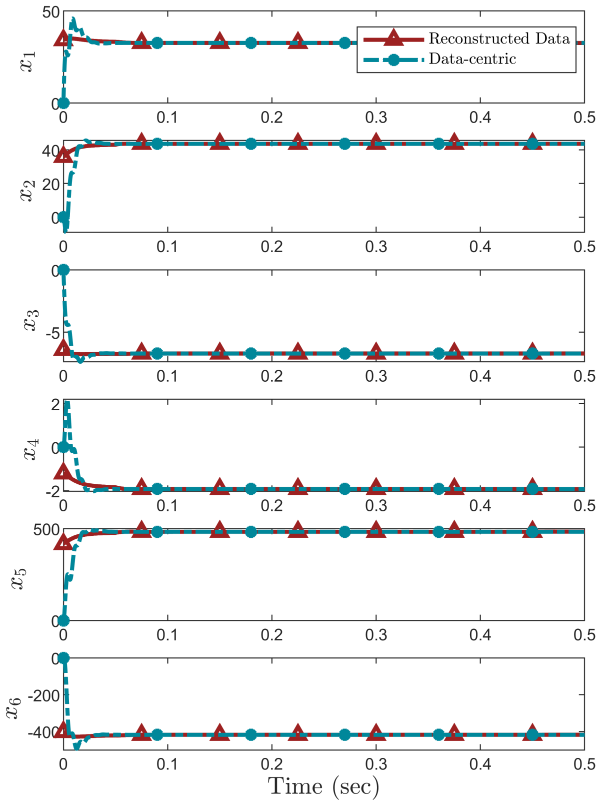

Fig. 3. Comparison between SVD and data-centric model.

inverter-based resources (IBRs). This innovative methodology facilitates a more robust, adaptable, and efficient handling of dynamic grid environments, significantly enhancing operational stability and reducing reliance on extensive data inputs. The case studies underscored the practical benefits of our approach, demonstrating not only its ability to reduce data dimension, but also its potential in improving system identification and response times. Moving forward, the adoption of such advanced data-centric models, supplemented by mathematical tools such as SVD, could facilitate model-free and data-centric control designs for inverter-dominated smart grids.

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