Estimation of the Equivalent Impedance of Lightning Return-Stroke Channel From Current Waveforms Measured at Tall Towers

Masahiro Hasegawa, Koki Kutsuna, Yoshihiro Baba, *Fellow, IEEE*, Naoto Nagaoka, Toshihiro Tsuboi, and Vladimir A. Rakov, *Fellow, IEEE*

***Abstract*—Two waveforms of current measured at a height of 497 m of the 634 m high Tokyo Skytree and three waveforms of current measured at a height of 533 m of the 540-m high Ostankino tower have been reproduced in simulations using the finite-difference time-domain method in the two-dimensional cylindrical coordinate system. The lightning return-stroke channel is represented by a dielectric-coated vertical conductor. The dielectric coating has high relative permeability, such as 20, 40, or 60, and serves to reduce the propagation speed of current wave along the channel-representing vertical conductor and to increase its equivalent impedance relative to that of the vertical conductor without dielectric coating. The channel-representing vertical conductor and the tall tower are excited by a voltage source inserted between them. A procedure to determine the output waveform of the voltage source from measured current waveforms has been developed. The magnitudes of measured initial current peaks ranged from 5 to 12 kA. The equivalent impedance of the lightning return-stroke channel has been inferred to be roughly 800 to 900 Ω.**

***Index Terms*—Finite-difference time-domain (FDTD) method, lightning equivalent impedance, lightning return-stroke channel, lightning return-stroke current, lightning surge, tall tower.**

I. Introduction

The equivalent impedance of the lightning return-stroke channel, which is defined as the input impedance seen from the channel attachment point, is one of the important parameters in simulations of lightning interaction with man-made systems such as electric power and communication systems or in lightning protection studies of these systems. This is because the equivalent impedance of the lightning channel influences the magnitude of current flowing in the system and the current reflection coefficient at the connection point between the lightning channel and the system. Although the equivalent impedance of the lightning return-stroke channel cannot be measured directly, waveforms of lightning return-stroke current measured at tall towers contain information about the equivalent impedance of the lightning return-stroke channel [1].

Different estimates of the equivalent impedance of the lightning return-stroke channel have been reported in the literature. Gorin and Shkilev [2] estimated values ranging from about 600 to 2500 Ω based on current waveforms measured at the 540-m high Ostankino tower. In their derivation, the characteristic impedance of the tower was assumed to be 300 Ω. Datsios et al. [3] reviewed previously reported values of the equivalent impedance of the lightning return-stroke channel, which range from 100 to 9000 Ω. They have recommended a value from 400 to 1000 Ω for lightning surge simulations.

In simulations of lightning interaction with tall strike objects using full-wave electromagnetic computation methods such as the method of moments (MoM) [4] and the finite-difference time-domain (FDTD) method [5], the output waveform of a voltage source inserted between the lightning return-stroke channel and the strike object, as well as an appropriate representation of the lightning return-stroke channel and strike object are needed. Baba and Rakov [6] modeled lightning strikes to the 160-m high Peissenberg tower and 553-m high CN Tower using the FDTD method. Saito et al. [7] represented a lightning strike to the 634-m high Tokyo Skytree using the MoM and a voltage source.

In this article, two waveforms of current measured at a height of 497 m of the 634 m high Tokyo Skytree and three waveforms of current measured at a height of 533 m of the 540-m high Ostankino tower are reproduced in simulations using the FDTD method in the two-dimensional (2-D) cylindrical coordinate system. The lightning return-stroke channel is represented by a dielectric-coated vertical conductor [8], [9], [10]. The dielectric coating has high relative permeability to reduce the propagation speed of current wave along the channel-representing vertical conductor and to increase its equivalent impedance relative to that of the vertical conductor without dielectric coating. This model belongs to the class of electromagnetic models [11], and can successfully yield plausible values of the equivalent impedance of the lightning return-stroke channel [2] and typical

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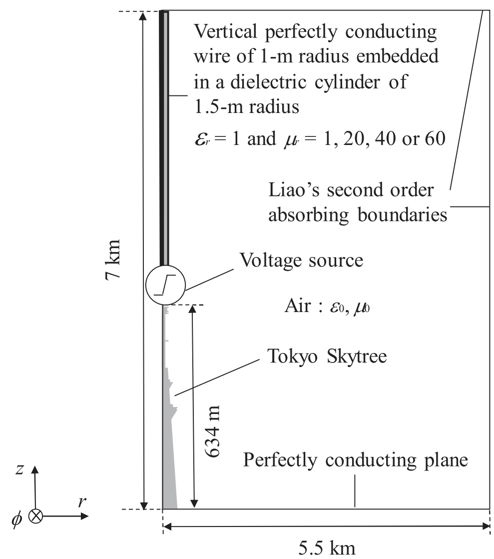


Fig. 1. Lightning strike to the top of the 634-m high Tokyo Skytree configured for simulations using the FDTD method in the 2-D cylindrical coordinate system. The lightning channel is represented by a vertical conductor coated by dielectric with high relative permeability.

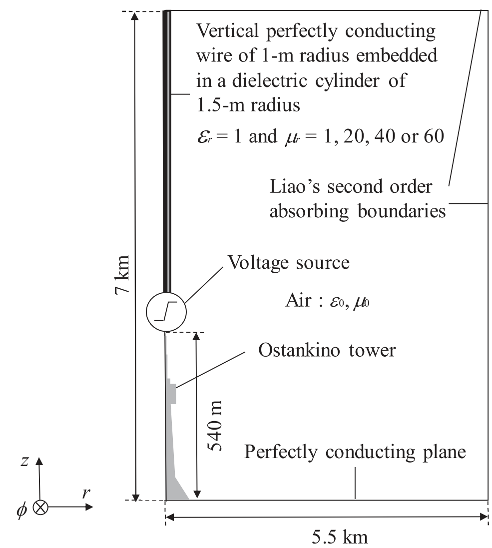


Fig. 2. Lightning strike to the top the 540-m high Ostankino tower configured for simulations using the FDTD method in the 2-D cylindrical coordinate system.

values of return-stroke speed [12]. The channel-representing vertical conductor and the tower are excited by a voltage source inserted between them. A procedure to determine the output waveform of the voltage source from measured current waveforms is presented. The equivalent impedance of lightning return-stroke channel is inferred and discussed.

II. Models of Lightning Strike to Tall Tower

Fig. 1 shows a lightning strike to the top of the 634-m high Tokyo Skytree configured for simulation using the FDTD method in the 2-D cylindrical coordinate system. The lightning return-stroke channel is represented by a 1-m-radius vertical perfect conductor with a 0.5-m thick dielectric coating with high relative permeability [10], although the estimated resistance per unit length of the return-stroke channel behind the return-stroke front is about 0.035 Ω/m and 3.5 Ω/m ahead of the front [13] (nonlinear resistance models [14], [15] have been also proposed). Influence of the above values of series resistance on the channel propagation characteristics is small at frequencies of ≥10 kHz and ≥100 kHz, respectively. The relative permittivity ε_r is set to 1, and the relative permeability μ_r is set to 20, 40, or 60 to reduce the propagation speed of current wave along the vertical conductor to a value ranging from about 1/3 to 1/2 of the speed of light [12]. Note that parameters of the lightning-channel model are assumed here to be independent of current or charge transfer. The radius of the tower model is 1 m at the top and 33.5 m at the base. The upper observatory, whose bottom radius is 10.5 m, top radius is 16.5 m, vertical dimension is 23 m, and bottom height is 423 m, and the lower observatory, whose bottom radius is 15.5 m, top radius is 28 m, vertical dimension is 44 m, and bottom height is 321 m, are also included in the model. The lightning channel and the tower are excited by a voltage source inserted between them. The ground is represented by a flat perfectly conducting plane. Since the Tokyo Skytree has a very large steel-reinforced underground basement in moist soil near rivers, the ground resistance is expected to be very low regardless of season. The conductor system is accommodated in a computational domain of 5.5 km × 7 km, which is divided uniformly into 0.5 m × 1 m rectangular cells. Liao's absorbing boundary condition [16] is applied to the top and right-side planes of the computational domain to suppress unwanted reflections there.

Similarly, Fig. 2 shows a lightning strike to the top of the 540-m high Ostankino tower in Moscow, Russia. The radius of the tower model is 0.5 m at the top and 23.5 m at the base. The observatory, whose radius is 11.5 m, vertical dimension is 36 m, and bottom height is 328 m, is also included in the model. The ground is represented by a flat perfectly conducting plane. The grounding resistance of the Ostankino tower is about 0.2 Ω [17], which is not expected to be materially influenced by season.

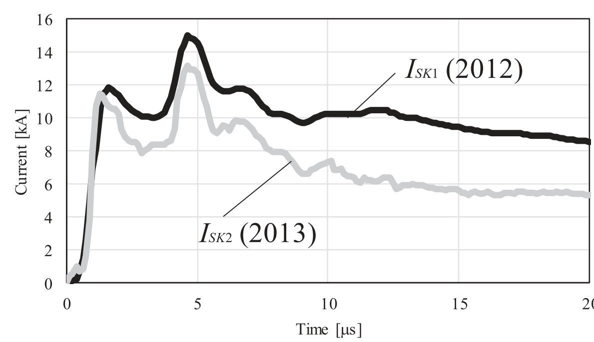


Fig. 3. Waveforms of lightning return-stroke current in the 634-m Tokyo Skytree measured at a height of 497 m by Saito et al. [7].

III. Measured Current Waveforms

Fig. 3 shows two waveforms of lightning return-stroke current in the 634-m high Tokyo Skytree measured at a height of 497 m

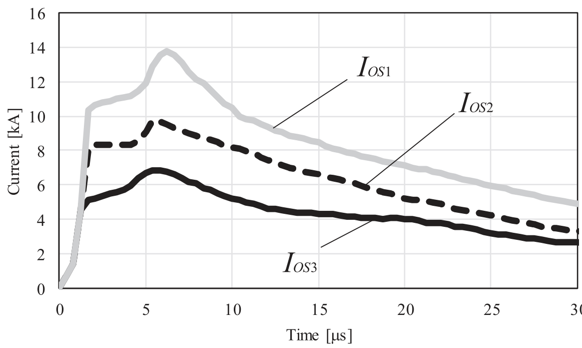


Fig. 4. Waveforms of lightning return-stroke current in the 540-m high Ostankino tower measured at a height of 533 m (essentially at the tower top) by Gorin et al. [17].

in 2012 and 2013 [7], which are denoted as I_{SK1} and I_{SK2}, respectively. The risetimes of I_{SK1} and I_{SK2} are about 0.8 μs and about 0.5 μs, respectively. The second peak in each waveform is due to the arrival of ground-reflected current wave (and its reflection from the bottom of lightning channel) at the current observation point below the connection point between the lightning channel and the tower top, which includes information about the equivalent impedance of the lightning channel. A schematic (conceptual) representation of the transient process in a tall tower is found in Figs. 4(a) and 5 of Rakov [1].

Fig. 4 shows three waveforms of lightning return-stroke current in the 540-m high Ostankino tower measured at a height of 533 m [17], which are denoted as I_{OS1}, I_{OS2}, and I_{OS3}. The risetime of all these currents is about 1.3 μs.

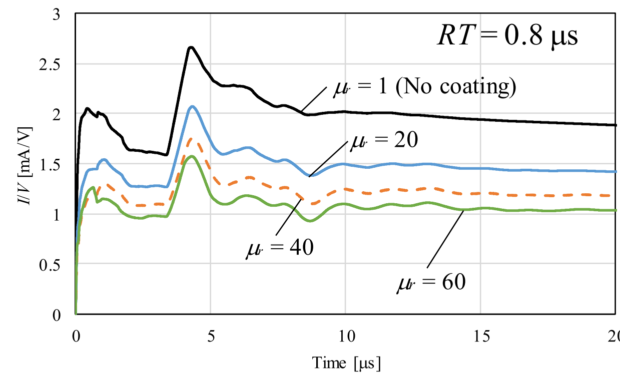
The magnitudes of initial current peaks (or shoulders) in Figs. 3 and 4 range from 5 to 12 kA.

IV. Analysis and Results

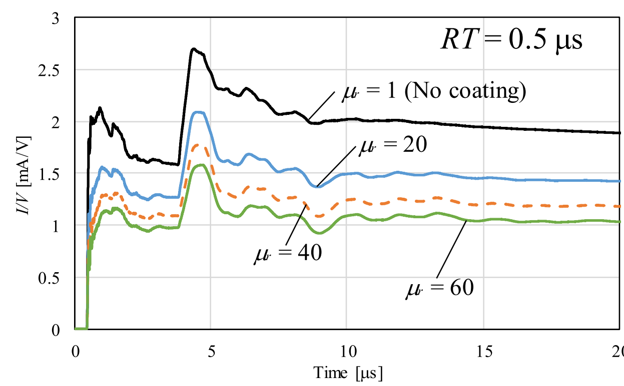
A. Tokyo Skytree

In this section, we present a procedure to determine the output waveform of the voltage source to reproduce the measured current waveforms shown in Fig. 3. Fig. 5(a) and (b) shows time variations of the ratio of the FDTD-computed current at a height of 497 m of the 634-m high tower shown in Fig. 1 to applied voltage linearly rising to a maximum value of 1 V and remaining constant afterwards for different values of the relative permeability of dielectric coating: $\mu_r = 1$, 20, 40, and 60. The applied voltage has the same risetime as the measured current waveform: 0.8 μs for I_{SK1} and 0.5 μs for I_{SK2} (see Fig. 3). Note that the time variations of the current-to-voltage ratio in Fig. 5 are obtained considering the time delay due to the distance between the voltage source at the tower top and the current observation point (0.457 μs = (634 m–497 m)/300 m/μs).

Table I gives ratios of the applied voltage to the current at the moment of the initial current peak, Z_1 (reciprocal of the ratio shown in Fig. 5(a) at 0.8 μs), and those at 20 μs, Z_2 (reciprocal of the ratio shown in Fig. 5(a) at 20 μs), for different values of μ_r, evaluated from Fig. 5(a). Similarly, Table II gives the reciprocal ratios evaluated from Fig. 5(b).



(a)



(b)

Fig. 5. Time variations of the ratio of the current at a height of 497 m of the 634-m high tower shown in Fig. 1 to applied ramp-like voltage having a risetime of (a) 0.8 μs and (b) 0.5 μs for different values of the relative permeability of dielectric coating: $\mu_r = 1$, 20, 40, and 60.

TABLE I
Ratios of the Source Voltage, Linearly-Rising to 1 V and Remaining Constant Afterwards, to the FDTD-Computed Initial Current Peak, Z_1, and Those At 20 μs, Z_2, for Different Values of μ_r, Evaluated From Fig. 5(a)

μ_r	1	20	40	60
Z_1, Ω	500	650	770	870
Z_2, Ω	530	700	850	970

TABLE II
Same as Table I, but Evaluated From Fig. 5(b)

μ_r	1	20	40	60
Z_1, Ω	470	640	770	880
Z_2, Ω	530	700	850	970

The initial part of the measured current waveform shown in Fig. 3 (I_{SK1}) or Fig. 6 (thick orange line) is approximated with the Heidler function [18], which is given as follows:

$$i\ (t) = I_0 \frac{(t/\tau_1)^n}{1 + (t/\tau_1)^n}\ \exp\left(-\frac{t}{\tau_2}\right). \quad (1)$$

Values of the parameters in (1) I_0, τ_1, τ_2, and n are obtained using the least-squares method. The current waveform approximated using (1) is plotted as a broken orange line in Fig. 6.

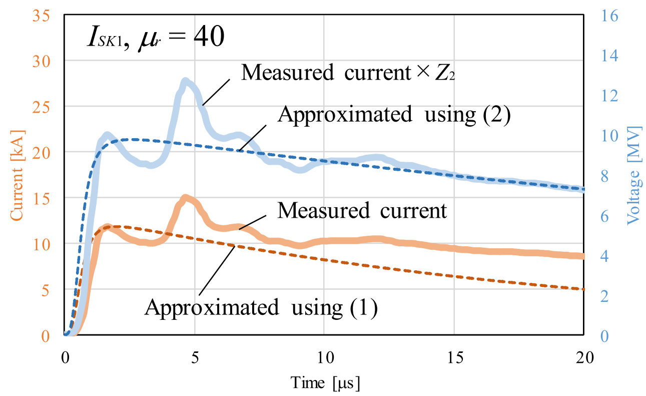


Fig. 6. Waveforms of source voltage at the tower top (634 m) and current at a height of 497 m. Orange color (left vertical axis) is used for current and blue color (right vertical axis) for voltage.

TABLE III
PARAMETERS OF THE HEIDLER FUNCTION FOR THE SOURCE VOLTAGE WAVEFORM GIVEN BY (2) TO REPRODUCE THE MEASURED CURRENT WAVEFORMS, I_{SK1}, FOR DIFFERENT VALUES OF μ_r

μ_r	1	20	40	60
V_0, MV	6.71	8.76	10.4	11.7
τ_1, μs	0.64	0.64	0.64	0.64
τ_2, μs	52.0	53.0	56.5	57.0
n	3	3	3	3

TABLE IV
SAME AS TABLE III, BUT FOR I_{SK2}

μ_r	1	20	40	60
V_0, MV	5.86	8.01	9.64	11.0
τ_1, μs	0.38	0.38	0.38	0.38
τ_2, μs	27.0	26.0	26.0	26.0
n	3	3	3	3

The thick blue waveform in Fig. 6 is the voltage obtained as the measured current multiplied by Z_2. The wavetail of the voltage waveform is approximated using the Heidler function, as follows:

$$v\ (t) = V_0 \frac{\left(\frac{t+t_d}{\tau_1}\right)^n}{1+\left(\frac{t+t_d}{\tau_1}\right)^n}\ \exp\left(-\frac{t+t_d}{\tau_2}\right) \tag{2}$$

where t_d is the time delay due to the distance between the voltage source at the tower top and the current observation point (0.457 μs), τ_1 and n are the same as those obtained for (1). The value of τ_2 is re-estimated and the value of V_0 is estimated under the constraint condition that the peak of the resultant voltage should be equal to the peak of the current waveform, approximated using (1), multiplied by Z_1.

Tables III and IV give values of parameters of the Heidler function for the voltage waveform given by (2) to reproduce the measured current waveforms I_{SK1} and I_{SK2}, respectively. Fig. 7(a) and (b) shows waveforms of the voltage represented using (2) with parameter values given in Tables III and IV to reproduce the measured current waveforms I_{SK1} and I_{SK2}, respectively. Fig. 8(a) and (b) shows measured current waveforms of I_{SK1} and I_{SK2}, respectively, and the corresponding FDTD-computed current waveforms. Table V gives cross-correlation coefficients between the measured and computed current waveforms shown in Fig. 8. It follows from Fig. 8 and Table V that the use of the relative permeability $\mu_r = 40$ or 60 yields best reproduction of the measured current waveforms, although the cross-correlation coefficients are higher than 0.9 for any value of μ_r considered.

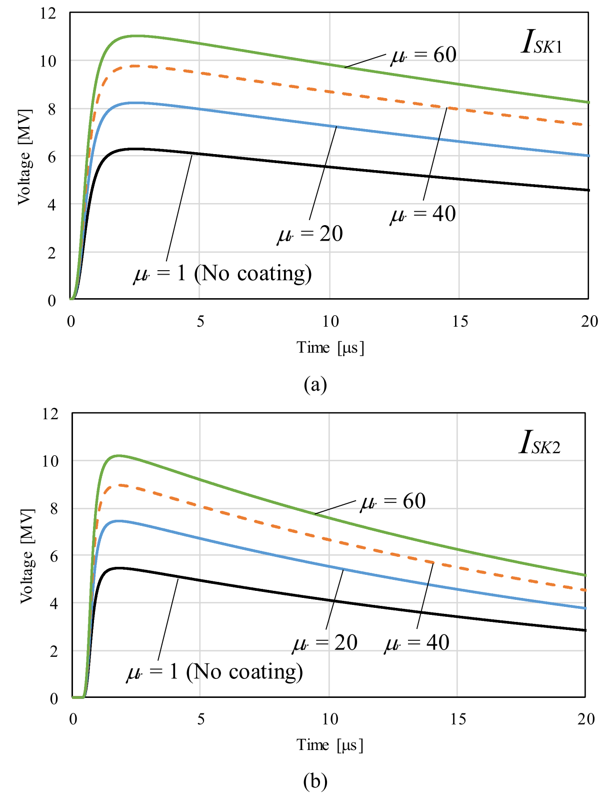


Fig. 7. Waveforms of the source voltage obtained using (2) with parameter values given in Tables III and IV to reproduce the measured current waveforms, (a) I_{SK1} and (b) I_{SK2}.

TABLE V
CROSS-CORRELATION COEFFICIENTS BETWEEN THE MEASURED AND COMPUTED CURRENT WAVEFORMS SHOWN IN FIG. 8

μ_r	1	20	40	60
I_{SK1}	0.92	0.94	0.95	0.95
I_{SK2}	0.94	0.97	0.97	0.97

TABLE VI
RATIOS OF THE SOURCE VOLTAGE, LINEARLY-RISING TO 1 V AND REMAINING CONSTANT AFTERWARDS, TO THE FDTD-COMPUTED INITIAL CURRENT PEAK, Z_1, AND THOSE AT 30 μS, Z_2, FOR DIFFERENT VALUES OF μ_r

μ_r	1	20	40	60
Z_1, Ω	550	690	800	890
Z_2, Ω	550	720	870	990

B. Ostankino Tower

Here, in the same manner as done in Section IV-A for current waveforms shown in Fig. 3, we obtain source voltage waveforms

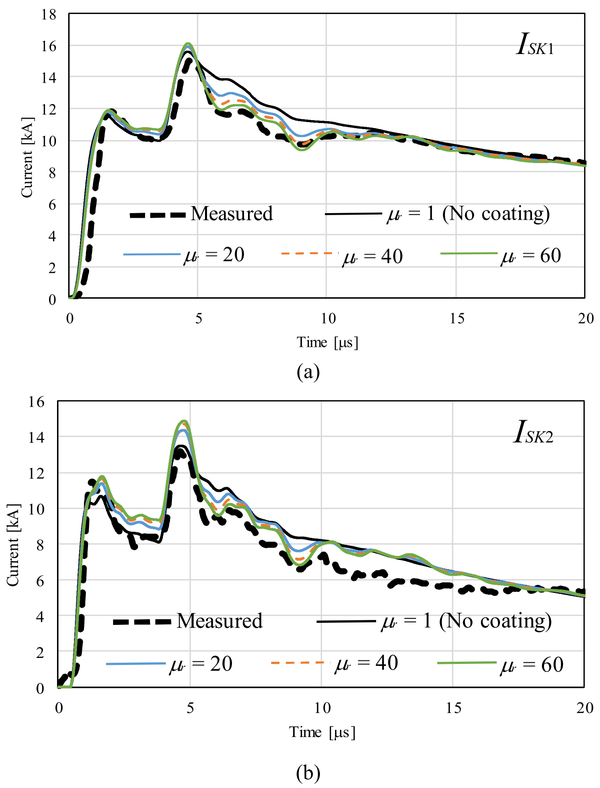


Fig. 8. Tokyo Skytree current waveforms measured at a height of 497 m (thick broken line) and the corresponding FDTD-computed current waveforms (thinner solid and broken lines): (a) I_{SK1} and (b) I_{SK2}.

TABLE VII
PARAMETERS OF THE HEIDLER FUNCTION FOR THE SOURCE VOLTAGE WAVEFORM GIVEN BY (2) TO REPRODUCE THE MEASURED CURRENT WAVEFORMS I_{OS1} FOR DIFFERENT VALUES OF μ_r

μ_r	1	20	40	60
V_0, MV	3.58	4.54	5.25	5.84
τ_1, μs	1.10	1.10	1.10	1.10
τ_2, μs	33.8	35.2	36.8	37.5
n	3	3	3	3

TABLE VIII
SAME AS TABLE VII, BUT FOR I_{OS2}

μ_r	1	20	40	60
V_0, MV	5.58	7.07	8.17	9.10
τ_1, μs	1.10	1.10	1.10	1.10
τ_2, μs	27.0	27.5	28.5	29.3
n	3	3	3	3

TABLE IX
SAME AS TABLE VII, BUT FOR I_{OS3}

μ_r	1	20	40	60
V_0, MV	7.22	9.15	10.6	11.8
τ_1, μs	1.10	1.10	1.10	1.10
τ_2, μs	30.5	31.8	33.2	34.0
n	3	3	3	3

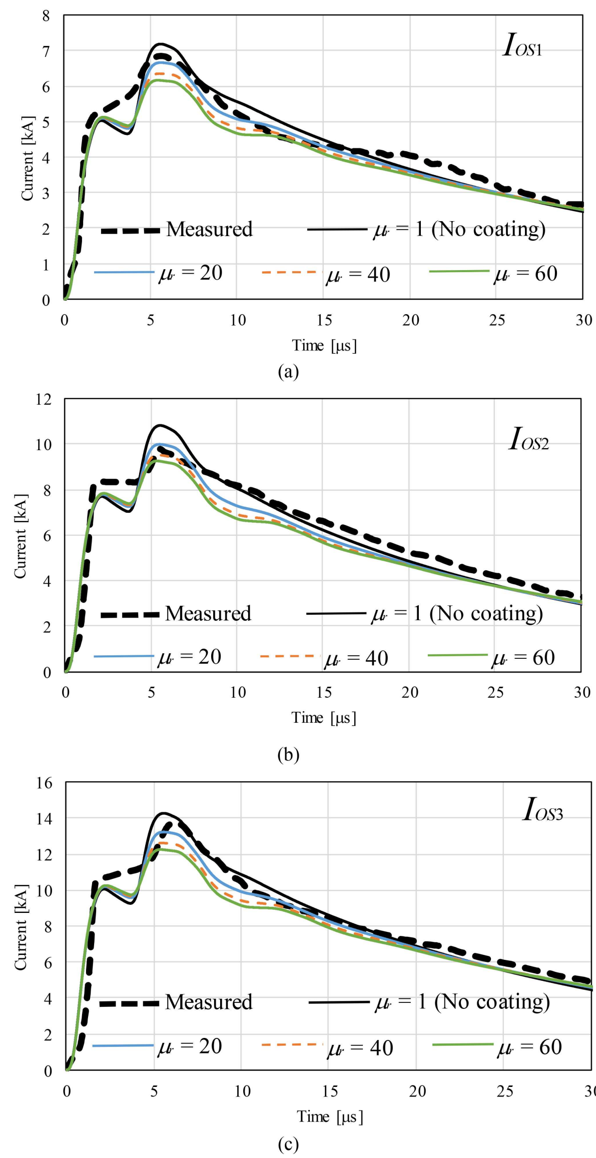


Fig. 9. Ostankino tower current waveforms measured at a height of 533 m (thick broken line) and the corresponding FDTD-computed current waveforms (thinner solid and broken lines): (a) I_{OS1}, (b) I_{OS2}, and (c) I_{OS3}.

to reproduce the measured current waveforms shown in Fig. 4. Table VI gives ratios of the source voltage (linearly rising to 1 V and remaining constant afterwards) to the FDTD-computed initial current peak, Z_1 (reciprocal of the *I*/*V* ratio at 1.3 μs), and those at 30 μs, Z_2 (reciprocal of the *I*/*V* ratio at 30 μs), for different values of μ_r.

Tables VII, VIII, and IX give values of parameters of the Heidler function for the source voltage waveform given by (2) to reproduce the measured current waveforms I_{OS1}, I_{OS2}, and I_{OS3}, shown in Fig. 4, respectively. Fig. 9(a), (b), and (c) shows measured current waveforms I_{OS1}, I_{OS2}, and I_{OS3}, respectively, and the corresponding FDTD-computed waveforms. Table X gives cross-correlation coefficients between the measured and computed current waveforms shown in Fig. 9. It follows from Fig. 9 and Table X that the relative permeability $\mu_r = 40$ corresponds to the best reproduction of the measured current waveforms, although the cross-correlation coefficients are higher than 0.9 for any value of μ_r considered.

TABLE X
CROSS-CORRELATION COEFFICIENTS BETWEEN THE MEASURED AND COMPUTED CURRENT WAVEFORMS SHOWN IN FIG. 9

μ_r	1	20	40	60
I_{OS1}	0.97	0.98	0.99	0.99
I_{OS2}	0.97	0.98	0.98	0.97
I_{OS3}	0.97	0.97	0.98	0.97

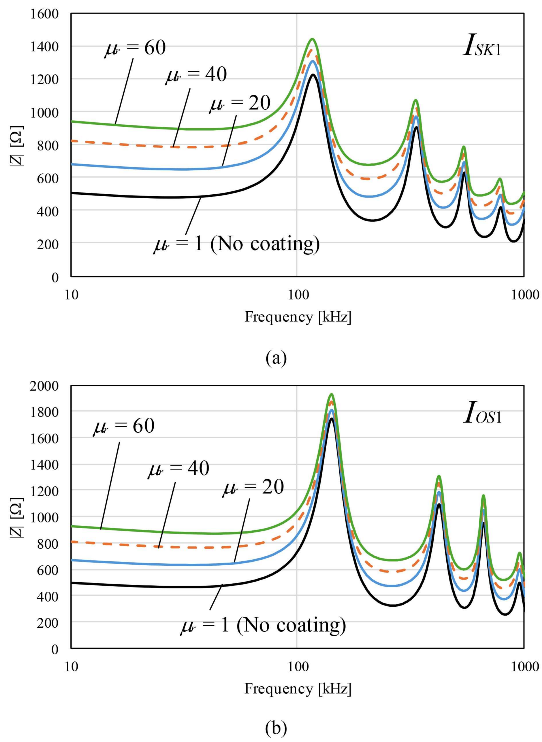


Fig. 10. Frequency dependence of the magnitude of the impedance based on the ratio of the source voltage to the current in the range from 10 kHz to 1 MHz: (a) I_{SK1} and (b) I_{OS1}. Note a slight increase in the impedance with decreasing frequency in the range of about 10 to 50 kHz.

V. DISCUSSION

Fig. 10(a) and (b) shows frequency dependences of the magnitude of the impedance based on the ratio of the source voltage at the tower top [Fourier-transform of the voltage waveform shown in Fig. 7(a) for Fig. 10(a)] to the current at the tower top [Fourier-transform of the tower-top current waveform; not shown here, but similar to the *I/V* waveform in Fig. 5(a) for Fig. 10(a)] in the range from 10 kHz to 1 MHz for the Tokyo Skytree (I_{SK1}) and the Ostankino tower (I_{OS1}), respectively. It appears from Fig. 10 that the impedance increases with increasing μ_r. This is because a higher permeability of the coating contributes to the increase of series inductance [10]. In the lower frequency region (10 to 100 kHz), the contribution of the tower decreases, and the impedance of the channel-representing dielectric-coated conductor becomes dominant. The impedance in the lower frequency range is about 800 Ω for $\mu_r = 40$ and about 900 Ω for $\mu_r = 60$. Thus, the equivalent impedance of a vertical conductor with 0.5-m thick dielectric coating whose relative permeability is $\mu_r = 40$ or 60 is within the 600 to 2500 Ω range reported for equivalent lightning channel impedance by Gorin et al. [2]. In simulations of both the Tokyo Skytree and the Ostankino tower, similar impedance characteristics are found in the frequency range from 10 to 100 kHz, as seen in Figs. 10(a) and (b). Therefore, the influence of detailed tower geometry can be viewed as insignificant. The height of tower is more important than its detailed geometry and characteristic impedance. This is because it is difficult to isolate the ground-reflected current wave from the total contaminated current waveform at or near the tower top if the tower height is small, such that the round-trip time of lightning current wave in the tower is shorter than the current risetime.

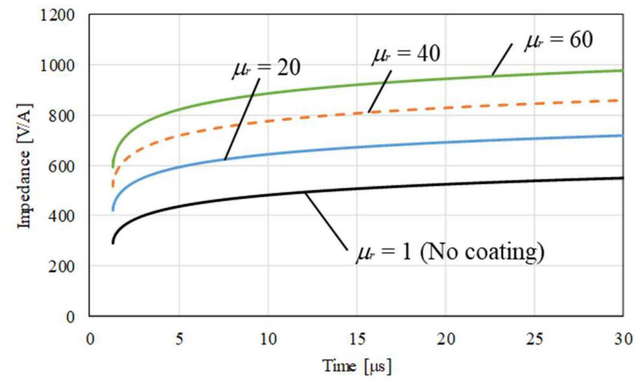


Fig. 11. Time variation of the ratio of the ramp-like source voltage to the FDTD-computed current injected into the channel-representing vertical conductor in the absence of tower.

Fig. 11 shows time variation of the impedance found as ratio of the source voltage (linearly rising to 1 V and remaining constant afterwards) to the FDTD-computed current (having the same risetime as that of the source voltage and decreasing with time afterwards) injected into the channel-representing vertical conductor attached to a flat perfectly conducting ground, in the absence of tower. The ratio increases with time from about 260 to 530 Ω, from 390 to 690 Ω, from 470 to 830 Ω, and from 540 to 950 Ω for $\mu_r = 1$, 20, 40, and 60, respectively. This increasing trend seems to correspond to the impedance slightly increasing with decreasing frequency in the range of about 10 to 50 kHz seen in Fig. 10. Also, it is similar to the gradual increase with time, derived theoretically in [19].

Also note that the reciprocal of the *I/V* ratio (see Fig. 5) at a time of about 3.8 μs (just before the moment of the arrival of ground-reflected current wave at a height of the current measurement point, 497 m) can be used for estimating the sum of the characteristic impedance of the 634-m tower and the equivalent impedance of the channel. It is 633 Ω (= 1/0.974 mA/V), 787 Ω (= 1/0.974 mA/V), 917 Ω (= 1/0.974 mA/V), and 1030 Ω (= 1/0.974 mA/V) for $\mu_r = 1$, 20, 40, and 60, respectively. Furthermore, from Fig. 11, the equivalent impedance of the same channel at the same time is 422, 576, 698, and 800 Ω for $\mu_r = 1$, 20, 40, and 60, respectively. The characteristic impedance of the tower estimated as the difference between those two impedances is 211, 211, 219, and 230 Ω, respectively.

Fig. 12 shows the dependence of the speed of current wave propagating along the dielectric-coated vertical conductor on μ_r. The current propagation speed is evaluated by monitoring the propagation time from the channel base (tower top) to a height of 1 km. As seen in Fig. 12, the propagation speed decreases significantly with increasing μ_r. For $\mu_r = 40$ or 60, it is about

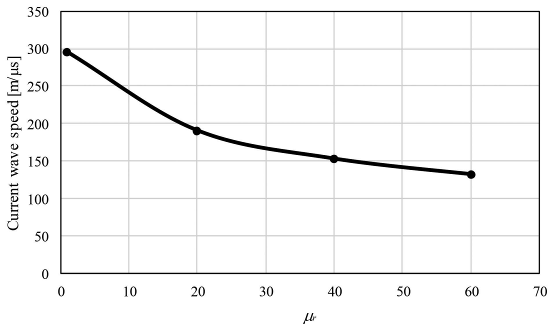


Fig. 12. Dependence of the speed of current wave propagating along the dielectric-coated vertical conductor on μ_r. For $\mu_r = 1$, the speed is equal to the speed of light $c = 300$ m/μs.

one-half of the speed of light, which is expected for lightning return strokes [12].

VI. Summary

Two waveforms of current measured at a height of 497-m of the 634-m high Tokyo Skytree and three waveforms of current measured at a height of 533-m of the 540-m high Ostankino tower have been reproduced in simulations using the FDTD method. The lightning return-stroke channel was represented by a dielectric-coated vertical conductor with relative permeability equal to 20, 40, or 60 to reduce the propagating speed of current wave along the channel-representing vertical conductor and to increase its equivalent impedance relative to that of the vertical conductor without dielectric coating. The channel-representing vertical conductor and the tall tower were excited by a voltage source inserted between them. A procedure to determine the output waveform of the voltage source from measured current waveforms has been developed. The magnitudes of measured initial current peaks ranged from 5 to 12 kA. The equivalent impedance of the lightning return-stroke channel has been inferred to be roughly 800 to 900 Ω.

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